SOLUTIONS A-03 APPLIED MECHANICS (June 2003)

Q.1 a. A The resultant of any two forces, would be in the plane of these two forces and must be equal, opposite and collinear with the third one.

b. D For a perfect plane truss, the relation between the number of members n and the number of joints k is $n = 2k - 3$.

c. A The total reaction must be vertically upward to balance the weight of the body acting vertically downward.

d. D The magnitude of the total acceleration $a = (a_c^2 + a_t^2)^{1/2}$, where centripetal acceleration $a_c = \omega^2 r$, tangential acceleration $a_t = \dot{\omega} r$.

e. B For the beam span l, the support reactions are $R_1 = - R_2 = M/l$. The B.M. at a distance x from the support is $R_1 x - M\langle x - l/2\rangle$. Maximum B.M. is at the centre of the beam $x = l/2$, i.e. $M/2$.

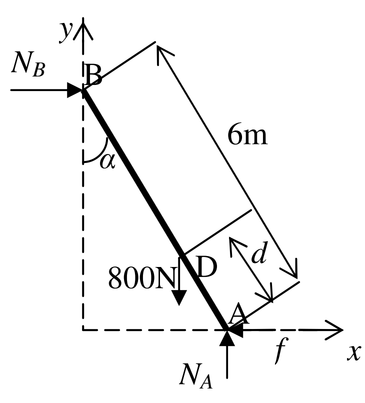
f. D The stiffness of a close-coiled spring $k = P/\delta$ is proportional to d^4. If the diameter d is doubled the stiffness would be $2^4 = 16$ times.

g. C The vacuum pressure is the pressure below the atmospheric pressure.

h. B The runner vanes of a reaction turbine are made adjustable for optimizing the efficiency at part loads.

Q.2.

The F.B.D. of the ladder AB with the man at point D, a distance d up along the ladder is shown in Fig.2. The normal reaction of the floor N_A and the friction force f act on the end A of the ladder. The normal reaction of the wall N_B is at the end B of the ladder. The 800 N weight of the man acts at D. The coefficient of friction $\mu = \tan 15$.



F.B.D. of Ladder

Fig.2

The equilibrium equations for the ladder give

$\Sigma F_x = 0 \rightarrow N_B - f = 0$ (1)

$\Sigma F_y = 0 \rightarrow N_A - 800 = 0$ (2)

$\Sigma M_A = 0 \rightarrow 800 d \sin\alpha - N_B \times 6\cos\alpha = 0$ (3)

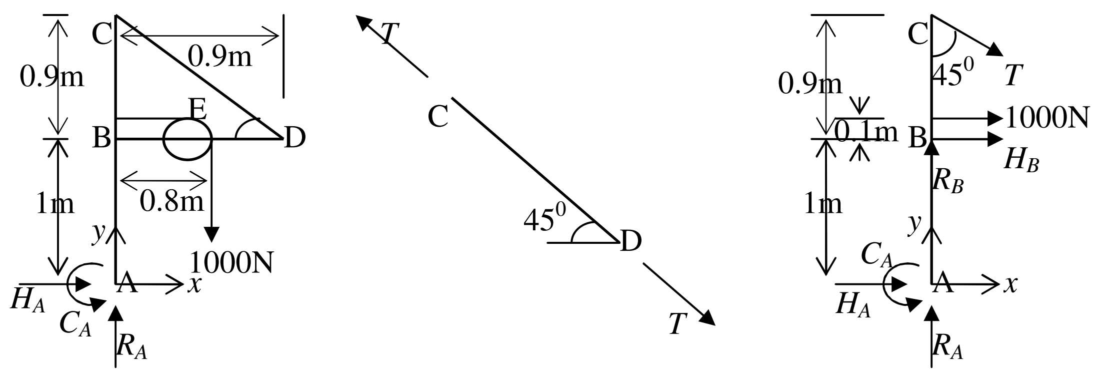
$f \leq \mu N_A = N_A \tan 15$ (4)

Solving equations (1) to (4), $d \leq 6 \tan 15/\tan\alpha$.

For $\alpha = 30$, maximum $d = 6 \tan 15/ \tan 30 =$ <u>2.78 m</u>.

For $d = 6$ m, $\tan\alpha \leq \tan 15$, i.e. $\underline{\alpha \leq 15^0}$.

Q.3.
The F.B.Ds. of the whole frame and members CD and ABC are shown in Fig.3.



F.B.D. of the Whole Frame F.B.D. of Member CD F.B.D. of Member ABC

Fig.3

As the end A of the frame is fixed, the reactions at A are the horizontal force H_A, the vertical force R_A and a couple C_A. From the equilibrium equations of the frame
$H_A = \underline{0}$, $R_A = \underline{1000\ \text{N}}$ and $C_A = 1000\times0.8 = \underline{800\ \text{Nm}}$.
The member CD is a two force member and hence the forces T at the ends C and D must be collinear with CD.
Considering the equilibrium of ABC and taking moment about B to eliminate the unknown reactions H_B, R_B at B from the equation,
$\Sigma M_B = 0 \rightarrow C_A - T\times0.9\sin45 - 1000\times0.1 = 0 \rightarrow T = \underline{1100\ \text{N}}$.

Q.4a.
A circular area A of radius R in the xy plane is shown in Fig.4a. Consider an infinitesimal element of area $dA = rd\theta dr$. The second moment of the area I of the circular area A about the z axis, normal to the area and passing through the centre O, would be

$$I = \int_A r^2 dA$$

$$= \int_0^R \int_0^{2\pi} r^2 (rd\theta dr)$$

$$= \int_0^R r^3 2\pi dr = \underline{\pi R^4 / 2}.$$

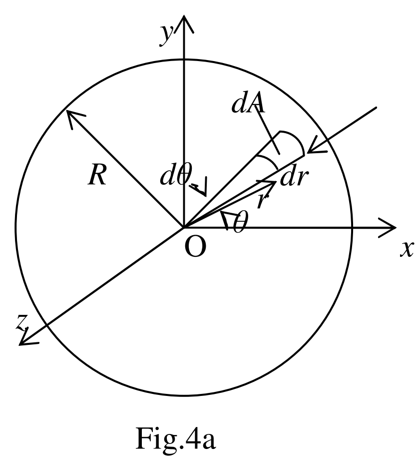


Fig.4a

Q.4b.

Let the superscripts 1 and 2 refer to the uniform thin disc of radius R and the hole of radius $R/2$, respectively. Then, the coordinates of the centroid C of the disc with the hole would be

$$x_c = \frac{\sum A_i x_i}{\sum A_i} = \frac{\pi R^2 \times 0 - \pi (R/2)^2 \times R/2}{\pi R^2 - \pi (R/2)^2} = \underline{-R/6}.$$

From symmetry about the axis x, $y_c = \underline{0}$.

Its second moment of area I^C_{zz} about an axis through C and parallel to the z axis would be

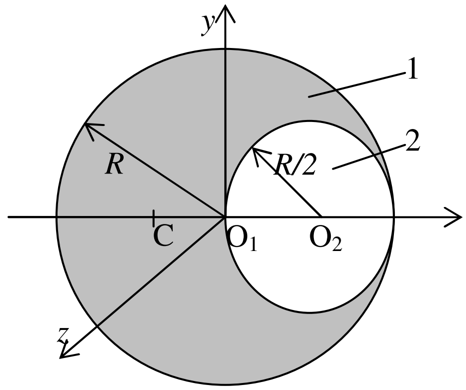


Fig. 4b

$$I^C_{zz} = \sum_i (I^C_{zz})_i$$

$$= [(I^{O_1}_{zz})_1 + A_1(x_C - x_1)^2] - [(I^{O_2}_{zz})_2 + A_2(x_C - x_2)^2]$$

$$= [\pi R^4/2 + \pi R^2(-R/6)^2] - [\pi (R/2)^4/2 + \pi (R/2)^2 \times (-R/6 - R/2)^2]$$

$$= 19\pi R^4/36 - 43\pi R^4/288 = \underline{37\pi R^4/96}.$$

Q.5.

The F.B.Ds. of the pulleys 1, 2 and masses A, B, C are shown in Fig.5. As the pulleys are light and frictionless, the tension in a string on both sides of a pulley would be the same. Also from the F.B.D. of the pulley 2,

$T_1 = 2T_2$ (1)

Let a_A, a_B, a_C be the accelerations of the masses A, B, C, respectively and a_2 the acceleration of the pulley 2. Then

$a_2 = - a_A$ (2)

$a_B - a_2 = - (a_C - a_2)$ (3)

The equations of motion for the masses A, B, C are

$60 - T_1 = 6a_A$ (4)

$30 - T_2 = 3a_B$ (5)

$20 - T_2 = 2a_C$ (6)

Solving equations (1) t0 (6),

$a_A = \underline{1.11 \text{ m/s}^2}$, $a_B = \underline{-1.11 \text{ m/s}^2}$ and $a_C = \underline{-1.11 \text{ m/s}^2}$.

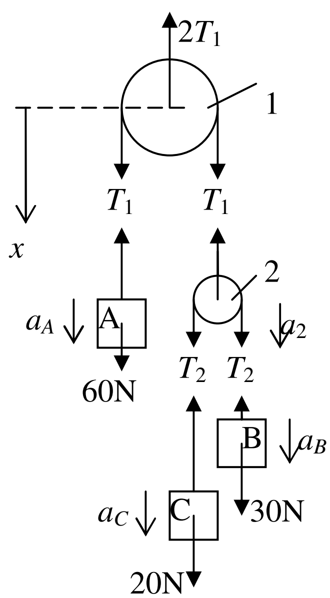


Fig.5

Q.6.

Let d be the diameter of the rod. From strength consideration,

$\sigma = P/A = 1000g/(\pi d^2/4) \leq \sigma_{\text{allowable}} = 150\times10^6 \rightarrow d \geq 0.0092$ m = 9.2 mm.

From stiffness consideration,

$\delta = PL/AE = 1000g\times5/[(\pi d^2/4)\times210\times10^9] \leq \delta_{\text{allowable}} = 3\times10^{-3} \rightarrow d \geq 0.01$ m = 10 mm.

Hence $\underline{d = 10\text{mm}}$.

Spring constant of the rod $k = P/\delta = 1000g/(3\times10^{-3}) = 10^7/3$ N/m.

The frequency $f = (1/2\pi)\sqrt{(k/m)} = (1/2\pi)\sqrt{[(10^7/3)/1000]} = \underline{9.19 \text{ Hz.}}$

Q.7.

The loading on the cantilever beam and the support reactions at the built in end are as shown in Fig. 7.

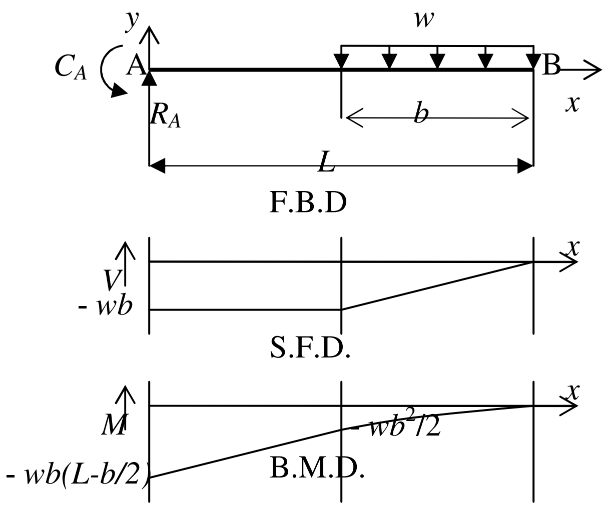


Fig.7

Considering the equilibrium of the cantilever, the reactions at the built in end A are

$R_A = wb$ and $C_A = wb(L\text{-}b/2)$.

