Code: AC10 Subject: DISCRETE STRU
PART - I, VOL – I

TYPICAL QUESTIONS & ANSWERS

OBJECTIVE TYPE QUESTIONS

Each Question carries 2 marks.

Choose correct or the best alternative in the following:

Q.1 Which of the following statement is the negation of the statement,
"2 is even and –3 is negative"?

(A) 2 is even and –3 is not negative.
(B) 2 is odd and –3 is not negative.
(C) 2 is even or –3 is not negative.
(D) 2 is odd or –3 is not negative.

Ans:D

Q.2 If $A \times B = B \times A$, (where A and B are general matrices) then

$A = \varphi$. **(B)** A = B'.
(C) B = A. **(D)** A' = B.

Ans:C

Q.3 A partial ordered relation is transitive, reflexive and

(A) antisymmetric. **(B)** bisymmetric.
(C) antireflexive. **(D)** asymmetric.

Ans:A

Q.4 Let N = {1, 2, 3, ….} be ordered by divisibility, which of the following subset is totally ordered,

(A) $(2, 6, 24)$. **(B)** $(3, 5, 15)$.
(C) $(2, 9, 16)$. **(D)** $(4, 15, 30)$.

Ans:A

Q.5 If B is a Boolean Algebra, then which of the following is true

(A) B is a finite but not complemented lattice.
(B) B is a finite, complemented and distributive lattice.
(C) B is a finite, distributive but not complemented lattice.
(D) B is not distributive lattice.

Ans:B

Q.6 In a finite state machine, $m = (Q, \sum, \delta, q_0, F)$ transition function δ is a function which maps

(A) $Q \to \sum \times Q$. **(B)** $Q \times \sum \to Q$.

(C) $Q \to F$. **(D)** $Q \times q_0 \to \sum$.

Ans:B

Q.7 If $f(x) = \cos x$ and $g(x) = x^3$, then $(f \circ g)(x)$ is

(A) $(\cos x)^3$. **(B)** $\cos 3x$.

(C) $x^{(\cos x)^3}$. **(D)** $\cos x^3$.

Ans:D

Q.8 $p \to q$ is logically equivalent to

(A) $\sim q \to p$ **(B)** $\sim p \to q$

(C) $\sim p \wedge q$ **(D)** $\sim p \vee q$

Ans:D

Q.9 Which of the following is not a well formed formula?

(A) $\forall x [P(x) \to f(x) \wedge x]$

(B) $\forall x_1 \forall x_2 \forall x_3 [(x_1 = x_2 \wedge x_2 = x_3) \Rightarrow x_1 = x_3]$

(C) $\sim (p \to q) \to q$

(D) $[T \vee P(a, b)] \to \exists z Q(z)$

Ans:B

Q.10 The number of distinguishable permutations of the letters in the word BANANA are,

(A) 60. **(B)** 36.

(C) 20. **(D)** 10.

Ans:A

Q.11 Let the class of languages accepted by finite state machines be L1 and the class of languages represented by regular expressions be L2 then,

(A) $L1 \subset L2$. **(B)** $L2 \subseteq L1$.

(C) $L1 \cap L2 = \Phi$. **(D)** L1 = L2.

Ans:D

Q.12 $[\sim q \wedge (p \to q)] \to \sim p$ is,

(A) Satisfiable. **(B)** Unsatisfiable.
(C) Tautology. **(D)** Invalid.

Ans:C

Q.13 The minimized expression of $AB\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + \overline{ABC}$ is
(A) $A + \overline{C}$. **(B)** $\overline{B}C$.
(C) $\overline{C}$. **(D)** C.

Ans:C

Q.14 Consider the finite state machine M = $(\{s_0, s_1, s_2, s_3\}, \{0,1\}, F)$, where $F = \{fx \mid x \in \{0,1\}\}$ is defined by the following transition table. Let for any $w = x_1 x_2 .....x_n$, let fw = $fx_n \circ fx_{n-1} \circ fx_{n-2} \circ ...fx_2 \circ fx_1$. List the values of the transition function, f_{10110}.

	0	1
s_0	s_0	s_1
s_1	s_2	s_1
s_2	s_2	s_3
s_3	s_0	s_3

(A) $fw(s_0) = s_0, fw(s_1) = s_0, fw(s_2) = s_2, fw(s_3) = s_2$.
(B) $fw(s_0) = s_0, fw(s_1) = s_1, fw(s_2) = s_2, fw(s_3) = s_3$.
(C) $fw(s_0) = s_2, fw(s_1) = s_0, fw(s_2) = s_3, fw(s_3) = s_1$.
(D) $fw(s_0) = s_3, fw(s_1) = s_3, fw(s_2) = s_1, fw(s_3) = s_1$.

Ans:B

Q.15 Which of the following pair is not congruent modulo 7?
(A) 10, 24 **(B)** 25, 56
(C) -31, 11 **(D)** -64, -15

Ans:B

Q.16 For a relation R on set A, let $M_R = [m_{ij}], m_{ij} = 1$ if $a_i R a_j$ and 0 otherwise, be the matrix of relation R. If $(M_R)^2 = M_R$ then R is,
(A) Symmetric **(B)** Transitive
(C) Antisymmetric **(D)** Reflexive

Ans:B

Q.17 In an examination there are 15 questions of type True or False. How many sequences answers are possible.

Ans:
Each question can be answered in 2 ways (True or False). There are 15 questions, so they all can be answered in 2^{15} different possible ways.

Q.18 Find the generating function for the number of r-combinations of {3.a, 5.b, 2.c}

Ans:
Terms sequence is given as r-combinations of {3.a, 5.b, 2.c}. This can be written as ${}^{r}C_{3.a}$, ${}^{r}C_{5.b}$, ${}^{r}C_{2.c}$. Now generating function for this finite sequence is given by

$$f(x) = {}^{r}C_{3.a} + {}^{r}C_{5.b}x + {}^{r}C_{2.c}x^2$$

Q.19 Define a complete lattice and give one example.

Ans:

A lattice (L, ≤) is said to be a complete lattice if, and only if every non-empty subset S of L has a greatest lower bound and a least upper bound. Let A be set of all real numbers in [1, 5] and ≤ is relation of 'less than equal to'. Then, lattice (A, ≤) is a complete lattice.

Q.20 For the given diagram Fig. 1 compute

(i) $\overline{A \cup B}$
(ii) $A \cap \overline{B \cap C}$

Ans: (i) $\overline{A \cup B}$ = u + v (ii) $A \cap \overline{B \cup C}$ = y + z

Q.21 The converse of a statement is: If a steel rod is stretched, then it has been heated. Write the inverse of the statement.

Ans: The statement corresponding to the given converse is "If a steel rod has not been heated then it is not stretched". Now the inverse of this statement is "If a steel rod is not stretched then it has not been heated".

Q.22 Is $(P \wedge Q) \wedge \sim (P \vee Q)$ a tautology or a fallacy?

Ans: Draw a truth table of the statement and it is found that it is a fallacy.

Q.23 In how many ways 5 children out of a class of 20 line up for a picture.

Ans: It can be performed in ${}^{20}P_5$ ways.

Q.24 If P and Q stands for the statement
P : It is hot
Q : It is humid,
then what does the following mean?
$P \wedge (\sim Q)$:

Ans: The statement means "It is hot and it is not humid".

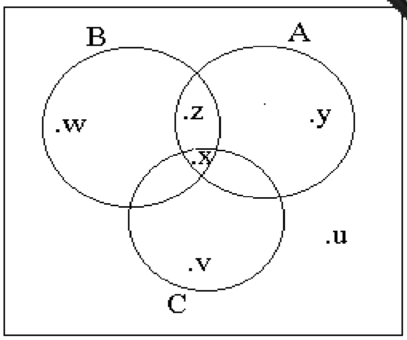
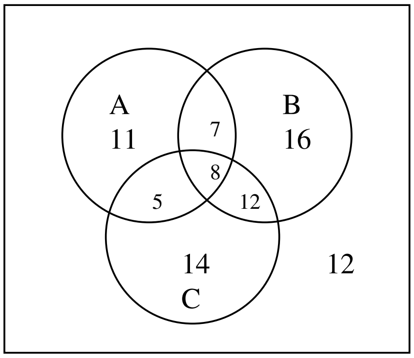


Fig.1

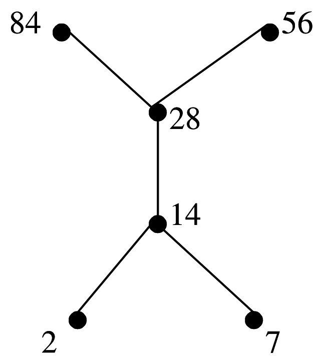
Q.25 In a survey of 85 people it is found that 31 like to drink milk, 43 like coffee and 39 like tea. Also 13 like both milk and tea, 15 like milk and coffee, 20 like tea and coffee and 12 like none of the three drinks. Find the number of people who like all the three drinks. Display the answer using a Venn diagram.

Ans: Let A, B and C is set of people who like to drink milk, take coffee and take tea respectively. Thus $|A| = 31$, $|B| = 43$, $|C| = 39$, $|A \cap B| = 15$, $|A \cap C| = 13$, $|B \cap C| = 20$, $|A \cup B \cup C| = 85 - 12 = 73$. Therefore, $|A \cap B \cap C| = 73 + (15 + 13 + 20) - (31 + 43 + 39) = 8$. The Venn diagram is shown below.



Q.26 Consider the set A = {2, 7, 14, 28, 56, 84} and the relation $a \leq b$ if and only if a divides b. Give the Hasse diagram for the poset $(A, \leq)$.

Ans: The relation is given by the set {(2,2), (2, 14), (2, 28), (2, 56), (2, 84), (7, 7), (7, 14), (7, 28), (7, 56), (7, 84), (14, 14), (14, 28), (14, 56), (14, 84), (28, 28), (28, 56), (28, 84), (56, 56), (84, 84)}. The Hasse Diagram is shown below:



Q.27 How many words can be formed out of the letters of the word 'PECULIAR' beginning wi
and ending with R?

(A) 100 **(B)** 120
(C) 720 **(D)** 150

Ans:C

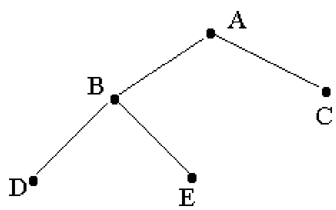
Q.28 Represent the following statement in predicate calculus: Everybody respects all the selfless leaders.

Ans: For every X, if every Y that is a person respects X, then X is a selfless leader. So in predicate calculus we may write:

$\forall X.\ \forall Y(\text{Person}(Y) \rightarrow \text{respect}(Y,X)) \rightarrow \text{selfless-leader}(X)$

Where person(Y): Y is a person
respect(Y,X): Y respects X
selfless-leader(X): X is a selfless leader.

Q.29 In which order does a post order traversal visit the vertices of the following rooted tree?



Ans: The post order traversal is in order (Right, Left, Root). Thus the traversal given tree is C E D B A.

Q.30 Consider a grammar G = ({S, A, B}, {a, b}, P, S). Let L (G)= $\left\{ (ab)^n \mid n \geq 1 \right\}$ then if P be the set of production rules and P includes S $\longrightarrow$ aB, give the remaining production rules.

Ans: One of the production rules can be: S→aB, B→bA, A→aB, B→b.

Q.31 Choose 4 cards at random from a standard 52 card deck. What is the probability that four kings will be chosen?

Ans: There are four kings in a standard pack of 52 cards. The 4 kings can be selected in $^{4}C_{4}$ = 1 way. The total number of ways in which 4 cards can be selected is $^{52}C_{4}$. Thus the probability of the asked situation in question is = 1 / $^{52}C_{4}$.

Q.32 How many edges are there in an undirected graph with two vertices of degree 7, four vertices of degree 5, and the remaining four vertices of degree is 6?

Ans: Total degree of the graph = 2 x 7 + 4 x 5 + 4 x 6 = 58. Thus number of edges in the g
is 58 / 2 = 29.

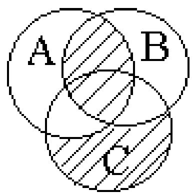
Q.33 Identify the flaw in the following argument that supposedly shows that n^2 is even when n is an even integer. Also name the reasoning:

Suppose that n^2 is even. Then $n^2 = 2k$ for some integer k. Let n = 2l for some integer l. This shows that n is even.

Ans: The flaw lies in the statement "Let n = 2l for some integer l. This shows that n is even". This is a loaded and biased reasoning. This can be proved by method of contradiction.

Q.34 The description of the shaded region in the following figure using the operations on set is,

(A) $C \cup (A \cap B)$
(B) $(C - ((A \cap C) \cup (C \cap B))) \cup (A \cap B)$
(C) $(C - (A \cap C) \cup (C \cap B)) \cup (A \cap B)$
(D) $A \cup B \cup C - (C \cup (A \cap B))$



Ans:B

Q.35 A 2×2 matrix $\begin{bmatrix} x & y \\ u & v \end{bmatrix}$ has an inverse if and only if,

(A) x, y, u and v are all nonzeros. **(B)** xy - uv $\neq$ 0
(C) xv - uy $\neq 0$ **(D)** all of the above.

`

Ans:C

Q.36 If x and y are real numbers then max (x, y) + min (x, y) is equal to

(A) 2x **(B)** 2y
(C) (x+y)/2 **(D)** x+y

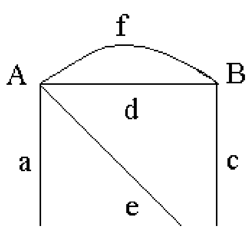
Ans:D

Q.37 The sum of the entries in the fourth row of Pascal's triangle is

(A) 8 **(B)** 4
(C) 10 **(D)** 16

Ans:A

Q.38 In the following graph



the Euler path is

(A) abcdef **(B)** abcf
(C) fdceab **(D)** fdeabc

Ans:C

Q.39 Let A = Z^+, be the set of positive integers, and *R* be the relation on A defined by *a R b* if and only if there exist a $k \in Z^+$ such that $a = b^k$. Which one of the following belongs to *R*?
(A) (8, 128) **(B)** (16, 256)
(C) (11, 3) **(D)** (169, 13)

Ans:D

Q.40 For the sequence defined by the following recurrence relation
$T_n = nT_{n-1},$ with initial condition $T_1 = 7$, the explicit formula for T_n is

(A) $T_n = n\,7^{n-1}$ **(B)** $T_n = 7.n!$

(C) $T_n = \frac{n!}{7}$ **(D)** $T_n = n! - 7$

Ans:B

Q.41 Which one of the following is not a regular expression?

(A) $\left[(a+b)^* - (aa+bb)\right]^*$ **(B)** $\left[(0+1) - (0b+a1)^*(a+b)\right]^*$

(C) $(01+11+10)^*$ **(D)** $(1+2+0)^*(1+2)$

Ans:B

Q.42 Which of the following statement is the negation of the statement "2 is even or –3 is negative"?
(A) 2 is even & -3 is negative **(B)** 2 is odd & -3 is not negative
(C) 2 is odd or –3 is not negative **(D)** 2 is even or –3 is not negative

Ans:B

Q.43 The statement $(p \wedge q) \Rightarrow p$ is a
(A) Contingency. **(B)** Absurdity
(C) Tautology **(D)** None of the above

Ans:C

Q.44 In how many ways can a president and vice president be chosen from a set of 30 candidates?
(A) 820 **(B)** 850
(C) 880 **(D)** 870

Ans:D

Q.45 The relation { (1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)} is
(A) Reflexive. **(B)** Transitive.
(C) Symmetric. **(D)** Asymmetric.

Ans:B

Q.46 Let L be a lattice. Then for every a and b in L which one of the following is correct?
(A) $a \vee b = a \wedge b$ **(B)** $a \vee (B \vee C) = (a \vee B) \vee C$
(C) $a \vee (b \wedge c) = a$ **(D)** $a \vee (b \vee c) = b$

Ans:B

Q.47 The expression $a + \bar{a}\,c$ is equivalent to
(A) $\bar{a}$ **(B)** a+c
(C) c **(D)** 1

Ans:B

Q.48 A partial order relation is reflexive, antisymmetric and
(A) Transitive. **(B)** Symmetric.
(C) Bisymmetric. **(D)** Asymmetric.

Ans:A

Q.49 If n is an integer and n^2 is odd ,then n is:
(A) even. **(B)** odd.
(C) even or odd. **(D)** prime.

Ans:B

Q.50 In how many ways can 5 balls be chosen so that 2 are red and 3 are black
(A) 910. **(B)** 990.
(C) 980. **(D)** 970.

Ans:B

Q.51 A tree with n vertices has _____ edges
(A) n **(B)** n+1
(C) n-2 **(D)** n-1

Ans:D

Q.52 In propositional logic which one of the following is equivalent to $p \rightarrow q$

(A) $\overline{p} \rightarrow q$ **(B)** $p \rightarrow \overline{q}$
(C) $\overline{p} V q$ **(D)** $\overline{p} V \overline{q}$

Ans:C

Q.53 Which of the following statement is true:
(A) Every graph is not its own sub graph.
(B) The terminal vertex of a graph are of degree two.
(C) A tree with n vertices has n edges.
(D) A single vertex in graph G is a sub graph of G.

Ans:D

Q.54 Pigeonhole principle states that $A \rightarrow B$ and $|A| > |B|$ then:
(A) f is not onto **(B)** f is not one-one
(C) f is neither one-one nor onto **(D)** f may be one-one

Ans:B

Q.55 The probability that top and bottom cards of a randomly shuffled deck are both aces is:
(A) $4/52 \times 4/52$ **(B)** $4/52 \times 3/52$
(C) $4/52 \times 3/51$ **(D)** $4/52 \times 4/51$

Ans:C

Q.56 The number of distinct relations on a set of 3 elements is:
(A) 8 **(B)** 9
(C) 18 **(D)** 512

Ans:D

Q.57 A self complemented, distributive lattice is called
(A) Boolean Algebra **(B)** Modular lattice
(C) Complete lattice **(D)** Self dual lattice

Ans:A

Q.58 How many 5-cards consists only of hearts?
(A) 1127 **(B)** 1287
(C) 1487 **(D)** 1687

Ans:B

Q.59 The number of diagonals that can be drawn by joining the vertices of an octagon is:

(A) 28 **(B)** 48
(C) 20 **(D)** 24

Ans:C

Q.60 A graph in which all nodes are of equal degrees is known as:
(A) Multigraph **(B)** Regular graph
(C) Complete lattice **(D)** non regular graph

Ans:B

Q.61 Which of the following set is null set?
(A) $\{0\}$ **(B)** $\{\ \}$
(C) $\{\phi\}$ **(D)** ϕ

Ans:B

Q.62 Let A be a finite set of size n. The number of elements in the power set of A x A is:
(A) 2^{2n} **(B)** 2^{n^2}
(C) $(2^n)^2$ **(D)** $(2^2)^n$

Ans:B

Q.63 Transitivity and irreflexive imply:
(A) Symmetric **(B)** Reflexive
(C) Irreflexive **(D)** Asymmetric

Ans:D

Q.64 A binary Tree T has n leaf nodes. The number of nodes of degree 2 in T is:
(A) log n **(B)** n
(C) n-1 **(D)** n+1

Ans:A

Q.65 Push down machine represents:
(A) Type 0 Grammar **(B)** Type 1 grammar
(C) Type-2 grammar **(D)** Type-3 grammar

Ans:C

Q.66 Let * be a Boolean operation defined by
$A * B = AB + \overline{A}\,\overline{B}$, then A*A is:

(A) A **(B)** B
(C) 0 **(D)** 1

Ans:D

Q.67 In how many ways can a party of 7 persons arrange themselves around a circular table?
(A) 6! **(B)** 7!
(C) 5! **(D)** 7

Ans:A

Q.68 In how many ways can a hungry student choose 3 toppings for his prize from a list of 10 delicious possibilities?
(A) 100 **(B)** 120
(C) 110 **(D)** 150

Ans:B

Q.69 A debating team consists of 3 boys and 2 girls. Find the number of ways they can sit in a row?
(A) 120 **(B)** 24
(C) 720 **(D)** 12

Ans:A

Q.70 Suppose v is an isolated vertex in a graph, then the degree of v is:
(A) 0 **(B)** 1
(C) 2 **(D)** 3

Ans:A

Q.71 Let p be "He is tall" and let q "He is handsome". Then the statement "It is false that he is short or handsome" is:
(A) $p \wedge q$ **(B)** $\sim(\sim p \vee q)$
(C) $p \vee \sim q$ **(D)** $\sim p \wedge q$

Ans:B

Q.72 The Boolean expression $XY + XY' + X'Z + XZ'$ is independent of the Boolean variable:
(A) Y **(B)** X
(C) Z **(D)** None of these

Ans:A

Q.73 Which of the following regular expression over $\{0,1\}$ denotes the set of all strings not containing 100 as sub string

(A) $0^*(1^*0)^*$. **(B)** 0^*1010^*.
(C) $0^*1^*01^*$. **(D)** $0^*(10+1)^*$.

