

Code: A-07
Time: 3 Hours

Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING

Max. Marks: 100

JUNE 2003

NOTE: There are 11 Questions in all.

- Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.
- Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or best alternative in the following: (2x8)

a. Using one-point Gauss-Chebyshev formula an approximate value of the

$$\text{integral} \int_{-1}^1 \frac{\cos x}{\sqrt{1-x^2}} dx \text{ is}$$

- (A) 0. (B) 2.40.
(C) 3.14. (D) 1.57.

b. The postfix notation for the following infix notation

$$A + (B * (C + (D * (E + F)))) \text{ is}$$

- (A) ABCDEF**+++ . (B) ABCDEF+*+*+ .
(C) +++**ABCD . (D) ABCD+++** .

c. The n-point Gauss Quadrature formula is exact for all polynomials of degree upto

- (A) n. (B) 2n.
(C) 2n - 1. (D) 2n + 1.

d. If `int s[5]` is a one dimensional array of integers, which of the following refers to the third element in the array?

- (A) `*(s + 2)`. (B) `*(s + 3)`.
(C) `s + 3`. (D) `s + 2`.

e. The interpolating polynomial that fits the data

x	-1	0	2	4	5
f(x)	1	-1	1	11	19

is

- (A) $x^2 - x - 1$. (B) $x^2 + x + 1$.
(C) $x^2 - x + 1$. (D) $x^2 + x - 1$.

- f. If λ is an eigenvalue of \mathbf{A} , then the eigenvalue of \mathbf{A}^{-1} is
- (A) $1/\lambda$. (B) $-1/\lambda$.
 (C) $-\lambda$. (D) λ^2 .
- g. Given that $k = 4$ and $m = 1$, the value of k after execution of the statement $k += k > m$ is
- (A) equal to 4. (B) equal to 8.
 (C) equal to 9. (D) cannot be found because the statement is invalid.
- h. The rate of convergence of the Newton-Raphson method for finding a simple root of the equation $f(x) = 0$ is
- (A) 1 (B) 3.
 (C) 5. (D) 2.

PART I

Answer any THREE Questions. Each question carries 14 marks.

- Q.2** Show that the following two sequences have convergence of second order with the same limit \sqrt{a}
- (i) $x_{n+1} = \frac{1}{2}x_n \left(1 + \frac{a}{x_n^2} \right)$ (ii) $x_{n+1} = \frac{1}{2}x_n \left(3 - \frac{x_n^2}{a} \right)$
- If x_n is a close approximation to \sqrt{a} , show that the error in the first formula is about one-third of that in the second formula and deduce that the formula $x_{n+1} = \frac{1}{8}x_n \left(6 + \frac{3a}{x_n^2} - \frac{x_n^2}{a} \right)$ gives a sequence with third-order convergence. (14)
- Q.3** a. Apply Cholesky's method to the matrix $\mathbf{A} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 5 \end{bmatrix}$ to obtain upper triangular matrix \mathbf{U} such that $\mathbf{A} = \mathbf{U}\mathbf{U}^T$. (7)
- b. Write a program in C to solve a system of linear equations using Jacobi's iteration method. (7)
- Q.4** a. Derive Euler's method for solving the initial value problem $y'(x) = f(x, y(x))$, $y(x_0) = y_0$. Use this method to obtain an approximate value of $y(0.4)$ for the equation $y' = x + y$, $y(0) = 1$ with $h = 0.1$. (5)

b. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 5 & 8 \\ 2 & 8 & 14 \end{bmatrix}$$

using Gauss-Jordan method. (5)

c. Find the linear least squares polynomial approximation for the data (4)

x_i	-2	-1	0	1
f_i	6	3	2	2

Q.5 Describe power method for finding the largest eigen value in magnitude of a square matrix. Perform four iterations of this method to find the largest eigen value in magnitude and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

Take the initial approximation to the eigenvector as $[1, 1, 1]^T$. (14)