Using singularity functions, the shear force V and the bending moment M at any section x are

$V = -\, wb + w\langle x - (L - b)\rangle$,

$M = -wb(L–b/2) + wbx – w\langle x - (L - b)\rangle^2/2$.

The S.F. and B.M. diagrams are also shown in Fig.7.

Their maximum values are at A, $x = 0$,

$\underline{V_{\max} = -wb,\ M_{\max} = -wb(L\text{-}b/2).}$

Let v be the deflection of the elastic line at x, $EId^2v/dx^2 = M$. Then,

$EId^2v/dx^2 = -\, wb(L–b/2) + wbx – w\langle x - (L - b)\rangle^2/2$

Integrating,

$EIdv/dx = -\, wb(L–b/2)x + wbx^2/2 – w\langle x - (L - b)\rangle^3/6 + C_1$

$EIv = -\, wb(L–b/2)x^2/2 + wbx^3/6 – w\langle x - (L - b)\rangle^4/24 + C_1x + C_2$.

Using the boundary conditions $v = 0$ and $dv/dx = 0$ at $x = 0 \rightarrow C_1 = 0$ and $C_2 = 0$.

The maximum deflection occurs at the free end B i.e. $x = L$,

$v_{max} = [-\, wb(L–b/2)L^2/2 + wbL^3/6 – w\langle L - (L - b)\rangle^4/24]/EI = \underline{-\, wb(L^3/3 – bL^2/4 + b^3/24)}$.

Q.8a.

The spring is under an axial pull P. Let R be the radius of the coil and d be the wire diameter. The F.B.D. of one part of the spring cut by a section with normal along the spring wire is shown in Fig.8a. Any coil section is subjected to a direct shear force P and a moment $T = PR$. For a close coiled spring the moment T is a twisting moment. Using the torsion formula $\tau = Tr/I_p$, the maximum shear stress due to torsion would be

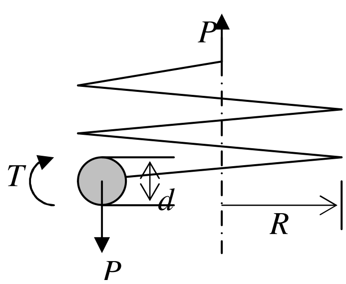


Fig.8a

$\tau_{\max} = PR(d/2)/(\pi d^4/32) = \underline{16PR/(\pi d^3)}$.

Let n be the number of turns, G the shear modulus of the wire material and δ t
deflection. Then the strain energy U would be
$U = P\delta/2 = T^2L/(2GI_p) = (PR)^2(2\pi Rn)/[2G(\pi d^4/32)] \rightarrow \underline{\delta = 64PR^3n/Gd^4}$.

Q.8b.
Let d be the diameter of the solid shaft, and d_o, d_i the outer and internal diameters, respectively of the hollow shaft. From the torsion formula, the torque transmitted T for the same maximum shear stress $\tau_{\max}$ in the shafts would be $T = \tau_{\max}I_p/r_{\max}$.
For the solid shaft $T_{\text{solid}} = \tau_{\max}\,(\pi d^4/32)/(d/2) = \tau_{\max}\,\pi d^3/16$.
For the hollow shaft $T_{\text{hollow}} = \tau_{\max}\,[\pi(d_o^4 - d_i^4)/32]/(d_o/2) = \tau_{\max}\pi(d_o^4 - d_i^4)/(16d_o)$.
As the shafts are of the same material length and weight, $d_o^2 - d_i^2 = d^2$.
Hence, the ratio $T_{\text{hollow}}/T_{\text{solid}} = (d_o^4 - d_i^4)/d^3d_o = (d_o^2 + d_i^2)/dd_o = \underline{d_o/d + d_i^2/dd_o} > 1$.

Q.9a.
A cube floating in water, with its sides vertical, is shown in Fig.9a. Let M be the metacentre, G the centre of gravity and B the centre of buoyancy. If h is the height of immersion in water, the weight of the water displaced equals the weight of the cube, i.e.
$1000hb^2 = 1000\gamma b^3 \rightarrow h = b\gamma$.
$BG = b/2 - h/2 = b/2 - b\gamma/2 = b(1 - \gamma)/2$
$BM = I/V = b(b^3/12)/\,b^2h = b/12\gamma$
$MG = BM - BG = b/12\gamma - b(1 - \gamma)/2 = 0$
$\rightarrow \gamma^2 - \gamma + 1/6 = 0 \rightarrow \gamma = (1 \pm \sqrt{3})/2 = \underline{0.789,\ 0.211}$.

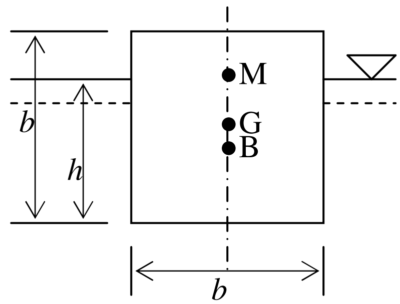


Fig.9a

Q.9b.
The velocity components $u = 2x - x^2y + y^3/3$ and $v = xy^2 - 2y + x^3/3$.
The continuity condition for an incompressible 2D flow is $\partial u/\partial x + \partial v/\partial y = 0$.
$\partial u/\partial x + \partial v/\partial y = (2 - 2xy) + (2xy - 2) = 0$. $\rightarrow$ It is a possible 2D flow.
The irrotational flow condition for a 2D flow is $\partial v/\partial x - \partial u/\partial y = 0$.
$\partial v/\partial x - \partial u/\partial y = (y^2 + x^2) - (-x^2 + y^2) = 2x^2 \neq 0$. $\rightarrow$ The flow is not irrotational.

Q.10a.
Consider a 2D inviscid steady flow in the xz plane. The gravity acts in the - z direction. A differential control volume with the forces acting on it is shown in Fig.10a.
The mass in the control volume $m = \rho dxdydz$.
The sum of the forces in the x direction,
$\sum F_x = -\,[p+(\partial p/\partial x)dx]dydz + pdydz$.
The total acceleration in the x direction,
$Du/Dt = u\partial u/\partial x + w\partial u/\partial z$.
The equation of motion $mDu/Dt = \sum F_x$ yields the Euler's equation in the x direction,
$\rightarrow \underline{u\partial u/\partial x + w\partial u/\partial z = -\,(1/\rho)\,\partial p/\partial x}$.
Similarly, $mDw/Dt = \sum F_z$ yields the Euler's equation in the z direction,
$\rightarrow \underline{u\partial w/\partial x + w\partial w/\partial z = -\,(1/\rho)\,\partial p/\partial z - g}$.

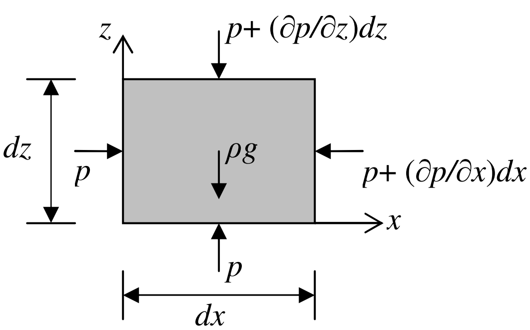


Fig.10a

Q.10b.
Let subscripts 1 and 2 refer to the inlet and outlet, respectively of the draft tube. The continuity equation yields the velocity at the outlet V_2 as
$V_2 = V_1A_1/A_2 = 5(\pi\times3^2/4)/(\pi\times5^2/4) = 1.8$ m/s.
The Bernoulli's equation between the inlet and outlet sections is
$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2$ + losses.
Hence the pressure head p_1/γ at the inlet would be
$(p_1/\gamma - p_2/\gamma) = (z_2 - z_1) + (V_2^2 - V_1^2)/2g$ + losses = $-5 + (1.8^2 - 5^2)/(2\times9.81) + 0.1 =$ <u>- 6.01 m</u>.

Q.11.
A bucket of a Pelton wheel with its inlet and outlet velocity diagrams is shown in Fig. 11. The bucket speed is v and the turning angle is θ. Let subscripts 1 and 2 refer to the inlet and outlet, respectively. Let u_1, u_2 be the absolute jet velocities and w_1, w_2 the relative velocities. As there is no friction,

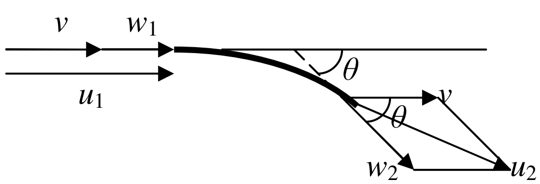


Fig.11

$w_2 = w_1 = u_1 - v$.
The peripheral jet velocity at the outlet is,
$v + w_2\cos\theta = v + (u_1 - v)\cos\theta$.
Force R on the jet would be
$R = \rho Q[v + (u_1 - v)\cos\theta - u_1]$
$= -\rho Q(u_1 - v)(1 - \cos\theta)$.
The force F on the bucket, $F = -R = \rho Q(u_1 - v)(1 - \cos\theta)$.
The power developed $P = F\times v = \rho Qv(u_1 - v)(1 - \cos\theta)$.
The input energy $E = \rho Q u_1^2/2$. The efficiency $\eta = P/E =$ <u>$2((v/u_1)(1 - v/u_1)(1 - \cos\theta)$</u>.
For maximum efficiency,
$d\eta/dv = 0. \rightarrow$ <u>$v = u_1/2$, i.e. the bucket speed must be half the absolute jet speed at inlet</u>.

SOLUTIONS A-03 APPLIED MECHANICS (December 2003)

Q.1. a. A Any horizontal section of the block is subjected to a shear force.

b. B The specific speed $N_s = N\sqrt{P}/H^{5/4}$ with speed N in rpm, power P in kW and head H in m of a Francis turbine is from 60 to 300.

c. C $T = \tau_{\max} I_p / r_{\max} \rightarrow T_{\text{hollow}}/T_{\text{solid}} = I_{phollow}/I_{psolid} = [d_o^4 - (d_o/2)^4]/d_o^4 = 15/16$.

d. A The slope and deflection under the load are $Wa^2/2EI$ and $Wa^3/3EI$. Free end deflection = $Wa^3/3EI + (l - a)(Wa^2/2EI) = (3l - a)Wa^2/6EI$.

e. B The first moment of area of a semicircle about its diameter D is

$$\int_0^{D/2}\int_0^{\pi} r\sin\theta(rd\theta dr) = D^3/12.$$

f. B A rigid body is in translation if all its points have the same velocity **V**(t) (which may change with time t). Hence, it can move along a straight or curved path.

g. D A point of the rigid body or its hypothetical extension, having zero velocity always exists for plane motion.

h. C Due to the phenomenon of surface tension, a quantity of liquid tries to minimize its free surface area.

Q.2a.

As the resultant of the three forces acting on the lever passes through O (refer Fig. 1 of Q.2a), the sum of their moments about O must be zero.

$\sum M_o = P\times 250\cos 20 - 120\times 200 - 80\times 400 = 0 \rightarrow P = \underline{238.4\text{ N}}$.

The expression for the moment $\sum M_o$ does not depend on the angle θ and consequently, the force P does not depend on the angle θ.

Q.2b.