Ans:D

Q.74 In an undirected graph the number of nodes with odd degree must be
(A) Zero **(B)** Odd
(C) Prime **(D)** Even

Ans:D

Q.75 Find the number of relations from A = {cat, dog, rat} to B = {male , female}
(A) 64 **(B)** 6
(C) 32 **(D)** 15

Ans:A

Q.76 Let P(S) denotes the powerset of set S. Which of the following is always true?
(A) $P(P(S)) = P(S)$ **(B)** $P(S) \cap S = P(S)$
(C) $P(S) \cap P(P(S)) = \{\varnothing\}$ **(D)** $S \in P(S)$

Ans:D

Q.77 The number of functions from an m element set to an n element set is:
(A) m^n **(B)** $m + n$
(C) n^m **(D)** $m * n$

Ans:A

Q.78 Which of the following statement is true:
(A) Every graph is not its own subgraph
(B) The terminal vertex of a graph are of degree two.
(C) A tree with n vertices has n edges.
(D) A single vertex in graph G is a subgraph of G.

Ans:D

Q.79 Which of the following proposition is a tautology?
(A) $(p \vee q) \to p$ **(B)** $p \vee (q \to p)$
(C) $p \vee (p \to q)$ **(D)** $p \to (p \to q)$

Ans:C

Q.80 What is the converse of the following assertion?
I stay only if you go.

(A) I stay if you go. **(B)** If you do not go then I do not stay
(C) If I stay then you go. **(D)** If you do not stay then you go.

Ans:B

Q.81 The length of Hamiltonian Path in a connected graph of n vertices is
(A) n–1 **(B)** n
(C) n+1 **(D)** n/2

Ans:A

Q.82 A graph with one vertex and no edges is:
(A) multigraph **(B)** digraph
(C) isolated graph **(D)** trivial graph

Ans:D

Q 83 If R is a relation "Less Than" from A = {1,2,3,4} to B = {1,3,5} then RoR^{-1} is
(A) {(3,3), (3,4), (3,5)}
(B) {(3,1), (5,1), (3,2), (5,2), (5,3), (5,4)}
(C) {(3,3), (3,5), (5,3), (5,5)}
(D) {(1,3), (1,5), (2,3), (2,5), (3,5), (4,5)}

Ans:C

Q.84 How many different words can be formed out of the letters of the word VARANASI?
(A) 64 **(B)** 120
(C) 40320 **(D)** 720

Ans:D

Q.85 Which of the following statement is the negation of the statement "4 is even or -5 is negative"?
(A) 4 is odd and -5 is not negative **(B)** 4 is even or -5 is not negative
(C) 4 is odd or -5 is not negative **(D)** 4 is even and -5 is not negative

Ans:A

Q.86 A complete graph of n vertices should have __________ edges.
(A) n-1 **(B)** n
(C) n(n-1)/2 **(D)** n(n+1)/2

Ans:C

Q.87 Which one is the contrapositive of $q \rightarrow p$?

(A) $p \rightarrow q$ **(B)** $\neg p \rightarrow \neg q$
(C) $\neg q \rightarrow \neg p$ **(D)** None of these

Ans:B

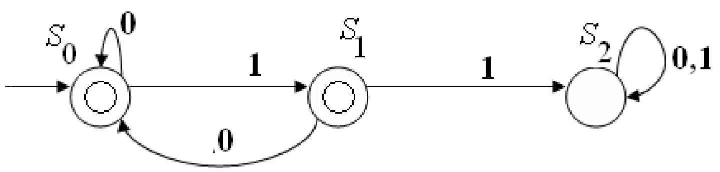
Q.88 A relation that is reflexive, anti-symmetric and transitive is a
(A) function **(B)** equivalence relation
(C) partial order **(D)** None of these

Ans:C

Q.89 A Euler graph is one in which
(A) Only two vertices are of odd degree and rests are even
(B) Only two vertices are of even degree and rests are odd
(C) All the vertices are of odd degree
(D) All the vertices are of even degree

Ans:D

Q.90 What kind of strings is rejected by the following automaton?



(A) All strings with two consecutive zeros
(B) All strings with two consecutive ones
(C) All strings with alternate 1 and 0
(D) None

Ans:B

Q.91 A spanning tree of a graph is one that includes
(A) All the vertices of the graph
(B) All the edges of the graph
(C) Only the vertices of odd degree
(D) Only the vertices of even degree

Ans:A

Q.92 The Boolean expression $A + AB + A\overline{B}$ is independent to
(A) A **(B)** B
(C) Both A and B **(D)** None

Ans:B

Q.93 Seven (distinct) car accidents occurred in a week. What is the probability that they all occurred on the same day?

(A) $1/7^6$ **(B)** $1/2^7$

(C) $1/7^5$ **(D)** $1/7^7$

Ans:A

DESCRIPTIVES

Q.1 Solve the recurrence relation

$$T(k) = 2\,T(k-1), \quad T(0) = 1\,.$$ **(5)**

Ans: The given equation can be written in the following form:

$$t_n - 2t_{n-1} = 0$$

Now successively replacing n by (n – 1) and then by (n – 2) and so on we get a set of equations.

The process is continued till terminating condition. Add these equations in such a way that all intermediate terms get cancelled. The given equation can be rearranged as

$t_n - 2t_{n-1} = 0$
$t_{n-1} - 2t_{n-2} = 0$
$t_{n-2} - 2t_{n-3} = 0$
$t_{n-3} - 2t_{n-4} = 0$
.
$t_2 - 2t_1 = 0$
$t_1 - 2t_0 = 0$ [we stop here, since $t_0 = 1$ is given]

Multiplying all the equations respectively by $2^0, 2^1, \ldots, 2^{n-1}$ and then adding them together, we get $t_n - 2^n t_0 = 0$

or, $t_n = 2^n$

Q.2 Find the general solution of

$$S_r - 4\,S_{r-1} + 4\,S_{r-2} = 2^r + r\,2^r$$ **(9)**

Ans: The general solution, also called **homogeneous solution**, to the problem is given by homogeneous part of the given recurrence equation. The homogeneous part of the given equation is

$$S_r - 4S_{r-1} + 4S_{r-2} = 0 \qquad \text{(1)}$$

The characteristic equation of (2) is given as

$$\begin{aligned} & x^2 - 4x + 4 = 0 \\ or, \quad & (x-2)(x-2) = 0 \\ or, \quad & x = 2, 2 \end{aligned}$$

The general solution is given by

$$S_r = A2^r + Br2^r \qquad \underline{\qquad\qquad\qquad\qquad}(2)$$

Q.3 Find the coefficient of X^{20} in $\left(X^3 + X^4 + X^5 + ...\right)^5$. **(7)**

Ans: The coefficient of x^{20} in the expression $(x^3 + x^4 + x^5 + \ldots)^5$ is equal to the coefficient of x^5 in the expression $(1 + x + x^2 + x^3 + x^4 + x^5 + \ldots)^5$ because x^{15} is common to all the terms in $(x^3 + x^4 + x^5 + \ldots)^5$. Now expression $(1 + x + x^2 + x^3 + x^4 + x^5 + \ldots)^5$ is equal to $\frac{1}{(1-x)^5}$. The required coefficient is

$$^{5+5-1}C_5 = {}^9C_5 = 126$$

Q.4 If $G = (\{S\}, \{0,1\}, \{S \rightarrow OS1, S \rightarrow \in\}, S)$, find L(G). **(7)**

Ans: The language of the grammar can be determined as below:

$S \rightarrow 0\ S\ 1$	[Apply rule $S \rightarrow 0\ S\ 1$]
$\rightarrow 0^n\ S\ 1^n$	[Apply rule $S \rightarrow 0\ S\ 1$ $n \geq 0$ times]
$\rightarrow 0^n\ 1^n$	[Apply rule $S \rightarrow \varepsilon$]

The language contains strings of the form $0^n\ 1^n$ where $n \geq 0$.

Q.5 By using pigeonhole principle, show that if any five numbers from 1 to 8 are chosen, then two of them will add upto 9. **(5)**

Ans: Let us form four groups of two numbers from 1 to 8 such that sum the numbers in a gr is 9. The groups are: (1, 8), (2, 7), (3, 6) and (4,5).
Let us consider these four groups as pigeonholes (m). Thus m = 4. Take the five numbers to be selected arbitrarily as pigeons i.e. n = 5. Take a pigeon and put in the pigeonhole as per its value. After placing 4 pigeons, the 5th has to go in one of the pigeonhole. i.e by pigeonhole principle has at least one group that will contain $\left\lfloor \frac{5-1}{4} \right\rfloor + 1\ numbers$. Thus two of the numbers, out of the five selected, will add up to 9.

Q.6 Test whether 101101, 11111 are accepted by a finite state machine M given as follows : **(9)**

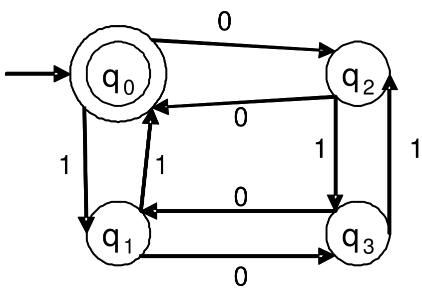
$$M = \{Q = \{q_0, q_1, q_2, q_3\}, \sum = \{0,1\}, \delta, \{q_0\}\}$$

Where

δ is

	Inputs	
States	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Ans: The transition diagram for the given FSM is as below:



The given FSM accepts any string in {0, 1} that contains even number of 0's and even number of 1's. The string "101101" is accepted as it contains even number of 0's and even number of 1's. However string "11111" is not acceptable as it contains odd number of 1's.

Q.7 Let L be a distributive lattice. Show that if there exists an a with
$a \wedge x = a \wedge y$ and $a \vee x = a \vee y$, then $x = y$. **(7)**

Ans: In any distributive lattice L, for any three elements a, x, y of L, we can write

$$\begin{aligned}
x &= x \vee (a \wedge x) & & \\
&= (x \vee a) \wedge (x \vee x) & &\text{[Distributive law]}\\
&= (a \vee x) \wedge x & &\text{[Commutative and Idempotent law]}\\
&= (a \vee y) \wedge x & &\text{[Given that } (a \vee x) = (a \vee y)\text{]}\\
&= (a \wedge y) \vee (y \wedge x) & &\text{[Distributive law]}\\
&= (a \wedge y) \vee (y \wedge x) & &\text{[Given that } (a \wedge x) = (a \wedge y)\text{]}\\
&= (y \wedge a) \vee (y \wedge x) & &\text{[Commutative law]}\\
&= y \wedge (a \vee x) & &\text{[Distributive law]}\\
&= y \wedge (a \vee y) & &\text{[Given that } (a \vee x) = (a \vee y)\text{]}\\
&= y & &
\end{aligned}$$

Q.8 Check the validity of the following argument :-

"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the labour market is not perfect." **(7)**

Ans: Let p: "Labour market is perfect"; q : "Wages of all persons in a particular employment will be equal". Then the given statement can be written as

$$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

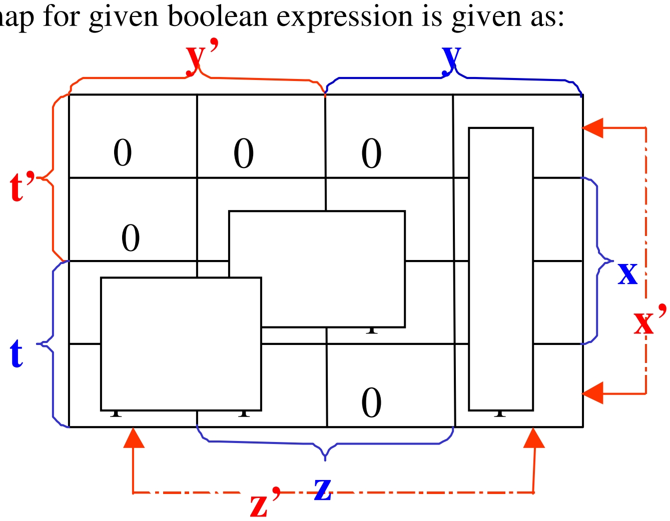
Now,
$$\begin{aligned}
[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p &= [(\sim p \vee q) \wedge \sim q] \rightarrow \sim p\\
&= [(\sim p \wedge \sim q) \vee (q \wedge \sim q)] \rightarrow \sim p\\
&= [(\sim p \wedge \sim q) \vee 0] \rightarrow \sim p\\
&= [\sim (p \vee q)] \rightarrow \sim p\\
&= \sim [\sim (p \vee q)] \vee \sim p\\
&= (p \vee q) \vee \sim p = (p \vee \sim p) \vee q = 1 \vee q = 1
\end{aligned}$$

A tautology. Hence the given statement is true.

Q.9 Let $E = xy + y'\,t + x'\,y\,z' + xy'\,zt'$, find

(i) Prime implicants of E, (ii) Minimal sum for E. **(7)**

Ans: K –map for given boolean expression is given as:



Prime implicant is defined as the minimum term that covers maximum number of min terms. In the K-map prime implicant y't covers four min terms. Similarly yz' and xz are other prime implicants. The minimal sum for E is **[y't + yz' + xz]**.

Q.10 What are the different types of quantifiers? Explain in brief.
Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$ **(7)**

Ans: There are two quantifiers used in predicate calculus: Universal and existential.

Universal Quantifier: The operator, represented by the symbol $\forall$, used in predicate calculus to indicate that a predicate is true for all members of a specified set.

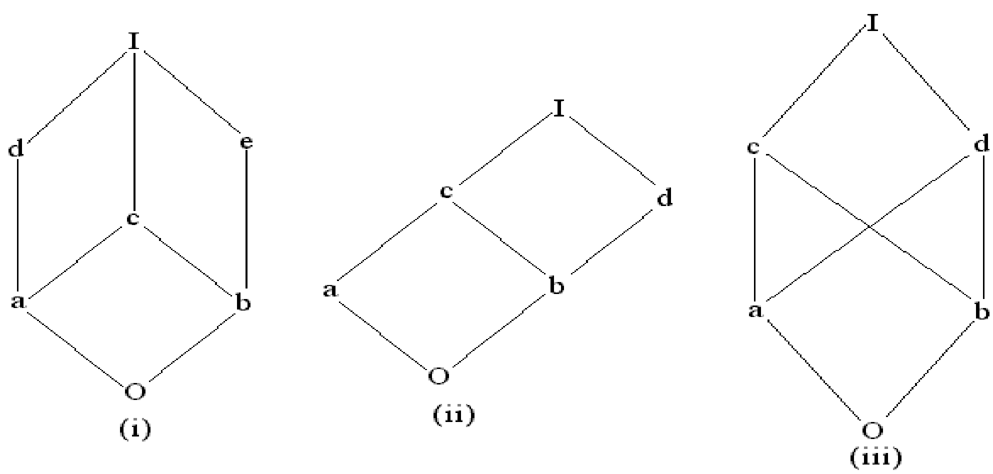
Some verbal equivalents are *"for each"* or *"for every"*.

Existential Quantifier: The operator, represented by the symbol $\exists$, used in predicate calculus to indicate that a predicate is true for at least one member of a specified set.

Some verbal equivalents are *"there exists"* or *"there is"*.

Now, $(\exists x)\ (P(x) \wedge Q(x)) \Rightarrow$ For some $(x = c)\ (P(c) \wedge Q(c))$
$\Rightarrow P(c) \wedge Q(c)$
$\Rightarrow \exists x\ P(x) \wedge \exists x\ Q(x)$

Q.11 Which of the partially ordered sets in figures (i), (ii) and (iii) are lattices? Justify your answer. **(7)**



(i) (ii) (iii)

Ans: Let (L, ≤) be a poset. If every subset {x, y} containing any two elements of L, has a **glb** (Infimum) and a **lub** (Supremum), then the poset (L, ≤) is called a **lattice**. The poset in (i) is a lattice because for every pair of elements in it, there a glb and lub in the poset. Similarly poset in (ii) is also a lattice. Poset of (iii) is also a lattice.

Q.12 Consider the following productions :

$S \rightarrow aB \mid bA$

$A \rightarrow aS \mid bAA \mid a$

$B \rightarrow bS \mid aBB \mid b$

Then, for the string a a a b b a b b b a, find the

(i) the leftmost derivation.
(ii) the rightmost derivation. **(7)**

Ans:
(i) In the left most derivation, the left most nonterminal symbol is first replaced with some terminal symbols as per the production rules. The nonterminal appearing at the right is taken next and so on. Using the concept, the left most derivation for the string aaabbabbba is given below:

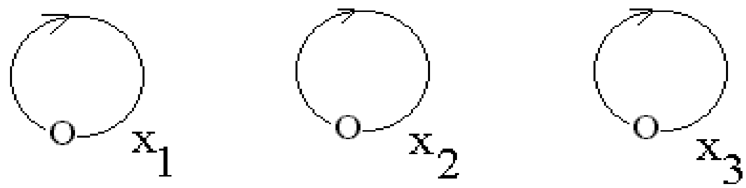
$S \rightarrow aB$	[Apply rule $S \rightarrow aB$]
$\rightarrow aaBB$	[Apply rule $B \rightarrow aBB$]
$\rightarrow aaaBBB$	[Apply rule $B \rightarrow aBB$]
$\rightarrow aaabBB$	[Apply rule $B \rightarrow b$]
$\rightarrow aaabbB$	[Apply rule $B \rightarrow b$]
$\rightarrow aaabbaBB$	[Apply rule $B \rightarrow aBB$]
$\rightarrow aaabbabB$	[Apply rule $B \rightarrow b$]
$\rightarrow aaabbabbS$	[Apply rule $B \rightarrow bS$]
$\rightarrow aaabbabbbA$	[Apply rule $S \rightarrow bA$]
$\rightarrow a\ a\ a\ b\ b\ a\ b\ b\ b\ a$	[Apply rule $A \rightarrow a$]

(ii) In the right most derivation, the right most nonterminal symbol is first replaced with some terminal symbols as per the production rules. The nonterminal appearing at the left is taken next and so on. Using the concept, the right most derivation for the string aaabbabbba is given below:

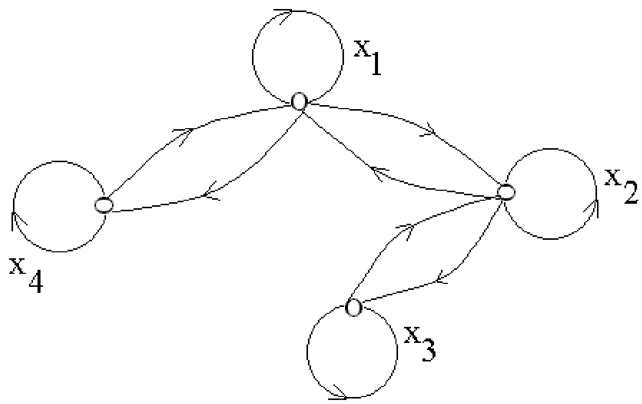
$S \rightarrow aB$	[Apply rule $S \rightarrow aB$]
$\rightarrow aaBB$	[Apply rule $B \rightarrow aBB$]
$\rightarrow aaBbS$	[Apply rule $B \rightarrow bS$]
$\rightarrow aaBbbA$	[Apply rule $S \rightarrow bA$]
$\rightarrow aaBbba$	[Apply rule $A \rightarrow a$]
$\rightarrow aaaBBbba$	[Apply rule $B \rightarrow aBB$]
$\rightarrow aaa\ B\ bbba$	[Apply rule $B \rightarrow b$]
$\rightarrow aaa\ bS\ bbba$	[Apply rule $B \rightarrow bS$]
$\rightarrow aaab\ bA\ bbba$	[Apply rule $S \rightarrow bA$]
$\rightarrow a\ a\ a\ b\ b\ a\ b\ b\ b\ a$	[Apply rule $A \rightarrow a$]

Q.13 Determine the properties of the relations given by the graphs shown in fig.(i) and (ii) and also write the corresponding relation matrices. **(7)**

(i)



(ii)



Ans8. (a) (i) The relation matrix for the relation is : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

The relation is **reflexive, Symmetric and transitive**. It is in fact **antisymmetric** as well.

The relation matrix for the relation is : $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.

The relation is **reflexive, Symmetric** but not transitive.

