# SOLUTIONS D-02 APPLIED MECHANICS (June 2003)

Q1. a. A The friction force is absent hence the total reaction is normal to the surface at the point of contact.

b. C For equilibrium of a body subjected to two forces, the forces must be equal in magnitude, opposite in direction and must have the same line of action.

c. B For a plane area, with the $x$,$y$ axes in the plane and $z$ axis normal to the plane, the polar moment of inertia $I_{zz} = I_{xx} + I_{yy}$.

d. B Let $d$ be the depth of the well and $t$ the time splash is heard. Then the relation $t = \sqrt{(2d/9.81)} + d/350 = 4$ is satisfied for $d = 70.77$ m.

e. A The normal acceleration = $V^2/R = (72\times1000/3600)^2/200 = 2$ m/s$^2$.

f. B The shear stress = Force/Area resisting shear = $F/(\pi Dt)$.

g. B The modulus of elasticity is defined as the ratio of direct stress and direct strain within elastic limit.

h. B The B.M. at any section at a distance $x$ from the load $P$ at the free end is $Px$. The maximum B.M. would be at the fixed end $x_{max} = L$.

Q.2.
Consider the I-section to be divided in three parts 1, 2 and 3 as shown in Fig.2. Let $x_i$, $y_i$ be the coordinates of the centroids of the $i^{th}$ area $A_i$. The centroid of the whole section is at C($x_C$, $y_C$).

As the section is symmetrical about the $y$ axis its centroid would lie on the $y$ axis, i.e. $x_C = \underline{0}$.

$y_C = \sum A_i y_i / \sum A_i$
$= (150\times2.5 + 75\times12.5 + 100\times22.5)/(150+75+100)$
$= \underline{10.96\text{ cm}}$.

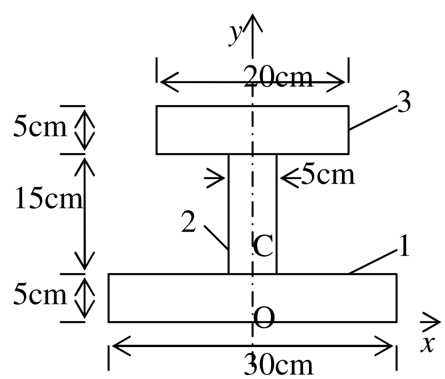
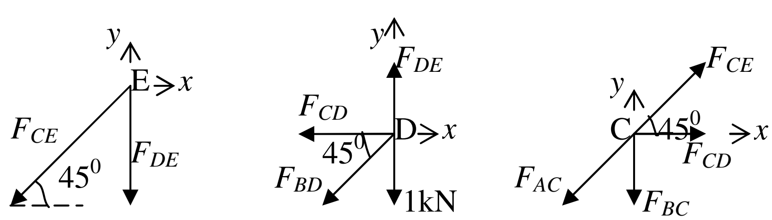


Fig.2

Q.3.
Consider the F.B.Ds. of the joints E, D and C shown in Fig.3. The F.B.Ds are shown assuming tensile forces in the various members.



Considering the equilibrium of the joint E:
$\sum F_x = - F_{CE}\cos45 = 0 \rightarrow F_{CE} = \underline{0}$, $\sum F_y = 0 \rightarrow - F_{DE} - \underline{F_{CE}}\sin45 = 0 \rightarrow F_{DE} = \underline{0}$.
Considering the equilibrium of the joint D:
$\sum F_y = F_{DE} - F_{BD}\sin45 - 1 = 0 \rightarrow \underline{F_{BD} = -\sqrt{2}\text{ kN}}$, i.e. $\underline{\sqrt{2}\text{ kN(C).}}$
$\sum F_x = - F_{CD} - F_{BD}\cos45 = 0 \rightarrow F_{CD} = \underline{1\text{ kN}}$, i.e. $\underline{1\text{ kN(T).}}$
Considering the equilibrium of the joint C:
$\sum F_x = F_{CD} + F_{CE}\cos45 - F_{AC}\cos45 = 0 \rightarrow F_{AC} = \sqrt{2}$ kN, i.e. $\underline{\sqrt{2}\text{ kN(T).}}$
$\sum F_y = - F_{BC} + F_{CE}\sin45 - F_{AC}\sin45 = 0 \rightarrow F_{BC} = -1$ kN, i.e. $\underline{1\text{ kN(C).}}$

Q.4.
A developed screw thread of pitch $p$, mean diameter $d$ is shown in Fig. 4. The helix angle with the horizontal $\theta = \tan^{-1} p/\pi d = \tan^{-1} 1.25/10\pi = 2.28^0$.
The F.B.D. of the weight $W$ while being lifted is also shown.
The friction angle $\varphi = \tan^{-1}\mu = \tan^{-1}0.2 = 11.31^0$.
The force $F$ at the end of the lever of length $R$ is equivalent to a horizontal force $P$ at the screw thread,
$P = F\times R/(d/2) = 50F/(10/2) = 10F$.

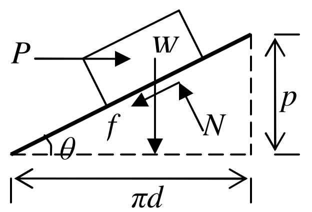


Fig.4

Case(i)
Consider the equilibrium of the load at impending slip upward as shown. The friction force $f = \mu N$.
$\sum F_V = N\cos\theta - f\sin\theta - W \rightarrow N = W/(\cos\theta + \mu\sin\theta)$.
$\sum F_H = P - N\sin\theta - f\cos\theta, \rightarrow P = N(\sin\theta + \mu\cos\theta) = W(\sin\theta + \mu\cos\theta)/(\cos\theta + \mu\sin\theta)$.
$\rightarrow P = W\tan(\theta + \varphi) \rightarrow F = W\tan(\theta + \varphi)/10 = 50\tan(2.28 + 11.31)/10 = \underline{1.209\text{ kN}}$.
Case(ii)
For impending motion down, the direction of friction force is reversed and hence,
$\rightarrow P = W\tan(\theta - \varphi) \rightarrow F = W\tan(\theta - \varphi)/10 = 50\tan(2.28 - 11.31)/10 = \underline{-0.795\text{ kN}}$.
Efficiency = effort without friction/effort with friction, i.e.
$\eta = \tan\theta/\tan(\theta + \varphi) = \tan2.28/\tan(2.28+11.31) = 0.165 = \underline{16.5\%}$.
As a force in the reverse direction is required to lower the load, <u>the jack would be self locking.</u>

Q.5.
The F.B.Ds. of the blocks are shown in Fig.5. The block on the horizontal surface is subjected to its weight $M_1g$, normal reaction $N$, friction force $\mu N$ and the tension $T$. The other block is subjected to its weight $M_2g$ and the tension $T$. The magnitude of acceleration of both the blocks is the same, i.e. $a$. The equations of motion for the blocks of mass $M_1$ and $M_2$ are,

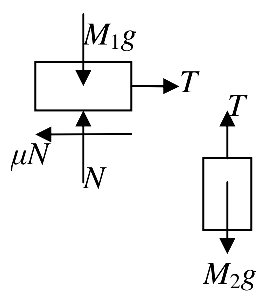


Fig.5

$N = M_1g$. (1)
$T - \mu N = M_1a$ (2)
$M_2g - T = M_2a$ (3)
From equations (1) to (3)
$a = (M_2 - \mu M_1)g/(M_1 + M_2) = (5 - 0.25\times10)\times9.81/(10 + 5) = \underline{1.635\text{ m/s}^2}$.
$T = M_2(g - a) = 5(9.81 - 1.635) = \underline{40.875\text{ N}}$.

Q.6.

Let $a_t$ be the uniform tangential acceleration of the car. If it starts from rest, the speed $V$ after time t would be, $V = a_t t$.

Then, $a_t = V_{60}/t = (18\times1000/3600)/60 = 1/12 = \underline{0.083 \text{ m/s}^2}$.

The speed after 30 s would be $V_{30} = a_t t = (1/12)\times30 = 2.5$ m/s.

The normal acceleration $a_n = V_{30}^2/R = 2.5^2/250 = \underline{0.025 \text{ m/s}^2}$.

Q.7.

As the cylinder rolls without slip on the horizontal plane AB, the instantaneous centre of rotation is at the point of contact O of the cylinder with the horizontal plane (see Fig.7). The velocity of any point on the cylinder is proportional to the distance from O and normal to the line joining the point to O.

Let $\omega$ be angular velocity of the cylinder. Then,

$\omega = V_C/\text{OC} = 20/1 = 20$ rad/s.

The velocity of point E, $V_E = \omega\text{OE} = 20\sqrt{2}$ m/s and is normal to the line OE as shown, i.e. $\mathbf{V}_E = \underline{20\mathbf{i} + 20\mathbf{j} \text{ m/s}}$.

The velocity of point F, $V_F = \omega\text{OF} = 20\times2 = 40$ m/s and is normal to the line OF as shown. i.e. $\mathbf{V}_F = \underline{40\mathbf{i} \text{ m/s}}$.

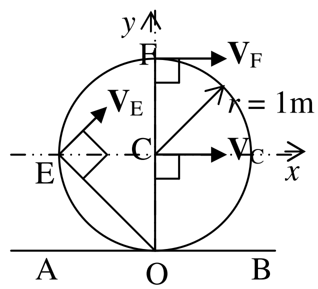



Fig.7

Q.8.