Let R_x, R_y be the x, y components, respectively of the resultant **R** of the three forces acting on the eye bolt (refer Fig. 2 of Q.2b.).

$R_x = \sum F_x = 6 + 8\cos 45 - 15\cos 30 = -1.33$ kN,

$R_y = \sum F_y = 8\sin 45 + 15\sin 30 = 13.16$ kN.

Hence $R = (R_x^2 + R_y^2)^{1/2} = [(-1.33)^2 + (13.16)^2]^{1/2} = \underline{13.23\text{ kN}}$.

The angle θ which **R** makes with + x axis is

$\theta = \cos^{-1}(R_x/R) = \cos^{-1}(-1.33/13.23) = \underline{95.8^0}$.

Q.3

Let H_A, R_A be the support reactions at A and R_D the support reaction at D as shown in Fig.3(i).

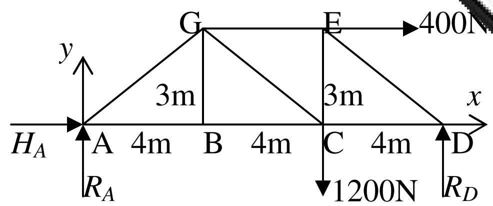


Fig3(i)

Considering the equilibrium of the whole truss,

$\sum F_x = 0 \rightarrow H_A + 400 = 0 \rightarrow H_A = -400$ N.

$\sum M_A = 0 \rightarrow 12R_D - 9600 - 1200 = 0 \rightarrow R_D = 900$ N.

$\sum F_y = 0 \rightarrow R_A + R_D - 1200 = 0 \rightarrow R_A = 300$ N.

The sides AG = GC = ED = $\sqrt{(4^2+3^2)} = 5$ m.

Imagine the truss to be cut by a section 1-1 and consider the equilibrium of the portion to the left of the section 1-1 as shown in Fig.3(ii). The forces shown in the members are tensile.

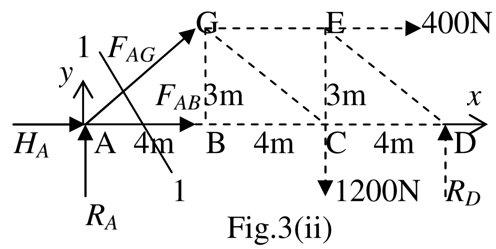


Fig.3(ii)

$\sum F_y = 0 \rightarrow F_{AG}(3/5) + R_A = 0.$

$\rightarrow F_{AG} = \underline{-500 \text{ N} = 500 \text{ N (C)}}.$

$\sum F_x = 0 \rightarrow F_{AG}(4/5) + F_{AB} + H_A = 0.$

$\rightarrow F_{AB} = \underline{800 \text{ N (T)}}.$

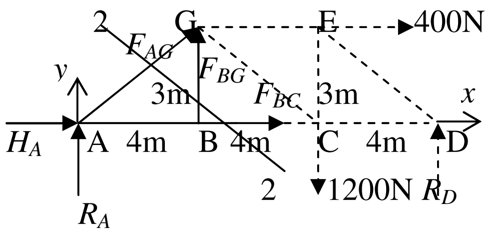


Fig.3(iii)

Imagine the truss to be cut by a section 2-2 as shown in Fig.3(iii). Consider the equilibrium of the portion to the left of the section 2-2.

$\sum F_x = 0 \rightarrow F_{AG}(4/5) + F_{BC} + H_A = 0.$

$\rightarrow F_{BC} = \underline{800 \text{ N (T)}}.$

$\sum F_y = 0 \rightarrow F_{AG}(3/5) + F_{BG} + R_A = 0.$

$\rightarrow F_{BG} = \underline{0}.$

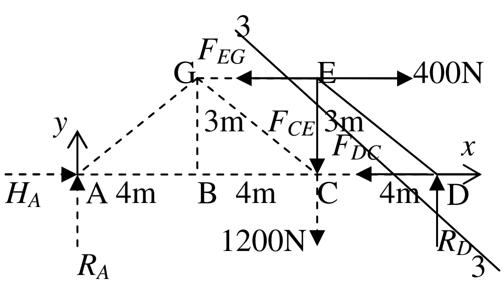


Fig.3(iv)

Imagine the truss to be cut by a section 3-3 and consider the equilibrium of the portion to the right of the section 3-3 as shown in Fig.3(iv).

$\sum F_y = 0 \rightarrow -F_{CE} + R_D = 0$

$\rightarrow F_{CE} = \underline{900 \text{ N (T)}}.$

$\sum M_E = 0 \rightarrow -F_{DC} \times 3 + R_D \times 4 = 0.$

$\rightarrow F_{DC} = \underline{1200 \text{ N (T)}}.$

$\sum F_x = 0 \rightarrow -F_{EG} - F_{DC} + 400 = 0.$

$\rightarrow F_{EG} = -800 \text{ N} = \underline{800 \text{ N (C)}}.$

Finally, imagine it to be cut by a section 4-4 and consider the equilibrium of the portion above the section 4-4 as shown in Fig.3(v).

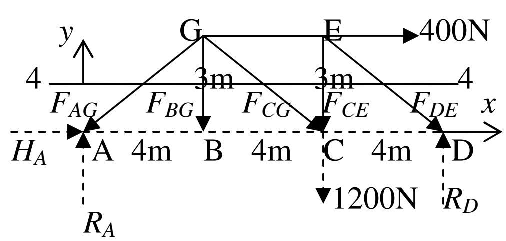


Fig.3(v)

$\sum M_G = 0 \rightarrow -[F_{DE}(3/5) + F_{CE}] \times 4 = 0.$

$\rightarrow F_{DE} = -1500 \text{ N} = \underline{1500 \text{ N (C)}}.$

$\sum M_E = 0 \rightarrow [F_{AG}(3/5) + F_{BG} + F_{CG}(3/5)] \times 4 = 0.$

$\rightarrow F_{CG} = \underline{500 \text{ N (T)}}.$

Q.4.

The F.B.Ds. of the bodies A, B and the weight W for impending motion of the bodies A and B down the planes are shown in Fig.4. This would correspond to the least magnitude of $W = W_{min}$.

From the equilibrium of body A,

$N_A = 1000 \cos20 = 939.7$ N.

$T_A = 1000 \sin20 – 0.2\, N_A = \underline{154.1 \text{ N}}$.

From the equilibrium of body B,

$N_B = 800 \cos30 = 692.8$ N.

$T_B = 800 \sin30 – 0.25\, N_B = \underline{226.8 \text{ N}}$.

From the equilibrium of weight W in the vertical direction

$W_{min} = T_A\sin45 + T_B\sin60 = \underline{305.4 \text{ N}}$.

For horizontal equilibrium, additional horizontal force is required.

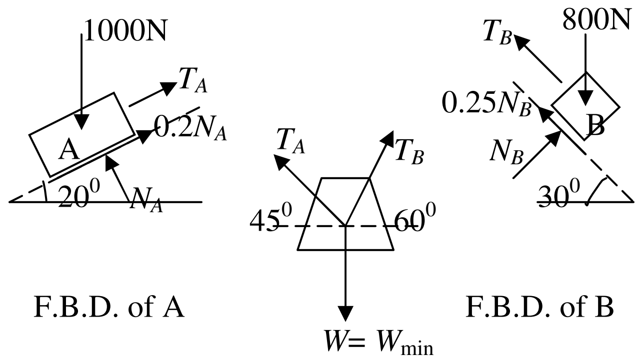


Fig.4

The impending motion of the bodies A and B up the planes correspond to the maximum magnitude of $W = W_{max}$. In this case, the direction of frictional forces on both the blocks would be reversed and must act down the planes. Considering the equilibrium of the bodies A and B, the normal reactions remain the same. Then,

$T_A' = 1000 \sin20 + 0.2\, N_A = \underline{530.0 \text{ N}}$.

$T_B' = 800 \sin30 + 0.25\, N_B = \underline{573.2 \text{ N}}$.

$W_{max} = T_A'\sin45 + T_B'\sin60 = \underline{871.2 \text{ N}}$.

Q.5.

Let subscripts 1 refer to the rectangular area ABGD, 2 to the triangular area DGC and 3 to the semicircular area EFB as shown in Fig.5. Then the given area A would be

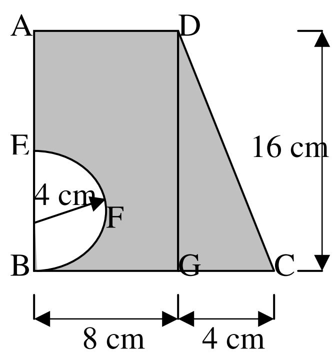


Fig.5

$A = A_1 + A_2 – A_3$.

The moment of inertia of area A_1, $(I_{BC})_1$ about BC and $(I_{AB})_1$ about AB, would be

$(I_{BC})_1 = 8\times16^3/12 + (8\times16)(16/2)^2 = 10922.7$ cm^4.

$(I_{AB})_1 = 16\times8^3/12 + (8\times16)(8/2)^2 = 2730.7$ cm^4.

The moment of inertia of area A_2, $(I_{BC})_2$ about BC and $(I_{AB})_2$ about AB, would be

$(I_{BC})_2 = 4\times16^3/36+ (4\times16/2)(16/3)^2 = 1365.3$ cm^4.

$(I_{AB})_2 = 16\times4^3/36 + (4\times16/2)(8+4/3)^2 = 2816$ cm^4.

The moment of inertia of area A_3, $(I_{BC})_3$ about BC and $(I_{AB})_3$ about AB, would be

$(I_{BC})_3 = (\pi\times4^4/4)/2 + (\pi\times4^2/2)\times4^2 = 502.7$ cm^4. $(I_{AB})_3 = (\pi\times4^4/4)/2 = 100.5$ cm^4.

The moments of inertia for the area A, I_{BC} about BC and I_{AB} about AB, would be

$I_{BC} = (I_{BC})_1 + (I_{BC})_2 - (I_{BC})_3 = 10922.7 + 1365.3 - 502.7 = \underline{11785.3 \text{ cm}^4}$.

$I_{AB} = (I_{AB})_1 + (I_{AB})_2 - (I_{AB})_3 = 2730.7 + 2816 - 100.5 = \underline{5446.2 \text{ cm}^4.}$

Q.6.

Let the common velocity after impact be V. The conservation of momentum yields,

$(800+500)V = 800\times12 + 500\times9 \rightarrow V = \underline{10.9\text{ m/s}}$.

The loss of kinetic energy (K.E.) due to impact would be

Initial K.E. – Final K. E. $= 800\times12^2/2 + 500\times9^2/2 - (800+500)(10.9)^2/2 = \underline{623.5\text{ J}}$.

Q.7.