Q.14 If f is a homomorphism from a commutative semigroup (S, *) onto a semigroup $(T, *')$, then show that $(T, *')$ is also commutative. **(7)**

Ans: The function f is said to be a ***homomorphism*** of S ***into*** T if

$$f(a * b) = f(a) *' f(b) \ \forall\ a, b \in S$$

i.e. f preserves the composition in S and T.
It is given that (S, *) is commutative, so

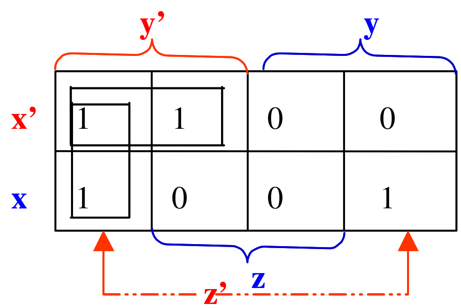
$$a*b = b*a \Rightarrow f(a*b) = f(b*a) \qquad \forall\ a, b \in S$$

$\Rightarrow$ f(a) *' f(b) = f(b)*' f(a)
$\Rightarrow$ (T,*') is also commutative.

Q.15 Construct K-maps and give the minimum DNF for the function whose truth table is shown below : **(9)**

x	y	z	f(x, y, z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Ans: The K-map for the given function (as per the truth table) is produced below.



The three duets are: y'z', x'y' and xz'. The **DNF (Disjunctive normal form)** is the Boolean expression for a Boolean function in which function is expressed as disjoint of the min terms for which output of function is 1. The minimum DNF is the sum of prime implicant. Thus the minimum DNF for the given function is

y'z'+ x'y' + xz'

Q.16 Define tautology and contradiction. Show that "If the sky is cloudy then it will rain and it will not rain", is not a contradiction. **(5)**

Ans: If a compound proposition has two atomic propositions as components, then the truth table for the compound proposition contains four entries. These four entries may be all T, may be all F, may be one T and three F and so on. There are in total 16 (2^4) possibilities. The possibilities when all entries in the truth table is T, implies that the compound proposition is always true. This is called **tautology.**
However when all the entries are F, it implies that the proposition is **never true**. This situation is referred as **contradiction.**

Let p: "Sky is cloudy"; q : "It will rain".
Then the given statement can be written as "$(p\rightarrow q) \wedge \sim q$". The truth table for the expression is as below:

p	q	$p\rightarrow q$	$(p\rightarrow q) \wedge \sim q$
0	0	1	1
0	1	1	0
1	0	0	0
1	1	1	0

Obviously**,** $(p\rightarrow q) \wedge \sim q$ is not a contradiction.

Q.17 How many words of 4 letters can be formed with the letters a, b, c, d, e, f, g and h, when
(i) e and f are not to be included and
(ii) e and f are to be included. **(7)**

Ans: There are 8 letters: a, b, c, d, e, f, g and h. In order to form a word of 4 letters, we have to find the number of ways in which 4 letters can be selected from 8 letters. Here order is important, so permutation is used.

(i) When **e** and **f** are **not** to be included, the available letters are 6 only. Thus we can select 4 out of six to form a word of length 4 in ${}^{6}P_{4}$ ways = $\frac{6!}{(6-4)!} = 360$. Thus 360 words can be formed.

(ii) When **e** and **f** are to be included, the available letters are all 8 letters. Thus we can select 4 out of 8 to form a word of length 4 in ${}^{8}P_{4}$ ways = $\frac{8!}{(8-4)!} = 1680$. Thus 1680 words can be formed.

Q.18 Explain chomsky classification of languages with suitable examples. **(7)**

Ans: Any language is suitable for communication provided the syntax and semantic of the language is known to the participating sides. It is made possible by forcing a standard on the way to make sentences from words of that language. This standard is forced through a set of rules. This set of rules is called grammar of the language. According to the **Chomsky** classification, a grammar $G = (N, \Sigma, P, S)$ is said to be of

Type 0: if there is no restriction on the production rules i.e., in $\alpha\rightarrow\beta$, where $\alpha, \beta \in (N \cup \Sigma)^*$. This type of grammar is also called an ***unrestricted grammar*** and language is called free language.

Type 1: if in every production $\alpha \to \beta$ of P, $\alpha, \beta \in (N \cup \Sigma)^*$ and $|\alpha| \le |\beta|$. Here $|\alpha|$ and $|\beta|$ represent number of symbols in string α and β respectively. This type of grammar is also called a ***context sensitive*** grammar (or ***CSG***) and language is called ***context sensitive.***

Type 2: if in every production $\alpha \to \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \Sigma)^*$. Here α is a single non-terminal symbol. This type of grammar is also called a ***context free*** grammar (or ***CFG***) and language is called ***context free.***

Type 3: if in every production $\alpha \to \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \Sigma)^*$. Here α is a single non-terminal symbol and β may consist of at the most one non-terminal symbol and one or more terminal symbols. The non-terminal symbol appearing in β must be the extreme right symbol. This type of grammar is also called a ***right linear*** grammar or ***regular*** grammar (or ***RG***) and the corresponding language is called regular language.

Q.19 Find a generating function to count the number of integral solutions to $e_1 + e_2 + e_3 = 10$, if for each I, $0 \le e_i$. **(7)**

Ans: According to the given constraints polynomial for variable e_1 can be written as

$1 + x + x^2 + x^3 + \ldots$ since $0 \le e_1$
Similarly polynomial for variable e_2 and e_3 can be written as
$1 + x + x^2 + x^3 + \ldots$ since $0 \le e_2$
$1 + x + x^2 + x^3 + \ldots$ since $0 \le e_3$
respectively.

Thus the number of integral solutions to the given equations under the constraints is equal to the coefficient of x^{10} in the expression

$$(1 + x + x^2 + x^3 + \ldots)(1 + x + x^2 + x^3 + \ldots)(1 + x + x^2 + x^3 + \ldots)$$

Thus the generating function $f(x) = \dfrac{1}{(1-x)^3}$.

Q.20. Prove by elimination of cases : if X is a number such that

$X^2 - 5X + 6 = 0$ then X = 3 or X = 2. **(7)**

Ans: Let a and b be two roots of the equation, then

$a + b = 5$ and $a * b = 6$

Now, $(a - b)^2 = (a + b)^2 - 4a*b$
$= 25 - 24 = 1$

∴ $a - b = 1$ or -1

Now using the method of elimination (adding) the two equation $a + b = 5$ and $a - b = 1$, we get,

$2a = 6$, or $a = 3$. Using this value in any of the equation, we get $b = 2$.

It is easy to verify that, if $a - b = -1$ is taken then we get $a = 2$ and $b = 3$. Thus two root of the given equations are $X = 3$ or $X = 2$.

Q.21 Consider the following open propositions over the universe $U = \{-4, -2, 0, 1, 3, 5, 6, 8, 10\}$

$P(x): x \geq 4$

$Q(x): x^2 = 25$

R(x) : s is a multiple of 2

Find the truth values of

(i) $P(x) \wedge R(x)$

(ii) $[\sim Q(x)] \wedge P(x)$ **(8)**

Ans: The truth values for each element of U under the given conditions of propositions is given in the following table:

x	P(x)	Q(x)	R(x)	$P(x) \wedge R(x)$	$\sim Q(x) \wedge P(x)$
-4	0	0	1	0	0
-2	0	0	1	0	0
0	0	0	1	0	0
1	0	0	0	0	0
3	0	0	0	0	0
5	1	1	0	0	0
6	1	0	1	1	1
8	1	0	1	1	1
10	1	0	1	1	1

Q.22 Let $n\zeta_r = (n+1).nP_{(r-1)}$, then compute $(n+1)\zeta_{(r-1)}, 7\zeta_4$. **(6)**

Ans: It is given that ${}^n\zeta_r$ is defined as $(n + 1)\, {}^nP_{r-1}$. Then we can write

$^{(n+1)}\zeta_{(r-1)} = (n + 2)\ {}^{n+1}P_{r-2} = \dfrac{(n+2)(n+1)!}{(n-r+1)!}$, and

${}^7\zeta_4 = 8 * {}^7P_3 = \dfrac{8*7!}{4!} = 1680$

Q.23 Translate and prove the following in terms of propositional logic

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$ **(5)**

Ans: Let SUB(A, B) stands for "A is subset of B" and EQL(A, B) stands for "A is equal to B". Then the given statement can be written as

$$\forall A, B\ [SUB(A, B) \wedge SUB(B, A)] \Leftrightarrow EQL(A, B)$$

Q.24 Determine the number of integral solutions of the equation

$x_1 + x_2 + x_3 + x_4 = 20$

where **(9)**

$2 \le x_1 \le 6,\ 3 \le x_2 \le 7,\ 5 \le x_3 \le 8,\ 2 \le x_4 \le 9$

Ans: According to the given constraints polynomial for variable x_1 can be written as

$x^2 + x^3 + x^4 + x^5 + x^6$ since $2 \le x_1 \le 6$

Similarly polynomial for variable x_2, x_3 and x_4 can be written as

$x^3 + x^4 + x^5 + x^6 + x^7$ since $3 \le x_2 \le 7$

$x^5 + x^6 + x^7 + x^8$ since $5 \le x_3 \le 8$

$x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9$ since $2 \le x_4 \le 9$

respectively.

Thus the number of integral solutions to the given equations under the constraints is equal to the coefficient of x^{20} in the expression

$(x^2 + x^3 + x^4 + x^5 + x^6)(x^3 + x^4 + x^5 + x^6 + x^7)(x^5 + x^6 + x^7 + x^8)(x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9)$

$= x^{12} (x^5 - 1)^2 (x^4 - 1)^2 (x^4 + 1) (x - 1)^{-4}$

$= x^{12} [1 - x^4 - 2x^5 - x^8 + 2x^9 + x^{10} + x^{12} + 2x^{13} - x^{14} - 2x^{17} - x^{18} + x^{22}] (1 - x)^{-4}$

In the coefficient of x^{20}, the terms that can contribute are from $(1 - x^4 - 2x^5 - x^8)$ only. All other terms when multiplied with x^{20}, yields higher degree terms. Therefore coefficients of x^{20} in the entire expression are equal to linear combinations of coefficients of x^8, x^4, x^3 and 1 in $(1 - x)^{-4}$ with respective constant coefficients 1, -1, -2, -1. This is equal to

$$^{11}C_8 - {}^7C_4 - 2\,{}^6C_3 - 1 = 165 - 35 - 40 - 1 = 89$$

Therefore number of integral solution is 89.

Q.25 Using BNF notation construct a grammar G to generate the language over an alphabet $\{0,1\}$ with an equal number of 0's and 1's for some integer $n \ge 0$. **(7)**

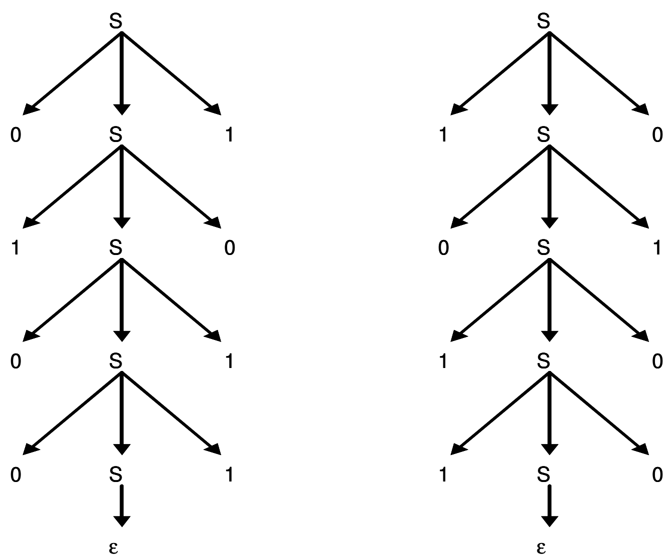
Ans: The given language is defined over the alphabet $\Sigma = \{1, 0\}$. A string in this language contains equal number of 0 and 1. Strings 0011, 0101, 1100, 1010, 1001 and 0110 are all valid strings of length 4 in this language. Here, order of appearance of 0's and 1's is not fixed. Let S be the start symbol. A string may start with either 0 or 1. So, we should have productions S $\rightarrow$

0S1, S → 1S0, S → 0A0 and S → 1B1 to implement this. Next A and B should be replaced such a way that it maintains the count of 0's and 1's to be equal in a string. This is achieved by the productions A → 1S1 and B → 0S0. The **null** string is also an acceptable string, as it contains **zero** number of 0's and 1's. Thus S → ε should also be a production. In addition to that a few Context sensitive production rules are to be included.

These are: A → 0AD0, 0D1 → 011, 0D0 → 00D, B→ 1BC1, 1C0 → 100, 1C1 → 11C. Therefore, the required grammar G can be given by (N, ∑, P, S), where N = {S, A, B}, ∑ = (1, 0} and P = {S → 0S1, S → 1S0, S → 0A0, S → 1B1, A → 1S1, B → 0S0, A → 0AD0, 0D1 → 011, 0D0 → 00D, B→ 1BC1, 1C0 → 100, 1C1 → 11C, S → ε}.

Q.26 Construct the derivation trees for the strings 01001101 and 10110010 for the above grammar. **(7)**

Ans: The derivation tree for the given string under the presented grammar is as below.



Q.27 Find the generating function to represent the number of ways the sum 9 can be obtained when 2 distinguishable fair dice are tossed and the first shows an even number and the second shows an odd number. **(5)**

Ans: Let x_1 and x_2 be the variable representing the outcomes on the two dice the x_1 has values 2, 4, 6 and x_2 has 1, 3, 5. Now polynomial for variable x_1 and x_2 can be written as $(x^2 + x^4 + x^6)$ and $(x + x^3 + x^5)$ respectively.

Thus the number of ways sum 9 can be obtained is equal to the coefficient of x^9 in the expression $(x^2 + x^4 + x^6)(x + x^3 + x^5)$. This is equal to 2.

Q.28 Prove that if ℓ_1 and ℓ_2 are elements of a lattice $< L; \vee, \wedge >$ then

$$(\ell_1 \vee \ell_2 = \ell_1) \leftrightarrow (\ell_1 \wedge \ell_2 = \ell_2) \leftrightarrow (\ell_2 \leq \ell_1)$$ **(9)**

Ans: Let us use b = L_1 and a = L_2. It is given that a∨b = b. Since a∨b is an upper bound of a, a ≤ a∨b. This implies a ≤ b.

Next, let a ≤ b. Since ≤ is a partial order relation, b ≤ b. Thus, a≤b and b≤b together implies that b is an upper bound of a and b. We know that a∨b is least upper bound of a and b, so a∨b ≤ b. Also b ≤ a∨b because a∨b is an upper bound of b. Therefore, a∨b ≤ b and b ≤ a∨b ⇔ a∨b = b by the anti-symmetry property of partial order relation ≤. Hence, it is proved that a∨b = b if and only if a ≤ b.

Q.29 Let $(S,*)$ be a semigroup. Consider a finite state machine M = (S, S F), where $F = \{fx \mid x \in S\}$ and fx (y) = x * y for all x, y ∈ S Consider the relation R on S, x R y if and only if there is some z ∈ S such that fz (x) = y. Show that R is an equivalence relation. **(14)**

Ans: Let M = (S, I, $\Im$, s_0, T) be a FSM where S is the set of states, I is set of alphabets, $\Im$ is transition table, s_0 is initial state and T is the set of final states. Let T^C = S – T. Then set S is decomposed into two disjoint subsets T and T^C. Set T contains all the terminal (acceptance) states of the M and T^C contains all the non-terminal states of M. Let w ∈ I* be any string. For any state s ∈ S, $f_w(s)$ belongs to either T or T^C. A*ny two states: s and t are said to be* ***w-compatible*** *if both* $f_w(s)$ *and* $f_w(t)$ *either belongs to T or to* T^C *for all w∈ I*.*

Now we have to show that if R be a relation defined on S as sRt if and only if both s and t are w-compatible the R is an equivalence relation.

Let s be any state in S and w be any string in I*. Then $f_w(s)$ is either in T or in T^C. This implies that sRs ∀ s∈ S. Hence R is reflexive on S. Let s and t be any two states in S such that sRt. Therefore, by definition of R both s and t w-compatible states. This implies that tRs. Thus, R is symmetric also. Finally, let s, t and u be any three states in S such that sRt and tRu. This implies that s t and u are w-compatible states i.e. s and u are w-compatible. Hence sRu. Thus, R is a transitive relation. Since R is reflexive, symmetric and transitive, it is an equivalence relation.

Q.30 Represent each of the following statements into predicate calculus forms :

(i) Not all birds can fly.
(ii) If x is a man, then x is a giant.
(iii) Some men are not giants.
(iv) There is a student who likes mathematics but not history. **(9)**

Ans:
(i) Let us assume the following predicates

bird(x): "x is bird"
fly(x): "x can fly".
Then given statement can be written as: $\exists x \text{ bird}(x) \wedge \sim \text{fly}(x)$

(ii) Let us assume the following predicates
man(x): "x is Man"
giant(x): "x is giant".

Then given statement can be written as: $\forall x\ (\text{man}(x) \rightarrow \text{giant}(x))$

(iii) Using the predicates defined in (ii), we can write
$\exists x \text{ man}(x) \wedge \sim \text{giant}(x)$

(iv) Let us assume the following predicates
student(x): "x is student."
likes(x, y): "x likes y". and ~likes(x, y) $\Rightarrow$ "x does not like y".

Then given statement can be written as:
$\exists x\ [\text{student}(x) \wedge \text{likes}(x, \text{mathematics}) \wedge \sim \text{likes}(x, \text{history})]$

Q.31 Let X be the set of all programs of a given programming language. Let R the relation on X be defined as
$P_1 \; R \; P_2$ if P_1 and P_2 give the same output on all the inputs for which they terminate.
Is R an equivalence relation? If not which property fails? **(5)**

Ans: R is defined on the set X of all programs of a given programming language. Now let us test whether R is an equivalence relation or not

Reflexivity: Let P be any element of X then obviously P gives same output on all the inputs for which P terminates i.e. P R P. Thus R is reflexive.

Symmetry: Let P and Q are any two elements in X such that P R Q. Then P and Q give same output on all the inputs for which they terminate => Q R P =>R is symmetric.

Transitivity: Let P, Q and S are any three elements in X such that P R Q and Q R S. It means P and Q gives the same out for all the inputs for which they terminate. Similar is the case for Q and S. It implies that there exists a set of common programs from which P, Q and S give the same output on all the inputs for which they terminate i.e. P R S => R is transitive.
Therefore R is an equivalence relation.

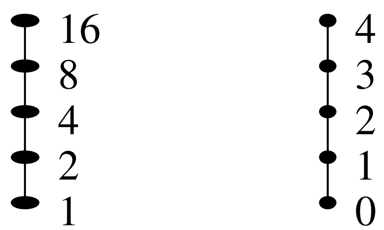
Q.32 Let $A = \{1, 2, 4, 8, 16\}$ and relation R_1 be partial order of divisibility on A. Let $A' = \{0, 1, 2, 3, 4\}$ and R_2 be the relation "less than or equal to" on integers. Show that (A, R_1) and (A', R_2) are isomorphic posets. **(7)**

Ans: Any two posets (A, R_1) and (A', R_2) are said to isomorphic to each other if there exists a function $f: A \rightarrow A'$ such that

- f is one to one and onto function; and
- For any $a, b \in A$, $a\, R_1\, b \Leftrightarrow f(a)\, R_2\, f(b)$, i.e. images preserve the order of pre-images.

Let $f: A \rightarrow A'$ is defined as $f(1) = 0$, $f(2) = 1$, $f(4) = 2$, $f(8) = 3$, $f(16) = 4$.

It is obvious from the definition that f is one to one and onto. Regarding preservation of order, it is preserved under the respective partial order relation. It is shown in the following hasse diagram.



Q.33 Let ∇ be defined as $R\nabla S = (R - S) \cup (S - R)$, then either prove or give counter example for the following,

$R \subseteq S \Rightarrow R \nabla T \subseteq S \nabla T$ **(7)**

Ans: Let R = {1, 2, 3, 4}; S = {1, 2, 3, 4, 5, 6} and T = {3, 6, 7}. Obviously $R \subseteq S$. However $R\nabla$ T = {1, 2, 4, 6, 7}and S ∇ T = {1, 2, 4, 5, 7}. Clearly, $(R\nabla T) \not\subseteq (S \nabla T)$. Hence it is disproved.

