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Code: C-09 / T-09 **Subject: NUMERICAL COMP**
Time: 3 Hours **Max. Marks:**

NOTE: There are 11 Questions in all.

- **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: **(2x8)**

a. Newton – Raphson method is to be applied to find the value of $\sqrt{N}$. Then, the formula can be written as

(A) $\frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$. **(B)** $\frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$.

(C) $x_n + \frac{N}{x_n}$. **(D)** $x_n - \frac{N}{x_n}$.

b. The divided difference $f[x_0, x_1, x_2]$ is equal to

(A) $\Delta^2 f_0$. **(B)** $\Delta^2 f_0 / h^2$.

(C) $\Delta^2 f_0 / (2h^2)$. **(D)** $\Delta^3 f_0 / (6h^3)$.

Where x_0, x_1, x_2 are equispaced points with spacing h and Δ is the forward difference operator.

c. Attempt is made to solve the system of equations $Ax = b$, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$ by the Gauss–Jacobi iteration method. Then, the iteration

(A) has rate of convergence 0.5634. **(B)** has rate of convergence 0.235.
(C) has rate of convergence 1.234. **(D)** diverges.

d. The interpolating polynomial that fits the data

x	1	2	3	4
f(x)	-1	-1	1	5

is

(A) $x^2 + 3x + 1$. **(B)** $x^2 - 3x + 1$.

(C) $2x^2 - 6x + 3$. **(D)** $x^2 - 5x + 1$.

e. The integral $I = \int_0^1 \frac{dx}{1+x^2}$ is evaluated by Simpson's rule using 3 points. Then, the value of I is equal to

(A) $\pi/4$. **(B)** $23/60$.
(C) $37/60$. **(D)** $47/60$.

f. The least squares straight line approximation to the data

x	1	2	3	4
f(x)	-1	1	3	5

is given by

(A) $2x - 3$. **(B)** $2x + 3$.
(C) $x + 4$. **(D)** $2x - 4$.

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g. A numerical differentiation formula for finding $f'(x)$ is given by
$f'(x) = [f(x+h) + a\ f(x) - f(x-h)]/(2h)$
Then, the value of a for which the method is of highest order is given by
(A) -2. **(B)** 0.
(C) -1. **(D)** 2.

h. The integration formula $\int_{-1}^{1} f(x)dx = \frac{1}{2} f(-1) + \frac{3}{2} f(a)$ is to be used. The value of a for which the method is of highest order, is given by
(A) 1. **(B)** 2/3.
(C) 1/3. **(D)** 1/2.

PART I
Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. The equation $x^2 - 2x - 3\cos x = 0$ is given.
(i) Locate the smallest root in magnitude, in an interval of length one unit.
(ii) Hence, find this root correct to 3 decimals using the secant method. **(6)**

b. A method for determining $N^{1/4}$, where N is a positive real number, is written as $x_{k+1} = a\ x_k + \frac{bN}{x_k^3} + \frac{cN^2}{x_k^7}$. Determine the values of the parameters a, b, c such that the order of the method is as high as possible. **(8)**

Q.3 a. The system of equations $x^2 + 3y^2 + 2xy = 2.51,\ 2x^2 + y^2 - 5xy = 12.83$ has a solution near x = 1.5, y = -1.0. Set up the Newton's method for solution and iterate once. **(7)**

b. Using the Cholesky method, solve the system of equations
$$\begin{bmatrix} 4 & 0 & 1 \\ 0 & 4 & 3 \\ 1 & 3 & 37/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 59 \end{bmatrix}.$$ **(7)**

Q.4 a. Solve the system of equations
$x_1 - 3x_2 + 2x_3 = 3$
$2x_1 + 6x_2 + 8x_3 = -1$
$4x_1 - 3x_2 + x_3 = 4.25$
using the Gauss elimination method with partial pivoting. **(7)**

b. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & -1 \\ 2 & 4 & 3 \end{bmatrix}$ using the Gauss – Jordan method. **(7)**

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Q.5 a. Perform 4 iterations of the Gauss – Seidel method for finding the solution of the linear system of equations

$$4x_1 - 2x_2 + x_3 = 4$$
$$x_1 + 2x_2 + x_3 = 0.75$$
$$3x_1 - 3x_2 + 5x_3 = 5.5$$