The F.B.D. of the beam is shown in Fig 7. Considering the equilibrium of the beam,

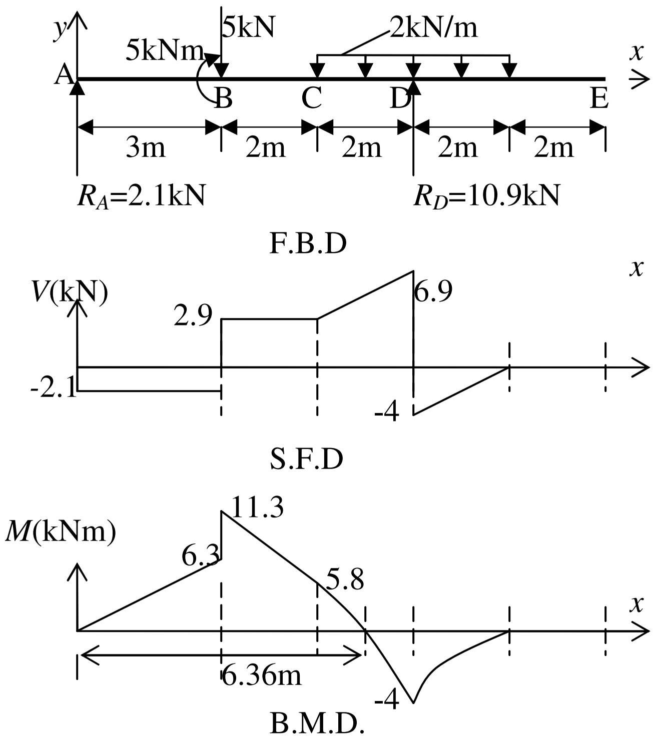


Fig.7

$\sum M_A = 7R_D - (4\times2)\times7 - 5\times3 - 5 = 0.$

$\rightarrow R_D = 76/7 = 10.9$ kN.

$\sum F_y = 0 \rightarrow R_A + R_D - 5 - 4\times2.$

$\rightarrow R_A = 15/7 = 2.1$ kN.

The S.F. V and B.M. M at various sections is:

$0 \le x \le 3$m

$V = -2.1$, $M = 2.1x$.

$3\text{m} \le x \le 5\text{m}$

$V = -2.1 + 5 = 2.9$,

$M = 2.1x + 5 - 5(x - 3)$.

$5\text{ m} \le x \le 7\text{ m}$

$V = -2(x - 9) + 10.9$,

$M = -2(9 - x)^2/2 + 10.9(7 - x)$.

$7\text{ m} \le x \le 9\text{ m}$

$V = -2(x - 9)$,

$M = -2(9 - x)^2/2$.

$9\text{ m} \le x \le 11\text{ m}$

$V = 0$, $M = 0$.

<u>The S.F. and B.M. diagrams are also shown in Fig.7</u>.

The maximum S.F.

$V_{max} = 6.9$ kN at D, i.e. $x = 7$ m.

The maximum B.M. $M_{max} = 11.3$ kNm at B, i.e. x = 3 m.

From, $M = -2(9 - x)^2/2 + 10.9(7 - x) = 0, \rightarrow x = 6.36$ m, is the point of contraflexure.

Q.8.

Let d_o be the outside diameter and $d_i = 0.6\, d_o$ the inside diameter of the shaft.

The polar moment of inertia $I_p = \pi\,(d_0^4 - d_i^4)/32 = \pi d_o^4(1 - 0.6^4)/32 = 0.0272\pi d_o^4$.

Using the torsion formula, from stiffness consideration,

$\theta = TL/GI_p = 25000\times3/[85\times10^9\times0.0272\pi d_o^4] \le 2.5\pi/180.$

$\rightarrow d_o^4 \ge 25000\times3\times180/[85\times10^9\times0.0272\pi\times2.5\pi] \rightarrow d_o \ge = 0.124$ m = 12.4 cm.

Using the torsion formula, from strength consideration,

$\tau_{max} = Tr_{max}/I_p = 25000(d_o/2)/[\,0.0272\pi d_o^4] \le 90\times10^6$

$\rightarrow d_o^3 \ge 25000/[2\times90\times10^6\times0.0272\pi] \rightarrow d_o \ge 0.118$ m = 11.8 cm.

Hence, $d_o = \underline{12.5\text{ cm}}$ should be selected. Then, $d_i = 0.6\, d_o = \underline{7.5\text{ cm}}$.

Q.9a.

Consider a vertical surface BD in the xz plane, submerged in a liquid with free surface at atmospheric pressure p_o as shown in Fig.9a.

The relation between the pressure p at a depth z in a static incompressible fluid of density ρ is

$p = p_o + \rho gz$.

The force dF on an elemental area dA would be $dF = pdA$.

The resultant force $F_R = \int_A pdA = = p_oA + \rho g\int_A zdA$.

If C is the centroid of the area A, $\int_A zdA = z_CA$.

The pressure at the centroid C, $p_C = p_o + \rho gz_C$. Then,

$F_R = p_oA + \rho gz_CA = (p_o + \rho gz_C)\,A = \underline{p_CA}$.

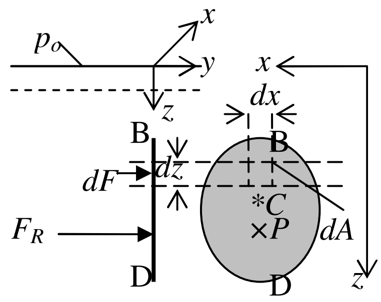


Fig.9a

The resultant force F_R acts at the centre of pressure $P(x_P, z_P)$ such that the moment of the resultant F_R about the x and z axes must be the same as the moment of the distributed pressure loading on the surface.

$z_PF_R = z_P\,\underline{p_CA} = \int_A zdF = \int_A zpdA = \int_A z(p_o + \rho gz)dA = p_oz_CA + \rho g\int_A z^2dA$

As $\int_A z^2dA = I_{xx}$, the moment of inertia of the area A about the x axis,

$z_P\,\underline{p_CA} = p_oz_CA + \rho gI_{xx}. \rightarrow \underline{z_P = (p_oz_CA + \rho gI_{xx})/p_CA}$.

$x_PF_R = x_P\,\underline{p_CA} = \int_A xdF = \int_A xpdA = \int_A x(p_o + \rho gz)dA = p_ox_CA + \rho g\int_A x\,zdA$

As $\int_A xzdA = I_{xz}$, the product of inertia about the x,z axes,

$x_P\,\underline{p_CA} = p_ox_CA + \rho gI_{xz}. \rightarrow \underline{x_P = (p_ox_CA + \rho gI_{xz})/p_CA}$.

Q.9b.

Consider an inclined surface BD in the xz plane at an angle θ to the horizontal, submerged in a liquid with free surface at atmospheric pressure p_o as in Fig.9b. The relation between the pressure p at a depth z in a static incompressible fluid of density ρ is

$p = p_o + \rho gh = \rho gz\sin\theta$.

The force dF on an element dA would be $dF = pdA$.

The resultant $F_R = \int_A pdA = p_oA + \rho g\sin\theta\int_A zdA$.

If C is the centroid of the area A, $\int_A zdA = z_CA$.

The pressure at the centroid C, $p_C = p_o + \rho gz_C\sin\theta$.

Then, $F_R = p_oA + \rho gz_C\sin\theta A = (p_o + \rho gh_C)\,A = \underline{p_CA}$.

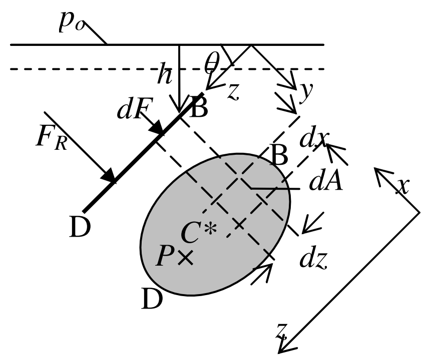


Fig.9b

The resultant force F_R acts at the centre of pressure $P(x_P, z_P)$ such that the moment of the resultant F_R about the x and z axes must be the same as the moment of the distributed pressure loading on the surface.

$z_PF_R = z_P\,\underline{p_CA} = \int_A zdF = \int_A zpdA = \int_A z(p_o + \rho gz\sin\theta)dA = p_oz_CA + \rho g\sin\theta\int_A z^2dA$

As $\int_A z^2dA = I_{xx}$, the moment of inertia of the area A about the x axis,

$z_P\,\underline{p_CA} = p_oz_CA + \rho g\sin\theta I_{xx}. \rightarrow \underline{z_P = (p_oz_CA + \rho g}z\underline{\sin\theta I_{xx})/p_CA}$.

$x_PF_R = x_P\,\underline{p_CA} = \int_A xdF = \int_A xpdA = \int_A x(p_o + \rho gz\sin\theta)dA = p_ox_CA + \rho g\sin\theta\int_A x\,zdA$

As $\int_A xzdA = I_{xz}$, the product of inertia about the x,z axes,

$x_P\,\underline{p_CA} = p_ox_CA + \rho g\sin\theta I_{xz}. \rightarrow \underline{x_P = (p_ox_CA + \rho g}\sin\theta\underline{I_{xz})/p_CA}$.

Q.10.
The stream function $\psi = 3x^2 - y^3$.
The velocity component u in the x direction, $u = \partial\psi/\partial y = \underline{-3y^2}$.
The velocity component v in the y direction, $v = -\partial\psi/\partial x = \underline{-6x}$.
The velocity components at the point $P(3,1)$ are $u_P = -3$ and $v_P = -18$.
Hence at the point (3,1), the velocity vector $\underline{\mathbf{v} = -3\mathbf{i} - 18\mathbf{j}}$.
Magnitude $v = \sqrt{(3^2 + 18^2)} = \underline{18.25,}$ inclination with x axis $\theta = \tan^{-}1(18/3) - 180 = \underline{-99.5^0}$.

The flow is derived from a stream function and hence is a possible 2D flow. The stream function $\psi = 3x^2 - y3$ does not satisfy the Laplace equation,
$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 = 6 - 6y \neq 0$. Therefore, the flow is not irrotational and the potential function would not exist for this flow.

Q.11.
The continuity equation between the inlet section 1 and the outlet section 2 is,
$Q = A_2V_2 = A_1V_1 = (\pi\times6^2/4)\times15 = 424.115$ m^3/s.
$\rightarrow V_2 = Q/A_2 = 424.115/(\pi\times4.8^2/4) = 23.4375$ m/s.
The Bernoulli's equation between the inlet sections 1 and the outlet section 2 would be
$P_2/\rho g + V_2^2/2g + z_2 = P_1/\rho g + V_1^2/2g + z_1$.
$\rightarrow P_2 = P_1 + \rho(V_1^2 - V_2^2)/2 + (z_1 - z_2)$
$= 282\times10^3 + 0.9\times10^3(15^2 - 23.4375^2)/2 = 136.1\times10^3$ Pa = 136.1 kPa.
The gage pressure at the inlet and outlet are,
$P_{g1} = 282 - 101.325 = 180.675$ kPa and $P_{g2} = 136.1 - 101.325 = 34.775$ kPa.
The momentum equation in the x direction yields:
$-F_x + P_{g1}A_1 - P_{g2}A_2\cos60 = \rho Q(V_2\cos60 - V_1)$.
$\rightarrow F_x = P_{g1}A_1 - P_{g2}A_2\cos60 - \rho Q(V_2\cos60 - V_1)$
$= 180.675\times10^3(\pi\times6^2/4) - 34.775\times10^3(\pi\times4.8^2/4)\cos60$
$- 0.9\times10^3\times424.115(23.4375\cos60 - 15) = 6046.3\times10^3$ N = 6046.3 kN.
The momentum equation in the y direction yields:
$F_y - P_{g2}A_2\sin60 = \rho QV_2\sin60$.
$\rightarrow F_y = P_{g2}A_2\cos60 + \rho QV_2\sin60$
$= 34.775\times10^3(\pi\times4.8^2/4)\sin60 + 0.9\times10^3\times424.115\times 23.4375\sin60$
$= 8292.6\times10^3$ N = 8292.6 kN.