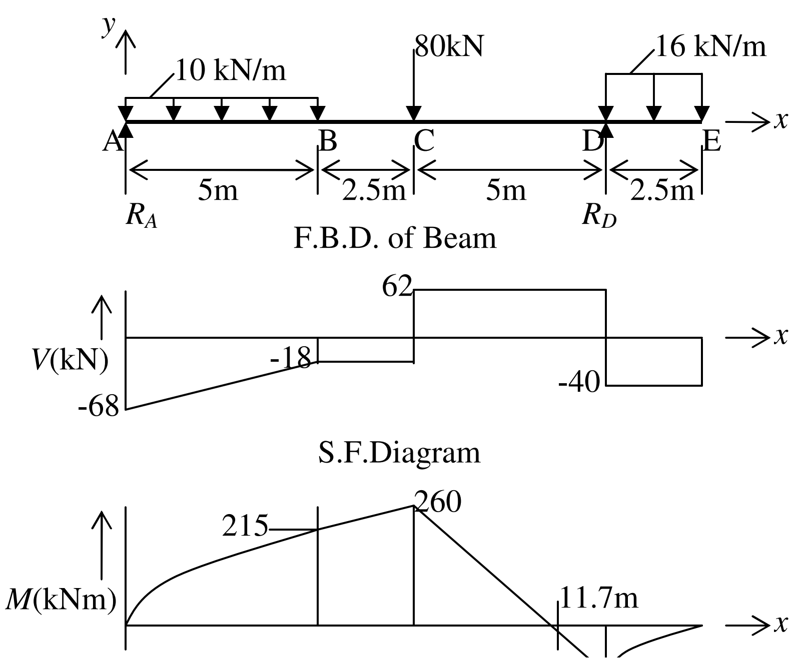
The F.B.D. of the beam is shown in Fig.8. Considering the equilibrium of the beam,

$\sum M_A = R_D\times12.5 - (10\times5)\times2.5 - 80\times7.5 - (16\times2.5)\times13.75 = 0 \rightarrow R_D = 102$ kN.

$\sum F_y = R_A + R_D - 10\times5 - 80 - 16\times2.5 = 0 \rightarrow R_A = 68$ kN.

The expressions for the S.F. $V$(in kN) and B.M. $M$(in kNm) between various sections are,

| | | |
|---|---|---|
| $0 \le x \le 5$ m | $V = -68 + 10x$, | $M = 68x - 10x^2/2$, |
| $5 \le x \le 7.5$ m | $V = -68 + 10\times5$, | $M = 68x - 10\times5(x-2.5)$, |
| $7.5 \le x \le 12.5$m | $V = -68 + 10\times5 + 80$, | $M = 68x - 10\times5(x-2.5) - 80(x-7.5)$, |
| $12.5 \le x \le 15$m | $V = -16(15 - x)$, | $M = -16(15 - x)^2/2$, |




<u>The S.F. and B.M. diagrams with values at important sections are also shown in Fig.8.</u>

The maximum S.F. is $V_{max} = -68$ kN, at A ($x = 0$).

The maximum B.M. is $M_{max} = 260$ kNm, at C ($x = 7.5$ m).

$7.5 \le x \le 12.5$m,

$M = -62x + 725 = 0$,

at $x = 725/62 = 11.7$ m which is a point of contraflexure.

Q.9.
The steel bar is loaded as shown in Fig.9.

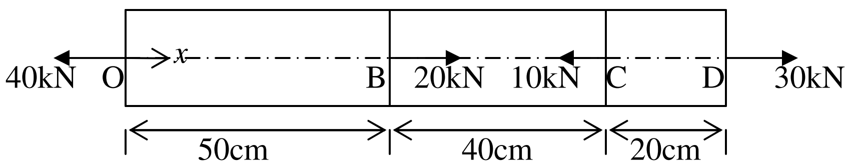



Fig.9

The cross-sectional area of the bar $A = \pi d^2/4 = \pi\times25^2/4 = 490.87$ mm$^2$.
The axial force $F$ in various portions of the bar OD would be,
Portion OB ($0 \le x \le 50$cm):
$P_{OB} = 40$ kN,
Hence, stress $\sigma_{OB} = P_{OB}/A = 40\times10^3/(490.87\times10^{-6}) = 81.5\times10^6$ Pa = $\underline{81.5 \text{ MPa}}$.
Portion BC (50 cm $\le x \le 90$cm):
$P_{BC} = 40 - 20 = 20$ kN,
Hence, stress $\sigma_{BC} = P_{BC}/A = 20\times10^3/(490.87\times10^{-6}) = 40.7\times10^6$ Pa. = $\underline{40.7 \text{ MPa}}$.
Portion CD ($90 \le x \le 110$cm):
$P_{CD} = 30$ kN,
Hence stress $\sigma_{CD} = P_{CD}/A = 30\times10^3/(490.87\times10^{-6}) = 61.1\times10^6$ Pa. = $\underline{61.1 \text{ MPa}}$.
The total elongation $\delta = \delta_{OB} + \delta_{BC} + \delta_{CD} = (\sigma_{OB}\text{OB} + \sigma_{OB}\text{BC} + \sigma_{OB}\text{CD})/E$
$= (81.5\times10^6\times0.5 + 40.7\times10^6\times0.4 + 61.1\times10^6\times0.2)/(210\times10^9) = 0.33\times10^{-3}$ m.
= $\underline{0.33 \text{ mm}}$.

Q.10.
Let a circular shaft of radius $r$ and length $l$ be subjected to a torque $T$ as shown in Fig.10.
The following basic assumptions are made while deriving the torsion formula:
The material is linearly elastic following Hooke's law.
A plane section normal to the axis of the shaft remains plane and normal during deformation.
A radial line in the section remains radial during deformation.

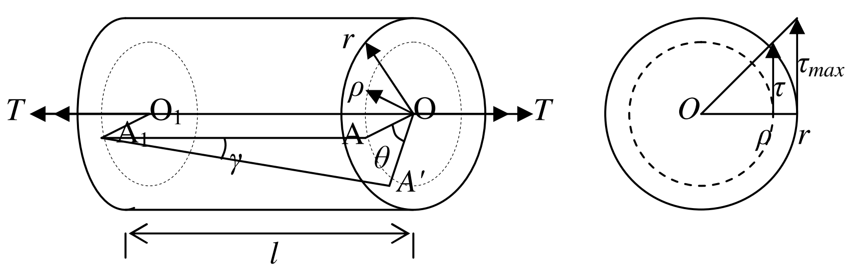



Fig.10

Consider a radial plane $OAA_1O_1$. After deformation the plane occupies the position $OA'A_1O_1$, where $\theta$ is the angle of twist over the length $l$.
Hence, the shear strain between circumferential and axial elements at radius $\rho$ is $\gamma = \rho\theta/l$.

As the material follows Hooke's law, the shear stress $\tau = G\gamma = G\rho\theta/l$.
Both the shear strain $\gamma$ and shear stress $\tau$ are proportional to the radial distance $\rho$.
The maximum shear stress $\tau_{max} = Gr\theta/l$ is at the outer fibers. (1)
The shear stress distribution over the cross-section is equivalent to the applied torque $T$

$$T = \int_A \rho \times \tau dA = \int_A \rho(G\rho\theta / l)dA = (G\theta / l)\int_A \rho^2 dA = (G\theta / l)J \quad (2)$$

where, $J = \int_A \rho dA = \pi r^4/2$ is the polar moment of inertia of the cross-sectional area $A$.
Combining the equations (1) and (2),

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau_{\max}}{r}.$$

Q.11(i).

<u>Complementary shear stresses</u>:

Consider an infinitesimal element of material at O, subjected to shear stresses $\tau_{xy}$ on the planes OB, AC and $\tau_{yx}$ on the planes OA, BC. Considering the moment equilibrium of the element, $\sum M_O = 0 \rightarrow \tau_{xy} = \tau_{yx}$.
The pair of shear stresses like $\tau_{xy}$, $\tau_{yx}$ are called complementary shear stresses. They act at mutually perpendicular planes at a point, both directed towards or away from the common edge and are equal in magnitude.

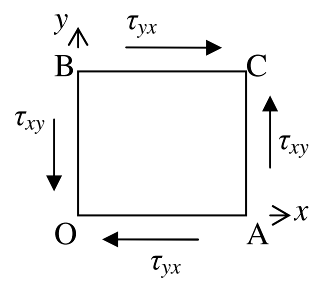


Fig.11(i)

Q.11(ii).