Assume the initial approximation as $x_1 = 0.6,\ x_2 = -0.2$ and $x_3 = 0.5$. Find the iteration matrix and hence determine the rate of convergence of the method. **(8)**

b. Find all the eigenvalues of the matrix $\begin{bmatrix} 2 & \sqrt{2} & 2 \\ \sqrt{2} & 4 & \sqrt{2} \\ 2 & \sqrt{2} & 2 \end{bmatrix}$, using the Jacobi method. (Use exact arithmetic) **(6)**

Q.6 a. Find the smallest eigenvalue in magnitude and the corresponding eigenvector of the matrix $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{bmatrix}$, using the inverse power method. Assume the initial approximation to the eigen vector as $[0.4 \quad -0.9 \quad 0.4]^T$. **(8)**

b. Transform the matrix $\begin{bmatrix} 1 & 4 & -4 \\ 4 & 1 & 2 \\ -4 & 2 & 1 \end{bmatrix}$ to tri-diagonal form using the Given's method. Set up the Sturm sequence and find the smallest eigenvalue in magnitude. **(6)**

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. A table of values is to be constructed for the function $f(x) = (1+x)^5$ on $[1, 2]$. If the linear interpolation is to be used on this table of values, find the largest step size that can be used so that the error is bounded by 5×10^{-4}. **(7)**

b. Obtain the unique polynomial $P(x)$ of degree 3 or less corresponding to a function $f(x)$, where $f(0) = 1,\ f'(0) = 2,\ f(1) = 5, f'(1) = 4$. **(7)**

Q.8 a. If $f(x) = u(x)v(x)$, find the divided difference $f[x_0, x_1]$ in terms of $u(x_0), v(x_0)$ and the divided differences $u[x_0, x_1], v[x_0, x_1]$. **(5)**

b. If Δ and ∇ are the forward and backward differences respectively, show that $\Delta + \nabla = (\Delta / \nabla) - (\nabla / \Delta)$. **(4)**

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c. Find the interpolating polynomial which fits the data **(3**

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.93	0.92	0.97	1.08	1.25	1.48

Q.9 a. Use the method of least squares to fit a function of the form $y = a + (b/x)$ to the following data **(7)**

x	1	2	3	4	5
y	5	3.5	3	2.7	2.5

b. The following table of values is given

x	0.2	0.3	0.4	0.5	0.6
y(x)	1.8054	1.5769	1.2834	0.9483	0.5981

Find all the possible approximations for $y''(0.4)$ using the differentiation formula. $y''(x) = \frac{1}{h^2}\left[y(x-h) - 2y(x) + y(x+h)\right]$.

Perform Richardson's extrapolation to obtain a better estimate. **(7)**

Q.10 a. The generalised trapezoidal rule

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] + p\,h^2\left[f'(x_0) - f'(x_1)\right]$$

where p is a constant and $h = x_1 - x_0$, is given. Find the value of the constant p. Deduce the composite rule for evaluating the integral

$$\int_a^b f(x)\,dx,\ a = x_0 < x_1 < x_2 < \dots < x_n = b.$$ **(8)**

b. Evaluate the integral $\int_2^3 \frac{\cos 4x}{1+\sin x}\,dx$, using the Gauss-Legendre two-point and three point integration rules. **(6)**

Q.11 a. Find approximations to $y(0.4)$ and $y'(0.4)$ with the Taylor's series method of second order and step length h = 0.2, where y(x) is the solution of the initial value problem

$y'' + 3y' + 2y = \cos x,\ y(0) = 1,\ y'(0) = 1.$ **(8)**

b. The initial value problem $y' = \frac{y+2x}{y+3x}$, $y(1) = 2$ is given. Find an approximation to y(1.2), when h = 0.2, using the Runge – Kutta method

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

$$k_1 = hf(x_n, y_n),\quad k_2 = hf\left(x_n + \frac{h}{2},\ y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf(x_n + h,\ y_n - k_1 + 2k_2).$$

for the solution of differential equation $y' = f(x, y)$. **(6)**

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Q.1 Choose the correct or best alternative in the following: (2x8)

a. An approximate value of π is given by $X_1 = \frac{22}{7} = 3.1428571$ and its true value is X = 3.1415926. Then, the absolute and relative errors are

(A) 0.0012645, 0.000402. **(B)** - 0.0012645, - 0.000402.
(C) 0.0012645, - 0.000402 . **(D)** - 0.0012645, 0.000402.