SOLUTIONS A-03 APPLIED MECHANICS (June 2004)

Q.1. a. C The resultant force magnitude $R = (P^2 + P^2 + 2PP\cos120)^{1/2} = P$. Hence, the acceleration magnitude = $R/m = P/m$.

b. C The simplest resultant of a system of parallel forces is either a force or a couple.

c. B The block is in equilibrium, i.e. $\sum F_h=0$. The frictional force must be equal and opposite to the applied force $P/2$.

d. D The second moment of area of a square area about any centroidal axis in the plane of the area is the same, i.e. $b^4/12$.

e. A The total distance traveled $d = 20 + 20 = 40$ km. the time to travel $t = 20/20 + 20/60 = 4/3$ h. average speed = $d/t = 40/(4/3) = 30$ km/h.

f. B The nominal stress = load/original area of cross-section is maximum at the ultimate load.

g. D The B.M. is constant. The curvature $d^2v/dx^2 = M/EI$ = constant. Hence, the deflection v would have a quadratic variation.

h. A A manometer connected to a pipeline is used to measure the static pressure.

Q.2.

The F.B.Ds. of the sphere B and the cylindrical tube C are as shown in Fig.2. The forces on the sphere B are its weight W, the radial reaction P from the tube C and the reaction Q from the sphere A along the common normal. From the geometry of the spheres inside the tube, $2R = 2r + 2r\cos\theta \rightarrow \cos\theta = (R - r)/R$.

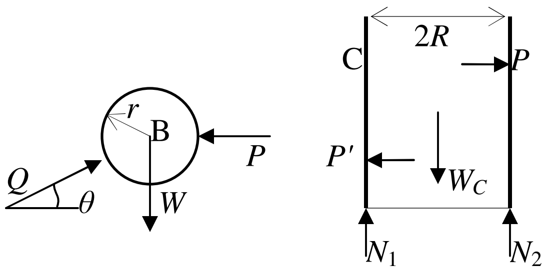
Considering the equilibrium of sphere B,

$P = Q\cos\theta$ and $W = Q\sin\theta \rightarrow P = W/\tan\theta$.

The tube C would be subjected to its weight W_C, the radial reactions P and P' from the spheres B and A, respectively and the vertical reactions N_1, N_2 from the horizontal table. From the force equilibrium equation in the horizontal direction,

$P' = P = W/\tan\theta$.

At impending clockwise tipping of the tube, the vertical reaction N_1 vanishes, i.e. $N_1 = 0$.



F.B.D of Sphere B F.B.D. of Tube B

Fig.2

Considering the moment equilibrium about the point of application of N_2,

$W_C \times R - P \times 2r\sin\theta < 0 \rightarrow \underline{r/R < (1 - W_C/2W)}$.

Q.3.

The F.B.D. of the truss is shown in Fig.3(i). As the support A is hinged, the reaction at A has both a horizontal component H_A and a vertical component, R_A. At the roller support C, the reaction R_C is vertical. The equilibrium equations of the truss,

$\sum F_x = H_A + 80 = 0 \rightarrow H_A = -80$ kN.

$\sum M_A = R_C \times 8 - 80\times 3 - 40\times 4 = 0 \rightarrow R_C = 50$ kN.

$\sum F_y = R_A + R_C - 40 = 0 \rightarrow R_A = -10$ kN.

Also $\tan\theta = 3/4. \rightarrow \sin\theta = 3/5, \cos\theta = 4/5$.

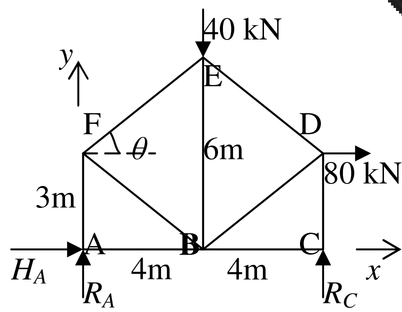


Fig.3(i)

The tensile force (T) in a member would be given a positive sign. Consider the equilibrium of the joints whose F.B.Ds are shown in Figs.3(ii) to (vi).

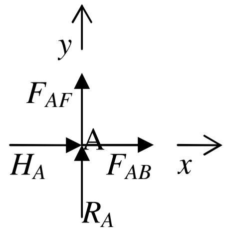


Fig.3(ii) Joint A

Consider Joint A:

$\sum F_x = F_{AB} + H_A = 0 \rightarrow F_{AB} = -H_A =$ <u>80 kN (T)</u>.

$\sum F_y = F_{AF} + R_A = 0 \rightarrow F_{AF} = -R_A =$ <u>10 kN (T)</u>.

Consider Joint F:

$\sum F_x = F_{EF}\cos\theta + F_{BF}\cos\theta = 0 \rightarrow F_{EF} = -F_{BF}$

$\sum F_y = -F_{AF} + F_{EF}\sin\theta - F_{BF}\sin\theta = 0$

$\rightarrow F_{EF} = F_{AF}/2\sin\theta =$ <u>8.3 kN(T)</u>,

$F_{BF} = -8.3$ kN, i.e. <u>8.3 kN(C).</u>

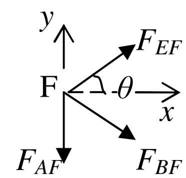


Fig.3(iii) Joint F

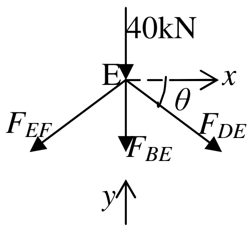


Fig.3(iv) Joint E

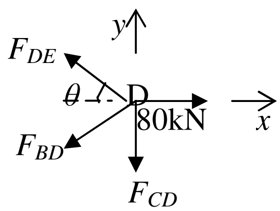


Fig.3(v) Joint D

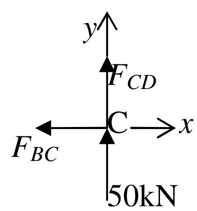


Fig.3(vi) Joint C

Consider Joint E:

$\sum F_x = F_{DE}\cos\theta - F_{EF}\cos\theta = 0. \rightarrow F_{DE} = F_{EF} =$ <u>8.3 kN(T).</u>

$\sum F_y = -F_{BE} - F_{DE}\sin\theta - F_{EF}\sin\theta - 40 = 0. \rightarrow F_{BE} = -50$ kN, i.e. <u>50 kN(C)</u>.

Consider Joint D:

$\sum F_x = -F_{DE}\cos\theta - F_{BD}\cos\theta + 80 = 0. \rightarrow F_{BD} = 275/3 =$ <u>91.7 kN(T)</u>

$\sum F_y = -F_{CD} + F_{DE}\sin\theta - F_{BD}\sin\theta = 0 \rightarrow F_{CD} = -50$ kN, i.e <u>50 kN(C)</u>.

Consider Joint C:

Considering the equilibrium equation of the joint C in the x direction, $\sum F_x = -F_{BC} = \underline{0}$. The member BC is a zero force member.

Q.4.

The unequal Z section is divided into three parts 1, 2, 3 as shown in Fig.4. The area of the Z section is A and x_c, y_c are the coordinates of its centroid. Let A_i refer to the area and x_i, y_i the coordinates of the centroid of its i^{th} part.

$x_c = \sum A_i x_i / \sum A_i = [20\times 5 + 24\times 1 + 12\times(-1)]/(20 + 24 + 12) = 112/56 =$ <u>2 cm</u>.

$y_c = \sum A_i y_i / \sum A_i = [20\times 1 + 24\times 8 + 12\times 15)]/(20 + 24 + 12) = 392/56 =$ <u>57/8 = 7 cm</u>.

Let I^c_{xx} and I^c_{yy} be the second moment of area of the Z section about centriodal axes through C parallel to the x,y axes.

$I^c_{xx} = \sum(I^c_{xx})_i = \sum[b_ih_i^3/12 + A_i(y_i - y_c)^2]$
$= 10\times2^3/12 + 20(1 - 7)^2$
$+ 2\times12^3/12 + 24(8 - 7)^2$
$+ 6\times2^3/12 + 12(15 - 7)^2$
$= 1810.67$ cm^4.

$I^c_{yy} = \sum(I^c_{yy})_i = \sum[h_ib_i^3/12 + b_ih_i(x_i - x_c)^2]$
$= 2\times10^3/12 + 20(5 - 2)^2$
$+ 12\times2^3/12 + 24(1 - 2)^2$
$+ 2\times6^3/12 + 12(-1 - 2)^2$
$= 522.67$ cm^4.

The polar moment of the area I^c_{zz} about an axis through C, would be

$I^c_{zz} = I^c_{xx} + I^c_{yy} = 1810.67 + 522.67 = \underline{2333.3 \text{ cm}^4}$.

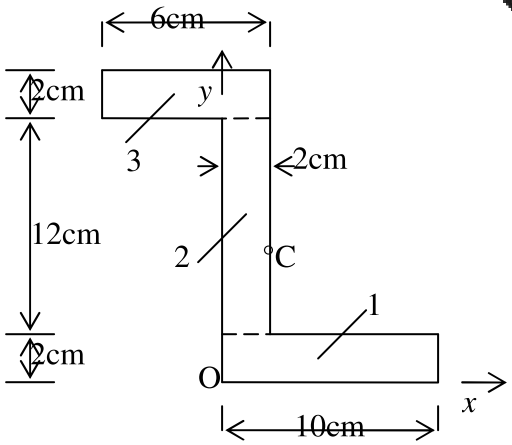


Fig.4

Q.5a.

The train starts from rest, i.e. initial speed $u = 0$. It moves with uniform tangential acceleration a_t and reaches a speed $v_1 = 36$ km/h in a distance $s_1 = 0.6$ km. Therefore, using the relation $v^2 = u^2 + 2a_ts$,

$a_t = v_1^2/2s_1 = 1080$ km/h^2.

The speed v_2 at the middle of the distance $s_2 = 0.3$ km, would be

$v_2 = \sqrt{(2a_ts_2)} = \underline{\sqrt{648} = 25.456 \text{ km/h}}$.

The centripetal acceleration a_{n2} at the mid-distance s_2 is $a_{n2} = v_2^2/R = 810$ km/h^2.

The total acceleration $a = \sqrt{(a_n^2 + a_t^2)} = \sqrt{(810^2 + 1080^2)} = \underline{1350 \text{ km/h}^2}$.

Q.5b.

Let v_1' and v_2' be the velocities of spheres of m_1 and m_2, respectively, just after impact.

The momentum is conserved,

$m_1v_1' + m_2v_2' = m_1v_1 + m_2v_2 \rightarrow m_2(v_2' - v_2) = m_1(v_1 - v_1')$ (1)

As the impact is perfectly elastic, the velocity of separation = the velocity of approach,

$v_2' - v_1' = v_1 - v_2 \rightarrow v_2' + v_2 = v_1 + v_1'$ (2)

Multiplying equations (1) and (2),

$m_2(v_2'^2 - v_2^2) = m_1(v_1^2 - v_1'^2) \rightarrow m_1v_1'^2 + m_2v_2'^2 = m_1v_1^2 + m_2v_2^2. \rightarrow (\text{K.E.})_{\text{final}} = (\text{K.E.})_{\text{initial}}$.

Thus, the kinetic energy is conserved.

Q.6.

The F.B.D. of the cylinder is shown in Fig. Q.6.The forces on the cylinder are the weight mg, normal reaction N and the frictional force f.

Let a_c be the acceleration of the centre C parallel to the plane and α the angular acceleration of the cylinder.