<u>Tensile stress strain curve for a ductile material</u>:

The tensile engineering stress $\sigma$ and the engineering strain $\varepsilon$ curve for a ductile material like mild steel is shown in Fig.11(ii).
The engineering stress = axial load/original area of cross-section, i.e. $\sigma = P/A_0$.
The engineering strain = change in length/original length, i.e. $\varepsilon = (L - L_0)/L_0$.
The important points on the curve are:

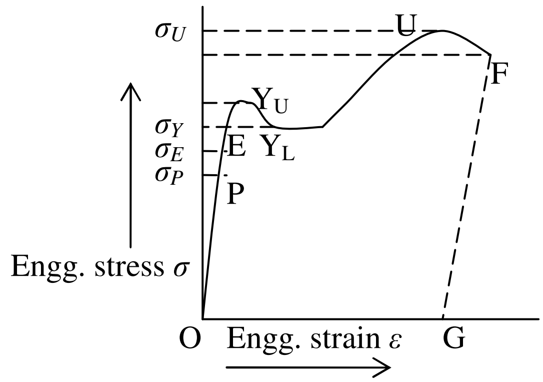


Fig.11(ii)

<u>The proportional Limit</u> P is the point upto which the stress is directly proportional to strain, i.e. follows Hooke's law.
<u>The elastic limit</u> E is the point upto which only elastic deformation takes place and the material returns to its original undeformed state on unloading. Generally, P and E are so close that they are indistinguishable.
Loading beyond the elastic limit also causes permanent deformation and the deformation continues with very little increment in the load. <u>The upper yield point</u> $Y_U$ and <u>the lower yield point</u> $Y_L$ correspond to upper and lower points on the kink (if present) in the stress strain curve beyond the elastic limit. Generally the lower yield point is more reliable and is taken as <u>the yield strength $\sigma_Y$</u> of the material. In the absence of a well defined yield point, the yield strength is taken as the stress which causes 0.2% permanent strain.

The ultimate point U corresponds to the maximum load the material can withstand. The stress $\sigma_U$ is called the ultimate strength of the material. The necking in the specimen initiates at the ultimate point.

The fracture point F corresponds to the fracture of the material. The stress $\sigma_F$ is called the fracture stress. As the original area of cross-section is used in stress calculation, the fracture stress is less than the ultimate stress.

Q.11(iii).

Law of polygon of forces:

The resultant of a number of coplaner forces can be found graphically using the law of polygon of forces. The law can be stated as follows:

If several coplaner forces are acting at a point such that they are represented in magnitude and direction by the sides of a polygon taken in the same order, their resultant is represented in magnitude and direction by the closing side of the polygon in the reverse direction.

If the polygon is closed, i.e. no closing side is required, the forces would be in equilibrium.

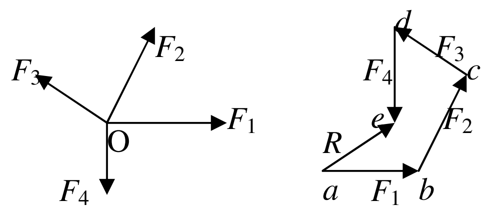


Fig.11(iii)

For example, consider coplaner forces $F_1$, $F_2$, $F_3$ and $F_4$ acting at a point O as shown in Fig.11(iii). The forces are represented in magnitude and direction by the sides *ab*, *bc*, *cd* and *de*, respectively. The closing side *ae* of the polygon *abcde* represents the resultant $R$ of the forces. If the points *a*, *e* are coincident, the resultant $R = 0$, and the forces would be equilibrium.

Q.11(iv).

General plane motion of a rigid body can be considered as the sum of a plane translation and a rotation about an axis perpendicular to the plane motion:

Consider a body undergoing plane motion in the *xy* plane from position 1 at time $t$ to position 2 at time $t + \Delta t$ as shown in Fig.11(iv). This general plane motion can be thought of as translation and rotation as follows:

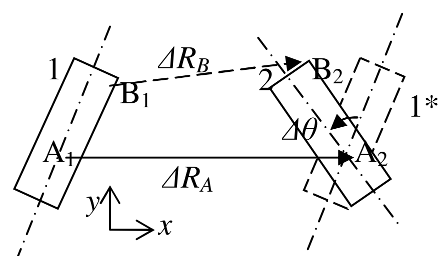


Fig. 11(iv)

(a) Select a point $A_1$ in the body and translate the whole body in the *xy* plane with displacement $\Delta R_A$ such that the point $A_1$ occupies its final destination $A_2$. The body is in position 1*.

(b) Rotate the body about the $z$ axis through the point $A_2$ by an amount $\Delta\theta$ to obtain the final position 2.

If instead of $A_1$, some other point $B_1$ is chosen for translation, the translation $\Delta R_B \neq \Delta R_A$, but the rotation $\Delta\theta$ would remain the same with the $z$ axis passing through the final position $B_2$ of the point $B_1$.

**SOLUTIONS D-02 APPLIED MECHANICS** (December 2003)

Q1. a. A If the forces are concurrent, the moment about any point would also be zero which may not be so in other cases.

b. D At impending slip the frictional force is equal to limiting friction.

c. A The efficiency $\eta$ = M.A./V.R. = (1000/90)/15 = 0.74 > 0.5.

d. C The altitude of the equilateral triangle which is also a median is $(\sqrt{3}/2)a$. One third of it is $a/(2\sqrt{3})$.

e. C In a compound lever, the simple leverages are all multiplied.

f. C The horizontal range of a projectile with projection angle $\theta$ is $(u^2/g)\sin 2\theta$ which is maximum for $\theta = 45^0$.

g. D The work done = torque×angle of rotation = $50\times 4\pi$ = 628 Nm.

h. C The point of contraflexure occurs in beams with two or more spans.

Q.2.
Let $M_1$ be the mass of the shot and $V$ its absolute velocity at an angle $\theta$ to the horizontal. The relative velocity of the shot $V_r$ with respect to the gun would be along the gun barrel at an angle $\alpha$ to the horizontal. Let $V_g$ be the recoil velocity of the gun in the opposite direction.
The equations relating the absolute and relative velocities of the shot and the recoil velocity of the gun are
$V_r\sin\alpha = V\sin\theta$. (1)
$V_r\cos\alpha = V\cos\theta + V_g$. (2)
The conservation of momentum in the horizontal direction yields,
$M_1V\cos\theta - mM_1V_g = 0$. (3)
From equations (1) to (3)
$\underline{\tan\alpha = m\tan\theta/(m+1)}$.

Q.3.
Assume that the block of weight $W$ is given a virtual displacement $\delta_W$ down along the plane. The corresponding upward virtual displacement $\delta_P$ of the effort P would be,
$\delta_P = \delta_W/2$. (1)
As the system is in equilibrium, according to the principle of virtual work, the virtual work done by all the forces must be zero. The forces which contribute to the virtual work are the effort $P$ and the component of the weight of the block along the plane $W\sin 20$.
$P\times \delta_P - W\sin 20\times\delta_W = 0$. (2)
From equations (1) and (2),
$P = \underline{2W\sin 20}$.

Q.4.

Let $V$ be the velocity of the stone required to hit the bird and $t$ the time at which it would be hit. Then,

$25 = V\cos30 \times t \rightarrow t = 25/V\cos30$ (1)

$10 = V\sin30 \times t - (½)gt^2$ (2)

Substituting for time $t$ from equation (1) in equation (2),

$10 = 25\tan30 - 4.905(25/V\cos30)^2$

$\rightarrow V = (25/\cos30)/\sqrt{((25\tan30 - 10)/4.905)} = \underline{30.4 \text{ m/s}}$.

Q.5.

The F.B.D. of the safe for possible slide down the plank of its own is shown in Fig.5. The safe is subjected to its 4000 N weight, the normal reaction $N$ of the inclined plane and the limiting friction force $\mu N$. The inclination of the plank to the horizontal is $\alpha = \tan^{-1}1.2/2.4 = 26.565^0$.

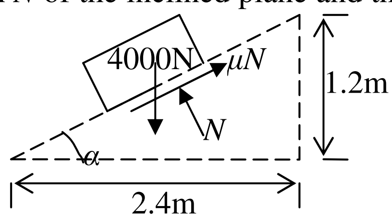



Fig.5

The forces normal to the plank must be in equilibrium,

$N = 4000\cos26.565 = 3577.7$ N.

The limiting friction $\mu N = 0.3 \times 3577.7 = 1073.3$ N.

The component of the weight of the safe parallel to the plank is $4000\sin3577.7 = 1788.9$ N

As the weight component parallel to the plank is more than the limiting friction force, the safe can slide down the plank of its own.

Q.6.

The F.B.Ds. of the joints F, D and E are shown in Fig.6. The forces are assumed tensile in the members and are assigned positive sign. All the inclined members are at $45^0$ to the horizontal.

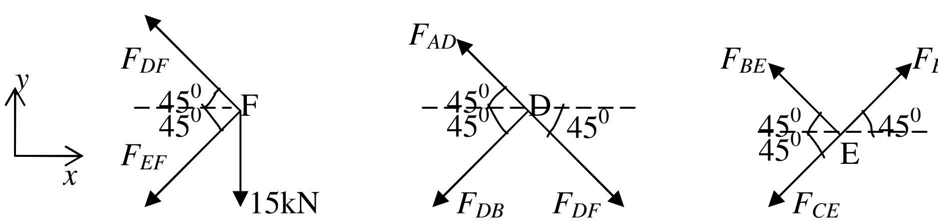



Fig.6

Considering the equilibrium of joint F:

$\sum F_x = -F_{DF}\cos45 - F_{EF}\cos45 = 0$, $\sum F_y = F_{DF}\sin45 - F_{EF}\sin45 - 15 = 0$

$\rightarrow F_{DF} = 15/\sqrt{2}$ kN, i.e. $\underline{10.6 \text{ kN(T)}}$ and $F_{EF} = -15/\sqrt{2}$ kN, i.e. $\underline{10.6 \text{ kN(C)}}$.