b. Consider a variation of Newton's method in which only one derivative is needed, that is, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_o)}$. Then, if $e_{n+1} = C e_n^s$, the value of s is

(A) 2. **(B)** 1.5.
(C) 1. **(D)** 0.5.

c. The spectral radius of the matrix $\begin{bmatrix} 0 & 1/3 & 1/4 \\ -1/3 & 0 & 1/2 \\ -1/4 & -1/2 & 0 \end{bmatrix}$ is

(A) 2. **(B)** 1.
(C) >2. **(D)** <1.

d. The maximum stepsize *h* that can be used in the tabulation of f (x) = sin x in the interval $[0, \pi/4]$ at equally spaced nodal points so that the truncation error of the linear interpolation is less than 5×10^{-8} is

(A) 0.075. **(B)** 0.0075.
(C) 0.01. **(D)** 0.00075.

e. The Jacobian matrix for the system of equations

$f_1(x, y) = x^2 + y^2 - x - y = 0$
$f_2(x, y) = x^2 - y^2 - y = 0$

at the point (1, 1) is given by

(A) $\begin{bmatrix} 1 & 2 \\ 1 & -3 \end{bmatrix}$. **(B)** $\begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$.

(C) $\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$. **(D)** $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.

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f. If f is a polynomial of degree k, then for n > k, $f[x_0, x_1, ..., x_n]$ equals

(A) n. **(B)** 1.
(C) 0. **(D)** k.

g. The Trapezoidal rule of numerical integration is exact for all polynomials of degree

(A) 2. **(B)** 3.
(C) 1. **(D)** >3.

h. The solution of the initial value problem $y' = -2xy^2, y(0) = 1$ at $x = 0.4$ by use of the mid-point method with h = 0.2 is given by

(A) 0.9580. **(B)** 0.8520.
(C) 0.7561. **(D)** 0.7021.

(You may calculate y_1 from the exact solution $y = 1/(1 + x^2)$.)

PART I

Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. Find analytically the solution of the following difference equation with the given initial values : $x_{n+1} = -0.2\,x_n + 0.99\,x_{n-1}, \quad x_0 = 1, \quad x_1 = 0.9$. Without actually computing the solution recursively, find whether such a computation would be stable or not. **(7)**

b. Find an interval of length 1, in which a real root of smallest magnitude, of 3x = cos x + 1 lies. Use Newton-Raphson method to find this root correct upto 3 decimal places. **(7)**

Q.3 a. Consider the following 2×2 system of linear equations given by $\begin{bmatrix} \varepsilon & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ where ε (small) is of the order of round-off error. Find the solution of the above system using

(i) Gauss elimination without pivoting.
(ii) Gauss elimination with partial pivoting.

Do you find any difference in the solutions of (i) and (ii) ? Justify. **(7)**

b. The system of equations Ax = b is to be solved iteratively by $x_{n+1} = M\,x_n + b$. Suppose $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}, k \neq \sqrt{2}/2$, k real. Find a necessary and sufficient condition on k for convergence of the Jacobi method. **(7)**

Q.4 a. Find the solution of the system of equations

$4x + y + z = 5$
$2x + 5y - 2z = 9$
$2x + 3y + 6z = 13$

using the Gauss elimination method. **(7)**

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b. Determine the rate of convergence of the Gauss-Seidel method for solving the system $\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$. **(7)**

Q.5 a. Starting with $x_1 = 0,\ x_2 = 1$, perform two iterations of the Newton's method on the following system of equations:

$$\begin{aligned} 4x_1^2 - x_2^2 &= 0 \\ 4x_1x_2^2 - x_1 &= 1 \end{aligned}.$$ **(7)**

b. Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$

using the Choleski method. **(7)**

Q.6 a. Find all the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$

using Jacobi's method. **(7)**

b. Use the Givens method to transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to tri-diagonal form. Find the largest eigenvalue, in magnitude, using the Sturm sequences. **(7)**