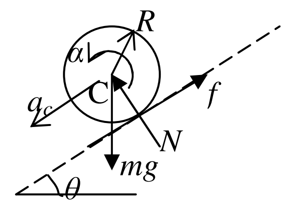


Fig.6

As there is no slip,
$a_c = \alpha R$. (1)
The equations of motion parallel to the plane and for rotation are
$mg\sin\theta - f = ma_c$. (2)
$fR = I\alpha = (mR^2/2)\alpha$. (3)
From equations (1) to (3), $\alpha = \underline{2g\sin\theta/3R}$, $a_c = \underline{2g\sin\theta/3}$, and $f = \underline{mg\sin\theta/3}$.
As the centre of mass C has no acceleration normal to the plane, $N = mg\cos\theta$ and the frictional force $f \le \mu N$,
$mg\sin\theta/3 \le \mu mg\cos\theta \rightarrow \underline{\tan\theta \le 3\mu}$.

Q.7a.
As the pin is in double shear, for determining the diameter d of the pin,
$\tau \le P_{max}/(2\pi d^2/4) \rightarrow d \ge (2P_{max}/\pi\tau)^{1/2} = [2\times78.5\times10^3/(\pi\times80\times10^6)]^{1/2} = 0.025$ m = 25 mm.
For the tension member,
$\sigma \le P_{max}/[(b - d)t] = P_{max}/(d\,t)$, as $b = 2d$.
$\rightarrow t \ge P_{max}/(\sigma\,d) = 78.5\times10^3/(157\times10^6\times0.025) = 0.020$ m = 20mm.

Q.7b.
Consider a V notch with an angle θ as shown in Fig. 7b. The liquid is at a level H above the base point. The discharge dQ through an elementary strip of depth dh at a depth h below the free liquid level would be
$dQ = VdA = \sqrt{(2gh)}bdh$.
The discharge Q through the whole notch would be

$$Q = \int_0^H \sqrt{(2gh)}bdh.$$

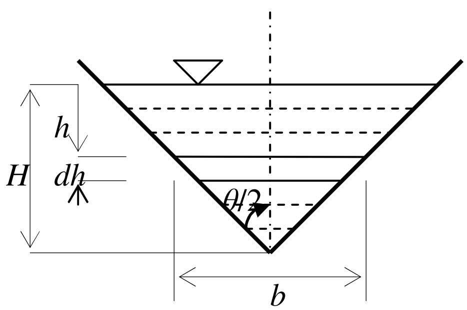


Fig.7b

For a V notch, $b = 2(H - h)\tan(\theta/2)$. Hence,

$$Q = 2\tan(\theta/2)\sqrt{2g}\int_0^H (H-h)h^{1/2}dh$$

$$Q = 2\tan(\theta/2)\sqrt{2g}[(2/3)Hh^{3/2} - (2/5)h^{5/2}]_0^H = \underline{(8/15)\tan(\theta/2)\sqrt{2g}H^{5/2}}.$$

Q.8.
Let d_i and d_o be the internal and external diameters, respectively of the shaft. The polar moment of the cross-sectional area would be $I_p = \pi(d_o^4 - d_i^4)/32$. (1)
Using the torsion formula, from stiffness consideration, $\theta = TL/GI_p$. (2)
Using the torsion formula, from strength consideration, $\tau_{max} = T(d_o/2)/I_p$. (3)
Eliminating I_p From equations (2) and (3),
$d_o = 2\tau_{max}L/(G\theta) = 2\times82\times10^6\times2.5/(82\times10^9\times2\pi/180) = 0.144$ m = 14.4 cm. (4)
Using equations, (1), (2) and (4),
$d_i^4 \le d_o^4 - 32I_p/\pi = (32TL/G\theta/\pi) = 32\times25000\times2.5/(82\times10^9\times\pi/90\times\pi)$
$\rightarrow d_i = 0.118$ m = 11.8 cm.

Let d be the diameter of the solid shaft. Then, $I_p = \pi d^4/32$.
From stiffness consideration, $\theta \leq TL/GI_p = 32TL/(G\pi d^4)$
$\rightarrow \pi/90 \leq 32\times25000\times2.5/(82\times10^9\times \pi d^4) \rightarrow d \geq .123\text{m} = 12.3\text{cm}$.
From strength consideration, $\tau_{max} \leq T(d_o/2)/I_p = 16T/(\pi d^3)$.
$82\times10^6 \leq 16\times25000/(\pi d^3) \rightarrow d \geq .116 \text{ m} = 11.6 \text{ cm}$.
Hence d = <u>12.3 cm.</u>
The % increase in weight $= 100[d^2 - (d_o^2 - d_i^2)]/(d_o^2 - d_i^2)$
$= 100[12.3^2 - (14.4^2 - 11.8^2)]/(14.4^2 - 11.8^2) = \underline{122.1}$

Q.9.
The beam with the loading and support reactions is shown in Fig.9. From the equilibrium equations of the beam,
$\sum M_B = R_A\times L - (wL/2)L/4 = 0 \rightarrow R_A = wL/8$.
$\sum F_y = R_A + R_B + wL/2 = 0 \rightarrow R_B = 3wL/8$.

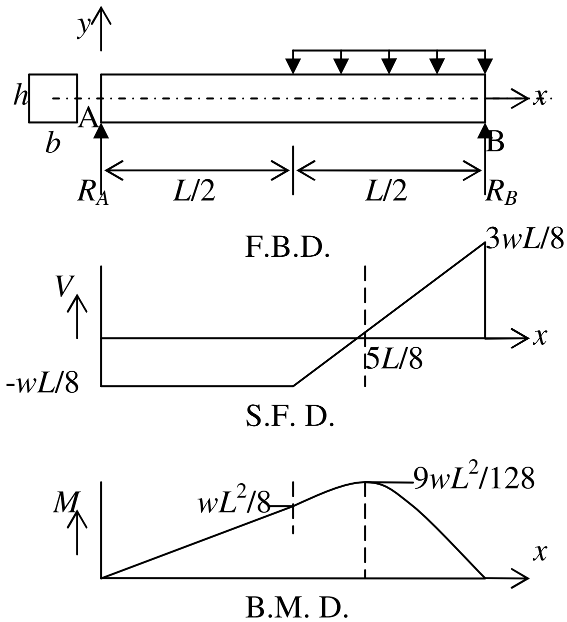


Fig.9

The S.F. V at any section x of the beam, using singularity functions would be,
$V = - wL/8 + w\langle x - L/2\rangle$.
The S.F. diagram is also shown in Fig. 9.
The maximum S.F.
$V_{max} = 3wL/8$ at the right support, $x = L$.
$V = - wL/2 + w\langle x - L/2\rangle = 0$ at $x = 5L/8$.

The B.M. M at any section x is
$M = (wL/8)x + w\langle x - L/2\rangle^2/2$.
The B.M.diagram is also shown in Fig.9.
The maximum B.M.
$M_{max} = 9wL^2/128$ at $x = 5L/8$.

The maximum bending stress σ_{max} in the beam would be at $x = 5L/8$ at the top and bottom fibers, $y = \pm h/2$.
$|\sigma_{max}| = M_{max}(h/2)/I$
$= (9wL^2/128)(\pm h/2)/(bh^3/12)$
$= \underline{27wL^2/(64bh^2)}$.

Q.10a.
The F.B.D. of the wooden block is shown in Fig. 10a. Assume the length of the block normal to the plane of paper to be unity. At the pivot A, it is subjected to the reactions R and H. The weight W acts at the centre of gravity G. It is also subjected to a linear pressure distribution on the left from 0 at D to p_B at B and a constant pressure distribution p_B at the bottom from B to A. Let γ be the specific gravity of the wood. Take the density of water $\rho = 1000$ kg/m^3. Then,

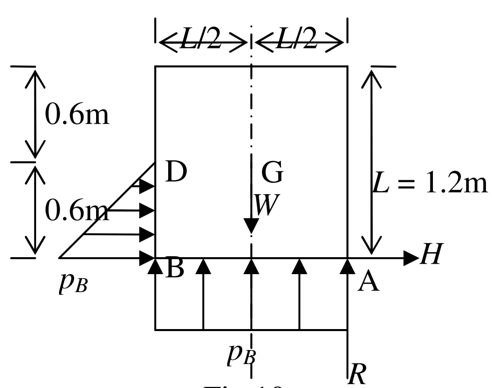


Fig.10a

$W = \gamma \rho g L^2 = 1000(1.2)^2 \gamma g = 1440\gamma g$.
$p_B = \rho g h = 1000(0.6)g = 600g$.
Considering the moment equilibrium about the pivot A, $\sum M_A = 0$.
$\rightarrow W\times0.6 - (p\times0.6/2)\times0.6/3 - (p\times1.2)\times0.6 = 0$.
$\rightarrow 1440\gamma g\times0.6 - (600g\times0.6/2)\times0.6/3 - (600g \times1.2)\times0.6 = 0. \rightarrow$ $\underline{\gamma = 0.542}$.

Q.10b.
Let the subscripts i and o refer to the nozzle inlet and outlet, respectively. Applying the continuity equation for incompressible flow,
$Q = A_iV_i = A_oV_o = 50\times0.02 = 1 \rightarrow V_i = A_oV_o/A_i = 0.02\times50/0.1 = 10$ m/s.
Now applying the Bernoulli's equation between the nozzle inlet and outlet,
$p_i/\rho g + V_i^2/2g + z_i = p_o/\rho g + V_o^2/2g + z_o$,
the gauge pressure $(p_i - p_o)$ at the inlet would be,
$(p_i - p_o) = \rho(V_o^2 - V_i^2)/2 + \rho g(z_o - z_i) = 1.23\times(50^2 - 10^2)/2 + 0 = 1476$ Pa = $\underline{1.476 \text{ kPa}}$.
If R is the axial force required to hold the nozzle in place,
$R + (p_i - p_o)\, A_i = \rho Q(V_o - V_i)$
$\rightarrow R = \rho Q(V_o - V_i) - (p_i - p_o)\, A_i = 1.23\times(50 - 10) - 1476\times0.1 =$ $\underline{-98.4 \text{ N}}$.

Q.11.
The inlet and outlet velocity triangles are as shown in Fig.11. Let subscripts 1 and 2 refer to the inlet and outlet diagrams, respectively. As water enters the runner blades in the radial direction and leaves the runner blades axially,
$V_{f1} = V_{r1}$ and $V_{f2} = V_2$.
From the inlet velocity triangle,
$u_1 = V_{f1}/\tan\alpha = 8/\tan15 = 29.856$ m/s $= V_{w1}$.
Let D_1 and D_2 be the inlet and outlet diameters of the runner.
As $u_1 = \pi D_1 N/60 \rightarrow D_1 = 60\times29.856/(\pi\times350) =$ $\underline{1.629 \text{ m}}$.
$D_2 = 0.6D_1 =$ $\underline{0.977 \text{ m}}$.
The head applied
$H = V_{w1}u_1/g + V_2^2/2g = (29.856)^2/9.81 + 8^2/(2\times9.81) =$ $\underline{48.69 \text{ m}}$.

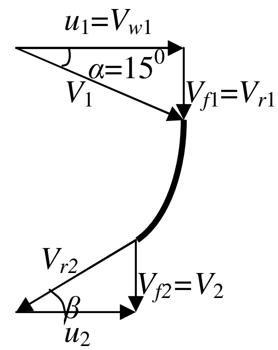


Fig.11

From the outlet velocity diagram, $\tan\beta = V_{f2}/u_2$.
The flow velocity is constant, $V_{f2} = 8$ m/s, and the blade velocity at the outlet $u_2 = 0.6u_1$.
Hence, the blade angle at outlet $\beta = \tan^{-1}[8/(0.6\times29.856)] =$ $\underline{24.06^0}$.
The discharge $Q = K(\pi D_1 b_1)V_{f1} = 0.95(\pi\times1.629\times0.1\times1.629)\times8 = 6.34$ m^3/s.
The power output $P = \rho Q V_{w1} u_1 = 1000\times6.34\times29.856\times29.856 = 5651000$ W = $\underline{5.651 \text{ MW}}$.