Q.34 Prove or disprove the following equivalence,

$\sim (p \leftrightarrow q) \equiv ((p \wedge \sim q) \vee (q \wedge \sim p))$ **(9)**

Ans: RHS of the given expression $((p \wedge \sim q) \vee (q \wedge \sim p))$ can be written as

$$
\begin{aligned}
\sim(\sim((p \wedge \sim q) \vee (q \wedge \sim p))) &= \sim((\sim(p \wedge \sim q)) \wedge (\sim(q \wedge \sim p))) \\
&= \sim((\sim p \vee q) \wedge (\sim q \vee p)) \\
&= \sim((p \rightarrow q) \wedge (q \rightarrow p)) \\
&= \sim(p \leftrightarrow q)
\end{aligned}
$$

Q.35 Solve the following recurrence relation and indicate if it is a linear homogeneous relation or not. If yes, give its degree and if not justify your answer. $t_n = t_{n-1} + n,\ t_1 = 4$ **(5)**

Ans: The given equation can be written in the following form:

$$t_n - t_{n-1} = n$$

Now successively replacing n by (n – 1) and then by (n – 2) and so on we get a set of equation. The process is continued till terminating condition. Add these equations in such a way that all intermediate terms get cancelled. The given equation can be rearranged as

$$t_n - t_{n-1} = n$$
$$t_{n-1} - t_{n-2} = n - 1$$
$$t_{n-2} - t_{n-3} = n - 2.$$
$$t_{n-3} - t_{n-4} = n - 3$$
.
$$t_2 - t_1 = 2 \qquad \text{[we stop here, since } t_1 = 4 \text{ is given]}$$

Adding all, we get $t_n - t_1 = [n + (n - 1) + (n - 2) + ... + 2]$

or, $t_n = [n + (n - 1) + (n - 2) + ... + 2 + 4]$

or, $t_n = \dfrac{n(n+1)}{2} + 3$

The given recurrence equation is linear and non homogeneous. It is of degree 1.

Q.36 State the relation between Regular Expression, Transition Diagram and Finite State Machines. Using a simple example establish your claim. **(6)**

Ans: For every regular language, we can find a FSM for this, because a FSM work as recognizer of a regular language. Whether a language is regular can be determined by finding whether a FSM can be drawn or not.

A machine contains finite number of states. To determine whether a string is valid or not, the FSM begins scanning the string from start state and moving along the transition diagram as per the current input symbol. The table showing the movement from one state to other for input symbols of the alphabets is called **transition table**.

Every machine has a unique transition table. Thus there is one to one relationship between Regular language and FSM and then between FSM and transition table.

Q.37 Identify the type and the name of the grammar the following set of production rules belong to. Also identify the language it can generate. **(8)**

$$S_1 \to S_1S_2$$
$$S_1 \to 1S_1$$
$$S_1 \to e$$
$$S_2 \to S_2S_3$$
$$S_2 \to 01S_2$$
$$S_2 \to 01$$
$$S_3 \to 0S_3$$
$$S_3 \to e$$

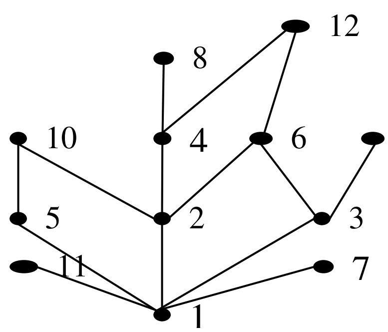
Ans: In the left hand side of the production rules, only single non terminal symbol is use. Th implies that the grammar is either of Type 2 or Type 3. In the RHS of some production rules, there are more than one non terminal. The null production can be removed by combining it with the other productions. Thus, according to the Chomsky hierarchy, the grammar is of type 2, and name of the grammar is **Context Free Grammar(CFG).** The language of the grammar can be determined as below:

$S_1 \to S_1S_2$
$\to S_1S_2S_3$ [Apply rule $S_2 \to S_2S_3$]
$\to S_1S_2 0^m S_3$ [Apply rule $S_3 \to 0S_3$ ($m \geq 0$) times]
$\to S_1S_2 0^m$ [Apply rule $S_3 \to e$]
$\to S_1 01S_2 0^m$ [Apply rule $S_2 \to 01S_2$]
$\to S_1 (01)^l (0^m)^k$ [Apply rule $S_2 \to S_2S_3$ and $S_2 \to 01S_2$ ($k \geq 0$, $l \geq 1$) times and then $S_2 \to 01$ once]
$\to 1^n [(01)^l (0^m)^k]^t$ [Apply rule $S_1 \to S_1S_2$, ($t \geq 0$) times, $S_1 \to 1S_1$ ($n \geq 0$) time and $S_2 \to e$ once]

That the language contains strings of the form $1^n [(01)^l (0^m)^k]^t$ where $l \geq 1$ and other values like n, m, k and t are ≥ 0.

Q.38 For any positive integer n, let $I_n = \{x | 1 \leq x \leq n\}$. Let the relation "divides" be written as $a | b$ **iff a divides b** or b = ac for some integer c. Draw the Hasse diagram and determine whether $[I_{12} ; |]$ is a lattice. **(8)**

Ans: Hasse diagram for the structure (I_{12}, |) is given below.



It is obvious from the Hasse diagram that there are many pairs of element, e.g (7, 11), (8,

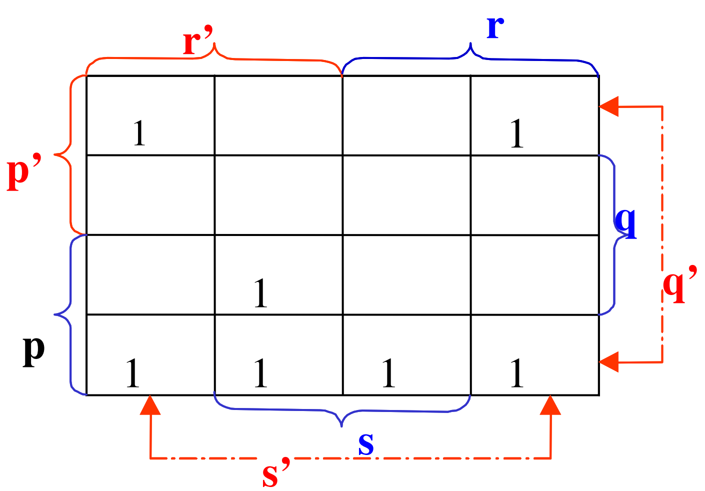
etc that do not have any join in the set. Hence (I_{12}, |) is not a lattice.

Q.39 Give the formula and the Karnaugh map for the minimum DNF, (Disjunctive Normal Form)

$$(p\wedge \sim q)\vee(\sim q\wedge \sim s)\vee(p\wedge \sim r\wedge s)$$ **(6)**

Ans: The K-map for the given DNF is shown below. The formula for the same is

pq' + s'q' + pr's



Q.40 If the product of two integers a and b is even then prove that either a is even or b is even. **(5)**

Ans: It is given that product of a and b is even so let a * b = 2n. Since 2 is a prime number and n is any integer (odd or even), either a or b is a multiple of 2. This shows that either a is even or b is even.

Q.41 Using proof by contradiction, prove that "$\sqrt{2}$ is irrational". Are irrational numbers countable? **(5)**

Ans:

$$Let \quad \sqrt{2} = s$$
$$So \quad s^2 = 2 \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)$$

If s were a rational number, we can write

$$s = \frac{p}{q} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(2)$$

Where p is an integer and q is any positive integer. We can further assure that the **gcd**(p, q) =1 by driving out common factor between p and q, if any. Now using equation (2), in equation (1), we get

$$p^2 = 2q^2 \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)$$

Conclusion: From equation (3), **2** must be a prime factor of $\mathbf{p^2}$. Since **2** itself is a prime, 2 must be a factor of **p** itself. Therefore, 2∗2 is a factor of $\mathbf{p^2}$. This implies that 2∗2 is a factor of 2∗$\mathbf{q^2}$.

⇒ 2 is a factor of $\mathbf{q^2}$
⇒ 2 is a factor of **q**

Since, 2 is factor of both **p** and **q** **gcd**(p, q) = 2. This is contrary to our assumption that **gcd**(p, q) = 1. Therefore, √2 is an irrational number. The set of irrational numbers is uncountable.

Q.42 Use quantifiers and predicates to express the fact that $\lim_{x \to a} f(x)$ does not exist. **(4)**

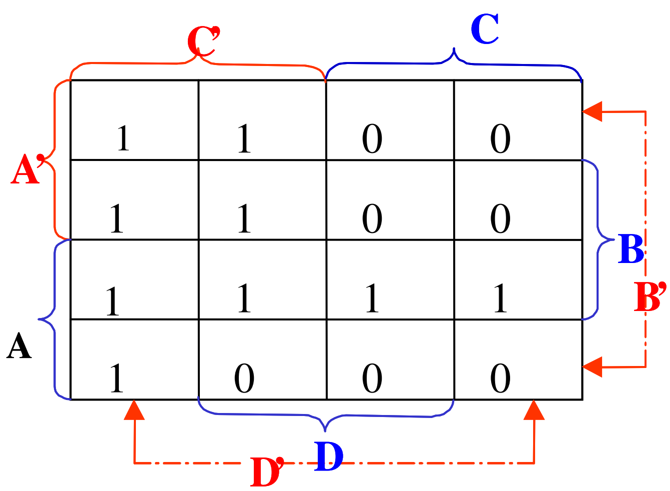
Ans: Let P(x, a) : $\lim\limits_{x \to a} f(x)\ exist.$ Then the given limit can be written as

∃a ¬P(x, a).

Q.43 Minimize the following Boolean expression using k-map method

$$F(A, B, C, D) = \pi\,(0, 1, 4, 5, 8, 12, 13, 14, 15) \qquad \textbf{(7)}$$

Ans:



The minimized boolean expression C'D' + A'C' + AB

Q.44 Find a recurrence relation for the number of ways to arrange flags on a flagpole n feet tall using 4 types of flags: red flags 2 feet high, white, blue and yellow flags each 1 foot high. **(7)**

Ans: Let A_n be the number of ways in which the given flags can be arranged on n feel tall flagpole. The red color flag is of height 2 feet, and white, blue and yellow color flag is of height 1 foot. Thus if n =1, flag can be arranged in 3 ways. If n = 2, the flags can be arranged in either 3x3 ways using 3 flags of 1 foot long each or in 1 way using 2 feet long red flag. Thus, flags on 2 feet long flagpole can be arranged in 10 ways. This can be written in recursive form as:

$A_n = 3A_{n-1}$; $A_n = 10A_{n-2}$; Adding them together, we have

$2A_n - 3A_{n-1} - 10A_{n-2} = 0$ and $A_1 = 3$; $A_2 = 10$.

Q.45 What is the principle of inclusion and exclusion? Determine the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. **(7)**

Ans:
This principle is based on the cardinality of finite sets. Let us begin with the given problem of finding all integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 or 7. Let A, B, C and D be two sets defined as:

$A = \{x \mid x \text{ is divisible by 2 and } 1 \le x \le 250\}$,

$B = \{x \mid x \text{ is divisible by 3 and } 1 \le x \le 250\}$,

$C = \{x \mid x \text{ is divisible by 5 and } 1 \le x \le 250\}$ and
$D = \{x \mid x \text{ is divisible by 7 and } 1 \le x \le 250\}$

Similarly, $A \cap B = \{x \mid x \text{ is divisible by 2 and 3 and } 1 \le x \le 250\}$

.

.

C ∩ D = {x | x is divisible by 5 and 7 and 1 ≤ x ≤ 250}
A∩B∩D = {x | x is divisible by 2, 3 and 7 and 1 ≤ x ≤ 250}

Here, problem is to find the number of elements in A∪B∪C∪D. Also, there are numbers that are divisible by two or three or all the four numbers taken together. A ∩ B, A ∩C, A ∩ D, B ∩ C, …, C ∩ D are not null set. This implies that while counting the numbers, whenever common elements have been included more than once, it has to be excluded. Therefore, we can write

| A ∪ B ∪ C ∪ D | = |A| + |B| + |C| + |D| – |A∩B| – |A∩C| – |A∩D| – |B∩C| – |B∩D| – |C ∩ D| + |A∩B∩C| + |A∩B∩D| + |A∩C∩D| + |B∩C∩D| - |A∩B∩C∩D|

The required result can be computed as below:

$$|A| = 125 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{250-2}{2} + 1 = 125\right]$$

$$|B| = 83 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{249-3}{3} + 1 = 83\right]$$

$$|C| = 50 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{250-5}{5} + 1 = 50\right]$$

$$|D| = 35 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{245-7}{7} + 1 = 35\right]$$

$$|\mathrm{A} \cap B| = 41 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{246-6}{6} + 1 = 41\right]$$

$$|\mathrm{A} \cap C| = 25 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{250-10}{10} + 1 = 25\right]$$

$$|\mathrm{A} \cap D| = 17 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{238-14}{14} + 1 = 17\right]$$

$$|\mathrm{B} \cap C| = 16 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{240-15}{15} + 1 = 16\right]$$

$$|\mathrm{B} \cap D| = 11 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{231-21}{21} + 1 = 11\right]$$

$$|\mathrm{C} \cap D| = 7 \qquad \left[n = \frac{l-a}{d} + 1 = \frac{245-35}{35} + 1 = 7\right]$$

$|A \cap B \cap C| = 8 \quad \left[\frac{240-30}{30} + 1 = 8\right]$

$|A \cap B \cap D| = 5 \quad \left[\frac{210-42}{42} + 1 = 5\right]$

$|A \cap C \cap D| = 3 \quad \left[\frac{210-70}{70} + 1 = 3\right]$

$|B \cap C \cap D| = 2 \quad \left[\frac{210-105}{105} + 1 = 2\right]$

$|A \cap B \cap C \cap D| = 1 \quad \left[\frac{210-210}{210} + 1 = 1\right]$

Therefore,

$|A \cup B \cup C \cup D| = 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7 + 8 + 5 + 3 + 2 - 1$
$= 311 - 118$
$= 193.$

Q.46 Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$. **(7)**

Ans: Let us find the truth table for $[(P\vee Q)\wedge(P\to R)\wedge(Q\to S)] \to (S\vee R)$.

P	Q	R	S	$(P\vee Q)$	$(P\to R)$	$(Q\to S)$	$(S\vee R)$	$[(P\vee Q)\wedge(P\to R)\wedge(Q\to S)] \to (S\vee R)$
0	0	0	0	0	1	1	0	1
0	0	0	1	0	1	1	1	1
0	0	1	0	0	1	1	1	1
0	0	1	1	0	1	1	1	1
0	1	0	0	1	1	0	0	1
0	1	0	1	1	1	1	1	1
0	1	1	0	1	1	0	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	0	1
1	0	0	1	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	1	0	0	0	1
1	1	0	1	1	0	1	1	1
1	1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1	1

Hence the implication is a tautology.

Q.47 Identify the regular set accepted by the Fig.2 finite state machine and construct the corresponding state transition table. **(7)**

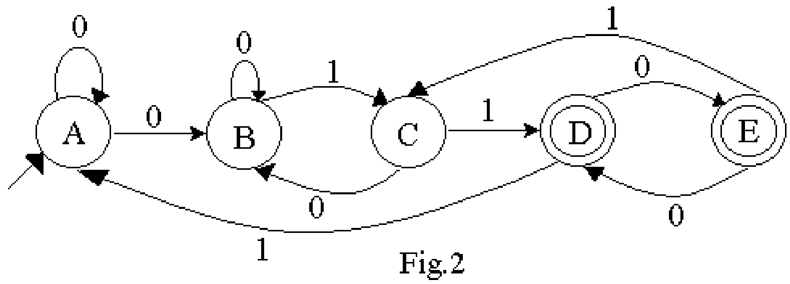


Fig.2

Ans: The transition table for the given FSM is as below:

State	0	1
A	{A, B}	{}
B	{B}	{C}
C	{B}	{D}
D	{E}	{A}
E	{D}	{C}

The language (Regular set) accepted by the FSM is any string on the alphabets {0, 1} that
(a) begins with ‘0’
(b) contains substring ‘011’ followed by zero or more 0’s, and
(c) does not contain 3 consecutive 1’s.

Q.48 Construct truth tables for the following:

$$[(p \vee q) \wedge (\sim r)] \leftrightarrow q$$ **(7)**

Ans: The required truth table for $(p\vee q) \wedge (\neg r) \leftrightarrow q$ is as below:

p	q	r	p∨q	(p∨q) ∧ (¬r)	(p∨q) ∧ (¬r) ↔ q
0	0	0	0	0	1
0	0	1	0	0	1
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	0	0

Q.49 Find the coefficient of x^{10} in $\left(1+x+x^2+.........\right)^2$ **(7)**

Ans: Let the given expression is written as

$$f(x) = (1 + x + x^2 + x^3 + \ldots + x^n)^2 = \left\{\frac{1}{1-x}\right\}^2$$

$$= (1 - x)^{-2}$$

The coefficient of x^{10} is equal to

$${}^{2+10-1}C_{10} = \frac{11!}{10!*1!} = 11.$$

Q.50 If $[L, \wedge, \vee]$ is a complemented and distributive lattice, then the complement $\bar{a}$ of any element $a \in L$ is unique. **(7)**

Ans: Let I and 0 are the unit and zero elements of L respectively. Let b and c be two complements of an element $a \in L$. Then from the definition, we have

$a \wedge b = 0 = a \wedge c$ and
$a \vee b = I = a \vee c$

We can write $b = b \vee 0 = b \vee (a \wedge c)$

$= (b \vee a) \wedge (b \vee c)$ [since lattice is distributive]
$= I \wedge (b \vee c)$
$= (b \vee c)$

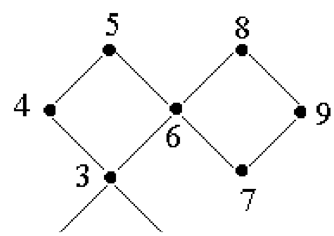
Similarly, $c = c \vee 0 = c \vee (a \wedge b)$

$= (c \vee a) \wedge (c \vee b)$ [since lattice is distributive]
$= I \wedge (b \vee c)$ [since $\vee$ is a commutative operation]
$= (b \vee c)$

The above two results show that b = c.

Q.51 Consider the following Hasse diagram Fig.3 and the set B= {3, 4, 6}. Find, if they exist,

(i) all upper bounds of B;
(ii) all lower bounds of B;
(iii) the least upper bound of B;
(iv) the greatest lower bound of B; **(7)**



Ans:
(i) The set of upper bounds of subset B = {3, 4, 6} in the given lattice is {5}. It is only element in the lattice that succeeds all the elements of B.
(ii) Set containing lower bounds is {1, 2, 3}
(iii) The least upper bound is 5, since it is the only upper bounds.
(iv) The greatest lower bound is 3, because it is the element in lower bounds that succeeds all the lower bounds.

Q.52 Solve the recurrence relation $S(k) - 4S(k-1) + 3S(k) = 3^k$. **(7)**

Ans:

The general solution, also called **homogeneous solution**, to the problem is given by homogeneous part of the given recurrence equation. The homogeneous part of the given equation is

$$S_k - 4S_{k-1} + 3S_{k-2} = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

The characteristic equation of (2) is given as

$$\begin{aligned} & x^2 - 4x + 3 = 0 \\ or, \quad & (x-1)(x-3) = 0 \\ or, \quad & x = 1 \quad and \quad x = 3. \end{aligned}$$

The homogeneous solution is given by

$$S_k = A1^k + B3^k \quad \text{------------------------}(2)$$

$$S_k = kC3^k$$

Now, particular solution is given by

Substituting the value of S_k in the given equation, we get

$$C\left[k3^k - 4(k-1)3^{k-1} + 3(k-2)3^{k-2}\right] = 3^k$$

$$or,\quad C\left[9k - 4(k-1)3 + 3(k-2)\right]3^{k-2} = 3^k$$

$$or,\quad C\left[9k - 12(k-1) + 3k - 6\right] = 9$$

$$or,\quad C\left[9k - 9k + 6\right] = 9$$

$$or,\quad 6C = 9 \qquad or,\quad C = \frac{3}{2}$$

$$\therefore\quad Particular\ solution\ is\quad S_k = \frac{3}{2}k3^k$$

The **complete,** also called **total**, solution is obtained by combining the homogeneous and particular solutions. This is given as

$$S_k = A + B3^k - \frac{3}{2}k3^k \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(3)$$

Since no initial condition is provided, it is not possible to find A and B,

Q.53 Prove that $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$. **(7)**

Ans: In order to prove this let x be any element of A ∪ B, then

$x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

$\Leftrightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$ or $(x \in A$ and $x \in B)$

$\Leftrightarrow (x \in A - B)$ or $(x \in B - A)$ or $(x \in A \cap B)$

$\Leftrightarrow x \in (A - B) \cup (B - A) \cup (A \cap B)$

This implies that

$A \cup B \subseteq (A - B) \cup (B - A) \cup (A \cap B)$ and

$(A - B) \cup (B - A) \cup (A \cap B) \subseteq A \cup B$

Thus $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Q.54 There are 15 points in a plane, no three of which are in a straight line except 6 all of which are in one straight line.

(i) How many straight lines can be formed by joining them?
(ii) How many triangles can be formed by joining them? **(7)**

Ans: There are 15 points out of which 6 are collinear. A straight can pass through any two points. Thus
(i) The number straight lines that can be formed is ${}^{15}C_2 - {}^{6}C_2 + 1 = 105 - 15 + 1 = 91$. ${}^{15}C_2$ is the number of ways two points can be selected. If the two points are coming from 6 collinear, then there can be only one straight line. Thus ${}^{6}C_2$ is subtracted and 1 is added in the total.

