EXAMINATION

21 April 2009 (am)

Subject CT1 — Financial Mathematics
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Describe the characteristics of Government Bills. [3]

2 Describe the characteristics of:

(a) an interest-only loan (or mortgage); and
(b) a repayment loan (or mortgage). [4]

3 A loan is to be repaid by an annuity payable annually in arrear. The annuity starts at a rate of £300 per annum and increases each year by £30 per annum. The annuity is to be paid for 20 years.

Repayments are calculated using a rate of interest of 7% per annum effective.

Calculate:

(i) The amount of the loan. [3]

(ii) The capital outstanding immediately after the 5^{th} payment has been made. [2]

(iii) The capital and interest components of the final payment. [2]

[Total 7]

4 (i) Explain what is meant by the "no arbitrage" assumption in financial mathematics. [2]

An investor entered into a long forward contract for £100 nominal of a security eight years ago and the contract is due to mature in four years' time. The price per £100 nominal of the security was £94.50 eight years ago and is now £143.00. The risk-free rate of interest can be assumed to be 5% per annum effective throughout the contract.

(ii) Calculate the value of the contract now if it were known from the outset that the security will pay coupons of £9 two years from now and £10 three years from now. You may assume no arbitrage. [5]

[Total 7]

5 A company's required return for a particular investment project can be expressed as a force of interest, $\delta(t)$. This force of interest is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t \quad 0 \le t \le 5$$
$$\delta(t) = 0.06 \quad 5 < t$$

The expenditure required for this project is a payment of £100,000 at $t = 0$ and a further payment of £80,000 at $t = 2$.

The income received from the project is a payment stream paid continuously from $t = 8$ to $t = 12$ under which the annual rate of payment at time t is $£100,000e^{0.001t}$.

Calculate the discounted payback period for this project. [8]

6 A pension fund purchased an office block nine months ago for £5 million.

The pension fund will spend a further £900,000 on refurbishment in two months time.

A company has agreed to occupy the office block six months from now. The lease agreement states that the company will rent the office block for fifteen years and will then purchase the property at the end of the fifteen year rental period for £6 million.

It is further agreed that rents will be paid quarterly in advance and will be increased every three years at the rate of 4% per annum compound. The initial rent has been set at £800,000 per annum with the first rental payment due immediately on the date of occupation.

Calculate, as at the date of purchase of the office block, the net present value of the project to the pension fund assuming an effective rate of interest of 8% per annum. [8]

7 A fund had a value of £150,000 on 1 July 2006. A net cash flow of £30,000 was received on 1 July 2007 and a further net cash flow of £40,000 was received on 1 July 2008. The fund had a value of £175,000 on 30 June 2007 and a value of £225,000 on 30 June 2008. The value of the fund on 1 January 2009 was £280,000.

(i) Calculate the time-weighted rate of return per annum earned on the fund between 1 July 2006 and 1 January 2009. [3]

(ii) Calculate the money-weighted rate of return per annum earned on the fund between 1 July 2006 and 1 January 2009. [4]

(iii) Explain why the time-weighted rate of return is more appropriate than the money-weighted rate of return when comparing the performance of two investment managers over the same period of time. [2]

[Total 9]

8 An insurance company has liabilities consisting of eleven annual payments of £1 million, with the first payment due to be made in 10 years' time and the last payment due to be made in 20 years' time. The rate of interest is 6% per annum effective.

(i) Show that the discounted mean term of these liabilities, to four significant figures, is 14.42 years. [3]

The insurance company holds two zero-coupon bonds, one paying £X in 10 years' time and the other paying £Y in 20 years' time.

(ii) Find values of X and Y such that Redington's first two conditions for immunisation from small changes in the rate of interest are satisfied. [6]

(iii) Explain, without making any further calculations, whether you would expect Redington's third condition for immunisation to be satisfied for the values of X and Y calculated in (ii). [2]

[Total 11]

9 Two bonds paying annual coupons of 5% in arrear and redeemable at par have terms to maturity of exactly one year and two years, respectively.

The gross redemption yield from the 1-year bond is 4.5% per annum effective; the gross redemption yield from the 2-year bond is 5.3% per annum effective. You are informed that the 3-year par yield is 5.6% per annum.

Calculate all zero-coupon yields and all one-year forward rates implied by the yields given above. [12]

10 A loan pays coupons of 11% per annum quarterly on 1 January, 1 April, 1 July and 1 October each year. The loan will be redeemed at 115% on any 1 January from 1 January 2015 to 1 January 2020 inclusive, at the option of the borrower. In addition to the redemption proceeds, the coupon then due is also paid.

An investor purchased a holding of the loan on 1 January 2005, immediately after the payment of the coupon then due, at a price which gave him a net redemption yield of at least 8% per annum effective. The investor pays tax at 30% on income and 25% on capital gains.

On 1 January 2008 the investor sold the holding, immediately after the payment of the coupon then due, to a fund which pays no tax. The sale price gave the fund a gross redemption yield of at least 9% per annum effective.

Calculate the following:

(i) The price per £100 nominal at which the investor bought the loan. [6]

(ii) The price per £100 nominal at which the investor sold the loan. [4]

(iii) The net yield per annum convertible quarterly that was actually obtained by the investor during the period of ownership of the loan. [5]

[Total 15]

11 An individual wishes to receive an annuity which is payable monthly in arrears for 15 years. The annuity is to commence in exactly 10 years at an initial rate of £12,000 per annum. The payments increase at each anniversary by 3% per annum. The individual would like to buy the annuity with a single premium 10 years from now.

(i) Calculate the single premium required in 10 years' time to purchase the annuity assuming an interest rate of 6% per annum effective. [5]

The individual wishes to invest a lump sum immediately in an investment product such that, over the next 10 years, it will have accumulated to the premium calculated in (i). The annual effective returns from the investment product are independent and $(1+i_t)$ is lognormally distributed, where i_t is the return in the tth year. The expected annual effective rate of return is 6% and the standard deviation of annual returns is 15%.

(ii) Calculate the lump sum which the individual should invest immediately in order to have a probability of 0.98 that the proceeds will be sufficient to purchase the annuity in 10 years' time. [9]

(iii) Comment on your answer to (ii). [2]

[Total 16]

END OF PAPER