SOLUTIONS A-03 APPLIED MECHANICS (December 2004)

Q.1.

a. D Force, velocity and Linear momentum all follow the parallelogram law of addition.

b. A At the top of the trajectory, the speed is $v\cos\theta$ and centripetal acceleration g. Hence radius of curvature $R = (v\cos\theta)^2/g$.

c. B As the impact is perfectly elastic the kinetic energy is conserved. The impulse from the fixed plane changes the momentum.

d. C Force = $md^2x/dt^2 = md^2(A\sin\omega t)/dt^2 = -\,mA\omega^2\sin\omega t$. Hence, the maximum force = $mA\omega^2$.

e. A Yield stress is a material property.

f. D As the bending moment is maximum under the load, the curvature is also maximum there.

g. C Froude number is (inertia force/gravity force)$^{1/2}$.

h. B The energy gradient represents the total head and the hydraulic gradient line the pressure and datum head only.

Q.2a.

$\mathbf{F}_R = (\Sigma F_x)\,\mathbf{i} + (\Sigma F_y)\mathbf{j} =$ <u>$100\,\mathbf{i} - 75\,\mathbf{j}$ **N**</u>.

Equating the moment of the resultant and the given force system about O,

$x_R\mathbf{i} \times \mathbf{F}_R = 50\mathbf{k} + 2.5\mathbf{i} \times (-75)\mathbf{j} + 0.4\mathbf{j} \times 100\mathbf{i}$

$\rightarrow -75x_R\mathbf{k} = 50\mathbf{k} - 187.5\mathbf{k} - 40\mathbf{k} = -177.5\mathbf{k} \rightarrow x_R =$ <u>2.37 m</u>.

Q.2b.

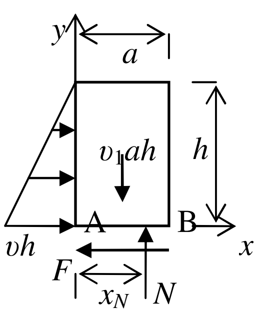


Fig.2b

The F.B.D. of the unit length of the dam is shown in Fig.2b. It is subjected to its own weight $\upsilon_1 ah$, the linearly increasing pressure on the left from 0 at the top to υh at the bottom, the shear force F and the normal reaction N from the foundation.

Considering the equilibrium of the dam,

$\Sigma F_x = (\upsilon h)h/2 - F = 0 \rightarrow F = \underline{\upsilon h^2/2}$.

$\Sigma F_y = N - \upsilon_1 ah = 0 \rightarrow N = \underline{\upsilon_1 ah}$.

$\Sigma M_A = Nx_N - (\upsilon_1 ah)a/2 - (\upsilon h^2/2)h/3 = 0$.

$\rightarrow x_N = \underline{a/2 + \upsilon h^2/(6\upsilon_1 a)}$.

Q.3a.

Consider the equilibrium of the portion of the truss to the right of the section XX as shown in Fig.3a. The forces acting on this portion are the 500 N loads at D, C and the tensile forces of the sectioned members F_{GF}, F_{BF} and F_{BC}.

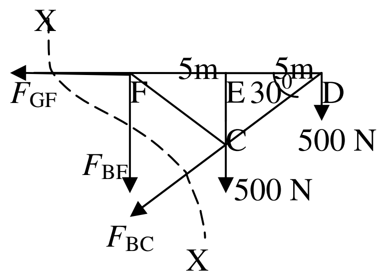


Fig.3a

Taking moment of all the forces about D

$\Sigma M_D = 10 \times F_{BF} + 5 \times 500 = 0$

$\rightarrow F_{BF} = -250$ N, i.e. 250 N (C).

Taking moment about F

$\Sigma M_F = -10\sin 30^0 \times F_{BC} - 5 \times 500 - 10 \times 500 = 0$

$\rightarrow F_{BC} = -1500$ N, i.e. 1500 N(C).

Q.3b.

There is inevitable play between the column and the collar and hence the collar will be in contact with the column at A and B. The F.B.D. of the collar is as shown in Fig.3b with load P, normal reactions N_A, N_B and frictional forces f_A, f_B.

At impending slip $f_A = \mu N_A, f_B = \mu N_B$.

Considering the equilibrium of the collar,

$\Sigma F_H = -N_A + N_B = 0 \rightarrow N_A = N_B = N$. Hence, $f_A = f_B = f = \mu N$.

$\Sigma M_C = N_B a - f_A(x + b/2) - f_B(x - b/2) = 0. \rightarrow Na - 2fx = 0$.

Hence $x = a/2\mu$.

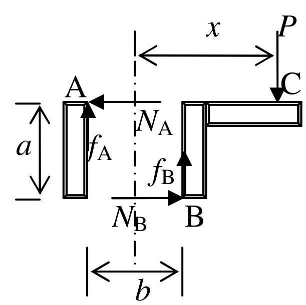


Fig.3b

Q.4.

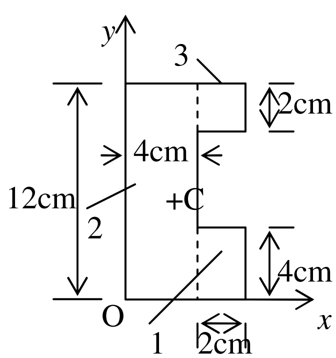


Fig.4

The channel section is divided in parts 1, 2, 3 as shown in Fig.4. Let a_i be the area and x_i, y_i the coordinates of the centroid C_i of the i^{th} part. C is the centroid of the Channel section. Then,

$x_C = (a_1x_1 + a_2x_2 + a_3x_3)/(a_1 + a_2 + a_3)$

$= (8 \times 5 + 48 \times 2 + 4 \times 5)/(8 + 48 + 4) = 2.6$ cm.

$y_C = (a_1y_1 + a_2y_2 + a_3y_3)/(a_1 + a_2 + a_3)$

$= (8 \times 2 + 48 \times 6 + 4 \times 11)/(8 + 48 + 4) = 5.8$ cm.

Let I^C_{xx} and I^C_{yy} be the second moments of area about the x, y axes through C. Then,

$I^C_{xx} = \sum[b_ih_i^3/12 + a_i(y_i - y_C)^2]$

$= (2 \times 4^3/12) + 8(2 - 5.8)^2 + (4 \times 12^3/12) + 48(6 - 5.8)^2 + (2 \times 2^3/12) + 4(11 - 5.8)^2$

$= 813.6$ cm^4.

$I^C_{yy} = \sum[h_ib_i^3/12 + a_i(x_i - x_C)^2]$

$= (4 \times 2^3/12) + 8(5 - 2.6)^2 + (12 \times 4^3/12) + 48(2 - 2.6)^2 + (2 \times 2^3/12) + 4(5 - 2.6)^2$

$= 154.4$ cm^4.

Polar moment of area about the axis through centroid $I^C_{zz} = I^C_{xx} + I^C_{yy} = 968$ cm^4

Q.5a.
Tangential acceleration in the positive x direction is $a_t = 3$ m/s^2.
Centripetal acceleration in the positive y direction is $a_n = V^2/R = 4^2/4 = 4$ m/s^2.
The total acceleration vector $\underline{\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} \text{ m/s}^2}$.
Magnitude $a = \sqrt{(3^2 + 4^2)} = \underline{5 \text{ m/s}^2}$ at angle $\theta = \tan^{-1}(4/3) = \underline{53.1}^0$ with the x axis.

Q.5b.
The initial velocity of the car is $V_{i1} = 8$ km/h $= 8\times1000/3600 = 20/9$ m/s.
As the impact with the rigid wall is perfectly plastic, the final velocity $V_{f1} = 0$.
Energy absorbed by the bumper during impact $E_b = mV_{i1}^2/2 = 1100(20/9)^2/2 = \underline{2716 \text{ J}}$.

Let U be the maximum initial speed of the moving car at which it can hit a similar stationary car without causing any damage. As the impact is perfectly plastic, the common velocity after impact would be V for both the cars.
From linear momentum conservation: $1100U = 1100V + 1100V \rightarrow V = U/2$.
Initial kinetic energy $KE_1 = 1100U^2/2$.
Kinetic energy after impact $KE_2 = (1100 + 1100)V^2/2 = 1100U^2/4$.
Energy to be absorbed by the bumpers during impact $= KE_1 - KE_2 = 1100U^2/4$.
The energy which can be absorbed by the two bumpers without damage is: $2E_b = 5432$ J.
Therefore, $1100U^2/4 = 5432 \rightarrow U = 4.444$ m/s $= \underline{16 \text{ km/h}}$.

Q.6a.
The reference xyz is fixed to the bent rod and at the instant of interest have the same orientation as the ground reference XYZ.
Unit vectors along x, y, z are **i**, **j**, **k** and along the X, Y, Z are **I, J, K,** respectively**.**
Angular velocity of the disc C,
$\boldsymbol{\omega}_C = \omega_1\mathbf{j} + \omega_2\mathbf{K} = 10\,\mathbf{j} + 5\,\mathbf{K}$ rad/s $= \underline{10\,\mathbf{J} + 5\,\mathbf{K}}$ rad/s at this instant.
Angular acceleration of the disc C
$\boldsymbol{\alpha}_C = (d\boldsymbol{\omega}_C/dt)_{XYZ} = (d\omega_1/dt)\mathbf{j} + \omega_1 d\mathbf{j}/dt + (d\omega_2/dt)\mathbf{K} + \omega_2 d\mathbf{K}/dt = \omega_1 d\mathbf{j}/dt = \omega_1(\omega_2\mathbf{K} \times \mathbf{j})$
$= \omega_1\omega_2\mathbf{i} = 50\mathbf{i}$ rad/s^2 $= \underline{50\mathbf{I} \text{ rad/s}^2}$ at this instant.

Q.6b.
As the string breaks, the F.B.D. of the rod is shown in Fig.6b.
The rod AB would start rotating about the pinned end A.
At this instant, its angular velocity $\omega = 0$, and angular acceleration is α.
The equation of motion for rotation is
$\Sigma M_A = I_A\alpha \rightarrow -mgL/2 = (mL^2/3)\alpha$
$\rightarrow \alpha = \underline{-3g/2L}$.

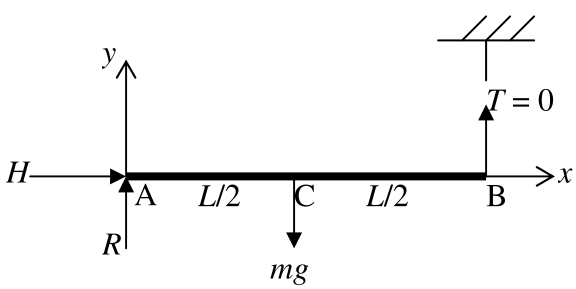


Fig.6b

Acceleration of the centre of mass C is $a_{Cx} = \underline{0}$ and $a_{Cy} = \alpha L/2 = \underline{-3g/4}$.
The equations of motion for the centre of mass give the reactions at the hinge A
$H = ma_{Cx} = \underline{0}$.
$R - mg = m\,a_{Cy} = m(-3g/4) = -3mg/4 \rightarrow R = \underline{mg/4}$.