Q.6 a. Find a bound on the truncation error in quadratic interpolation based on equispaced points with spacing h . Determine the largest step size h that can be used in tabulation of $f(x) = e^{-x}$ in $[0, 1]$ so that error in quadratic interpolation is less than 5×10^{-4} . (8)

b. Write a program in C that reads a variable x and computes the sum of series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ containing 25 terms. (6)

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Write a program in C using a recursive function for calculating values of Chebyshev polynomial $T_n(x)$ by

$$T_n(x) = \begin{cases} 1 & \text{for } n = 0 \\ x & \text{for } n = 1 \\ 2x T_{n-1}(x) - T_{n-2}(x) & \text{for } n > 1 \end{cases} \quad (7)$$

b. For the given data, use Newton divided difference interpolation to find $f(2)$ and $f(8)$: (7)

x	4	5	7	10
$f(x)$	48	100	294	900

Q.8 a. Find a quadratic factor of the polynomial $P(x) = x^4 + 5x^3 + 3x^2 - 5x - 9 = 0$ starting with $p_0 = 3$ and $q = -5$ using one iteration of the Bairstow's method. (7)

b. Use the classical Runge-Kutta fourth order method to obtain an approximate value of $y(1.2)$ for the initial value problem $y' = x^2 + y^2$, $y(1) = 2$ with $h = 0.2$. (7)

Q.9 a. Use the two point Gauss-Legendre quadrature formula to evaluate $\int_5^{12} \frac{1}{x} dx$. (7)

b. Write a program in C that reads a text file and then computes the total number of words and sentences in it. (7)

Q.10 a. Calculate the n^{th} divided difference of $f(x) = \frac{1}{x}$ based on the points x_0, x_1, \dots, x_n . (7)

b. Write the differences between a structure and union in C. Give an example of each. (7)

Q.11 a. Determine p and q such that the order of the iteration method $x_{n+1} = p x_n + \frac{q a}{x_n^2}$ for calculating $(a)^{1/3}$ becomes as high as possible. Find the order and the error constant of this method. (8)

b. Gauss – Seidel iteration method is used to solve the system of equations $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix}$ Find the iteration matrix. Show that the method converges. (6)

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Q.1 Choose the correct or best alternative in the following:

(2x8)

a. $f(x) = \frac{(e^{\sin x} - e^x)}{x^3}$. Then $\lim_{x \rightarrow 0} f(x)$ is

- (A) $\frac{1}{2}$. (B) $-\frac{1}{2}$.
 (C) $+\frac{1}{6}$. (D) $-\frac{1}{6}$.

b. The value of $\sum_{k=0}^n \ell_k(x)$, where $\ell_k(x)$ are Lagrange fundamental polynomials is

- (A) 0. (B) 1.
 (C) x. (D) x^n .

c. The rate of convergence of the Regula – Falsi method for finding a simple root of the equation $f(x) = x^2 + 4x + 4 = 0$ is

- (A) 1. (B) 1.5.
 (C) 2.0. (D) 2.25.

d. If $f(x) = e^{ax}$, then $\Delta^n f(x)$ is

- (A) $n(e^{ah} - 1)e^x$. (B) $(e^{ah} - 1)^n e^{ax}$.
 (C) $(e^{a^2h} - 1)^n e^x$. (D) $(e^{ah} + 1)^n e^{ax}$.

e. The result of mode bit noreplace is

- (A) error when opening if file does not already exist.
 (B) error when opening for output if file already exists unless ate or app is already set.
 (C) error when opening for input if file already exists, unless ate or app is set.
 (D) error when closing if file does not already exist.

f. The meaning of bit 0x01 is

- (A) No bit set, no errors. (B) at EOF.
 (C) Hard error. (D) Read or write failed.