Considering the equilibrium of joint D:

$\sum F_x = F_{DF}\cos45 - F_{AD}\cos45 - F_{DE}\cos45 = 0$, $\sum F_y = F_{AD}\sin45 - F_{DE}\sin45 - F_{DF}\sin45 = 0$.

$\rightarrow F_{AD} = 15/\sqrt{2}$ kN, i.e. $\underline{10.6 \text{ kN(T)}}$ and $F_{DB} = \underline{0}$.

Considering the equilibrium of joint E:

$\sum F_x = F_{EF}\cos45 - F_{BE}\cos45 - F_{CE}\cos45 = 0$, $\sum F_y = F_{EF}\sin45 + F_{BE}\sin45 - F_{CE}\sin45 = 0$.

$\rightarrow F_{CE} = -15/\sqrt{2}$ kN, i.e. $\underline{10.6 \text{ kN(C)}}$ and $F_{BE} = \underline{0}$.

Q.7.

A vehicle of mass $m$ is negotiating a level curve of radius $R$ with uniform speed $V$ as shown in Fig.7. The C.G. of the vehicle C is at a height $h$ above the ground and the distance between the inner and outer wheels is $b$.

The normal acceleration of C, $a_n = V^2/R$.

At impending overturning, the normal reaction $N^*$ and friction force $f^*$ on the inner wheels are zero.

Hence for vertical equilibrium,

$\sum F_V = N - mg = 0 \rightarrow N = mg$. (1)

The equations of motion in the radial direction is,

$\sum F_n = ma_n \rightarrow f = m\, V^2/R$. (2)

The moment equilibrium about C yields,

$\sum M_C = 0 \rightarrow N \times b/2 - f \times h \geq 0$. (3)

Substituting for $N$ and $f$ from equations (1) and (2) into inequality (3),

$V \leq \sqrt{(bgR/2h)}$. Hence, $V_{max} = \underline{\sqrt{(bgR/2h)}}$.

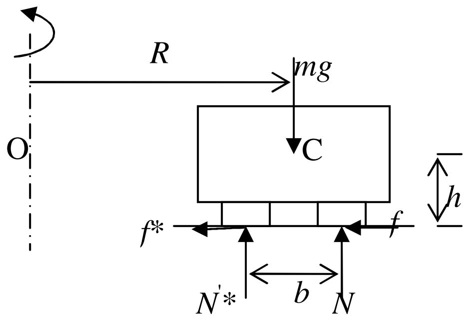



Fig.7

Q. 8.

The F.B.D. of the structure is shown in Fig.8. The vertical reactions $R_A$ and $R_C$ act at the roller supports A and C, respectively.

Considering the equilibrium of the beam,

$\sum M_A = 120 \times 20 - R_C \times 60 + (4 \times 30) \times 75 = 0$.

$\rightarrow R_C = 190$ kN.

$\sum F_y = R_A - 120 + R_C - 4 \times 30 = 0$.

$\rightarrow R_A = 50$ kN

The expressions for S.F. $V$ in kN and B.M. $M$ in kNm at various sections are

$0 \leq x \leq 20$m:

$V = -50$, $M = 50x$.

$20 \text{ m} \leq x \leq 60$m:

$V = -50+120 = 70$, $M = 50x - 120(x - 20)$.

$60 \text{ m} \leq x \leq 90$m:

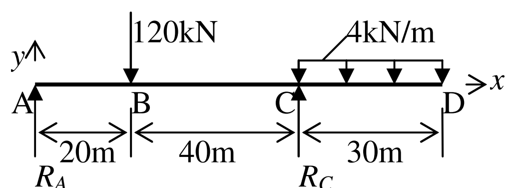
$V = -4(90 - x)$, $M = -4(90 - x)^2/2$.

<u>The S.F. and B.M. diagrams are also shown in Fig.8</u>.

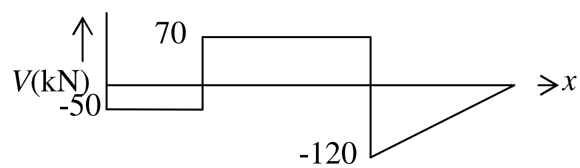
The maximum S.F. $V_{max} = -120$ kN occurs at the right support, $x = 60$m.

The maximum B.M. $M_{max} = -1800$ kNm also occurs at the right support $x = 60$ m.

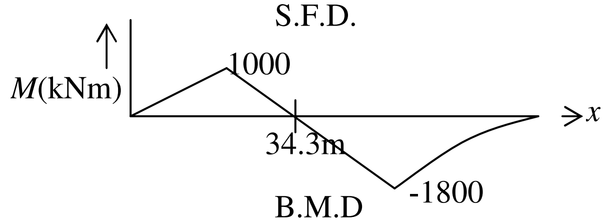
For $20 \text{ m} \leq x \leq 60$m, $M = 50x - 120(x - 20) = 0$ at $x = 34.3$ m which is a point of contraflexure.




F.B.D. of structure




S.F.D.




B.M.D

Fig.8

Q.9.

As the material is the same, the allowable shear stress $\tau_o$ is the same for the shafts.

From the torsion formula for shafts, the torque $T_{solid} = \tau_0 J/r_{max}$.

For a solid shaft of diameter $d$: The polar moment of inertia $J_{solid} = \pi d^4/32$.

$T_{solid} = \tau_0 J/r_{max} = \tau_0(\pi d^4/32)/(d/2) = \tau_0\pi d^3/16.$ (1)
For a hollow shaft of internal diameter $d_i$ and external diameter $d_o$:
The polar moment of inertia, $J_{hollow} = \pi(d_o^4 - d_i^4)/32$.
$T_{hollow} = \tau_0 J/r_{max} = \tau_0(\pi(d_o^4 - d_i^4)/32)/(do/2) = 16\tau_0(\pi(d_o^4 - d_i^4)/do)$ (2)
For the shafts of the same length and weight, the cross-sectional area must be equal, i.e.
$d_o^2 - d_i^2 = d^2 \rightarrow d_o/d \quad = \sqrt{}(1 + d_i^2/d^2)$ (3)
From equations (1) to (3),
$T_{hollow}/T_{solid}$
$= (d_o^4 - d_i^4)/(d_od^3) = (d_o^2 + d_i^2)/(d_od) = d_o/d + d_i^2/(d_od) > 1$. (proved).

Q.10.
Consider a uniform bar of original cross-sectional area $A$ and length $L_0$ subjected to equal and opposite axial forces $P$ at each end. The final deformed length of the bar is $L_f$. Then,
Stress $\sigma$ is defined as the force per unit area, i.e. $\sigma = P/A$. This stress is known as the normal stress and may be tensile or compressive.
Strain $\varepsilon$ is defined as the change in length per unit length, i.e. $\varepsilon = (L_f - L_0)/L_0$. This strain is known as the normal strain and may be tensile or compressive.
Young's modulus of elasticity $E$ is defined as the ratio of stress to strain within elastic limit, i.e. $E = \sigma/\varepsilon$, $\sigma < \sigma_{\text{elastic}}$.

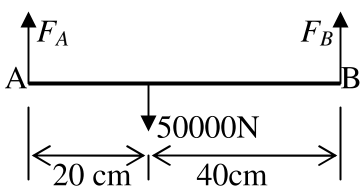


Fig.10

The F.B.D. of the rigid horizontal bar is shown in Fig.10.
From the equilibrium equations of the bar,
$\sum F_V = F_A + F_B - 50000 = 0.$
$\sum M_B = F_A\times 60 - 50000\times 40 = 0.$
$\rightarrow F_A = 100000/3$ N, $F_B = 50000/3$ N.
Let $A_B$ be the cross-sectional area and $\sigma_B$ the stresss in the rod B. Then, $A_B = F_B/\ \sigma_B = (50000/3)/50 = \underline{333.3 \text{ mm}^2}$.
As the rods are of the same length and their lengths remain equal after deformation, the strains in the rods must be equal, $\varepsilon_A = \varepsilon_B = \sigma_B/E_B = 50/90000 = 1/1800$.
Hence, the stress in rod A, $\sigma_A = E_A\varepsilon_A = 200000/1800 = \underline{1000/9 \text{ N/mm}^2}$.
The cross-sectional are of the rod A, $A_A = F_A/\sigma_A = (100000/3)/(1000/9) = \underline{300 \text{ mm}^2}$.

Q.11.
Let $R_A$ and $R_B$ be the support reactions. As the loading is symmetrical about the centre, $R_A = R_B = P/2$.
The expression for the B.M. $M$ between A and D is, $M = R_Ax = Px/2$, where $x$ is measured from A along the beam.
Using the moment curvature relation, the deflection $v$ is obtained as follows.
$EId^2v/dx^2 = - M = - Px/2$
Integrating, $EIdv/dx = - Px^2/2 + C_1$
Due to symmetry, the slope $dv/dx = 0$ at $x = l/2$, $C_1 = Pl^2/8$.
Integrating once more,
$EIv = - Px^3/6 + C_1x + C_2 = - Px^3/6 + Pl^2x/8 + C_2$
As the deflection $v = 0$ at the support $x = 0$, $C_2 = 0$.
Hence, the deflection at D ($x = l/2$), $v_D = \underline{Pl^3/(24EI)}$.