(ii) The number Triangles that can be formed is ${}^{15}C_3 - {}^{6}C_3 = 455 - 20 = 425$. ${}^{15}C_3$ is the number of ways three points can be selected to form a triangle. If the three points are coming from 6 collinear, then no triangle is formed. Thus ${}^{6}C_3$ is subtracted from the total.

Q.55 Prove the following Boolean expression:

$$(x \vee y) \wedge (x \vee \sim y) \wedge (\sim x \vee z) = x \wedge z$$ **(7)**

Ans: In the given expression, LHS is equal to:

$$\begin{aligned}
(x\vee y)\wedge(x\vee \sim y)\wedge(\sim x \vee z) &= [x\wedge(x\vee \sim y)] \vee [y\wedge(x\vee \sim y)] \wedge(\sim x \vee z) \\
&= [x\wedge(x\vee \sim y)] \vee [y\wedge(x\vee \sim y)] \wedge(\sim x \vee z) \\
&= [(x\wedge x) \vee (x\wedge\sim y)] \vee [(y\wedge x)\vee (y\wedge\sim y)] \wedge(\sim x \vee z) \\
&= [x \vee (x\wedge\sim y)] \vee [(y\wedge x)\vee 0] \wedge(\sim x \vee z) \\
&= [x \vee (y\wedge x)] \wedge(\sim x \vee z) \quad [x \vee (x\wedge\sim y) =x] \\
&= x \wedge(\sim x \vee z) \quad [x \vee (x\wedge y) =x] \\
&= [x \wedge\sim x)] \vee (x \wedge z) \quad [x \vee (x\wedge\sim y) =x] \\
&= 0 \vee (x \wedge z) = (x \wedge z) = RHS
\end{aligned}$$

Q.56 Find how many words can be formed out of the letters of the word DAUGHTER such that
(i) The vowels are always together.
(ii) The vowels occupy even places. **(7)**

Ans: In the word DAUGHTER, there are three vowels: A, E and U. Number of letters in the word is 8.

(i) When vowels are always together, the number of ways these letters can be arranged (5 +1)! * 3!. The 5 consonants and one group of vowels. Within each such arrangement, the three vowels can be arranged in 3! Ways. The required answer is 4320.

(ii) When vowel occupy even position, then the three vowels can be placed at 4 even positions in 3*2*1 ways. The three even positions can be selected in ${}^{4}C_3$ ways. And now the 5 consonants can be arranged in 5! Ways. Therefore the required number is = 3! * ${}^{4}C_3$ *5! = 2880

Q.57 Let R be the relation on the set of ordered pairs of positive integers such tha $((a, b), (c, d)) \in R$ if and only if ad = bc. Determine whether R is an equivalence relation or a partial ordering. **(8)**

Ans: R is defined on the set P of cross product of set of positive integers Z+ as (a, b) R (c, d) iff a*d =b*c. Now let us test whether R is an equivalence relation or not

Reflexivity: Let (x, x) be any element of P, then since a*a = a*a , we can say the (a, a) R (a, a).Thus R is reflexive.

Symmetry: Let (a, b) and (c, d) are any two elements in P such that (a, b) R (c, d). Then we have a*d = b*c => c*b = d*a => (c, d) R (a, b) => R is symmetric.

Transitivity: Let (a, b), (c, d) and (e, f) are any three pairs in P such that (a, b) R (c, d) and (c, d) R (e, f). Then we have a*d = b*c and c*f = d*e => a/e = b/f => a*f = b*e => (a, b) R (e, f) => R is transitive.
Therefore R is **an equivalence relation**.

Q.58 Prove that the direct product of any two distributive lattice is a distributive. **(6)**

Ans: Let $(L_1, \leq_1)$ and $(L_2, \leq_2)$ be two distributive lattices then we have to prove that $(L, \leq)$ is also a distributive lattice, where $L = L_1 \times L_2$ and $\leq$ is the product partial order of $(L_1, \leq_1)$ and $(L_2, \leq_2)$. It will be sufficient to show that L is lattice, because distributive properties shall be inherited from the constituent lattices.

Since $(L_1, \leq_1)$ is a lattice, for any two elements a_1 and a_2 of L_1, $a_1 \vee a_2$ is join and $a_1 \wedge a_2$ is meet of a_1 and a_2 and both exist in L_1. Similarly for any two elements b_1 and b_2 of L_2, $b_1 \vee b_2$ is join and $b_1 \wedge b_2$ is meet of b_1 and b_2 and both exist in L_2. Thus, $(a_1 \vee a_2, b_1 \vee b_2)$ and $(a_1 \wedge a_2, b_1 \wedge b_2) \in L$. Also (a_1, b_1) and $(a_2, b_2) \in L$ for any $a_1, a_2 \in L_1$ and any $b_1, b_2 \in L_2$.

Now if we prove that

$$(a_1, b_1) \vee (a_2, b_2) = (a_1 \vee a_2, b_1 \vee b_2) \text{ and}$$

$$(a_1, b_1) \wedge (a_2, b_2) = (a_1 \wedge a_2, b_1 \wedge b_2)$$

then we can conclude that for any two ordered pairs (a_1, b_1) and $(a_2, b_2) \in L$, its join and meet exist and thence $(L, \leq)$ is a lattice.

We know that $(a_1, b_1) \vee (a_2, b_2) =$ **lub** $(\{(a_1, b_1), (a_2, b_2)\})$. Let it be (c_1, c_2). Then,

$$(a_1, b_1) \leq (c_1, c_2) \text{ and } (a_2, b_2) \leq (c_1, c_2)$$

$$\Rightarrow a_1 \leq_1 c_1,\ b_1 \leq_2 c_2;\ a_2 \leq_1 c_1,\ b_2 \leq_2 c_2$$

$$\Rightarrow a_1 \leq_1 c_1,\ a_2 \leq_1 c_1;\ b_1 \leq_2 c_2,\ b_2 \leq_2 c_2$$

$$\Rightarrow a_1 \vee a_2 \leq_1 c_1;\ b_1 \vee b_2 \leq_2 c_2$$

$\Rightarrow(a_1 \vee a_2 , b_1 \vee b_2) \leq (c_1, c_2)$

This shows that $(a_1 \vee a_2 , b_1 \vee b_2)$ is an upper bound of $\{(a_1, b_1), (a_2, b_2)\}$ and if there is any other upper bound (c_1, c_2) then $(a_1 \vee a_2 , b_1 \vee b_2) \leq (c_1, c_2)$. Therefore, **lub** $(\{(a_1, b_1), (a_2, b_2)\}) = (a_1 \vee a_2, b_1 \vee b_2)$ i.e.

$$(a_1, b_1) \vee (a_2, b_2) = (a_1 \vee a_2, b_1 \vee b_2)$$

Similarly, we can show for the meet.

Q.59 Given n pigeons to be distributed among k pigeonholes. What is a necessary and sufficient condition on n and k that, in every distribution, at least two pigeonholes must contain the same number of pigeons?
Prove the proposition that in a room of 13 people, 2 or more people have their birthday in the same month. **(8)**

Ans: The situation when all the k pigeonholes may contain unique number of pigeons out of n pigeons, if $n \geq (0 + 1 + 2 + 3 + \ldots + (k - 1)) = \frac{k(k-1)}{2}$. Thus, as long as n (number of pigeons) is less than equal to $\frac{k(k-1)}{2}$, at least two of the k pigeonholes shall contain equal number of pigeons.

Let 12 months are pigeonholes and 13 people present in the room are pigeons. Pick a man and make him seated in a chair marked with month corresponding his birthday. Repeat the process until 12 people are seated. Now even in the best cases, if all the 12 people have occupied 12 different chairs, the 13th person will have to share with any one, i.e. there is at least 2 persons having birthday in the same month.

Q.60 Define a grammar. Differentiate between machine language and a regular language. Specify the Chowsky's classification of languages. **(6)**

Ans: Any language is suitable for communication provided the syntax and semantic of the language is known to the participating sides. It is made possible by forcing a standard on the way to make sentences from words of that language. This standard is forced through a set of rules. This set of rules is called **grammar** of the language. A grammar $G = (N, \Sigma, P, S)$ is said to be of

Type 0: if there is no restriction on the production rules i.e., in $\alpha \rightarrow \beta$, where $\alpha, \beta \in (N \cup \Sigma)^*$. This type of grammar is also called an ***unrestricted grammar***.

Type 1: if in every production $\alpha \rightarrow \beta$ of P, $\alpha, \beta \in (N \cup \Sigma)^*$ and $|\alpha| \leq |\beta|$. Here $|\alpha|$ and $|\beta|$ represent number of symbols in string α and β respectively. This type of grammar is also called a ***context sensitive*** grammar (or ***CSG***).

Type 2: if in every production $\alpha \to \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \Sigma)^*$. Here α is a single non-terminal symbol. This type of grammar is also called a ***context free*** grammar (or ***CFG***).
Type 3: if in every production $\alpha \to \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \Sigma)^*$. Here α is a single non-terminal symbol and β may consist of at the most one non-terminal symbol and one or more terminal symbols. The non-terminal symbol appearing in β must be the extreme right symbol. This type of grammar is also called a ***right linear*** grammar or ***regular*** grammar (or ***RG***).

For any language a acceptor (a machine) can be drawn that could be DFA, NFA, PDA, NPDA, TM etc. Language corresponding to that machine is called machine language. In particular a language corresponding to Finite automata is called Regular language.

Q.61 Determine the closure property of the structure $[5\times 5 \text{ Boolean matrices}, \wedge, \vee, (\bullet)]$ with respect to the operations $\wedge, \vee$ and $(\bullet)$. Illustrate using examples and give the identity element, if one exists for all the binary operations. **(9)**

Ans: Let A and B are any two 5x5 Boolean matrices. Boolean operations join ($\vee$), meet ($\wedge$) and product [($\bullet$)] on set of 5x5 Boolean matrix M is defined as

$A \vee B = [c_{ij}]$, where $c_{ij} = \max\{a_{ij}, b_{ij}\}$ for $1 \le i \le 5$, $1 \le j \le 5$
$A \wedge B = [c_{ij}]$, where $c_{ij} = \min\{a_{ij}, b_{ij}\}$ for $1 \le i \le 5$, $1 \le j \le 5$

$$A\,(\bullet)\,B = [c_{ij}], \text{ where } c_{ij} = \sum_{k=1}^{5} a_{ik}(\bullet)b_{kj}\} \text{ for } 1 \le i \le 5,\ 1 \le j \le 5$$

It is obvious that for any two matrices A and B of M, ($A \vee B$), ($A \wedge B$) and ($A\,(\bullet)\,B$) are also in M. Hence M closed under $\vee$, $\wedge$ and ($\bullet$). The identity element for the operations $\vee$, $\wedge$ and ($\bullet$) are

$$\begin{bmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}, \begin{bmatrix} 1&1&1&1&1\\ 1&1&1&1&1\\ 1&1&1&1&1\\ 1&1&1&1&1\\ 1&1&1&1&1 \end{bmatrix} \text{ and } \begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 0&0&0&0&1 \end{bmatrix} \text{ respectively.}$$

Q.62 Prove that if x is a real number then

$$[2x] = [x] + \left[x + \frac{1}{2}\right]$$ **(5)**

Ans: Let x be any real number. It has two parts: integer and fraction. Without loss of any generality, fraction part can always be made +ve. For example, -1.3 can be written as –2 + 0.7. Let us write x = a + b, and [x] = a (integer part only of the real x). The fraction part b needs to considered in two cases:

$$0 < b < 0.5 \text{ and } 0.5 \le b < 1.$$

Case 1: $0 < b < 0.5$; In this case $[2x] = 2a$, and $[x] + [x + .5] = a + a = 2a$

Case 2: $0.5 \le b < 1$; In this case $[2x] = 2a + 1$, and $[x] + [x + .5] = a + (a + 1) = 2a + 1$

Hence $[2x] = [x] + [x + .5]$

Q.63 Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. For n a positive integer and f_n the n^{th} Fibonacci number show that

$$A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$ **(5)**

Ans: The Fibonacci number is given as 1, 1, 2, 3, 5, 8, ... and so on and defined as

$$f_n = f_{n-1} + f_{n-2}$$

For $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, we have $A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} f_3 & f_2 \\ f_2 & f_1 \end{bmatrix}$. Let us suppose that it is true for n = k i.e. $A^k = \begin{bmatrix} f_{k+1} & f_k \\ f_k & f_{k-1} \end{bmatrix}$. Then we prove that it is true for n = k + 1 as well and hence the result shall follow from mathematical induction.

$$A^{k+1} = \begin{bmatrix} f_{k+1} & f_k \\ f_k & f_{k-1} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} f_{k+1} + f_k & f_{k+1} \\ f_k + f_{k-1} & f_k \end{bmatrix} = \begin{bmatrix} f_{k+2} & f_{k+1} \\ f_{k+1} & f_k \end{bmatrix}$$

Therefore we have the result, $A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$

Q.64 Let the universe of discourse be the set of integers. Then consider the following predicates:

$P(x): x^2 \ge 0$

$Q(x): x^2 - 5x + 6 = 0$

$R(x, y): x^2 = y$

Identify from the following expressions ones which are not well formed formulas, if any, and determine the truth value of the well-formed-formulas:

(i) $\forall_x [P(x) \wedge Q(x)]$
(ii) $\forall_x \forall_y [R(P(x), y) \rightarrow P(x)]$
(iii) $\forall_X P(x) \wedge \exists Y\, Q(y)$
(iv) $\forall_y \forall_x [P(x) \rightarrow R(x, y)]$
(v) $\left[\forall_y \exists_x R(y, x)\right] \vee [\forall_Z Q(z)]$ **(9)**

Ans:
(i) It is wff and **not true** for every x. P(x) is true for all x whereas Q(x) is true only for x = 2, 3 and not for all x.

(ii) It is **not a wff** because argument of R is predicate and not a variable as per the definition.
(iii) It is **true** because square of an integer is always ≥ 0 and there are y (= 2, 3) such that Q (y) is true.
(iv) It is **false**. Because there exists x = 3 and y =5 (there are infinite many such examples) such that P(3) is true but R(3, 5) is not true.
(v) $\forall y\ \exists x\ R(y, x)$ is always true because square of an integer is an integer so we can always find an x for every y such that R(y, x) is satisfied. However $\forall z Q(z)$ is not true. Since two statements are connected with $\vee$, it is **TRUE**.

Q.65 What is the minimum number of students required in a class to be sure that at least 6 will receive the same grade if there are five possible grades A, B,C, D and F? **(4)**

Ans: Let us consider 5 pigeonholes corresponding to five grades. Then, our problem is to find the number of pigeons (students) so that when placed in holes, one of the holes must contain at least 6 pigeons (students). In the theorem above, m = 5, then we have to find n such that

$$\left\lfloor \frac{n-1}{5} \right\rfloor + 1 = 6 \qquad or, \quad n-1 = 25 \qquad or, \quad n = 26$$

Therefore, at least 26 students are required in the class.

Q.66 Let n be a positive integer. Then prove that $\sum_{k=0}^{n} (-1)^k C(n, r) = 0$ **(4)**

Ans: $\sum_{k=0}^{n} (-1)^k C(n,r) = {}^nC_0 + {}^nC_1(-1) + {}^nC_2(-1)^2 + \ldots + {}^nC_n(-1)^n = (1 + (-1))^n = 0$

Q.67 Suppose E is the event that a randomly generated bit string of length 4 begins with a 1 and F is the event that this bit string contains an even number of 1's. Are E and F independent if the 16 bit strings of length 4 are equally likely? **(6)**

Ans: Number of 4 bit strings that begins with 1 is 8, thus P(E) = .5

Number of 4-bit string having even number of 1's is also 8 [C(4, 0) + C(4, 2) + C(4, 4)]. Therefore P(F) = .5

Now Number of 4-bit string that begins with 1 and contains even number of 1's is 4 [1 + C(3, 0)]. Thus $P(E \cap F) = .25$.

Clearly $P(E \cap F) = P(E)*P(F)$. Thus E and F are independent.

Q.68 Find the sum-of-products expression for following function,

$$F(x, y, z) = y + \bar{z}$$ **(6)**

Ans: The sum of the product expression for the given function f is **DNF (disjunctive normal form)** expression. The truth table for f is as below.

x	y	z	F(x, y, z) = y + z'
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The required sum of product (min term) notation is

F(x, y, z) = x'y'z' + x'yz' + x'yz + xy'z' + xyz' + xyz

Q.69 Using other identities of Boolean algebra prove the absorption law,

x + xy = x

Construct an identity by taking the duals of the above identity and prove it too. **(8)**

Ans: We have to prove that x + xy = x.

x + xy = x (y + y') + xy

$= xy + xy' + xy$
$= (xy + xy) + xy'$
$= xy + xy'$
$= x (y + y') = x.1 = x$

The dual of the given identity is $x(x+ y) = x$. Now
$x(x + y) = xx + xy$
$= x + xy$
$= x$ [From above theorem]

Q.70 Define grammar. Differentiate a context-free grammar from a regular grammar. **(6)**

Ans: Any language is suitable for communication provided the syntax and semantic of the language is known to the participating sides. It is made possible by forcing a standard on the way to make sentences from words of that language. This standard is forced through a set of rules. This set of rules is called **grammar** of the language.

A grammar $G = (N, \sum, P, S)$ is said to be of Context Free if in every production $\alpha \rightarrow \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \sum)^*$. Here α is a single non-terminal symbol.

On the other hand, in a regular grammar, every production $\alpha \rightarrow \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \sum)^*$. Here α is a single non-terminal symbol and β may consist of at the most one non-terminal symbol and one or more terminal symbols. The non-terminal symbol appearing in β must be the extreme right (or left) symbol. This type of grammar is also called a ***right linear*** grammar or ***regular*** grammar.

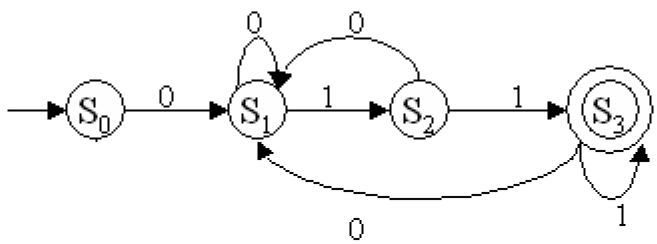
Q.71 Let L be a language over {0, 1} such that each string starts with a 0 and ends with a minimum of two subsequent 1's. Construct,

(i) the regular expression to specify L.
(ii) a finite state automata M, such that M (L) = L.
(iii) a regular grammar G, such that G(L) = L. **(8)**

Ans:

(i) The regular expression for the language is $0^+(0 \vee 1)^* 11$.

(ii) The finite automata M is as below:

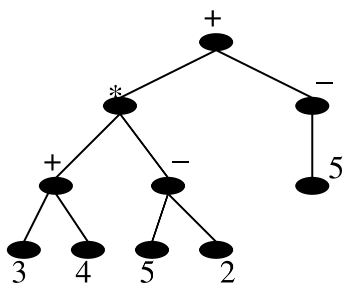


(iii) Now the regular grammar G is:

S → 0A,
A → 0A |1B,
B → 1 | 1C | 0A,
C →1C| 0A |ε

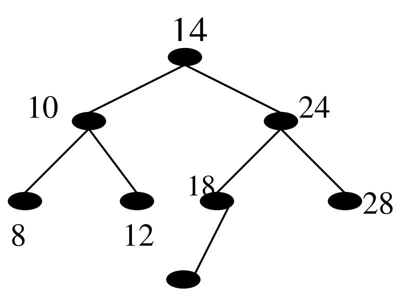
Q.72 Define an ordered rooted tree. Cite any two applications of the tree structure, also illustrate using an example each the purpose of the usage. **(7)**

Ans: A **tree** is a graph such that it is connected, it has no loop or circuit of any length and number of edges in it is one less than the number of vertices.



If the downward slop of an arc is taken as the direction of the arc then the above graph can be treated as a directed graph. In degree of node + is zero and of all other nodes is one. There are nodes like 3, 4, 5, 2 and 5 with out degree zero and all other nodes have out degree > 0. A node with in degree zero in a tree is called ***root*** of the tree. *Every tree has one and only one root and that is why a directed tree is also called a* ***rooted tree***. The position of every labeled node is fixed, any change in the position of node will change the meaning of the expression represented by the tree. Such type of tree is called ordered tree.

A tree structure is used in evaluation of an arithmetic expression (parsing technique) The other general application is search tree. The tree presented is an example of expression tree. An example of binary search tree is shown below.



Q.73 Determine the values of the following prefix and postfix expressions. ($\uparrow$ is exponentiation.)