g. Consider 3-point Gauss-Legendre formula:

$$\int_{-1}^1 f(x) dx = \lambda_0 f(x_0) + \lambda_1 f(x_1) + \lambda_2 f(x_2)$$

Then λ_i 's and x_i 's for $i=0,1,2$ are given by

(A)	$\begin{cases} x_0 = x_2 = \sqrt{\frac{3}{5}}, & x_1 = 0 \\ \lambda_0 = \lambda_2 = \frac{5}{11}, & \lambda_1 = \frac{8}{9} \end{cases}$	(B)	$\begin{cases} x_0 = x_2 = \sqrt{\frac{3}{5}}, & x_1 = 0 \\ \lambda_0 = \lambda_2 = \frac{5}{9}, & \lambda_1 = \frac{8}{9} \end{cases}$
(C)	$\begin{cases} x_0 = -x_2 = -\sqrt{\frac{3}{5}}, & x_1 = 0 \\ \lambda_0 = \lambda_2 = \frac{5}{9}, & \lambda_1 = \frac{8}{9} \end{cases}$	(D)	$\begin{cases} x_0 = -1, & x_1 = 0, & x_2 = 1 \\ \lambda_0 = \frac{1}{3}, & \lambda_1 = \frac{4}{3}, & \lambda_2 = \frac{1}{3} \end{cases}$

h. The order of the numerical differentiation method

$$f''(x_k) = \frac{1}{12h^2} [-f_{k-2} + 16f_{k-1} - 30f_k + 16f_{k+1} - f_{k+2}]$$

where $f_i = f(x_i)$, for all i , is

- | | |
|--------|--------|
| (A) 1. | (B) 2. |
| (C) 3. | (D) 4. |

PART I

Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. Using Cholesky factorisations method, solve the system of equations

$$\begin{bmatrix} 4 & 2 & 3 \\ 2 & 2 & 5 \\ 3 & 5 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix} \tag{7}$$

b. For the Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, to find a simple root of

$$f(x) = 0, \text{ find } C \text{ and } s \text{ such that } e_{n+1} = C e_n^s \tag{7}$$

Q.3 a. What conditions must be placed on a , b and c so that the following system of equations has a solution?

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 6y - 11z &= b. \\ x - 2y + 7z &= c \end{aligned} \tag{7}$$

b. The system of equations

$$\begin{aligned} 3x^2 + y^2 + 2xy &= 11 \\ x^2 + 2y^2 - 5xy &= -1 \end{aligned}$$

has a solution near $x = 0.8, y = 1.5$. Set up the Newton's iteration scheme for finding the solution and iterate once. (7)

Q.4 a. A program in C contains the following statements

```
float a = 0.001;
float b = 0.003;
float c, *pa, *pb;
pa = &a;
*pa = 2 * a;
```

```
pb = & b;
c = 3* (*pb - *pa);
```

Suppose each floating-point number occupies 4 bytes of memory. If the value assigned to a begins at (hexadecimal) address 1130 the value assigned to b begins at address 1134 & c at 1138, then

- (i) what value is assigned to &a, &b and &c?
- (ii) what value is represented by *pa, & (*pa)? (7)

b. Find the interpolating polynomial for the following data :

x	1	$\frac{3}{2}$	0	2
f(x)	3	$\frac{13}{4}$	3	$\frac{5}{3}$

(7)

Q.5 a. Describe the output generated by the following program:

```
* include <stdio.h>
int a = 100, b = 200;
main()
{
    int count, c, d; int funct1(int a, int b);
    for (count = 1; count <=5; ++count){
        c = 20 *(count - 1);
        d = 4 * count * count;
        printf(“%d %d”, funct1(a, c), funct1(b, d));
    }
    funct1 (int x, int y)
    { return(x - y);
    }
}
```

(7)

b. For the generalized Simpson’s rule of the form

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{3} (a f_0 + b f_1 + af_2) + ph^2 (f'_0 - f'_2) \quad \text{where } a, b, p \text{ are}$$

constants and $f_k = f(x_k), f'_k = f'(x_k)$, find the constants a, b, p and the error term. (7)

Q.6 a. Determine the least squares approximation of the type $ax^2 + bx + c$ to the function 2^x at the points $x_i = 0, 1, 2, 3, 4$. (7)

b. Write a program with a macro to find the roots of a quadratic equation $ax^2 + bx + c = 0$ where a, b, c are any real numbers. (7)