# SOLUTIONS D-02 APPLIED MECHANICS (June 2004)

Q.1. a. D If a two force body is in equilibrium, the forces must be equal, opposite and collinear.

b. C $(\pi R^2/2)\overline{y} = \int_0^R\int_0^\pi (rd\theta dr) r\sin\theta = \int_0^R 2r^2 dr = 2R^3/3.$

c. A The limiting force of friction $F = \mu R$.

d. C The force required to move the load $W$ down is $W\tan(\alpha - \varphi)$. For $\alpha < \varphi$, the load would not move down for zero force.

e. B The moment of inertia of a semicircular section about its diameter $I = (1/2)(\pi R^4/4) = 32\pi$ cm$^4$.

f. B $V = dS/dt = 14t + 10$. At $t = 0$, $V = 10$ m/s.

g. C The point of contra-flexure is a point where the beam curvature changes sign and hence bending moment changes sign.

h. C Torsional rigidity of a shaft is given by $GJ$.

Q.2.
The F.B.D. of the wheel when it is just about to roll over the block is shown in Fig.2. Just when the wheel begins to roll over, there is no force from the ground on the wheel at B. The wheel is subjected to its own weight of 1000 N, the force $P$ and the reaction $R$ of the block.
As the wheel is in equilibrium under three forces, $W$, $P$ and $R$ only, the forces are concurrent and pass through the point D.
From the geometry of the figure,
cosECA = CE/CA = 15/30 =1/2 $\rightarrow 2\theta = 60^0$.
angleCDA = (1/2)angleECA = $\theta = 30^0$.
From Lame's theorem,
$P/\sin\theta = W/\sin(90-\theta) \rightarrow P = W\tan\theta = 1000\tan30 = \underline{1000/\sqrt{3} = 577.4 \text{ N}}$.

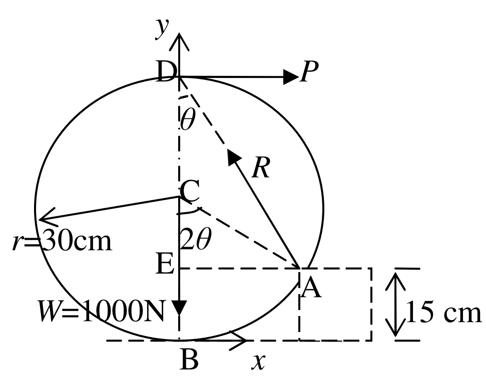


Fig.2

Q.3.
Consider the T section to consist of two rectangular parts 1 and 2 as shown in Fig.3.
Let C($x_C$, $y_C$) be the centroid of the T section. Let $A_i$ be the area and $x_i$, $y_i$ the cooordiantes of the centroid of the $i^{th}$ part.

The $y$ coordinate of the centroid is obtained as
$y_C = \sum A_i y_i / \sum A_i$
$= [(6\times4)\times3 + (2\times8)\times7]/(6\times4 + 2\times8)$
$= 184/40 = 4.6$ cm.
The moment of inertia $I^C_{xx}$ of the T section about an axis parallel to the $x$ axis through C would be
$I^C_{xx} = \sum[(b_i h_i^3)/12 + (b_i h_i)(y_C - y_i)^2]$
$= 4\times6^3/12 + 4\times6(4.6 - 3)^2$
$+ 8\times2^3/12 + 8\times2(4.6 - 7)^2$
$=$ <u>231 cm$^4$</u>.

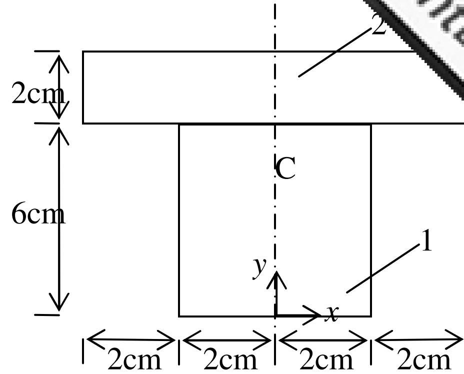
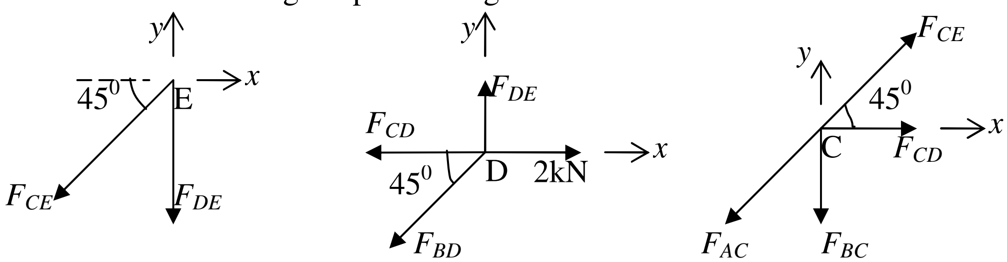


Fig.3

Q.4.
The F.B.Ds. of the joints E, D and C are shown in Fig.4. The forces are assumed tensile in the members and are assigned positive sign.



Joint E  Joint D  Joint C

Fig.4

Consider the equilibrium of joint E:
$\sum F_x = -F_{CE}\cos45 = 0 \rightarrow F_{CE} = \underline{0}$, $\sum F_y = F_{DE} - F_{CE}\sin45 = 0 \rightarrow F_{DE} = \underline{0}$.
Consider the equilibrium of joint D:
$\sum F_y = -F_{BD}\sin45 + F_{DE} = 0 \rightarrow F_{BD} = \underline{0}$, $\sum F_x = -F_{CD} - F_{BD}\cos45 + 2 = 0 \rightarrow F_{CD} = \underline{2\text{ kN(T)}}$.
Consider the equilibrium of joint C:
$\sum F_x = F_{CD} + F_{CE}\cos45 - F_{AC}\cos45 = 0 \rightarrow F_{AC} = \underline{2\sqrt{2}\text{ kN(T)}}$.
$\sum F_y = F_{CE}\sin45 - F_{AC}\sin45 - F_{BC} = 0 \rightarrow F_{BC} = -2$ kN, i.e. <u>2 kN(C)</u>.

Q.5.
The F.B.D. of the ladder is shown in Fig.5. The man is at D (AD = $d$) for the ladder to start slipping. The ladder is subjected to its own weight $W$ at C, the weight of the man $W/2$ at D, the normal reaction $N_A$ and the limiting friction force $N_A/2$ from the floor at A and the normal reaction $N_B$ and limiting friction force $N_B/3$ from the wall at B.
From the force equlibrium equations,
$\sum F_x = N_B - N_A/2 = 0$ and $\sum F_y = N_A + N_B/3 - W - W/2 = 0$.
$N_A = 9W/7$ and $N_B = 9W/14$.
From moment equilibrium about A,
$\sum M_A = -N_B\times7/\sqrt{2} - (N_B/3)\times7/\sqrt{2} + W\times3.5/\sqrt{2} + (W/2)d/\sqrt{2} = 0$.
Substituting for $N_B$ in the moment equation, <u>$d = 5$m</u>.

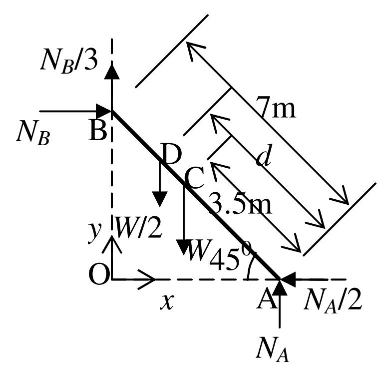


Fig.5

Q.6.
Let the effort $P$ be given a virtual displacement $\delta_P$ downward, then the virtual displacement of the load $W$ would be $\delta_W = \delta_P/2$ upward.
If the system is in equilibrium, the sum of the virtual work must be zero, i.e.
$P\delta_P - W\delta_W = 0. \rightarrow P = W/2 = \underline{500\text{ N}}$.

Q.7.
Let $a_t$ be the uniform tangential acceleration of the car. If it starts from rest, the speed $V$ after time t would be,
$V = a_t t$.
$\rightarrow a_t = V_{60}/60 = (18\times1000/3600)/60 = 1/12 = \underline{0.083\text{ m/s}^2}$.
The speed after 30 s would be $V_{30} = a_t\times30 = (1/12)\times30 = 2.5$ m/s.
The normal acceleration $a_n = V_{30}^2/R = 2.5^2/250 = \underline{0.025\text{ m/s}^2}$.

Q.8.
The stress $\sigma = P/A = P/(\pi d^2/4) = 100\times10^3/(\pi d^2/4) = 100\times10^6$.
Hence, $d = \sqrt{(4/1000\pi)} = 0.03568\text{ m} = \underline{3.568\text{ cm}}$.
The total elongation $\delta = (P/E)[L_1/A_1 + L_2/A_2 + L_3/A_3]$
$= 100\times10^3/(290\times10^9)[0.1/(\pi\times0.03568^2/4) + 0.15/(\pi\times0.1^2/4) + 0.15/(\pi\times0.08^2/4)]$
$= 0.0514\times10^{-3}\text{ m} = \underline{0.0514\text{ mm}}$.