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. The equation $f(x) = x - 9^{-x} = 0$ has a solution in (0, 1). Find the interpolation polynomial for the function f(x), using the points $(x_0, f_0), (x_1, f_1), (x_2, f_2)$ where $x_0 = 0, x_1 = 0.5, x_2 = 1$. By setting the interpolation polynomial equal to zero and solving the equation, find an approximate solution to the equation. **(7)**

b. The polynomial $p(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1)$ interpolates the first four points in the table :

x	–1	0	1	2	3
y	2	1	2	–7	10

By adding one additional term to p(x), find a polynomial that interpolates the whole of table. **(7)**

Q.8 a. A person runs the same race track for five consecutive days and is timed as follows :

day(x)	1	2	3	4	5
time(y)	15	14	13	12	11

Make a least squares fit to the above data using a function $y = a + (b/x)$. **(7)**

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b. Determine the constants a, b, c and d such that the interpolation polynomial $y_s = y(x_0 + sh) = ay_0 + by_1 + h^2(cy_0'' + dy_1'')$ becomes correct to the highest possible order. **(7)**

Q.9 a. Simpson's rule of numerical integration has the asymptotic error in the form

$$\text{Error} = c_1 h^4 + c_2 h^6 + ---$$

where Error = I (exact) – I (numerical solution).
Obtain the Romberg formula for improving the numerical solution. Use it to evaluate

$$I = \int_0^1 \frac{x\ dx}{1 + x + x^2}$$

with h = 0.5 and 0.25. **(7)**

b. Obtain a generalized trapezoidal rule of the form

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}(f_0 + f_1) + ph^2(f_0' - f_1')$$

by finding the constant p and the error term. Deduce the composite rule for integrating $\int_a^b f(x)\,dx,\ \ a = x_0 < x_1 < x_2 < \ldots\ldots < x_n = b$. **(7)**

Q.10 a. A differentiation rule of the form

$$f'(x_0) = \alpha_0 f_0 + \alpha_1 f_1 + \alpha_2 f_2,\ \ (x_k = x_0 + kh)$$

is given. Find the values of α_0, α_1 and α_2 so that the rule is exact for all polynomials of degree less than or equal to 2. Find the error term. **(7)**

b. Use Gauss-Chebyshev two point and three point formulas to evaluate

$$\int_{-1}^{1} \left(1 - x^2\right)^{1/2} x\ \sin x\, dx$$ **(7)**

Q.11 a. Solve the initial value problem $y' = -2xy^2, y(0) = 1$ with h = 0.2 on [0, 1] using the Euler method. **(6)**

b. Derive the second order Runge – Kutta method for solving the initial value problem: $y' = f(x, y),\quad y(x_0) = \eta_0,\ x \in [x_0, b]$ in the form

$$y_{n+1} = y_n + w_1 k_1 + w_2\, k_2$$

$$k_1 = h\,f\,(x_n, y_n), k_2 = h\,f\,(x_n + c_2 h, y_n + a_2\, k_1).$$ **(8)**

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Q.1 Choose the correct or best alternative in the following: **(2x8)**

a. For Simpson's $\frac{1}{3}$rd rule of numerical integration

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})]$$

where $h = \frac{b-a}{n}$, the value of

(A) n must be even. **(B)** n must be odd.
(C) n can be any integer. **(D)** n is multiple of 3.

b. An approximation for the solution of the initial value problem $y' = xy,\ y(0) = 1$ is

(A) 1. **(B)** $x^2/2$.
(C) $1 + \frac{x^2}{2}$. **(D)** None of these.

c. If Newton-Raphson method is used for solving the equation f (x) =0, the order of convergence is

(A) 1. **(B)** 3.
(C) 5. **(D)** None of these.

d. The equation $x^4 - 2x^3 + 2x^2 - 2x + 1 = 0$ has a multiple root $x = 1$ of multiplicity 2. The other two roots will be

(A) both real but different. **(B)** a pair of equal real roots.
(C) a complex pair. **(D)** one real and one complex.

e. The spectral radius of the matrix $\begin{bmatrix} 0 & 1/3 & 1/4 \\ -1/3 & 0 & 1/2 \\ -1/4 & -1/2 & 0 \end{bmatrix}$ is

(A) 2. **(B)** 1.
(C) > 1. **(D)** < 1.

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f. The highest degree polynomial for which 4 point Gauss Legendre integration formula gives exact result is equal to

(A) 2. (B) 3.
(C) 4. (D) 7.

g. The order of the method $y_{n+1} = y_{n-1} + 2h\, f_n$ for solving the initial value problem $y' = f(x, y), y(x_0) = y_0$ is

(A) second. (B) first.
(C) third. (D) fourth.

h. Point out the false statement

(A) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{\left(1 + \frac{\delta^2}{4}\right)}$. (B) $\Delta = E + 1$.