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Determine the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$ using partition method. (7)

b. Find the largest eigenvalue in magnitude and the corresponding eigenvector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using power method. Start

the iteration using the unit vector as an approximation to the eigenvector and obtain two successive iterations. (7)

Q.8 a. Find all the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using Jacobi method. Perform only two iterations. (7)

b. Write a program in C which sorts a list of strings into alphabetical order using an array of pointers. (7)

Q.9 a. Obtain a bound on the truncation error in linear interpolation based on the nodal points x_0 and x_1 . Determine the maximum step-size $h = x_1 - x_0$ that can be used in the tabulation of $f(x) = e^x$ in $[0, 1]$ so that the error in linear interpolation will be less than 5×10^{-4} . (6)

b. Determine the constants a, b, c α, β, γ in the following integration rules :

(i) $\int_0^1 f(x) dx \approx a f(0) + b f\left(\frac{1}{2}\right) + c f(1)$

(ii) $\int_0^1 f(x) dx \approx \alpha f\left(\frac{1}{4}\right) + \beta f\left(\frac{1}{2}\right) + \gamma f\left(\frac{3}{4}\right)$

Which of these two rules will give more accurate results? Justify. (8)

Q.10 a. Write a loop that will generate every third integer, beginning with $i = 2$ and continuing for all integers that are less than 100. Calculate the sum of those integers that are evenly divisible by 5. Use the following two different methods:

- (i) use the conditional operator ($?:$).
- (ii) use an if-else statement. (7)

b. The Gauss-Seidal iterations method is used to solve the linear system of

equations $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 6 & -2 \\ 4 & -3 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$. Find the iteration matrix and show

that the iterations converge. (7)

Q.11 a. Find an approximate value of $y(1.2)$ for the initial value problem $y' = x + y$, $y(1) = 2$ using the fourth order classical Runge Kutta method with $h = 0.1$. (9)

b. State the theorem concerning the existence and uniqueness of the solution of the initial value problem. Prove that the initial value problem

$y' = (x + \sin y)^2$, $y(0) = 3$ has a solution on $-1 \leq x \leq 1$. (5)

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Q.1 Choose the correct or best alternative in the following: (2x8)

- a. The integration rule $\int_{-1}^1 f(x)dx = \frac{1}{2} \left[f(-1) + 3f\left(\frac{1}{3}\right) \right]$ is given. The error is of the form $(c/6)f'''(\xi)$. Then, the value of c is given by
 (A) $\frac{1}{2}$. (B) $\frac{2}{9}$.
 (C) $\frac{4}{9}$. (D) $\frac{4}{27}$.
- b. The iteration method $x_{k+1} = (2x_k^3 + q)/(3x_k^2)$, q is a positive constant, is being used to evaluate a certain quantity. If the iteration converges, then the method is finding the value of
 (A) $q^{1/3}$. (B) q^3 .
 (C) $2/(3q^2)$ (D) $1/q$.
- c. An integral I is being evaluated by the composite trapezoidal rule. The values of I for two different step lengths $h = 1/3$ and $h = 1/4$ are obtained as $I(1/3) = 0.6123$, $I(1/4) = 0.6011$. A better approximation using Romberg integration is
 (A) 0.5974. (B) 0.6004.
 (C) 0.1893. (D) 0.5867.
- d. Using the data $f(0.1) = 0.9950$ and $f(0.2) = 0.9801$, an approximate value of $f(0.15)$ is determined by Lagrange interpolation. If $M_2 = \max|f''(x)|$ over $[0.1, 0.2]$, then the bound on the error in determining $f(0.15)$ is given by CM_2 where the value of C is
 (A) 0.0025. (B) 0.00125.
 (C) 0.0004. (D) 0.00375.
- e. The least squares straight line approximation to the data
- | | | | | |
|------|---|---|----|----|
| x | 1 | 2 | 3 | 4 |
| f(x) | 6 | 9 | 14 | 21 |
- is given by
 (A) $5x + 3$. (B) $6x + 1$.
 (C) $5x$. (D) $3x + 1$.
- f. Gauss-Jacobi iteration method is used to solve the system of equations $x + 4y = 3$, $4x + y = 4.5$ with arbitrary initial approximation. Then, the system
 (A) diverges. (B) has rate of convergence 0.6253,
 (C) has rate of convergence 0.2342. (D) has rate of convergence 0.4532.