Q.9.
The F.B.D. of the girder is reproduced in Fig.9. The support reactions are determined by considering its equilibrium,
$\sum M_A = 9R_B - 180\times4.5 - 30\times6 - 40\times7.5 = 0$.
$\rightarrow R_B = 143.33$ kN.
$\sum F_x = R_A + R_B - 20\times9 - 30 - 40 = 0$.
$\rightarrow R_A = 106.67$ kN.
The S.F. $V$ and the B.M. $M$ are,
$0 \le x \le 6$m:
$V = -106.67 + 20x$.
$M = 106.67x - 20x^2/2$.
$6\text{ m} \le x \le 7.5$m:
$V = -106.67 + 20x + 30$.
$M = 106.67x - 20x^2/2 + 30(x - 6)$.
$7.5\text{ m} \le x \le 9$m:
$V = 143.33 - 20(9 - x)$.
$M = 143.33(9 - x) - 20(9 - x)^2/2$.
The S.F. and B.M. diagrams are also shown in Fig.9.
The maximum S.F. $V_{max} = 143.33$ kN at the right support, i.e. $x = 9$ m.
The S.F. changes sign at x = 5.33 m.
The maximum B.M. is at $x = 5.33$ m
$M_{max}$=284.4 kNm.

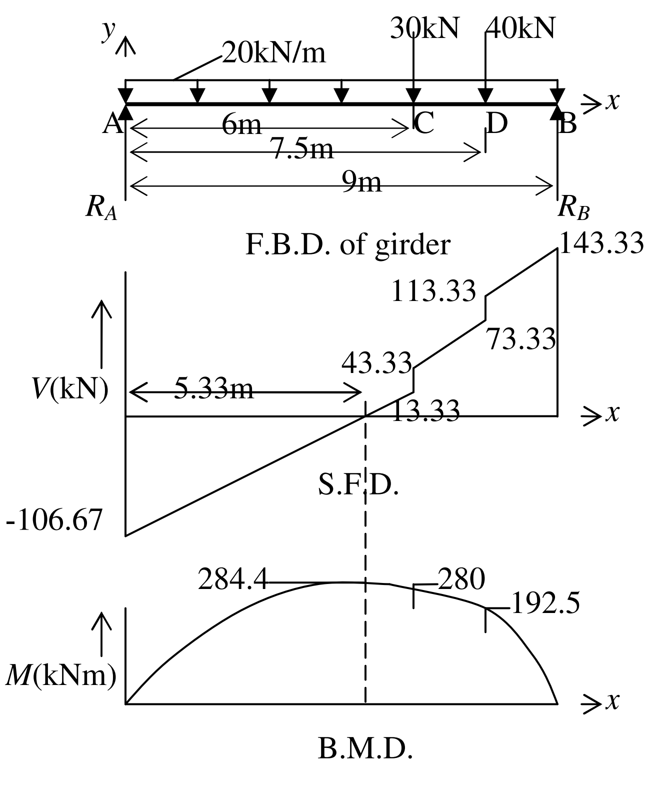



Fig.9

Q.10.

The F.B.D. of the girder is shown in Fig.10. $R_A$ and $R_B$ are the support reactions. From the equilibrium equations,

$\sum M_A = R_B \times L - P \times a = 0. \rightarrow R_B = Pa/L.$

$\sum F_y = R_A + R_B - P = 0. \rightarrow R_A = P(L - a)/L.$

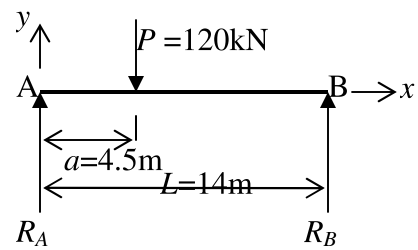



Fig.10

The expression for the B.M. at any section $x$ using singularity functions is:

$M = - R_A x + P < x - a> = - P(L - a)x/L + P < x - a>.$

If $v$ is the deflection of the elastic line of the beam,

$EI\, d^2v/dx^2 = M = - P(L - a)x/L + P < x - a>.$

Integrating twice,

$EI\, dv/dx = - P(L - a)x^2/2L + P < x - a>^2/2 + C_1$

$EI\, v = - P(L - a)x^3/6L + P < x - a>^3/6 + C_1 x + C_2$

At $x = 0$, $v = 0 \rightarrow C_2 = 0$.

At $x = L$, $v = 0 \rightarrow C_1 = Pa(L - a)(2L - a)/6L$.

Hence, $v = \underline{- P(L - a)x^3/6L + P < x - a>^3/6 + Pa(L - a)(2L - a)x/6L}$.

The deflection $v_P$ under the load $P$ at $x = a$,

$v_P = Pa^2(L - a)^2/(3EIL)$

$= 120\times10^3\times4.5^2\,(14 - 4.5)^2/(3\times210\times10^9\times16\times10^{-4}\times14) = 1.55\times10^{-2}$ m = $\underline{1.55 \text{ cm}}$.

Q.11.

A shaft of diameter $d$ is subjected to a torque $T$. The shear modulus of the material is $G$. The shear stress $\tau$ at any radius $r$ and the angle of twist $\theta$ over a length $L$ is given by the torsion formula for circular shafts as

$\tau/r = T/J = G\theta/L,$

where the polar moment of inertia of the cross-section $J = \pi d^4/32$.

(i) The maximum shear stress $\tau_{max}$ would be in the outer fibers at $r_{max} = d/2$.

$\tau_{max} = T(d/2)/J = 16T/(\pi d^3) = 16\times560/(\pi\times0.03^3) = 105.63\times10^6$ N/m$^2$ =$\underline{105.63 \text{ MPa}}$.

(ii) The angle of twist $\theta$ over a length $L$ = 1m would be

$\theta = TL/GJ = 32TL/(G\pi d^4) = 32\times560\times1/(82\times10^9\times\pi\times0.03^4) = 0.0859$ rad. = $\underline{4.92^0}$.

(iii) The shear stress $\tau$ at $r$ = 0.01m would be

$\tau = Tr/J = 32Tr/(\pi d^4) = 32\times560\times0.01/(\pi\times0.03^4) = 70.42\times10^6$ N/m$^2$ = $\underline{70.42 \text{ MPa}}$.

# SOLUTIONS D-02 APPLIED MECHANICS (December 2004)

Q1. a. C Displacement magnitude = $[(12\times1/2)^2+(8\times1)^2]^{1/2}$ = 10 km.

b. A As the body moves in the horizontal plane, the work done by weight is zero.

c. D The distance of the centroid from each leg is $b/4$ and hence from the corner is $b/\sqrt{8}$.

d. B The mechanical efficiency = output/input = 120×10/(20× 80) = 0.75. or 75%.

e. D Simple harmonic motion is always in a straight line.

f. C The kinetic energy = $I\omega^2/2 = (mL^2/3)\omega^2/2 = mL^2\omega^2/6$.

g. A The maximum shear strain at the outermost fibers $\gamma_{\max} = \theta r/L$.

h. B In pure bending, the bending moment is constant and so the curvature is constant.

Q.2a.
Resultant force in the x direction, $F_x = 8 - 16\cos 120 = 8 - 16/2 = 0$.
Resultant force in the y direction $F_y = 16\sin 120 = 16\sqrt{3}/2 = 8\sqrt{3}$ N
Acceleration the $x$ direction $a_x = F_x/m = \underline{0}$.
Acceleration the $y$ direction $a_y = F_y/m = 8\sqrt{3}/2 = 4\sqrt{3} = \underline{6.93 \text{ m/s}^2}$.

Q.2b.

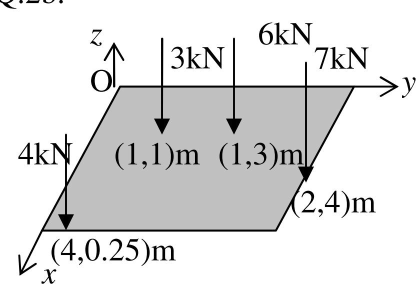



Fig.2b

Let the resultant $R$ act downward at a point $(x_R, y_R)$.
$\sum F_z = 0 \rightarrow R = 3 + 6 + 4 + 7 = \underline{20 \text{ N}}$.

$\sum M_{Oy} = 0 \rightarrow x_R R = 1\times3 + 1\times6 + 4\times4 + 2\times7 = 39$
$\rightarrow x_R = 39/20 = \underline{1.95 \text{ m}}$.
$\sum M_{Ox} = 0 \rightarrow y_R R = 1\times3 + 3\times6 + 0.25\times4 + 4\times7 = 50$
$\rightarrow y_R = 50/20 = \underline{2.5 \text{ m}}$.

Q.3a.
The F.B.Ds. of ball A and ball B are shown in Fig.3a.

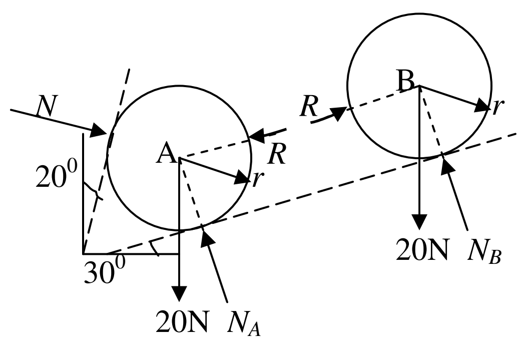



Fig.3a

Q.3b.