(C) $\delta^3 Y_{1/2} = Y_2 - 3Y_1 + 3Y_0 - Y_{-1}$. (D) $\Delta\nabla = \nabla\Delta = \Delta - \nabla$.

PART I
Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. Show that the following system of equations

$$x + y + 2z = 1$$
$$2x + y - 3z = 0$$
$$-3x - y + 8z = A$$

is inconsistent except for one value of A. Find this value. **(7)**

b. Use Gauss elimination method to solve

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$$
$$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = 0$$
$$\frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 = 0$$

(7)

Q.3 Derive Gauss-Legendre two-point integration method to evaluate the integral $\int_{-1}^{1} f(x)\,dx$. Use this method to compute

$$I = \int_{-2}^{2} e^{-x/2}\,dx.$$ **(14)**

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Q.4 a. Perform three iterations of the False position method to find the root of the equation $f(x) = x^2 - x - 2 = 0$ starting with the initial approximations $x_0 = 1$ and $x_1 = 3$. **(6)**

b. Find the Lagranges interpolation polynomial which fits the following data.

i	0	1	2	3
x_i	0	1	2	3
$f(x_i)$	0	1.7183	6.3891	19.0855

and use the same to estimate the value of f (1.5). **(8)**

Q.5 a. Using Runge-Kutta classical fourth order method, find the approximate value of y (1.2) for the initial value problem $y' = x + y^2, y(1) = 2$ with the step size h = 0.1. **(10)**

b. Determine the order of convergence of the iterative method

$$x_{k+1} = \frac{(x_0 f(x_k) - x_k f(x_0))}{(f(x_k) - f(x_0))}$$ **(4)**

Q.6 a. Find the approximate value of $I = \int_0^{1/2} \frac{x}{\sin x} dx$ using composite Trapezoidal rule with h = 1/2 , 1/4, 1/8 and the Romberg integration. **(8)**

b. Prove the following :

(i) $\Delta\left(\frac{f_i}{g_i}\right) = \frac{g_i \, \Delta f_i - f_i \, \Delta g_i}{g_i g_{i+1}}$.

(ii) $\Delta\left(\frac{1}{f_i}\right) = -\frac{\Delta f_i}{f_i f_{i+1}}$. **(6)**

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 Describe the inverse power method for determining the smallest eigenvalue in magnitude of a square matrix. Perform three iterations of this method to find the smallest eigenvalue in magnitude of the matrix

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Take the initial approximation to the eigenvector as $[0, \ 0, \ 0]^T$. **(14)**

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Q.8 Describe Jacobi's method to find all the eigen values of the real symmetric matrix and hence find all the eigen values of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. **(14)**

Q.9 a. Obtain the least squares approximation of degree two which fits the data

X	1	2	3	4	5	6
Y	1200	900	600	200	110	50

(7)

b. Use Taylor's series method of order four to obtain the approximate value of y (0.2) for the initial value problem

$$\frac{dy}{dx} = 1 - 2\,xy,\ y(0) = 0\,.$$

Take the step size h = 0.1. **(7)**

Q.10 Show that the following two sequences have convergence of the second order with the same limit $\sqrt{a}$.

(i) $x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right)$ (ii) $x_{n+1} = \frac{1}{2} x_n \left(3 - \frac{x_n^2}{a}\right)$

If x_n is a suitably close approximation to $\sqrt{a}$, show that the error in the first formula for x_{n+1} is about one-third of that in the second formula, and deduce that the formula

$$x_{n+1} = \frac{1}{8} x_n \left(6 + \frac{3a}{x_n^2} - \frac{x_n^2}{a}\right)$$

gives a sequence with third-order convergence. **(14)**

Q.11 a. Determine the truncation error and the order of the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h}$$

Find the optimal value of the step size h such that

| round-off error | = | truncation error | **(7)**

b. Using Choleski method solve the linear system of equations

$$x + 2y + z = 0$$
$$2x + 5y = -3$$
$$x + 13z = 14$$

(7)

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- **keep four decimal digits in your arithmetic calculations.**