(i) +, -, $\uparrow$, 3, 2, $\uparrow$, 2, 3, /, 6, –, 4, 2

(ii) 9, 3, /, 5, +, 7, 2, -, * **(7)**

Ans:

(i) +, -, $\uparrow$, 3, 2, $\uparrow$, 2, 3, /, 6, -, 4, 2 $\Leftrightarrow$ +, -, $\uparrow$, 3, 2, $\uparrow$, 2, 3, /, 6, (4 – 2)
$\Leftrightarrow$ +, -, $\uparrow$, 3, 2, $\uparrow$, 2, 3, (6 / 2)
$\Leftrightarrow$ +, -, $\uparrow$, 3, 2, (2$\uparrow$3), 3
$\Leftrightarrow$ +, -, (3 $\uparrow$ 2), 8, 3
$\Leftrightarrow$ +, (9 – 8), 3
$\Leftrightarrow$ (1 + 3) = 4

(ii) 9,3, /, 5, +, 7, 2, -, * $\Leftrightarrow$ (9 / 3), 5, +, 7, 2, -, *
$\Leftrightarrow$ (3 + 5), 7, 2, -, *
$\Leftrightarrow$ 8, (7 – 2), *
$\Leftrightarrow$ 8* 5 = 40

Q.74 Let m be a positive integer with m>1. Determine whether or not the following relation is an equivalent relation.

$R = \{(a, b) \mid a \equiv b (\mathrm{mod}\ m)\}$ **(6)**

Ans: Relation R is defined as $\equiv_m$ (congruence modulo m) on the set of positive integers. Let us check whether it is an equivalence relation.

Reflexivity: Let $x \in Z_+$ be any integer, then $x \equiv_m x$ since both yields the same remainder when divided by m. Thus, $(x, x) \in R\ \forall\ x \in Z$. $\therefore$R is a reflexive relation.

Symmetry*:* Let x and y be any two integers and $(x, y) \in R$. This shows that $x \equiv_m y$ and hence $y \equiv_m x$. Thus, $(y, x) \in R$. $\therefore$ R is a symmetric relation.

Transitivity*:* Let x, y and z be any three elements of Z such that (x, y) and $(y, z) \in R$. Thus, we have $x \equiv_m y$ and $y \equiv_m z$. It implies that (x–y) and (y–z) are divisible by m. Therefore, (x – y) + (y – z) = (x – z) is also divisible by m i.e. $x \equiv_m z$.

$\therefore$ (x, y) and $(y, z) \in R \Rightarrow (x, z) \in R$. i.e. R is a transitive relation.

Hence R is **an equivalence relation**.

Q.75 Let A = {1, 2, 3, 4} and, R = {(1, 1), (1, 4), (2, 1), (2, 2), (3, 3), (4, 4)}.
Use Warshall's algorithm to find the transitive closure of R. **(6)**

Ans: The matrix for R is $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Step 1: Call this matrix as initial Warshall matrix and denote it by W_0.

Step 2: For K = 1 to 4, compute W_k –called Warshall's matrix at k^{th} stage. To compute W_k from W_{k-1}, we proceed as follows:

Step 2.1: Transfer all 1's from W_{k-1} to W_k,

Step 2.2: List the rows in column K of W_{k-1} where, the entry in W_{k-1} is 1. Say these rows p_1, p_2, p_3, …;
Similarly, list the columns in row K of W_{k-1} where, the entry in W_{k-1} is 1. Say these columns q_1, q_2, q_3, …;

Step 2.3: Place 1 at the locations (p_i, q_j) in W_k if 1 is not already there.

W_1, W_2, W_3, and W_4 are computed below.

For K = 1. We have to find W_1.
Step 2.1: Transfer all 1's from W_0 to W_1.
Step 2.2: Here K = 1. Thus in W_0, we have '1' in column 1 at row = 1, 2; and '1' in row 1 at column =1, 4. Therefore, W_1 will have an additional '1'at (2, 4). The Warshall 's matrix W_1 is given in the right side.

$$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For K = 2. We have to find W_2.
Step 2.1: Transfer all 1's from W_1 to W_2.
2: Here K = 2. Thus in W_1, we have '1' in column 2 at row = 2; and '1' in row 2 at column =1, 2, 4. Therefore, W_2 will have no additional '1'. The Warshall 's matrix W_2 is given in the right side.

$$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For K = 3. We have to find W_3.
Step 2.1: Transfer all 1's from W_2 to W_3.
2:Here K = 3. Thus in W_2, we have '1' in column 3 at row = 3; and '1' in row 3 at column =3. Therefore, W_3 will have no additional '1' The W_3 is given in the right side.

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For K = 4. We have to find W_4.

Step 2.1: Transfer all 1's from W_3 to W_4.

Step 2.2: Here K = 4. Thus in W_3, we have '1' in column 4 at row = 1, 2, 4; and '1' in row 4 at column = 4. Therefore, W_4 will have no additional '1'. The W_4 is shown in the right side matrix.

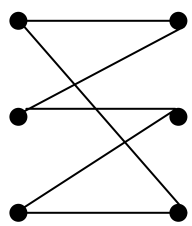
$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, W_4 is the matrix for the transitive closure of the given relation R.

R = {(1, 1), (1, 4), (2,1), (2, 2), (2, 4), (3, 3), (4, 4)}

Q.76 The graph C_n, $n \geq 3$ consists of n vertices and n edges making a cycle. For what value of n is C_n a bipartite graph? Draw the bipartite graph of C_n to justify your answer. **(2)**

Ans: For n = 2k, k = 2, 3, 4, …, C_n is a bipartite graph. C_6 is drawn below which is bipartite graph.



Q.77 Give an account of the two matrix representations for graphs. **(4)**

Ans: Two forms of representation used to represent a graph are: ***adjacency matrix*** and ***incidence matrix***. Let G = (V, E) be a graph, with no loops or multiple edges. Let |V| = m and |E| = n. An incidence matrix for G is an m×n matrix $M_I = (\mathbf{a_{ij}})$, such that

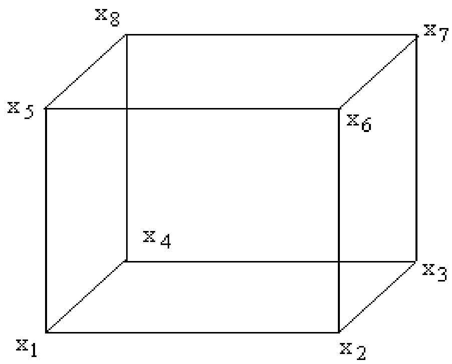
$$\mathbf{a_{ij}} = \begin{cases} 1 & \text{if } v_i \text{ is an incident to } e_j \\ 0 & \text{otherwise} \end{cases}$$

Where $v_1, v_2, v_3, \ldots, v_m$ are m vertices and $e_1, e_2, e_3, \ldots, e_n$ are n incidents (arcs) of G. An incidence matrix is also called *edge incidence matrix*.

Similarly, an adjacency matrix, also called *vertex adjacency matrix*, for G is an m×m matrix $M_A = (\mathbf{a_{ij}})$, such that

$$\mathbf{a_{ij}} = \begin{cases} 1 & \text{if } (v_i, v_j) \text{ is an edge of } G \\ 0 & \text{otherwise} \end{cases}$$

Q.78 Define a Hamilton path. Determine if the following graph has a Hamilton circuit. **(4)**



Ans: A path is called a ***Hamiltonian path*** if it contains every vertex of the graph exactly once. If a Hamiltonian path is a circuit, it is called a ***Hamiltonian circuit***.

Starting from node x_1, we can go to x_2 and then x_3. Proceeding in that way we get a Hamiltonian circuit, $\mathbf{x_1, x_2, x_3, x_7, x_6, x_5, x_8, x_4, x_1}$.

Q.79 Using generating function solve the recurrence relation

$a_k = 3a_{k-1}, k = 1, 2, 3......$

and the initial condition $a_0 = 2$ **(6)**

Ans:

Let f(x) be the binomial generating function for the given recurrence equation, $a_k = 3a_{k-1}; \quad a_0 = 2$. Then we have $f(x) = \sum_{k=0}^{\infty} a_k x^k = a_0 + \sum_{k=1}^{\infty} 3a_k x^k$.

$$or, \quad f(x) = a_0 + 3x \sum_{k=0}^{\infty} a_k x^k = a_0 + 3xf(x)$$

$$or, \quad (1-3x)f(x) = a_0 \qquad or, \quad f(x) = \frac{a_0}{1-3x} = \frac{2}{1-3x} \qquad [\because a_0 = 2]$$

The value of a_k is given by the coefficient of x^k in f(x). Therefore,

$$Coeff\ of\ x^k = \left[Coeff\ of\ x^k\ in\ \frac{1}{(1-3x)}\right] * 2$$

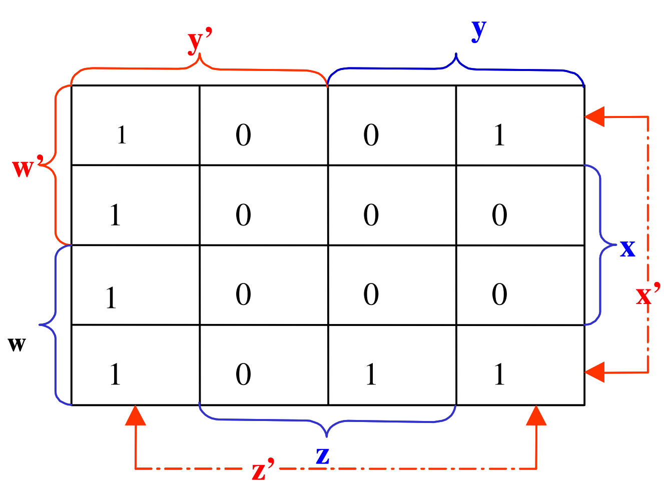
$$= {}^{1+k-1}C_k\, 3^k * 2 = {}^{k}C_k\, 3^k * 2$$

$$or, \quad a_k = 2 * 3^k$$

Q.80 Use Karnaugh map to simplify the following Boolean expression

$$w\,x\,\bar{y}\,\bar{z} + w\,\bar{x}\,y\,z + w\,\bar{x}\,y\,\bar{z} + w\,\bar{x}\,\bar{y}\,\bar{z} + \bar{w}\,x\,\bar{y}\,\bar{z} + \bar{w}\,\bar{x}\,y\,\bar{z} + \bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$$ **(6)**

Ans: The K-map for the given Boolean expression is presented below.



The two duets and one quad are wx'y, x'yz' and y'z' respectively. Thus the simplified Boolean expression is

wx'y + x'yz' + y'z'

Q.81 Consider the poset $(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\},\leq)$.

(i) Find the maximal elements.

(ii) Find the minimal elements.

(iii) Is there a least element.

(iv) Find the least upper bound of $\{\{2\},\{4\}\}$, if it exists. **(8)**

Ans:

(i) Maximal element in a poset is defined as element that is not succeeded by any other element in the poset. The maximal elements are {1, 2}, {1, 3, 4} and {2, 3, 4}

(ii) Maximal element in a poset is defined as element that is not preceded by any other ele in the poset. The minimal elements are {1}, {2} and {3}

(iii) There is no least element in the poset, as there exist no element x such that x precede every element of the poset. For example neither {1} precede {2} nor {2} precede {1}.

(iv) The upper bound of {{2}, {4}} are {2, 4} and {2, 3, 4}. The least of the upper bounds is {2, 4}.

Q.82 How many permutations of the letters A B C D E F G H contain string DEF? **(3)**

Ans: It is the problem of finding number of words that can be formed with the given 8 letters in which **DEF** come together in that order. There will be six letters to be used to form words: A, B, C, DEF, G, H. They can be permuted in **6!** = 720 ways. Thus required number of words is 720.

Q.83 Determine if the relation represented by the following Boolean matrix is partially ordered. **(7)**

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

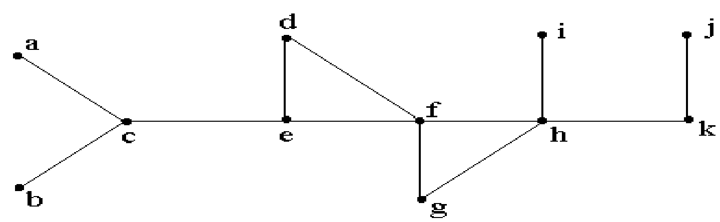
Ans: Let the given relation R is defined on set A = {x, y, z}. In order to test whether relation R is partially ordered, we have to test whether R is reflexive, anti symmetric and transitive.

Reflexivity: Since all elements in the principal diagonal is '1', R is reflexive.

Anti Symmetry: In the given relation, we do not have any pairs (x, y) & (y, x) such that x ≠ y i.e. for (x, y) & (y, x) in R, x = y. Thus R is anti symmetric.

Transitivity: The relation is not transitive because for (y, x) & (x, z) in R, (y, z) is not in R. Therefore R is not partially ordered.

Q.84 Apply depth-first-search to find the spanning tree for the following graph with vertex d as the starting vertex. **(4)**



Ans: Let us start with node 'd'. Mark d as visited. Node 'd' has two child 'e' and 'f'. Visit n 'e' and mark it as visited. Select edge (d, e). and add it to spanning tree T. Thus, T = {(d, e)} Now e has e has two children: c and f. Visit c, add (e, c) to T, and mark c as visited. Then visit a and then b. Mark them visited and add arcs (c, a) and (c, b) to T. Up to here

T = {(d, e), (e, c), (c, a), (c, b)}

Here c has one more child e, which is already visited, so exit recursion and go up to e which one more unvisited child f. Visit it, mark it as visited and we add (e, f) to T. f has three children: d, g and h. d is visited so leave it. Visit g, and doing the basic work of marking as visited and adding the arc used to visit the node in T, we finally get T as

T = {(d, e), (e, c), (c, a), (c, b), (e, f), (f, g), (g, h), (h, i), (h, k), (k, j)}

Q.85 A set contains (2n+1) elements. If the number of subsets of this set which contain at most n elements is 8192. Find the value of n. **(7)**

Ans: The given set has (2n + 1) elements. The number of subsets of this set having at the most n elements is given by formula

$^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots {}^{2n+1}C_n = \frac{1}{2}2^{2n+1} = 2^{2n}$.

This figure is given to be 8192, thus $2^{2n} = 8192 \Leftrightarrow 2^{2n} = 2^{13} \Leftrightarrow 2n = 13 \Leftrightarrow n = 6.5$
Therefore n = 6.5.

Q.86 Solve for a_n, the recurrence relation

$a_n - 2a_{n-1} - 3a_{n-2} = 0$, $n \geq 2$, with $a_0 = 3$ and $a_1 = 1$. **(7)**

Ans: The given recurrence equation is homogeneous recurrence equation. The

The characteristic equation is given as

$$\begin{aligned} & x^2 - 2x - 3 = 0 \\ or, \quad & (x+1)(x-3) = 0 \\ or, \quad & x = -1, 3 \end{aligned}$$

The general solution is given by

$$a_n = A(-1)^n + B3^n \qquad \underline{\hspace{6cm}}(2)$$

It is given that $a_0 = 3$ and $a_1 = 1$, so substituting n =0 and n = 1 in equation (2), we get

3 = A + B, and
1 = -A +3B

Solving the simultaneous equation we have A = 2 and B = 1. Therefore the solution to recurrence equation is

$$a_n = 2(-1)^n + 3^n \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

Q.87 Consider an 8×8 chess board. It contains sixty-four 1×1 squares and one 8×8 square. What is the total number of all the $n\times n$ squares for $1 \le n \le 8$, it contains? **(6)**

Ans: The number of 1x1 square is obviously 64 = 8^2. Number of 8x8 square is 1 = 1^2. If we now take 2x2 square, then there are 7 squares in taking two rows of square. The 7 squares so formed can be moved upward in 7 ways totaling the number of 2x2 square to 7^2. Proceeding in the same way, we can find that number of n x n squares ($1 \le n \le 8$) is

$$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = \frac{8(8+1)(2*8+1)}{6} = \frac{8*9*17}{6}$$

$$= \frac{8*9*17}{6} = 204$$

Q.88 Show that the following statements are equivalent:

P_1 : n is even integer.

$P_2 : n-1$ is an odd integer.

$P_3 : n^2$ is an even integer. **(8)**

Ans: In order to show the equivalence, we proceed to prove

(i) $P_1 \Rightarrow P_2$

(ii) $P_2 \Rightarrow P_3$

(iii) $P_3 \Rightarrow P_1$

(i) The statement P_1 stand for “n is even integer” and P_2 stands for “n – 1 is an odd integer”. Now

$P_1 \Rightarrow$ n is an even integer

$\Rightarrow \exists$ an integer m such that n = 2m

$\Rightarrow$ [(n – 1) = (2m – 1)] is an odd integer $\Rightarrow P_2$

(ii) The statement P_2 stand for “(n – 1) is an odd integer” and P_3 stands for “n^2 is an even integer”. Now

$P_2 \Rightarrow$ (n – 1) is an odd integer

$\Rightarrow (n-1)^2$ is an odd integer

$\Rightarrow (n^2 - 2n + 1)$ is an odd integer

$\Rightarrow n^2$ is an even integer because (2n – 1) is odd and once an odd is taken from odd, the result is even.

$\Rightarrow P_3$

(iii) The statement P_3 stands for “n^2 is an even integer”. and Now

$P_3 \Rightarrow n^2$ is an even integer
$\Rightarrow n^2 - 1$ is an odd integer
$\Rightarrow (n - 1)(n + 1)$ is an odd integer
$\Rightarrow (n - 1)$ and $(n + 1)$ are odd integers
$\Rightarrow (n - 1) + 1$ and $(n + 1) - 1$ are even integers
$\Rightarrow$ n is even integer $\Rightarrow P_1$

Q.89 State DeMorgan's law. Prove it using the truth table. **(4)**

Ans: DeMorgan's law state that

(i) $(x \vee y)' = x' \wedge y'$
(ii) $(x \wedge y)' = x' \vee y'$

Let us draw truth table for the two statements

1	2	3	4	5	6
x	y	$(x \vee y)'$	$x' \wedge y'$	$(x \wedge y)'$	$x' \vee y'$
0	0	1	1	1	1
0	1	0	0	1	1
1	0	0	0	1	1
1	1	0	0	0	0

It is obvious from the truth table that column (3) is equal to column (4). Similarly column(5) is equal to column (6).

Q.90 Consider the function $f : N \to N$, where N is the set of natural numbers, defined by $f(n) = n^2 + n + 1$. Show that the function f is one-one but not onto. **(6)**

Ans: In order to prove that f is one to one, it is required to prove that for any two integers n and m, if f(n) = f(m) then n = m.

$$f(n) = f(m) \Leftrightarrow n^2 + n + 1 = m^2 + m + 1$$
$$\Leftrightarrow n^2 + n = m^2 + m$$
$$\Leftrightarrow n(n + 1) = m(m + 1)$$

$\Leftrightarrow$ n = m. Because product of consecutive natural numbers starting from m and n are equal iff m = n.

Next f is **not onto** because for any n (odd or even) $n^2 + n + 1$ is odd. This implies that there are even elements in N that are not image of any element in N.

Q.91 Let T be a binary tree with n vertices. Determine the number of leaf nodes in tree. **(4)**

Ans: Let the height of the tree be r. The number of leave nodes will maximum 2^r. Total of nodes are then given by $[1+ 2 + 2^2 + \ldots + 2^{r-1} + 2^r]$. And this is equal to n. i.e.

$$\frac{(2^{r+1}-1)}{2-1} = 2^{r+1}-1; \quad i.e. \quad n = 2^{r+1}-1 \Rightarrow r = \log_2(n+1)-1$$

Therefore maximum number of leaf node is $2^{\log_2(n+1)-1}$.

Q.92 Construct the grammar G that generates the language over an alphabet {0, 1, 2}, $L_{012} = \{0^n1^n2^n : n \geq 1\}$ and prove that $L(G) = L_{012}$. **(10)**

Ans: The production rule of the grammar for the language L_{012} is as below:

1. $S \rightarrow 0SA2$
2. $S \rightarrow \varepsilon$
3. $0A \rightarrow 01$
4. $1A \rightarrow 11$
5. $2A \rightarrow A2$

Now let us show that the grammar produces the language L_{012}.

$S \rightarrow 0SA2$	[Apply rule 1 once]
$\rightarrow 0^n S (A2)^n$	[Apply rule 1 optionally n – 1 time]
$\rightarrow 0^n (A2)^n$	[Apply rule 2 once]
$\rightarrow 0^{n-1} (0A)\ 2\ (A2)^{n-1}$	[rearrangement]
$\rightarrow 0^{n-1} (01)\ 2\ (A2)^{n-1}$	[Apply rule 3 once]
$\rightarrow 0^n\ 1\ 2\ (A2)\ (A2)^{n-2}$	[rearrangement]
$\rightarrow 0^n\ 1A\ 2^2\ (A2)^{n-2}$	[Apply rule 5 once]
$\rightarrow 0^n\ 1^2\ 2^2\ (A2)^{n-2}$	[Apply rule 4 once]
…	
$\rightarrow 0^n\ 1^n\ 2^n$	

Therefore, $L(G) = L_{012}$.