g. What will be the output of the following program.

```
main()
{
    static int a [4] = {1, 2, 3, 4};
    int *p, i;
    p = a;
    printf("%d", *p);
    p = p*a;
    printf ("%d", *p);
}
```

- (A) 1 2 3 4 (B) 4 3 2 1
 (C) 4 2 3 1 (D) Error.

h. What will be the output of the following program.

```
main()
{
    struct name{
        char aname [10];
        char *bname;
    } p1 = { " Numerical", "Analysis"};
    printf ("%c..% c\n", p1.aname [0], *p1.bname);
    printf ("%s..%s\n", p1.aname, p1.bname);
}
```

- (A) N..n Numerical..Analysis (B) N..A Numerical..Analysis
 (C) N..s Numerical.. (D) N..A .. Analysis

PART I

Answer any THREE Questions. Each question carries 14 marks.

- Q.2** a. Locate the negative root of smallest magnitude, in an interval of length 1, for the equation $7x^3 + 8x^2 + 8x + 1 = 0$. Take the middle point of this interval as the starting approximation and find the root correct to three decimals using the Newton-Raphson method. (7)
- b. Write a C program to find a simple root of $f(x) = 0$, using the Newton-Raphson method. Read the initial approximation as $x_{initial}$, maximum number of iterations as n and error tolerance as ϵ . Define $f(x)$ and df/dx using functions. Assume $f(x)$ is defined as in part (a). (7)
- Q.3** a. Using BairStow's method, extract a quadratic factor of the form $x^2 + px + q$ for the equation $x^3 + 4.5x^2 + 3x + 4 = 0$. Assume the initial approximation as $p = 0.4$, $q = 1.1$ and perform one iteration. (7)

b. Find the inverse of the matrix $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 4 \end{bmatrix}$ using the Choleski

factorization method, in which the factorization is taken as $A = UU^T$, where U is an upper triangular matrix. (7)

Q.4 a. Solve the following system of equations using the Gauss-seidel iteration method

$$6x_1 + 3x_2 + x_3 = 9$$

$$2x_1 - 5x_2 + 2x_3 = -5$$

$$3x_1 + 2x_2 + 8x_3 = -4$$

Take the initial approximation as $x_1 = 1.0$, $x_2 = 1.0$ and $x_3 = -1.0$.

Perform two iterations. Find the iteration matrix and hence find the rate of convergence of the method. (8)

c. Find the largest eigen value in magnitude and the corresponding eigen-

vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 20 & -2 \\ 0 & -2 & 1 \end{bmatrix}$ using the power method. Assume the

initial approximation to the eigen vector as $v = [1 \ 1 \ 1]^T$. Perform five iterations. (6)

Q.5 a. Show that the order of the secant method for finding a simple root of the equation $f(x) = 0$ is 1.62. (7)

b. The following formulas for finding the yearly depreciation for some depreciable items like a car or machine are used.

(i) Straight line method: Depreciation each year = x/n .

(ii) Sum of the year's digits method: Depreciation = $\frac{(n-i+1)x}{[n(n+1)/2]}$

Where x = original value of the item, i = i th year and n = number of years.

Write a C program using switches to choose one of the depreciation methods. Output the year, depreciation and current value. (7)

Q.6 a. Find the smallest eigen value in magnitude, correct to two decimals, of the

matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ using the inverse power method. Assume the initial

eigen-vector as $[-0.8 \ -0.3 \ 1]^T$. (7)

b. Reduce the matrix $\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ to tridiagonal form by the Givens method.