Ans.7(a)

The bar is imagined to be cut by a plane at 45^0 to the cross-section and the F.B.D. of the portion to the left is shown in Fig.7a.

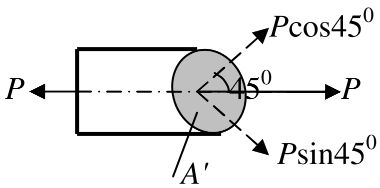


Fig.7a

The area of the inclined section $A' = A/\cos45^0 = A\sqrt{2}$.

The axial force P can be resolved into components normal to the area A' and in the plane of the area A'.

Normal Force $P_n = P\cos45^0 = P/\sqrt{2}$

$\rightarrow$Normal stress $= P_n/A' = (P/\sqrt{2})/A\sqrt{2}$ = $\underline{P/2A}$.

Shear force $P_t = P\sin45^0 = P/\sqrt{2}$

$\rightarrow$ Shear stress $= P_t/A' = (P/\sqrt{2})/A\sqrt{2}$ = $\underline{P/2A}$.

Q.7b.

Hoop stress $\sigma_{\theta\theta} = pd/2t = 0.8\times10^6\times2000/2\times10 = 80\times10^6$ Pa.

Axial stress $\sigma_{zz} = pd/4t = .8\times10^6\times2000/4\times10 = 40\times10^6$ Pa.

Hoop strain $\varepsilon_{\theta\theta} = (\sigma_{\theta\theta} - \nu\sigma_{zz})/E = (80\times10^6 - 0.25\times40\times10^6)/200\times10^9 = 35\times10^{-5}$.

Change in diameter $\Delta d = \varepsilon_{\theta\theta}d = 35\times10^{-5}\times2000$ = $\underline{0.7\text{ mm}}$.

Q.8.

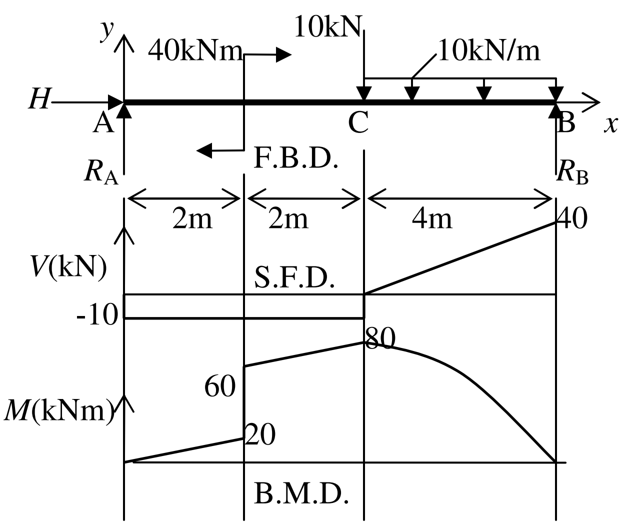


Fig.8

The F.B.D. of the beam is shown in Fig.8. From equilibrium of the beam,

$\Sigma F_x = 0 \rightarrow H = \underline{0}$.

$\Sigma M_B = 0 \rightarrow 8R_A = -40 + 10\times4 + 40\times2 = 80$

$\rightarrow \underline{R_A = 10\text{ kN}}$

$\Sigma F_y = 0 \rightarrow R_B = 50 - R_A = \underline{40\text{ kN.}}$

The S.F. at a section x is

$V = R_A - 10\langle x-4\rangle$

$\underline{V_{max} = 40\text{ kN at the right support B}}$.

The B.M. at a section x is

$M = R_A x + 40\langle x - 2\rangle^0 - 10\langle x - 2\rangle$

$-10\langle x - 2\rangle^2/2$.

$\underline{M_{max} = 80\text{ kNm at the centre C}}$.

<u>The S.F. and B.M. diagrams are also shown in Fig.8</u>.

Q.9a.

Let D be the diameter of the solid shaft in mm.

The polar moment of the cross-section $I_p = \pi D^4/32$.

If τ_{shaft} is the maximum shear stress in the shaft, the torque transmitted

$T = \tau_{shaft} I_p/(D/2) = \tau_{shaft}\,\pi D^3/16$. (1)

Number of bolts $n = 8$, diameter of bolts $d = 12.5$ mm, pitch circle radius $R = 115$ mm.

If τ_{bolt} is the average shear stress in a bolt, the torque transmitted

$T = n\times\tau_{bolt}(\pi d^2/4)\times R = 8\times\tau_{bolt}(\pi\times12.5^2/4)\times115$ (2)

As the torque transmitted T is the same and $\tau_{shaft} = \tau_{bolt}$, from equations (1) and (2)

$\pi D^3/16 = 8\times(\pi\times12.5^2/4)\times115 \rightarrow \underline{D = 83.2\text{ mm}}$.

Q.9b.

The inlet and outlet velocity diagrams are shown in Fig.9b. The subscripts 1 and 2 refer to the inlet and outlet conditions. Bucket speed $U_2 = U_1 = 15$ m/s. Inlet jet velocity

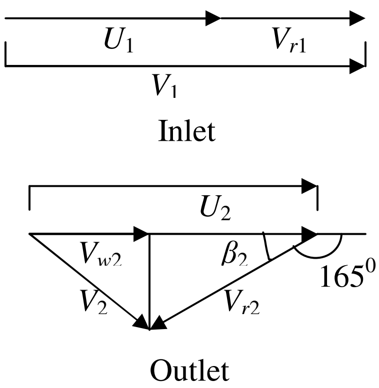


Fig.9b

$V_1 = C_v\sqrt{(2gH)} = 0.985\sqrt{(2\times9.81\times42)} = 28.27$ m/s.
From inlet velocity triangle
$V_{r1} = V_1 - U_1 = 28.57 - 15 = 13.27$ m/s.
$V_{w1} = V_1 = 28.27$ m/s.
The blade outlet angle $\beta_2 = 180^0 - 165^0 = 15^0$.
Neglecting frictional losses
$V_{r1} = V_{r2} = 13.27$ m/s.
From outlet velocity triangle
$V_{w2} = U_2 - V_{r2} \cos \beta_2 = 15 - 13.27 \cos 15^0 = 2.18$ m/s.
Power developed $P = \rho Q(V_{w1} - V_{w2})U_1$
$= 1000\times1\times(28.27 - 2.18)\times15 = 3913500$ W = <u>391.35 kW</u>.
Available Power = ρgH
$= 1000\times9.81\times42 = 41202$ W = 412.02 kW.
Turbine efficiency η = Power developed/available power = 391.35/412.02 = 0.95 = <u>95%</u>.

Q.10a.

At the section 6 m below the throat, i.e. section 1
Pressure $p_1 = 5$ atm $= 5\times10.33 = 51.65$m of water, velocity V_1 and datum $z_1 = 0$.
At the throat, i.e. section 2
Pressure $p_2 = 10.33 + 0.20 = 10.53$ m of water, velocity V_2 and datum $z_1 = 6$.
Applying Bernoulli's equation between sections 1 and 2
$51.65 + V_1^2/2g + 0 = 10.53 + V_2^2/2g + 6 \rightarrow V_2^2 - V_1^2 = 35.12\times2\times9.81 = 689$ (m/s)^2.
Area of section 1, $A_1 = \pi\times0.15^2/4 = 0.0177$ m^2,
Area of section 2, $A_2 = \pi\times0.07^2/4 = 0.00385$ m^2.
Using the continuity equation, discharge $Q = A_1V_1 = A_2V_2$.
$V_1 = Q/A_1 = Q/0.0177 = 56.6Q$ and $V_2 = Q/A_2 = Q/0.00385 = 260$.
Hence $V_2^2 - V_1^2 = (260^2 - 56.6^2)Q^2 = 689 \rightarrow Q =$ <u>0.1034 m^3/s</u>.

Q.10b.

Let the subscripts m and p denote the model and prototype, respectively.
The inertial and viscous forces are important. Hence, the Reynolds number must be identical in the model and prototype flow.
$R_e = (\rho VL/\mu)_m = (\rho VL/\mu)_p$
As the fluid is the same ρ and μ of the model and prototype are the same, Hence
$(VL)_m = (VL)_p \rightarrow V_m = V_mL_p/L_m = 60\times6 =$ <u>360 km/h</u>.
The non-dimensional term for the drag force F and inertia force ρV^2L^2 is $(F/\rho V^2L^2)$ and would be the same for the model and prototype, i.e. $(F/\rho V^2L^2)_p = (F/\rho V^2L^2)_m$
Hence, prototype drag $F_p = F_m (V_m^2/V_p^2)(L_m^2/L_p^2) = 510\times(360/60)^2(1/6)^2 =$ <u>510 N</u>.

Q.11a.

$V_r = (\partial\psi/\partial\theta)/r = \underline{V(1 - R^2/r^2)\cos\theta}$, $V_\theta = -\partial\psi/\partial r = \underline{-V(1 + R^2/r^2)\sin\theta}$.

For the stagnation points in the flow $V_r = 0 \rightarrow r = R$ and $V_\theta = 0 \rightarrow \theta = 0, \pi$.

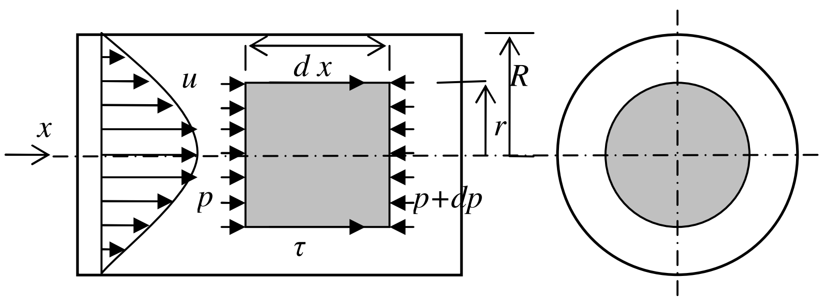
Hence the two stagnation points are $\underline{(R, 0) \text{ and } (R, \pi)}$.

The velocity on the surface of the cylinder is $V_r = 0$ and $V_\theta = -2V\sin\theta$.

As the flow is given to be irrotational, Bernoulli's equation can be applied between a point on the surface of the cylinder $r = R$ and a point far upstream in the uniform flow where the velocity is V and pressure p_∞. If p is the pressure on a point on the cylinder,

$p/\rho + (-2V\sin\theta)^2/2 = p_\infty/\rho + V^2/2 \rightarrow p = \underline{p_\infty + \rho(1 - 4\sin^2\theta)V^2/2}$.

Q.11b.



A fully developed laminar flow through a horizontal pipe of radius R is shown in Fig.11b. The axial equilibrium of a cylinder of fluid of radius r and length dx is considered.

Fig.11b

$(p + dp)\,\pi r^2 - p\pi r^2 = \tau\, 2\pi r dx \rightarrow \tau = (r/2)\, dp/dx$

According to Newton's law of viscosity $\tau = \mu du/dr$. Hence, $\mu du/dr = (r/2)dp/dx$

$\rightarrow du/dr = (1/2\mu)r dp/dx$

On integrating $u = (1/4\mu)r^2 dp/dx + C$

At $r = R$, $u = 0$. Hence, $C = -(1/4\mu)R^2 dp/dx$.

Substituting for C, $\underline{u = (1/4\mu)(R^2 - r^2)dp/dx.}$

This is a parabolic distribution. The maximum velocity is at the centre-line $r = 0$.

$u_{max} = \underline{(1/4\mu)\, R^2 dp/dx}$.