Q.93 Let Q(x,y) denote **x + y = 0**. Determine the truth values of $\forall x \exists y\, Q(x, y)$ and $\exists y \forall x\, Q(x, y)$. **(4)**

Ans: The statement $\forall x\ \exists y\ Q(x, y) \Rightarrow$ For every x, there exists y such that x + y = 0. The statement is true for the universe where 0 is identity element for the binary operation '+' and inverse for every element exist in the universe for every element x. The universe is not defined in the question, hence truth ness shall vary from universe to universe.

Next the statement $\exists x\ \forall y\ Q(x, y) \Rightarrow$ there exists x such that for every x, x + y = 0. The statement is ***cannot be true*** for the universe where 0 is identity element for the binary operation

'+' and inverse for every element exist in the universe for every element x. Even if the unive is not defined in the question, the statement is false.

Q.94 Let $A \subseteq Z$ and $f : A \rightarrow N$ be a one-one function where Z is a set of integers and N is a set of natural numbers. Let R be a relation on A defined as under:

$(x, y) \in R$ if and only if $f(y) = k\, f(x)$ where $k \in N$

Prove that R is a partial order relation on A. **(8)**

Ans: R is defined on the subset A of all integers. In order to show that R is partial order relation, it is to be shown that R is reflexive, anti-symmetric and transitive.

Reflexivity: Let x be any element of A then f(x) = 1.f(x) ⇒ (x, x) ∈ R. Thus R is reflexive.

Anti-symmetry: Let x, y ∈ A, such that (x, y) and (y, x) ∈ R.

Now (x, y) ∈ R ⇒ there exists a natural number k such that f(y) = kf(x)

Similarly, (y, x) ∈ R ⇒ there exists a natural number m such that f(x) = mf(y).

From the above two result, we can write f(x) = mk f(x). This implies that mk = 1, and product of any two natural numbers is 1 iff both are 1. Thus m = k = 1. Hence f(x) = f(y). Since f is one to one, we can conclude that x = y. Therefore, R is anti symmetric.

Transitivity: Let x, y, z ∈ A, such that (x, y) and (y, z) ∈ R.

Now (x, y) ∈ R ⇒ there exists a natural number k such that f(y) = kf(x)

Similarly, (y, z) ∈ R ⇒ there exists a natural number m such that f(z) = mf(y).

From the above two result, we can write f(z) = mk f(x). => (x, z) ∈ R. R is transitive.
Therefore R is a partial order relation.

Q.95 A speaks truth in 60% of the cases and B in 90% of the cases. Find the percentage of cases when they are likely to contradict each other in stating the same fact. **(6)**

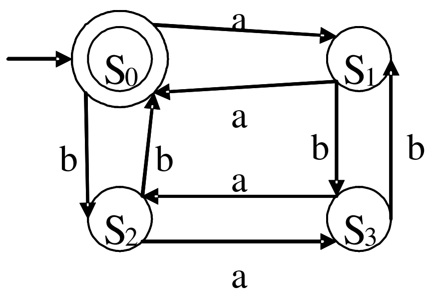
Ans:
Probability that A speaks **Truth: P(A=T)**= .6
Probability that A speaks **False: P(A=F)** = .4
Probability that B speaks **Truth**: **P(B=T)**= .9
Probability that A speaks **False: P(B =f)** = .1

Now probability that A and B likely to contradict is

P(A=T) x **P(B =F) + P(B=T)** x **P(A =F)**
= .6 x .1 + .9 x .4 = .06 + .36 = .42

Q.96 Give the regular expression of the set of all even strings over the alphabet {a, b} with at least one of the two substrings aa or bb. Also construct the finite automata that can accept the above language. **(8)**

Ans: Let us first draw the finite automata that accepts set of all even strings over alphabets



{a, b}. The regular expression for class of strings accepted by the automata is

(aa + bb + abba + abab + baba + baab)*

Q.97 Compute A (2, 1) when $A: N \times N \to N$, where N is the set of natural numbers, is defined by

$$A(0, y) = y + 1$$
$$A(x+1, 0) = A(x, 1)$$
$$A(x+1, y+1) = A(x+1, y) \qquad \textbf{(3)}$$

Ans:

A(2, 1) = A(1+1, 0+1)
= A(1+1, 0)
= A(1, 1)
= A(1, 0)
= A(0, 1) = 1 + 1 = 2.

Q.98 Express the following statement as a disjunction (in DNF) also using quantifiers:

There does not exit a woman who has taken a flight on every airline in the world. **(3)**

Ans: Let us define propositions.

Woman (x): "x is woman"
Flight (x, y): "x has taken flight on every airline y in world".

Now the given statement can be written as

~[∃x Woman (x) ∧ ∀y Flight (x, y)]
= ~∀y ∃x [Woman (x) ∧ Flight (x, y)]
= ∃y ∀x [~Woman (x) ∨ ~Flight (x, y)]

Q.99 Let L be a bounded distributive lattice. Show that if a complement exists it is unique. **(7)**

Ans: Let I and 0 are the unit and zero elements of B respectively. Let b and c be two complements of an element $a \in B$. Then from the definition, we have

$a \wedge b = 0 = a \wedge c$ and
$a \vee b = I = a \vee c$

We can write $b = b \vee 0 = b \vee (a \wedge c)$
$= (b \vee a) \wedge (b \vee c)$ [since lattice is distributive]
$= I \wedge (b \vee c)$
$= (b \vee c)$

Similarly, $c = c \vee 0 = c \vee (a \wedge b)$
$= (c \vee a) \wedge (c \vee b)$ [since lattice is distributive]
$= I \wedge (b \vee c)$ [since $\vee$ is a commutative operation]
$= (b \vee c)$

The above two results show that b = c.

Q.100 Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 20 computers can directly access 20 different printers. Justify your answer. **(7)**

Ans: The 100 computers and 20 printers can be connected using concept of a complete bipartite graph $K_{100,\ 20}$ to ensure that using minimum number of cable requirement. This arrangement will require 2000 cables and will ensure that at any time 20 computers can directly access the 20 computers.

Q.101 Prove that a simple graph is connected if and only if it has a spanning tree. **(7)**

Ans: First suppose that a simple graph G has a spanning tree T. T contains every node of G. By the definition of a tree, there is a path between any two nodes of T. Since T is a subgraph of G, there is a path between every pair of nodes in G. Therefore G is connected.

Now let G is connected. If G is a tree then nothing to prove. If G is not a tree, it must contain a simple circuit. Let G has n nodes. We can select (n – 1) arcs from G in such a way that they not form a circuit. It results into a subgraph having all nodes and only (n – 1) arcs. Thus by definition this subgraph is a spanning tree.

Q.102 Let $A = \{1, 2, 3, 4\}$ and let R and S be the relations on A described by $M_R = \begin{bmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

and $M_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Use Warshall's algorithm to compute the transitive closure of $R \cup S$. **(7)**

Ans: The matrix for $R \cup S$ is $M_R + M_S = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

Step 1: Call this matrix as initial Warshall matrix and denote it by W_0.

$$W_0 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Step 2: For K = 1 to 4, compute W_k –called Warshall's matrix at k^{th} stage. To compute W_k from W_{k-1}, we proceed as follows:

Step 2.1: Transfer all 1's from W_{k-1} to W_k,

Step 2.2: List the rows in column K of W_{k-1} where, the entry in W_{k-1} is 1. Say these rows p_1, p_2, p_3, …;
Similarly, list the columns in row K of W_{k-1} where, the entry in W_{k-1} is 1. Say these columns q_1, q_2, q_3, …;

Step 2.3: Place 1 at the locations (p_i, q_j) in W_k if 1 is not already there.

W_1, W_2, W_3, and W_4 are computed below..

For K = 1. We have to find W_1.

Step 2.1: Transfer all 1's from W_0 to W_1.

Step 2.2: Here K = 1. Thus in W_0, we have '1' in column 1 at row = 1; and '1' in row 1 at column =1, 2, 4. Therefore, W_1 will have no additional '1'. The Warshall 's matrix W_1 is given in the right side.

$$W_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

For K = 2. We have to find W_2.

Step 2.1: Transfer all 1's from W_1 to W_2.

Step 2.2: Here K = 2. Thus in W_1, we have '1' in column 2 at row = 1, 2, 3, 4; and '1' in row 2 at column = 2. Therefore, W_2 will have no additional '1'. The Warshall 's matrix W_2 is given in the right side.

$$W_2 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

For K = 3. We have to find W_3.

Step 2.1: Transfer all 1's from W_2 to W_3.

Step 2.2: Here K = 3. Thus in W_2, we have '1' in column 3 at row = 3, 4; and '1' in row 3 at column =2, 3. Therefore, W_3 will have no additional '1' The W_3 is given in the right side.

$$W_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

For K = 4. We have to find W_4.

Step 2.1: Transfer all 1's from W_3 to W_4.

Step 2.2: Here K = 4. Thus in W_3, we have '1' in column 4 at row = 1, 4; and '1' in row 4 at column = 2, 3, 4. Therefore, W_4 will have additional '1' at (1, 3). The W_4 is shown in the right side matrix.

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Therefore, W_4 is the matrix for the transitive closure of the given relation.

Q.103 Express the negation of the statement $\forall x \exists y (xy = 1)$ so that no negation precedes a quantifier. **(3)**

Ans: The negation of the given statement is written as $\sim[\forall x \ \exists y \ (xy = 1)]$. And

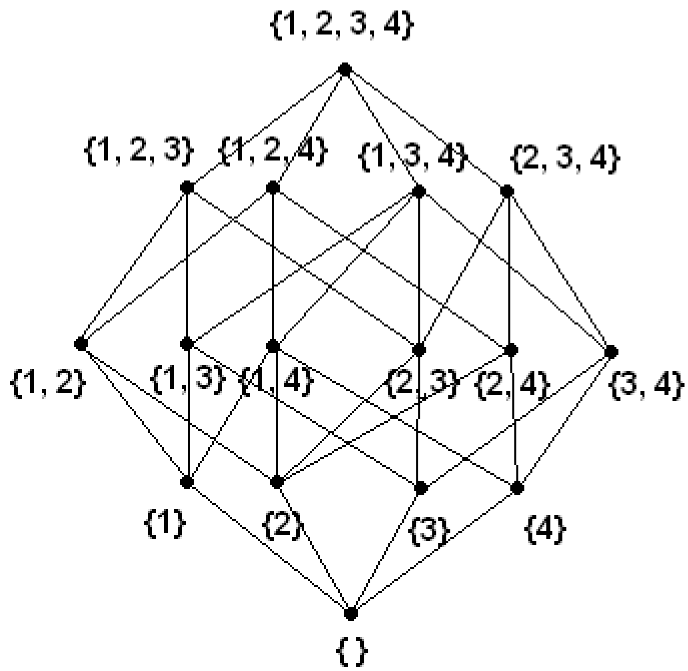
$$\begin{aligned} \sim[\forall x \ \exists y \ (xy = 1)] &= \exists x \ [\sim\exists y \ (xy = 1)] \\ &= \exists x \ \forall y \ [\sim(xy = 1)] \\ &= \exists x \ \forall y \ (xy \neq 1) \end{aligned}$$

Q.104 Let P(n) be the statement $\mathbf{n^2 + n}$ **is an odd number for** $\mathbf{n \in Z^+}$. Is P (n) true for all n? Explain. **(3)**

Ans: Sum of two even positive integers is even. Sum of two odd positive integers is even. Square of even is even and that of odd is odd. Thus for any positive integer n, n^2 +n is always an even positive integer. Therefore the statement P(n) is never true.

Q.105 Draw the Hasse diagram for the poset $(\wp(A), \subset)$ where $A = \{1, 2, 3, 4\}$ and $\wp(A)$ is the power set of A. **(8)**

Ans: The Hasse diagram for the poset is as below.



Q.106 Show that a positive logic NAND gate is equivalent to negative logic NOR gate. **(4)**

Ans:

- **Positive logic** represents True or 1 with a high voltage and False or 0 with a low voltage.
- **Negative logic** represents True or 1 with a low voltage and False or 0 with a high voltage.

The problem is equivalent to proving the DeMorgan's rule of complementation i.e.

$$(x \cdot y)' = x' + y'$$

Let us find now truth table for positive NAND and negative NOR.

Negative NOR					
x	**y**	**x'**	**y'**	**(x'+y')'**	**((x'+y')')'**
0	**0**	**1**	**1**	**0**	**1**
0	**1**	**1**	**0**	**0**	**1**
1	**0**	**0**	**1**	**0**	**1**
1	**1**	**0**	**0**	**1**	**0**

Positive NAND		
x	y	(x . y)’
0	0	1
0	1	1
1	0	1
1	1	0

Obviously, positive NAND gate is equivalent to negative NOR gate.

Q.107 Suppose $P(n) = a_0 + a_1 n + a_2 n^2 + ...a_m n^m$. Prove that $P(n) = O(n^m)$. **(5)**

Ans: Let f and g are functions whose domains are subsets of Z^+. We say f is O(g) if there exists constants c and k such that $|f(n)| \le c. |g(n)|\ \ \forall\ n \ge k$. Now,

$$\begin{aligned} P(n) &= a_0 + a_1 n + a_2 n^2 + \dots + a_m n^m \\ &\le a_0 n^m + a_1 n^m + a_2 n^m + \dots + a_m n^m \quad \forall\ n \ge 1 \\ &= [a_0 + a_1 + a_2 + \dots + a_m]\ n^m\ \forall\ n \ge 1 \end{aligned}$$

$\therefore\ P(n) = O(n^m)$

Q.108 Show that among any n+1 positive integers whose value does not exceed 2n, there must be an integer that divides one of the other integers. **(5)**

Ans: Every possible integer n can be written as $n = 2^k * m$, where m is odd part and $k \ge 0$. Here m is called odd part of n. In any two numbers having same odd part, one is multiple of other (and hence one divides other).

Mark 1, 3, 5, … 2n –1 (all (n – 1) odd numbers) as pigeonholes. Take a number and place it according to its odd part. There are n + 1 numbers and n pigeonholes, so one of the pigeonhole will have at least two numbers i.e. there is at least two integers having same odd parts. Hence one will divide the other.

Q.109 If A and B are two subsets of a universal set then prove that $A\text{-}B = A \cap \overline{B}$. **(8)**

Ans: In order to prove this let x be any element of (A – B) then
$x \in A - B \Leftrightarrow x \in A$ and $x \notin B$
$\Leftrightarrow x \in A$ and $x \in B^C$
$\Leftrightarrow x \in A \cap B^C$
This implies that
$A - B \subseteq A \cap B^C$ and

$A \cap B^C \subseteq A - B$

Thus $A - B = A \cap B^C$

Q.110 In a class of 60 boys, 45 boys play cards and 30 boys play carrom. How many boys play both games? How many play cards only and how many play carrom only? **(8)**

Ans: Let A and B be the set of boys who play cards and carom respectively, then as per the information available
$|A| = 45$, $|B| = 30$ and $|A \cup B| = 60$. Now $|A \cap B| = |A| + |B| - |A \cup B| = 45 + 30 - 60 = 15$.
i.e. Number of boys who play both the games = 15.

Number of boys who plays cards only = $|A| - |A \cap B| = 45 - 15 = 30$.
Number of boys who plays carrom only = $|B| - |A \cap B| = 30 - 15 = 15$.

Q.111 How many words can be obtained by arranging the letters of the word 'UNIVERSAL' in different way? In how many of them

(i) E,R,S occur together

(ii) No two of the letters E,R,S occur together. **(10)**

Ans: There are nine letters in the word 'UNIVERSAL' and none is repeated. Thus they are permuted in 9! ways.

(i) When E, R, S occur together, there will be 6 other letters and one (ERS). They are permuted in 7! ways and so there will be 7! * 3! words having E,R,S together.

(ii) Any two letters of E, R, S can come together in ${}^3P_2 = 6$ ways. When two letters (out of 9) come together then 7 +1 = 8 letters can be arranged in 8! ways. Thus there will be 8!* 6 words in which any two of E, R, S come together.

$\therefore$ Number of words in which no two of E,R,S come together = 9! - 8!*6

Q.112 Define symmetric, asymmetric and antisymmetric relations. **(6)**

Ans:

Symmetric Relation

A relation R defined on a set A is said to be a ***symmetric*** relation if for any $x, y \in A$, if $(x, y) \in R$ then $(y, x) \in R$ An examples of a symmetric relation is:

Let A = {1, 2, 3} be a set and R be a relation on A defined as {(1, 2), (2, 1), (3, 1), (1, 3)} then R is a symmetric relation.

Asymmetric Relation

A relation R on a set A is called an ***asymmetric*** relation

$$\text{if } (x, y) \in R \Rightarrow (y, x) \notin R \text{ for } x \neq y$$

i.e. presence of pair (x, y) in R excludes the possibility of presence of (y, x) in R.

Anti-Symmetric Relation

A relation R on a set A is called an ***anti-symmetric*** relation if for x, y$\in$ A

$$(x, y) \text{ and } (y, x) \in R \Leftrightarrow x = y$$

i.e. $x \neq y \Rightarrow$ either x ~R y or y ~R x or both.

Q.113 Prove by mathematical induction that if a set A has n elements, then P(A) has 2^n elements. **(8)**

Ans: We have to show that for a set A having n elements, its power set P(A) contains 2^n elements.

For n = 1, the result is obvious as P(A) = {ϕ, A}
Suppose the result is true for n = k i.e. when $|A| = k$ then $|P(A)| = 2^k$. Include one more element x to A. Now $|A \cup\{x\}| = k + 1$. All the subsets of A are now subsets of $|A \cup\{x\}|$. In addition to this there will be 2^k subsets that will contain element x. i.e.

$$|P(A \cup\{x\})| = 2^k + 2^k = 2^{k+1}.$$

This shows that result is true for n = k+1, whenever it is true for n = k.

Q.114 Let L be a lattice then for every a and b in L
$a \vee b = b$ if and only if a<=b **(8)**

Ans: Let $a \vee b = b$. Since $a \vee b$ is an upper bound of a, $a \leq a \vee b$. This implies $a \leq b$.

Next, let $a \leq b$. Since $\leq$ is a partial order relation, $b \leq b$. Thus, $a \leq b$ and $b \leq b$ together implies that b is an upper bound of a and b. We know that $a \vee b$ is least upper bound of a and b, so $a \vee b \leq b$. Also $b \leq a \vee b$ because $a \vee b$ is an upper bound of b. Therefore, $a \vee b \leq b$ and $b \leq a \vee b \Leftrightarrow a \vee b = b$ by the anti-symmetry property of partial order relation $\leq$. Hence, it is proved that $a \vee b = b$ if and only if $a \leq b$.

Q.115 Using Boolean algebra show that
$abc + ab\bar{c} + a\bar{b}c + \bar{a}bc = ab + ac + bc$ **(8)**

Ans: Let us consider the LHS of the given expression.

$$abc + ab\bar{c} + a\bar{b}c + \bar{a}bc = abc + ab\bar{c} + a\bar{b}c + \bar{a}bc + abc + abc$$

$$= (abc + ab\bar{c}) + (a\bar{b}c + abc) + (\bar{a}bc + abc)$$

$$= ab(c + \bar{c}) + a(\bar{b} + b)c + (\bar{a} + a)bc$$

$$= ab + ac + bc = RHS$$

Q.116 Explain extended pigeonhole principle and show that if 7 colors are used to paint 50 bicycles , at least 8 bicycles will be the same color. **(4, 4)**

Ans: The extended or generalized pigeonhole principle states that "If **n** objects (pigeons) are placed in **m** places (pigeonholes) for m < n, then one of the places (pigeonholes) must contains at least $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ objects (pigeons).

It can be proved in very simple way. Let us assume that none of the places (pigeonholes) contains more than **floor((n – 1)/m)** objects (pigeons). Then there are at the most **m** ∗ **floor((n – 1)/m)** ≤ **m** ∗ **(n – 1)/m = n-1** objects. This is contrary to the given fact that there are **n** objects. This contradiction is because of our assumption that "none of the places contains more than **floor((n – 1)/m)** objects". Thus our assumption is wrong. Therefore, one of the pigeonholes must contain at least $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ objects.

Let m = 7 colors are pigeonholes and n = 50 bicycles are pigeon, then one of the color (pigeonhole) has at least $\left\lfloor \frac{n-1}{m} \right\rfloor + 1\ objects$. Thus at least $\left\lfloor \frac{50-1}{7} \right\rfloor + 1 = 8$ bicycles will have same color.