Form the sturm sequence and locate the largest eigen value in magnitude in an interval of length one unit. (7)

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Determine the step size that can be used in the tabulation of $f(x) = (3 + 2x)^4$ in the interval $[0, 1]$, at equally spaced nodal points using quadratic interpolation such that $|\text{Error}| \leq 10^{-6}$. (7)

b. Construct an interpolation polynomial that fits the data

x	-1	0	1	2	3	4
f(x)	1.2	1.0	3.8	9.6	18.4	30.2

Interpolate the value at 0.5. (7)

Q.8 a. Evaluate $\int_0^2 \frac{dx}{x^2 + 2x + 10}$ using the Simpson's rule with three and five points. Improve the value of the integral using the Romberg integration. (7)

b. Write a C program to evaluate the integral $\int_a^b f(x)dx$ using trapezoidal rule. Input a, b and the number of subintervals N. Define the integrand as a function. Assume $f(x) = 1/(x^2 + 2x + 10)$. Output a, b, n and the value of the integral. (7)

Q.9 a. Construct the least squares approximation of the form $y = a + (b/x)$ to the data. (7)

x	0.1	0.2	0.4	0.5	1	2
y	21	11	6	5	3	5

b. Construct the Hermite interpolation polynomial that fits the following data.

X	f(x)	f'(x)
1	9	23
2	87	163

Interpolate f(x) at x = 1.5. (7)

Q.10 a. Find the uniform (Chebyshev) linear approximation $P(x) = a + bx$ to the function $f(x) = x^2 + 5x + 3$ on $[0, 1]$. (7)

b. The Fibonacci numbers are generated using the recurrence formula. $F_i = F_{i-1} + F_{i-2}$, $i = 3, 4, \dots$ with $F_1 = F_2 = 1$. Write a C program that generates the first n Fibonacci numbers, where n is specified by the user. Write a function called Fibonacci to generate the numbers. Use this function in the main program. The Fibonacci numbers are to be represented as long integers. (7)

Q.11 a. The initial value problem $y' = 2x^2 + 3y^2, y(1) = 1$ is given. Use the second order Taylor series method to find an approximation to y (1.2) with $h = 0.1$. (7)

b. Find an approximate value of y (0.4) for the initial value problem $y' = x^2 + y^2, y(0) = 1$ using the Runge-Kutta method of order four with $h=0.2$. (7)

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Q.1 Choose the correct or best alternative in the following: (2x8)

- a. The number 0.015625 is rounded as 0.0156. Then, the relative error in this approximation is
 (A) -0.0016. (B) 0.0016.
 (C) 0.016. (D) 0.000025.
- b. A real root of $f(x) = 0$ lies in the interval $[0, 1]$. Bisection method is applied to find this root. If the permissible error in the approximation is ϵ , then the number of iterations required is greater than or equal to
 (A) $-\log \epsilon / \log 2$. (B) $\log \epsilon / \log 2$.
 (C) $-\log \epsilon$. (D) $-\log \epsilon \log 2$.
- c. Gauss-Seidel method is applied to solve the system of equations $\begin{bmatrix} 1 & -p \\ -p & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, p real constant. The method converges for
 (A) $|p| > 1$. (B) all p .
 (C) $|p| \leq 1$. (D) $|p| < 1$.
- d. The divided difference $f[x_1, x_2, x_3]$ for the function $f(x) = x^3$ is given by
 (A) 6. (B) $2(x_1 + x_2 + x_3)$.
 (C) $x_1 + x_2 + x_3$. (D) $x_1 x_2 x_3$.
- e. The least squares approximation to the data
- | | | | | |
|--------|---|---|----|----|
| x | 1 | 2 | 3 | 4 |
| $f(x)$ | 6 | 9 | 14 | 21 |
- is given as $f(x) = 5x$. Then, the least squares error is given as
 (A) 0.04. (B) 4.
 (C) 6. (D) 0.004.
- f. The error in the numerical differentiation formula $f''(x_k) = \frac{1}{h^2} [f(x_{k-1}) - 2f(x_k) + f(x_{k+1})]$ is given by $Mf^{(4)}(\xi)$, where the value of M is
 (A) $h^2/12$. (B) $h^4/24$.
 (C) $h^4/12$. (D) $h/12$.
- g. The value of the integral $\int_0^1 \frac{x}{1+x^2} dx$ using Simpson's rule is
 (A) $\log \sqrt{2}$ (B) 13/40
 (C) 7/20 (D) 7/10