Q.1 Choose the correct or best alternative in the following: (2x8)

a. A root of the equation $x^4 - x - 10 = 0$ is obtained using the secant method with initial approximations $x_0 = 1.8$ and $x_1 = 1.9$. The root obtained after two iterations is

(A) 1.8535. **(B)** 1.8555.
(C) 1.8635. **(D)** 1.8655.

b. The iteration method $\underline{x}^{(n+1)} = \begin{bmatrix} 0 & 1/2 & 0 \\ 0 & 1/4 & 1/2 \\ 0 & 1/8 & 1/4 \end{bmatrix} \underline{x}^{(n)} + \begin{bmatrix} 7/2 \\ 9/4 \\ 15/8 \end{bmatrix}, n = 0, 1.....$ is used to solve the linear system of equations $\underline{A}\,\underline{x} = \underline{b}$. The spectral radius of the iteration matrix is

(A) $1/\sqrt{2}$. **(B)** $-\frac{1}{2}$.
(C) $\frac{1}{2}$. **(D)** 1.

c. The function f (x) is given in tabular form as

x	-3	-1	0	1	2
F(x)	7	1	1	3	7

Using interpolation, the value of f (-2) is obtained as

(A) 2. **(B)** 3.
(C) 4. **(D)** 5.

d. The truncation error in the method

$$f''(x_k) = \frac{1}{12h^2}\left[-30f(x_k) + 16\{f(x_{k-1}) + f(x_{k+1})\} - \{f(x_{k-2}) + f(x_{k+2})\}\right]$$

is written in the form ch^p. The value of p is

(A) 1. **(B)** 2.
(C) 3. **(D)** 4.

e. The integration rule $\int_{-1}^{1} f(x)dx = a\,f(-1) + b\,f(0) + a\,f(1)$ is of the highest possible order if the value of b is

(A) $\frac{1}{3}$. **(B)** $\frac{2}{3}$.
(C) 1. **(D)** $\frac{4}{3}$.

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f. The value of the integral $I = \int_{-1}^{1} \left(1-x^2\right)^{3/2} dx$ using Gauss-Chebyshev two-point method is

(A) $\frac{\pi}{4}$. **(B)** $\frac{\pi}{2\sqrt{2}}$.

(C) $\frac{\pi}{2}$. **(D)** $\frac{\pi}{\sqrt{2}}$.

g. The least square polynomial approximation of degree 1 to the function $f(x) = x^{1/3}$ on $[0,1]$ is

(A) $\frac{1}{14}(6+9x)$. **(B)** $\frac{1}{14}(6-9x)$.

(C) $\frac{1}{14}(9+6x)$. **(D)** $\frac{1}{14}(9-6x)$.

h. Euler's method is used to solve the initial value problem $y' = \sqrt{x+y}$, $y(1) = 2$ with the step size h =0.1. The approximate value of y (1.2) is

(A) 2.1732. **(B)** 2.3464.

(C) 2.3541. **(D)** 2.4541.

PART I

Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. The method $x_{n+1} = x_n - \alpha \frac{f(x_n)}{f'(x_n)}$, $n = 0,1....$ is used to determine a multiple root of multiplicity 3 of the equation f (x) =0. Find the value of α so that the method has highest rate of convergence. Obtain the rate of convergence and the asymptotic error constant. **(7)**

b. Perform two iterations of the method

$$x_n^* = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n^* - \frac{f(x_n^*)}{f'(x_n)}, \quad n = 0,1,....$$

to obtain a root of the equation $x^3 + x^2 + x + 4 = 0$ starting with the initial approximation $x_0 = -1.5$. **(7)**

Q.3 a. Set up the Newton's iteration method in matrix form to solve the system of equations

$$\begin{aligned} 2x^3 + 4y^3 &= 20 \\ 4x^2 + 5y^2 &= 21 \end{aligned}.$$

Perform two iterations of the method, starting with $x_0 = 1.9,\ y_0 = 0.9$. **(7)**

b. Gauss-Jacobi iteration method is used to solve the system of equations

$$\begin{aligned} 3x - 6y + 2z &= 23 \\ -4x + y - z &= -15 \\ x - 3y + 7z &= 16 \end{aligned}$$

Determine the iteration matrix. Find the rate of convergence of the method, if it converges. **(7)**