The F.B.Ds. of the joints C, B, D and E assuming tensile forces in the members, are shown in Fig.3b.

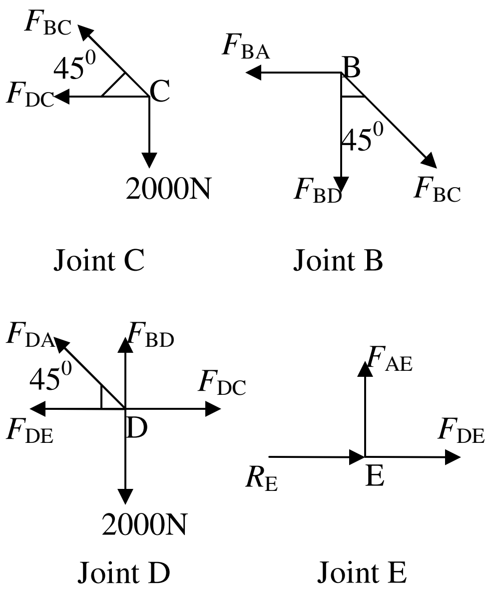



Fig.3b

Considering the equilibrium of joint C:

$\sum F_V = - F_{BC}\sin 45 – 2000 = 0$

$\rightarrow F_{BC} = 2000\sqrt{2}$ N = $\underline{2828.4 \text{ N(T)}}$.

$\sum F_H = - F_{DC} + F_{BC}\sin 45 = 0$

$\rightarrow F_{DC} = -2000$ N = $\underline{2000 \text{ N(C)}}$.

Considering the equilibrium of joint B:

$\sum F_H = - F_{BA} + F_{BC}\sin 45 = 0$

$\rightarrow F_{BA} = 2000$ N = $\underline{2000 \text{ N(T)}}$.

$\sum F_V = - F_{BD} - F_{BC}\cos 45 = 0$

$\rightarrow F_{BD} = - 2000$ N = $\underline{2000 \text{ N(C)}}$.

Considering the equilibrium of joint D:

$\sum F_V = F_{DA}\sin 45 + F_{BD} -2000 = 0$

$\rightarrow F_{DA} = 4000\sqrt{2}$ N = $\underline{5656.9 \text{ N(T)}}$.

$\sum F_H = -F_{DE} + F_{DC} – F_{DA}\cos 45 = 0$

$\rightarrow F_{DE} = - 6000$ N = $\underline{6000 \text{ N(C)}}$.

At support E, only normal reaction $R_E$ is possible.

Therefore $F_{AE} = \underline{0}$.

Q.4.

The I section is divided in parts 1, 2, 3 as shown in Fig.4. Let $x_C$, $y_C$ be the coordinates of its centroid. Let $A_i$ be the area and $x_i$, $y_i$ the coordinates of the centroid of the $i^{\text{th}}$ part.

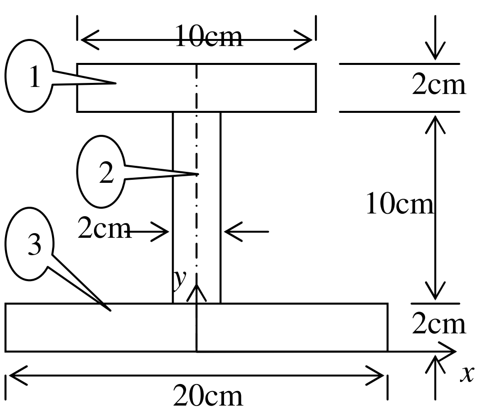



Fig.4

By symmetry $\rightarrow x_C = \underline{0}$.

$y_C = \sum A_i x_i / \sum A_i$

$= (20\times13 + 20\times7 + 40\times1)/(20 + 20 + 40) = \underline{5.5 \text{ cm}}$.

Let $I_{xx}$ and $I_{yy}$ be the second moments of area about the coordinate axes.Then,

$I_{xx} = \sum I_{xxi} = \sum A_i y_i^2 + \sum(b_i h_i^3/12)$

$\sum A_i y_i^2 = 20\times13^2 + 20\times7^2 + 40\times1^2 = 4400 \text{ cm}^4$

$\sum b_i h_i^3/12 = (10\times2^3 + 2\times10^3 + 20\times2^3)/12 = 186.67 \text{ cm}^4$

$\rightarrow I_{xx} = 4400 + 740/3 = 4586.67 \text{ cm}^4$

$I_{yy} = \sum A_i x_i^2 + \sum(h_i b_i^3/12)$

$= 0 + (2\times10^3 + 10\times2^3 + 2\times20^3)/12 = 1506.67 \text{ cm}^4$.

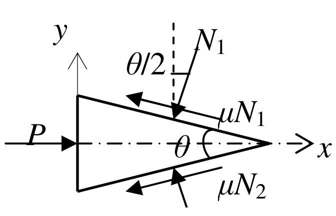
Let $I_{Cxx}$ and $I_{Cyy}$ be second moments of area about the centroidal axes parallel to O$x$, O$y$.

$I_{Cxx} = I_{xx} – (\sum A_i)\, y_C^2 = 4586.67 - 80\times5.5^2 = 2166.67 \text{ cm}^4$

$I_{Cyy} = I_{yy} – (\sum A_i)\, x_C^2 = 1506.67 – 80\times0^2 = 1506.67 \text{ cm}^4$

The polar moment about the axis through centroid $I_{Czz} = I_{Cxx} + I_{Cyy} = \underline{3673.3 \text{ cm}^4}$.

Q.5.




The F.B.D. of the wedge being driven in the wood is shown in Fig.5. Considering the equilibrium at impending motion,

$\sum F_y = \mu N_1 \sin(\theta/2) – N_1\cos(\theta/2) + N_2\cos(\theta/2) - \mu N_2 \sin(\theta/2) = 0$.

$\rightarrow (N_2 – N_1)[\cos(\theta/2) - \mu \sin(\theta/2)] = 0. \rightarrow N_1 = N_2 = N$.

$\Sigma F_x = P - N_1\sin(\theta/2) - \mu N_1\cos(\theta/2) - N_2\sin(\theta/2) - \mu N_2 \cos(\theta/2) = 0.$
$\rightarrow P = 2N[\sin(\theta/2) + \mu\cos(\theta/2)].$
For the wedge, depending on the coefficient of friction $\mu$, a force $P_1$ may be required to keep the wedge in place without being squeezed out. The friction forces are reversed and the force $P_1$ is obtained from the expression for $P$ by changing the sign of $\mu$.
$P_1 = 2N[\sin(\theta/2) - \mu\cos(\theta/2)].$
The wedge is self locking if $P_1 \leq 0$, $\rightarrow \mu \geq \tan(\theta/2)$.

Q.6a.
Horizontal velocity of the plane = 200 km/h = 200×1000/3600 = 55.56 m/s
At the time of firing, the plane is vertically above the gun. Therefore, for the shell to hit the plane, the horizontal velocity of the shell must be the same as that of the plane.
Let $\theta$ be the inclination of the gun to the horizontal. Then,
$300\cos\theta = 55.56 \rightarrow \theta = \cos^{-1} = 55.56/300 = 79.3^0$.
The shell hits the plane at time $t$ and travels a vertical distance of 1000 m as a projectile.
$300\sin 79.3\ t - 9.81\ t^2/2 = 1000 \rightarrow t^2 - 60.1\ t + 203.9 = 0 \rightarrow$ t = 3.6s, 56.5s
The shell hits the plane at $t$ = 3.6s.
The horizontal distance of the plane from the gun = 55.56×3.61 = 200.6 m.

Q.6b.
The centripetal acceleration $a_r = V^2/R = \omega^2 R = 2^2 \times 0.5 = 2$ m/s$^2$.
The tangential acceleration $a_t = (dV/dt) = \alpha R = 3 \times 0.5 = 1.5$ m/s$^2$.
The total acceleration = a = $\surd(a_r^2 + a_t^2) = \surd(2^2 + 1.5^2) = 2.5$ m/s$^2$.

Q.7a.
The F.B.D. of the 10 kg mass and 50 kg drum are shown in Fig.7a.
The equation of motion for downward acceleration $a_m$ of the mass is
$10g - T = 10a_m$ (1)
The equation of motion for the clockwise angular acceleration $\alpha$ of the drum is
$0.4T = I\alpha = 50\times(0.3)^2\alpha$ (2)
The relation between angular acceleration of the drum and linear acceleration of the mass is
$a_m = 0.4\alpha$ (3)
From equations (1) to (3) $\rightarrow \alpha = 6.43$ rad/s$^2$ and $T = 72.3$ N.

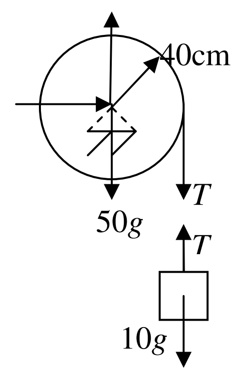


Fig.7a

Q.7b.
The hammer falls freely. Hence the velocity just before impact with the plate,
$V_1 = \surd(2gh) = \surd(2\times9.81\times2.5) = 7$ m/s
As the impact is perfectly plastic, both the hammer and pile move together after impact.
Their common velocity $V$ just after impact is obtained from momentum conservation.
$(50 + 20)\times v = 50\times7 \rightarrow V = 5$ m/s.
Let $R$ be the average resistance of the ground. Then from the work-energy principle
Work done against the resistance = Loss of Kinetic Energy + Loss of Potential Energy
$\rightarrow 0.1\times R = (50 + 20)\times5^2/2 + 0.1\times (50 + 20) \times9.81 \rightarrow R = 9436.7$ N.