- h. The following C program is given
- ```
include <stdio.h>
main()
{
 switch (choice = toupper(getchar()))
 {
 case 'B':
 printf("BLUE");
 break;
 case 'P':
 printf("PINK");
 break;
 case 'G':
 printf("GREEN");
 break;
 default:
 printf("ERROR");
 }
}
```
- If the character 'g' is entered, the output is
- (A) ERROR                                      (B) GREEN  
 (C) PINK                                      (D) green

**PART I**

**Answer any THREE Questions. Each question carries 14 marks.**

**Q.2** a. Solve the system of equations 
$$\begin{bmatrix} 2 & 1 & -4 & 1 \\ -4 & 3 & 5 & -2 \\ 1 & -1 & 1 & -1 \\ 1 & 3 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ 2 \\ -1 \end{bmatrix}$$
 using the Gauss elimination method. (8)

b. Using the Choleski method, find the solution of the following system of equations 
$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 8 & -3 \\ 0 & -3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 49/16 \\ 1/2 \end{bmatrix}.$$
 (6)

- Q.3** a. The error in the Newton-Raphson method for finding a simple root of  $f(x)=0$ , can be written as  $\epsilon_{k+1} = c\epsilon_k^p$ . Determine the values of c and p. What is the order of the method? (6)
- b. The equation  $f(x)=0$  has a simple root in the interval (1, 2). The function  $f(x)$  is such that  $|f'(x)| \geq 10$  and  $|f''(x)| \leq 1$  for all x in (1, 2). Assuming that the Newton-Raphson's method converges for all initial approximations in (1, 2), find the number of iterations required to obtain the root correct to  $5 \times 10^{-7}$ . (8)

- Q.4** a. Locate the negative root of smallest magnitude of the equation  $7x^4 + x^3 + 6x^2 + 2x - 16 = 0$  in an interval of length 1. Taking the end points of this interval as the initial approximations to the root, perform five iterations using secant method (use five decimal places). (7)
- d. Write a C program to find a simple root of  $f(x) = 0$  by the secant method. Input (i) two initial approximations to the root as a and b, (ii) maximum number of iterations m, that the user wants to be done. (iii) error tolerance epsilon. Evaluate  $f(x)$  as a function. Output (i) number of iterations taken to obtain the root, (ii) the value of the root, (iii) value of  $f(\text{root})$ . If the iterations m, are not sufficient, output that “Number of iterations given are not sufficient”. (7)

**Q.5** a. Solve the system of equations 
$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -2.75 \\ 1.75 \\ -1.25 \end{bmatrix}$$
 using the Gauss-Jacobi method, with the starting approximations taken as  $x_1 = 0.4, x_2 = -0.6, x_3 = 0.3, x_4 = -0.3$ . Perform three iterations. (5)

- b. For the system in 5 (a) above, write the Gauss-Jacobi method in matrix form. Hence, find the rate of convergence of the method. (9)
- Q.6** a. The system of equations  $x^2 + y^2 = 4.82, xy + yz + zx = 0.59, yz^2 + y^2z = -1.33$  has a solution near  $(x, y, z) = (1, 2, -0.5)$ . Derive the Newton’s method for solving this system. Iterate once using the given initial approximations. (7)

- b. Write a C program to rearrange a given set of integer numbers into ascending order. Use the following:
- Define initially an array f as an 100 element array.
  - Read n, the number of given integer numbers followed by the numerical values.
  - Write a function prototype called “reordering”, whose arguments are n and f.
  - The program for reordering in ascending order is to be given in “reordering”. (7)