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Q.4 a. Obtain the upper triangular matrix $\underline{u}$ such that the matrix

$$\underline{A} = \begin{bmatrix} 14 & -7 & 15 \\ -7 & 5 & -10 \\ 15 & -10 & 25 \end{bmatrix}$$

can be written in the form $\underline{A} = \underline{U}\underline{U}^T$. Hence, obtain the matrix A^{-1}. **(9)**

b. Solve the linear system of equations

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

using Gauss-elimination method with partial pivoting. **(5)**

Q.5 a. Determine all the eigenvalues of the matrix $\underline{A} = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 5 & -2 \\ 4 & -2 & 1 \end{bmatrix}$, using Jacobi's method. (Use exact arithmetic). **(8)**

b. Perform three iterations of the inverse power method to find an eigenvalue which is nearest to 3 for the matrix $\underline{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. Take the initial approximate vector as $\underline{V}^{(0)} = [1, \ -1, \ 1]^T$. **(6)**

Q.6 a. Reduce the matrix $\underline{A} = \begin{bmatrix} 2 & \sqrt{2} & 4 \\ \sqrt{2} & 6 & \sqrt{2} \\ 4 & \sqrt{2} & 2 \end{bmatrix}$ to tri-diagonal form using Given's method. Determine the characteristic equation of $\underline{A}$ using Sturms sequence. **(7)**

b. Find the inverse of the matrix $\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$ using Gauss-Jordan method. **(7)**

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Obtain the least square approximation of the form $y = a\sqrt{x} + \frac{b}{x}$ to the function $f(x) = x^2$ on $[1, 4]$. **(7)**

b. Using Newton's backward interpolation formula for the given data $(x_i, f_i), i = 0,1,2,........., n$, show that

$$f'(x_n) = \frac{1}{h}\left[\nabla f_n + \frac{1}{2}\nabla^2 f_n + \frac{1}{3}\nabla^3 f_n + ... + \frac{1}{n}\nabla^n f_n\right].$$ Hence, determine $f'(5)$, where f (x) is given by **(7)**

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x	1	2	3	4	5
f(x)	1	3	7	13	21

Q.8 a. Obtain the Lagrange interpolation polynomial which fits the data

x	-3	-1	0	1
f(x)	-29	-1	1	3

Also find an approximate value of f (0.5). **(7)**

b. Find the nth order divided difference of $f(x)=\frac{1}{x}$ based on the nodal points $x_0, x_1, \ldots, x_n$. **(7)**

Q.9 a. Find the error term in the method $f'(x_0)=\frac{1}{h}[f(x_0+h)-f(x_0)]$ as a power series in h. Derive the corresponding Richardson's extrapolation scheme. Using this method and the Richardson's extrapolation, obtain the best value of $f'(1)$ when $f(x)$ is given by **(7)**

x	1	2	3	5
f(x)	1	8	27	125

b. For the method $f'(x_0)=\frac{1}{2h}[-3f(x_0)+4f(x_0+h)-f(x_0+2h)]$ determine the optimal value of h, using the criterion

$\max|RE| + \max|TE| = \min imum$

where $\in > 0$ is the maximum round off error in function values and M_i is the maximum value of $\left|f^{(i)}(x)\right|$ in a given interval. **(7)**

Q.10 a. Determine the constants λ_0, λ_1 and λ_2 in the method

$$\int_{-1}^{1} f(x)dx = \lambda_0 f\left(-\sqrt{\frac{3}{5}}\right)+\lambda_1 f(0)+\lambda_2 f\left(\sqrt{\frac{3}{5}}\right)$$ so that the method is of highest possible order. Obtain the order and the error term of the method.

Find the value of $I=\int_0^1 \frac{dx}{1+x}$ using this method. **(8)**

b. The integral $I=\int_1^2 \frac{dx}{x^2-x+1}$ is evaluated using the trapezoidal rule with h=1, $\frac{1}{2}$ and $\frac{1}{4}$. Find the improved value of I using Romberg integration. **(6)**

Q.11 a. Using third order Taylor's series method with step size h=0.2, obtain an approximate value of y (1.4) for the initial value problem

$y' = x + y^2,\ y(1)=1$. **(7)**

b. Using classical fourth order Runge-Kutta method with step size h=0.1, obtain an approximate value of y(1.2) for the initial value problem

$y' = \sqrt{x+2y},\ y(1)=2$ **(7)**

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