Q.8a.

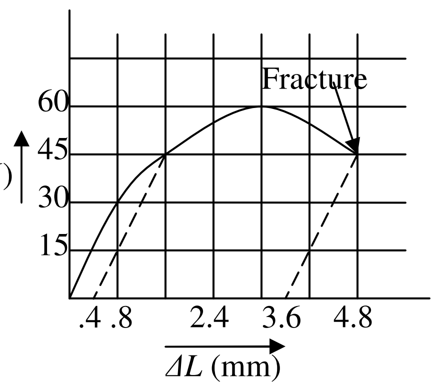


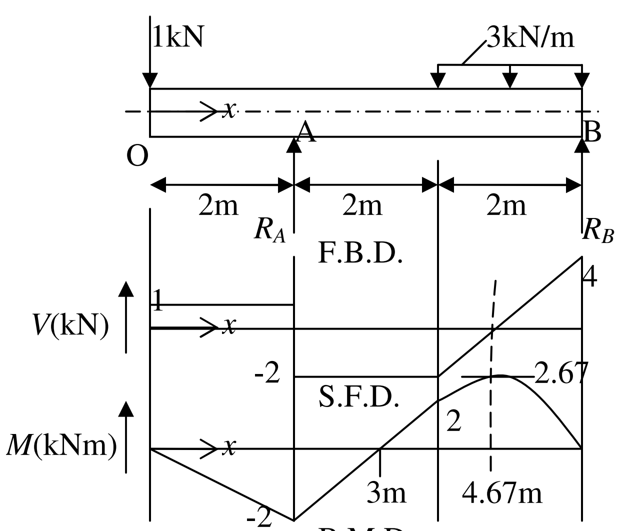

Fig.8a

The tensile load extension diagram is as shown in Fig.8a. The elongation is 0.8 mm at the elastic limit load of 30 kN. Young's modulus $E = (P/A)/(\Delta L/L)$ for $P \leq 30$ kN.
$\rightarrow$ E = $(30\times10^3/100\times10^{-6})/(0.8/200)$
$= 75\times10^9$ Pa = 75 GPa.
For 0.2% proof stress $\sigma_0$ the permanent strain must be .002, i.e. $\Delta L = 200\times0.2/100 = 0.4$ mm. This 0.4 mm permanent elongation is obtained by unloading from the 45 kN load.
Hence, $\sigma_0 = 45\times10^3/100\times10^{-6}$ Pa = 450 MPa.
Ultimate stress $\sigma_u$ = Max. load /Area of section.
$\rightarrow \sigma_u = 60\times10^3/100\times10^{-6}$ Pa = 600 MPa.
The permanent elongation after fracture = 3.6 mm. The permanent strain =3.6/200.
Hence, the % elongation = $(3.6/200)\times100$ = 1.8%.

Q.8b.
The area $A$ resisting the shear is $A = \pi dt$.
The punch force $P = \tau \times A = \tau\, \pi dt$.

Q.9.
The relation between power $P$, torque $T$ and rpm $N$ is $P = T\times2\pi N/60$.
$\rightarrow T = 628\times10^3\times60/(2\times3.14\times200) = 30000$ Nm.
Let $d_o$ be the outer diameter and $d_i$ the inner diameter of the shaft.
The polar moment of area of the shaft section is $J = \pi(d_o^4 - d_i^4)/32$.
The torsion formula for a hollow shaft is $\tau_{max}/(d_0/2) = T/J = G\theta/L$.
From $\tau_{max}/(d_0/2) = G\theta/L$,
$\rightarrow d_0 = 2\tau_{max}L/G\theta = 2\times80\times10^6\times4/[80\times10^9\times(3\times3.14/180)] = 0.153$ m = 153 mm.
From $T/J = G\theta/L$ and $J = \pi(d_o^4 - d_i^4)/32$,
$\rightarrow J = \pi(d_o^4 - d_i^4)/32 = TL/G\theta \rightarrow d_i^4 = d_o^4 - (32/3.14)\times TL/G\theta$
$\rightarrow d_i = [d_o^4 - (32/3.14)\times30000\times4/[80\times10^9\times(3\times3.14/180)]]^{1/4}$ = 0..126 m = 126 mm.

Q.10.
The F.B.D. of the beam is shown in Fig.10. From beam equilibrium,
$\sum M_B = -4R_A + 6\times1 + 1\times(3\times2) = 0$.
$\rightarrow R_A = 3$ kN.
$\sum F_y = R_A + R_B - 1 - 3\times2 = 0$
$\rightarrow R_B = 4$ kN
The S.F. $V$ ( kN) and B.M. $M$ (kNm) are:
$0 \leq x \leq 2$ m
$V = 1$, $M = -x$.
2 m $\leq x \leq$ 4 m
$V = 1 - 3 = -2$, $M = -x + 3(x - 2)$.



$4\text{ m} \le x \le 6\text{ m}$
$V = 1 - 3 + 3(x - 4)$, $M = -x + 3(x - 2) - 3(x - 4)^2/2$.
The S.F. and B.M. diagrams are also shown in Fig.10.
The maximum shear $V_{max} = 4$ kN at the right support, i.e. $x = 6$ m.
$V = 1 - 3 + 3(x - 4) = 0$ at $x = 4.67$ m. Hence, $M_{\max} = \underline{2.67\text{ kNm}}$ at $x = \underline{4.67\text{ m}}$.
The B.M. $M = 1 - 3 + 3(x - 4) = 0$ at x = 3 m which is a point of contraflexure.

Q.11a.
Let A and B be points on the elastic curve of a beam of flexural rigidity $EI$. The slopes of the tangents to the elastic curve at A and B are $\theta_A$ and $\theta_B$, respectively. The tangential deviation $t_{B/A}$ is the displacement of point A from the tangent at B in a direction normal to the undeformed elastic curve. Let $M$ be the bending moment at any section.

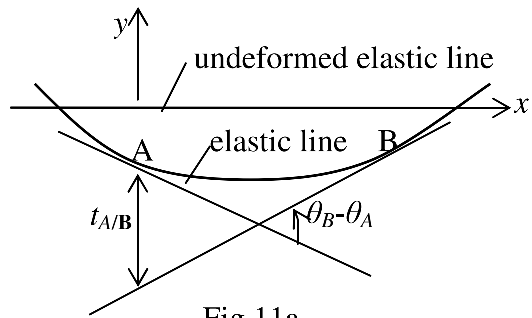


Fig.11a

The first theorem: The rotation between the tangents at points A and B is equal to the area of the ($M/EI$) diagram between these points.

$$\rightarrow \theta_B - \theta_A = \int_A^B (M / EI)dx$$

The second theorem: The tangential deviation of point A from the tangent at B, $t_{A/B}$ is equal to the first moment of the area of ($M/EI$) diagram about an axis normal to the undeformed elastic curve through A.

$$t_{A/B} = \int_A^B (M / EI)xdx$$

Q.11b.

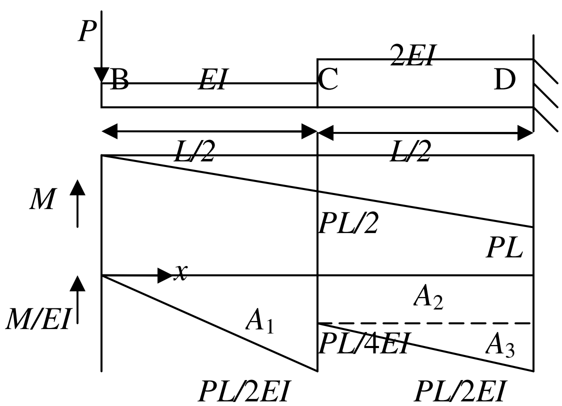


Fig.11b

The B.M. at any section $x$, $M = -Px$.
The B.M. $M$ and ($M/EI$) diagrams are as shown in Fig.11b.
As the tangent to the elastic curve at D is horizontal $\theta_D = 0$, the slope at B,
$\theta_B$= - area ($M/EI$) diagram B to D= $-\sum A_i$.
$A_1 = (PL/2EI)(L/2)/2 = -PL^2/8EI$,
$A_2 = (PL/4EI)L/2 = -PL^2/8EI$,
$A_3 = (PL/4EI)(L/2)/2 = -PL^2/16EI$
$\rightarrow \theta_B = \underline{5PL^2/16EI}$.
Distances $x_i$ of the centroids of $A_i$ from the vertical axis through B are
$x_1 = L/3$, $x_2 = 3L/4$, $x_3 = 5L/6$
The deflection $\delta_B = t_{B/D} = \sum A_i x_i$.
$\rightarrow \delta_B = -(PL^2/8EI)L/3 - (PL^2/8EI)\times 3L/4 - (PL^2/16EI)(5L/6) = \underline{-3PL^3/16EI}$.