**PART II**

**Answer any THREE Questions. Each question carries 14 marks.**

- Q.7** a. Construct an interpolating polynomial that fits the data

|      |      |      |      |       |       |        |
|------|------|------|------|-------|-------|--------|
| x    | 0    | 1    | 2    | 5     | 7     | 10     |
| f(x) | -2.5 | -0.5 | 10.5 | 187.5 | 515.5 | 1502.5 |

Hence, or otherwise interpolate the value of f (8). (7)

- b. A table of values for  $f(x) = e^{x+1}$  in  $[0, 1]$  is to be constructed with step size  $h = 0.1$ . Find the maximum total error if quadratic interpolation is to be used to interpolate in this interval. (7)

- Q.8** a. A mathematical model of a periodic process in an experiment is taken as  $f(t) = a + b \cos(t)$  and a data of  $N$  points  $(x_i, f_i)$ ,  $i = 1, 2, \dots, N$  is given. If the parameters  $a$  and  $b$  are to be determined by the method of least squares, find the normal equations. Use these equations to find  $a, b$  for the following data (keep four decimal accuracy). **(3+5)**

|            |        |        |        |         |         |
|------------|--------|--------|--------|---------|---------|
| t(radians) | 0.5    | 1.0    | 1.5    | 2.0     | 2.5     |
| f(t)       | 0.9082 | 0.6552 | 0.3031 | -0.0621 | -0.3509 |

- b. Write a C program for interpolation using Lagrange interpolation. Input the following (i) Limit to number of points as 10. (ii) Number of points for any application as  $n$ . (iii) (Abscissas, Ordinates) =  $(x_i, y_i)$ . (iv) The value of  $x$  for which interpolation is required. Output  $x$  and the interpolated value. **(6)**
- Q.9** a. A differentiation rule of the form  $hf'(x_2) = af(x_0) + bf(x_1) + cf(x_2)$ ;  $x_j = x_0 + jh, j=1,2$  is given. (i) Determine  $a, b, c$  so that the rule is exact for polynomials of degree 2. (ii) Find the error term. (iii) If the roundoff errors in computing  $f(x_0), f(x_1), f(x_2)$  are  $\epsilon_1, \epsilon_2, \epsilon_3$  where  $|\epsilon_i| \leq \epsilon, i = 1, 2, 3$ , then obtain the expression for the bound of roundoff error in computing  $f'(x_2)$ . **(8)**

- b. Use the formula  $f''(x_0) = \frac{1}{h^2} [f(x_0 - h) - 2f(x_0) + f(x_0 + h)]$ , to compute  $f''(0.6)$  from the following table of values with  $h=0.4$  and  $h=0.2$ . Perform Richardson extrapolation to compute a better estimate for  $f''(0.6)$ . **(6)**

|      |        |        |        |        |     |
|------|--------|--------|--------|--------|-----|
| X    | 0.2    | 0.4    | 0.6    | 0.8    | 1.0 |
| f(x) | 1.3016 | 2.5256 | 3.8296 | 5.3096 | 7.1 |

- Q.10** a. Find the values of  $a, b, c$  such that the numerical integration formula  $\int_{-1}^1 f(x) dx = af(-1) + bf(c)$  is of as high order as possible. Find the error term. **(7)**
- b. Write a C program for solving the initial value problem  $y' = f(x, y), y(x_0) = y_0$ , by Euler's method. (i) Input the initial values  $x_0, y_0$ ; the final value  $x = xf$  and step length  $h$ . (ii) Use a subprogram for evaluating  $f(x, y) = x^2 + y^2$ . (iii) Create a file named "result" and put the computed values, for each value of  $x$ , in it. **(7)**

- Q.11** a. Evaluate the integral  $\int_{-1}^1 (1 - x^2) \cos x dx$  by (i) two point Gauss-Legendre formula, (ii) two point Gauss-Chebyshev formula. **(6)**
- b. An approximate value of  $u(0.2)$  for the initial value problem  $u' = u^2 + t^2, u(0) = 1$ ; with  $h = 0.2$  is to be obtained. Find this value using (i) Euler's method, (ii) Taylor series method of order four. **(8)**