Q.117 If a graph G has more than two vertices of odd degree, then prove that there can be no Euler path in G. **(8)**

Ans: If G is not connected then result is obvious. Let us suppose that G is a connected graph and there are three vertices a, b and c of odd degrees. We know that if G has exactly two vertices of odd degree then G has an Euler path in which one of the odd degree node is starting node and other is end node. Let us assume that these two nodes are a and b. Let c has degree 2n + 1. Then starting from a or b, c can be reached in along n arcs and reached out along another n arcs. Next either the $(2n + 1)^{th}$ arc is left not travelled or in the process of travelling this arc, path is trapped at c. In either case no Euler path is possible.

Q.118 Show that

$$\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q) \Leftrightarrow (\neg P \vee Q) \qquad \textbf{(8)}$$

Ans: Let us consider the LHS of the given equivalent expression

$$\sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) = \sim(P \wedge Q) \rightarrow (\sim P \vee Q) \quad [\textit{absorbtion law}]$$
$$= \sim(\sim(P \wedge Q)) \vee (\sim P \vee Q)$$
$$= (P \wedge Q) \vee (\sim P \vee Q)$$
$$= (P \vee (\sim P \vee Q)) \wedge (Q \vee (\sim P \vee Q))$$
$$= ((P \vee \sim P) \vee Q) \wedge (Q \vee (\sim P \vee Q))$$
$$= (1 \vee Q) \wedge (Q \vee \sim P)$$
$$= (Q \vee \sim P) = (\sim P \vee Q) = RHS$$

$$\therefore \sim(P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Leftrightarrow (\sim P \vee Q)$$

Q.119 What is minimum spanning tree of a graph? Write down Prim's and Kruskal's algorithms and execute them by taking a suitable example. **(2,7,7)**

Ans: Let G = (V, E) be a connected graph. Any connected spanning subgraph H = (V, E_1) of G is called ***spanning tree*** if H is a tree i.e. $|E_1| = |V| - 1$. The concept of a spanning tree is used basically in communication network design for remote switch installation" Where to place all the required remote switches so that total cables requirement (cost) should be minimum". This requires finding of minimum spanning tree in a weighted graph. A spanning tree of a weighted graph is called ***minimum spanning tree*** if the sum of edges in the spanning tree is minimum.

The stepwise Prim's algorithm to find a minimum spanning tree in a weighted graph is given below. Let G = (V, E) be graph and S = (V_S, E_S) be the spanning tree to be found from G.

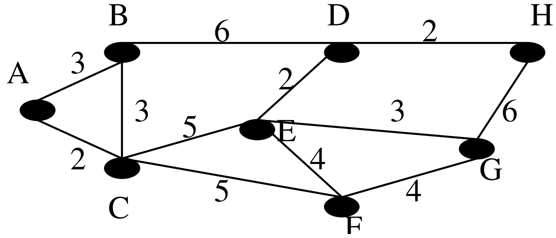
Step 1: Select a vertex v_1 of V and initialize

$V_S = \{v_1\}$ and
$E_S = \{\}$

Step 2: Select a nearest neighbor of v_i from V that is adjacent to some $v_j \in V_S$ and that edge (v_i, v_j) does not form a cycle with members edge of E_S. Set

$V_S = V_S \cup \{v_i\}$ and
$E_S = E_S \cup \{(v_i, v_j)\}$
Step 3: Repeat step2 until $|E_S| = |V| - 1$.

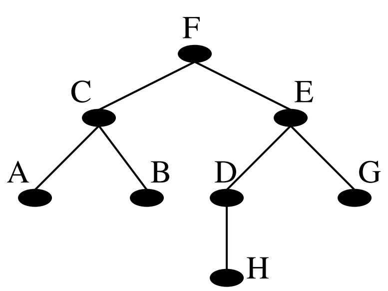
Let us consider the following graph for example.



Let us begin with the node A of the graph. Initialize $V_S = \{A\}$ and $E_S = \{\}$. There are eigh nodes so the spanning tree will have seven arcs. The iterations of algorithm applied on the graph are given below. The number indicates iteration number.

1. Nodes B and C are neighbors of A. Since node C is nearest to the node A we select C. Thus, we have $V_S = \{A, C\}$ and $E_S = \{(A, C)\}$.
2. Now node B is neighbor of both A and C and C has nodes E and F as its neighbor. We have AB = 3, CB = 3, CE = 5 and CF = 5. Thus, the nearest neighbor is B. We can select either AB or CB. We select CB. Therefore, $V_S = \{A, C, B\}$ and $E_S = \{(A, C), (C, B)\}$.
3. Now D, E, F are neighbor of nodes in V_S. An arc AB is still to be considered. This arc forms cycle with arcs AC and CB already in E_S so it cannot be selected. Thus we have to select from BD = 6, CE = 5, CF = 5. We may take either CE or CF. We select CF. Therefore, $V_S = \{A, C, B, F\}$ and $E_S = \{(A, C), (C, B), (C, F)\}$.
4. Now we have to select an arc from BD = 6, CE = 5, FE = 4, FG = 4. We select FE. Therefore, $V_S = \{A, C, B, F, E\}$ and $E_S = \{(A, C), (C, B), (C, F), (F, E)\}$.
5. The selection of arc CE is ruled out as it forms a cycle with the edges CF and FE. Thus, we have to select an arc from BD = 6, ED = 2, FG = 4. We select ED. Therefore, $V_S = \{A, C, B, F, E, D\}$ and $E_S = \{(A, C), (C, B), (C, F), (F, E), (E, D)\}$.
6. Now BD is ruled out as it forms cycle with CB, CF, FE and ED. Thus we have to consider DH = 2, EG = 3, FG = 4. We select DH. Therefore, $V_S = \{A, C, B, F, E, D, H\}$ and $E_S = \{(A, C), (C, B), (C, F), (F, E), (E, D), (D, H)\}$.
7. Now left over arcs are EG = 3, HG = 6, FG = 4. We select EG. Therefore, $V_S = \{A, C, B, F, E, D, H, G\}$ and $E_S = \{(A, C), (C, B), (C, F), (F, E), (E, D), (D, H), (E, G)\}$.

Since number of edges in E_S is seven process terminates here. The spanning tree so obtained is shown below.



Kruskal's Algorithm

Let $G = (V, E, \gamma)$ be graph and $S = (V_S, E_S, \gamma_S)$ be the spanning tree to be found from G. Let $|V| = n$ and $E = \{e_1, e_2, e_3, \ldots, e_m\}$. The stepwise algorithm is given below.

Step 1: Select an edge e_1 from E such that e_1 has least weight. Replace

$E = E - \{e_1\}$ and

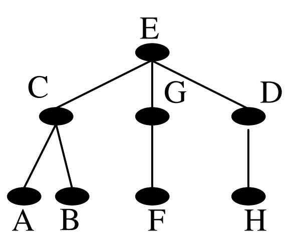
$E_S = \{e_1\}$
Step 2: Select an edge e_i from E such that e_i has least weight and that it does not form a cycle with members of E_S. Set

$E = E - \{e_i\}$ and
$E_S = E_S \cup \{e_i\}$
Step 3: Repeat step2 until $|E_S| = |V| - 1$.

Let us consider the same graph to demonstrate the algorithm.

1. Since arcs AC, ED and DH have minimum weight 2. Since they do not form a cycle, we select all of them and set E_S = {(A, C), (E, D), (D, H)} and E = E − {(A, C), (E, D), (D, H)}.
2. Next arcs with minimum weights 3 are AB, BC and EG. We can select only one of the AB and BC. Also we can select EG. Therefore, E_S = {(A, C), (E, D), (D, H), (B, C), (E, G)} and E = E − {(B, C), (E, G)}.
3. Next arcs with minimum weights 4 are EF and FG. We can select only one of them. Therefore, E_S = {(A, C), (E, D), (D, H), (B, C), (E, G), (F, G)} and E = E − {(F, G)}.
4. Next arcs with minimum weights 5 are CE and CF. We can select only one of them. Therefore, E_S = {(A, C), (E, D), (D, H), (B, C), (E, G), (F, G),(C, E)} and E = E − {(C, E)}.

Since number of edges in E_S is seven process terminates here. The spanning tree so obtained is as below.



Q.120 Show that is R_1 and R_2 are equivalence relations on A, then $R_1 \cap R_2$ is an equivalence relation. **(8)**

Ans: It is given that R_1 and R_2 are equivalence relation on set A. In order to prove that $R_1 \cap R_2$ is an equivalence relation on A, it is required to prove that $R_1 \cap R_2$ is reflexive, symmetric and transitive on A.

Reflexivity: Let $x \in A$ be any element, then pairs $\forall x$ (x, x) are in R_1 and R_2 and so all such pairs are in $R_1 \cap R_2$ also. Hence $R_1 \cap R_2$ is reflexive.

Symmetry: Let (x, y) be any pair in $R_1 \cap R_2$. Then (x, y) is in R_1 and in R_2. Since both the relations are symmetric Pair (y, x) is in Both R_1 and R_2. This implies that (y, x) is $R_1 \cap R_2$ and therefore it is symmetric as well.

Transitivity: Let (x, y) and (y, z) be any two pairs in $R_1 \cap R_2$. Therefore both these pairs are also in R_1 and R_2. Since both are transitive relation so (x, z) is in both R_1 and R_2. Hence (x, z) is in $R_1 \cap R_2$ and it is transitive. This shows that $R_1 \cap R_2$ is an equivalence relation.

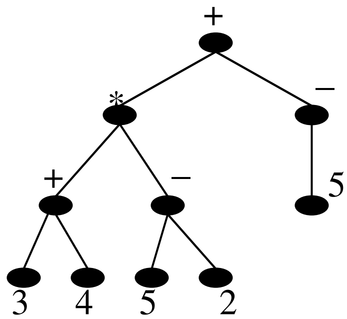
Q.121 Prove that the Digraph of a partial order has no cycle of length greater than 1. **(8)**

Ans: Suppose that there exists a cycle of length $n \geq 2$ in the digraph of a partial order $\leq$ on a set A. This implies that there are n distinct elements $a_1, a_2, a_3, \ldots, a_n$ such that $a_1 \leq a_2, a_2 \leq a_3, \ldots, a_{n-1} \leq a_n$ and $a_n \leq a_1$. Applying the transitivity n–1 times on $a_1 \leq a_2, a_2 \leq a_3, \ldots, a_{n-1} \leq a_n$, we get $a_1 \leq a_n$. Since relation $\leq$ is anti-symmetric $a_1 \leq a_n$ and $a_n \leq a_1$ together implies that $a_1 = a_n$. This is contrary to the fact that all $a_1, a_2, a_3, \ldots, a_n$ are distinct. Thus, our assumption that there is a cycle of length $n \geq 2$ in the digraph of a partial order relation is wrong.

Q.122 How do you traverse a Binary Tree? Explain Preorder, Inorder and Postorder traversals with example. **(10)**

Ans: Traversal of tree means tree searching for a purpose. The purpose may be for searching or sorting of the items contained in a tree. A tree may contain an item at its node as a label. Traversing a tree is a recursive procedure.

To implement this, a tree is considered to have three components: root, left subtree and right subtree. These three components can be arranged in six different ways: *(left, root, right), (root, left, right), (left, right, root), (right, left, root), (right, root, left) and (root, right, left).* The first three are used whereas the last three combinations are of no use as it alters the positions of a node in a positional tree.



Inorder Traversal: In this form of traversal, a tree is traversed in the sequence: Left subtre Root, Right subtree.

In the above expression, begin at the root node marked, +. Since first we have to traverse its left subtree, so move to the root of left subtree i.e. node marked, *. Again it has a left subtree with root node marked +, visit it. This subtree has a node labeled 3, which has no left subtree, so out put 3. Then root of this subtree i.e. '+' and then right subtree which is again a node labeled with 4, so output it. Thus we have expression obtained till here is 3 + 4. Proceeding this way we get **(3+4)*(5–2)+(-5)**. Parentheses signify both precedence and portion of the sub tree to which this sub-expression corresponds.

Preorder Traversal: In this form of traversal a tree is traversed in the sequence: Root, Left subtree, Right subtree. Apply the algorithm recursively till all nodes have been visited, we get **+ * + 3 4 – 5 2 –5**.

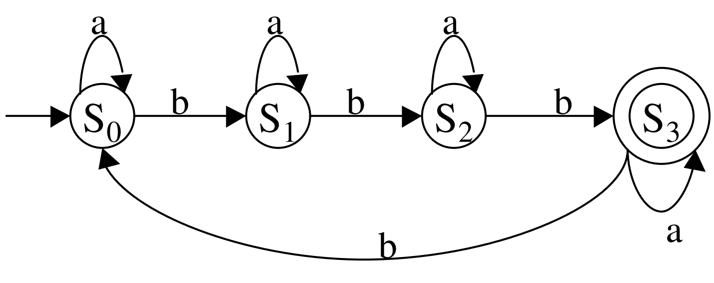
Postorder Traversal: In this form of traversal a tree is traversed in the sequence: Left subtree, Right subtree, Root. We get **3 4 + 5 2 – * 5 – +**.

Q.123 Consider the finite state machine whose state transition table is :

	a	b
S0	S0	S1
S1	S1	S2
S2	S2	S3
S3	S3	S0

Draw the graph for it. **(6)**

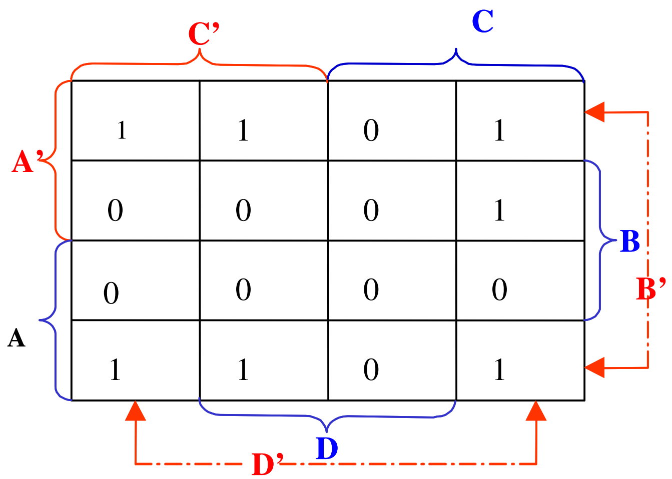
Ans: The graph for the automata as per the transition table is drawn below. Since no start state and no final state is specifies, using the convention, first state S_0 is shown as start state and last state S_3 is shown as final state.



Q.124 Simplify the Boolean function:

$$F = \overline{A}\overline{B}\overline{C} + \overline{B}C\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\overline{C}$$ **(8)**

Ans: The K-map for the given Boolean function: $\overline{A}\overline{B}\overline{C} + \overline{B}C\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\overline{C}$ is as below.



The simplified expression is sum of expressions for to two quads and one duet i.e.

$$\overline{B}\overline{C} + \overline{B}\overline{D} + \overline{A}C\overline{D}$$

Q.125 Is the following argument valid? If valid, construct a formal proof, if not explain why. "If wages increases, then there will be inflation. The cost of living will not increase if there is no inflation. Wages will increase. Therefore, the cost of living will increase.
(Let p = wages increase, q = inflation, r = cost of living will increase) **(8)**

Ans: The given statement can be represented using propositions and connectives as

$p \to q$, $\neg q \to \neg r$ and p.

Now, $p \to q$
p
----
q

And, contra positive statement corresponding to $\neg q \to \neg r$ is $r \to q$. Here $r \to q$ and q does not imply truthness of r. 'r' may be true or false for "$r \to q$ and q" to be true. Therefore it can not be said that "the cost of living will increase".

Q.126 A speaks truth in 80% of the cases and B speaks truth in 60% of the cases. Find the probability of the cases of which they are likely to contradict each other in stating the same fact. **(8)**

Ans:
Probability that A speaks **Truth: P(A=T)**= .8
Probability that A speaks **False: P(A=F)** = .2
Probability that B speaks **Truth**: **P(B=T)**= .6
Probability that A speaks **False: P(B =f)** = .4

Now probability that A and B likely to contradict is

P(A=T) x **P(B =F) + P(B=T)** x **P(A =F)**
=.8 x .4 + .6 x .2 = 32 + .12 = .44

Q.127 Let $A = \{-2,-1,0,1,2\}, B = \{0,1,4\}$ and $f : A \to B$ is defined as $f(x) = x^2$ is a function. If so find that whether it is one to one or bijection? **(8)**

Ans: For defined function **f: A→B** defined as f(x) = x^2 elements -2 and 2 have same image 4 in B. Similarly elements -1 and 1 of A has same image 1 in B. Thus f is not one to one. Next each element y of B is image of some element x of A hence f is onto function. Therefore f is neither one-to-one nor bijection but it is a surjective.

Q.128 Prove by induction
1.1! + 2.2! + …….+n.n! = (n+1)! – 1 **(8)**

Ans: The result is obvious for n = 1. When n = 1, we have 1.1! = 2! -1
Next suppose that the given result is true for n = k. i.e.

1.1! + 2.2! + 3.3! +…..+ k.k! = (k+1)! – 1

Now when n = k + 1, we get
1.1! + 2.2! + 3.3! +…..+ k.k! + (k + 1).(k+1)! = (k+1)! – 1+(k + 1).(k+1)!
= (k+1)!{1 + k +1} – 1
= (k+2)! – 1

This shows that whenever result is true for n = k, it is true for n = k + 1 as well. Hence by induction it true for all n.

Q.129 In the word 'MANORAMA'

(i) Find the number of permutations formed taking all letters.
(ii) Out of these the number of permutations with all A's together.
(iii) Find the number of permutations which start with A and end with M. **(8)**

Ans: In the word MANORAMA, M appears 2 times and A appears 3 times. All other character appears only once.

1. Number of permutations formed taking all letters = $\dfrac{8!}{2! * 3!} = 3360$.

2. When all A's are taken together, the three A's are counted as one. This results int number of 6 letters to be permuted. Number of permutations formed in this cas $\frac{6!}{2!} = 360$.
3. When permutations start with A and end in M, the six left over places are to be filled with 6 letters in which A has count 2. Number of permutations formed in this case = $\frac{6!}{2!} = 360$.

Q.130 Prove that a graph G is a tree iff G has no cycles and $|E| = |V| - 1$. **(8)**

Ans: Let G is tree then we have to prove that G has no cycle and it has |V| - 1 edge. Let |V| = n.

From the definition of a tree a root has indegree zero and all other nodes have indegree one. There must be (n – 1) incoming arcs to the (n – 1) non-root nodes. If there is any other arc, this arc must be terminating at any of the nodes. If the node is root, then its indegree will become one and that is in contradiction with the fact that root always has indegree zero. If the end point of this extra edge is any non-root node then its indegree will be two, which is again a contradiction. Hence there cannot be more arcs. Therefore, a tree of n vertices will have exactly (n – 1) edges and no cycle.

Next suppose that G has no cycle and it has n – 1 edge. Then we have to show that G is a tree.

A connected graph in n nodes having (n-1) edge is a tree.We prove that G is connected, then work is done.We prove it by induction.

For n=2 & |E| =1, the graph is connected if it is acyclic.

Let G_k be an acyclic graph with n=k & |E| = k-1 and it is connected. We add one node and arc e to G_k. If e is added between any two nodes of G_k, then it introduces a cycle in G_k. Thus to preserve the cyclic property of G_k & G_{k+1} e is added between any node of G_k to the new node. The modified graph G_{k+1} has k+1 nodes & k arcs.Thus Gn having n nodes & n-1 arcs is connected if it is acyclic.

Q.131 If A and B are two subsets of a universal set then prove that

$$A - B = A \cap B^C$$

Ans: In order to prove this let x be any element of (A – B) then

$$x \in A - B \Leftrightarrow x \in A \text{ and } x \notin B$$
$$\Leftrightarrow x \in A \text{ and } x \in B^C$$
$$\Leftrightarrow x \in A \cap B^C$$

This implies that

$A - B \subseteq A \cap B^C$ and

$A \cap B^C \subseteq A - B$

Thus $A - B = A \cap B^C$

Q.132 A man has four friends. In how many ways can he invite one or more friends to tea party? **(8)**

Ans: A friend may be called or may not be called. There are two possibilities for each frie Since one or more friends are to be invited, so total possible number of ways in which they ca be invited = $2^4 - 1 = 15$. 1 is subtracted from the possible way to remove the possibilities when none is invited.

Q.133 If R is a relation $N \times N$ defined by $^{(a,b)}R_{(c,d)}$ iff a + d = b + c, show that R is an equivalence relation. **(8)**

Ans: A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.

In order to prove that R is an equivalence relation, it needs to be proved that R is reflexive, symmetric and transitive.

***Reflexivity**:* Let (a, b) be any pair in N x N, then obviously a + b = b + a. This implies that $^{(a,b)}R_{(a,b)}$ $\forall(a, b) \in$ N x N. R is reflexive.

***Symmetry**:* For any pair (a, b) and (c, d) in N x N, if $^{(a,b)}R_{(c,d)}$ then

$$a + d = b + c \Leftrightarrow c + b = d + a \Leftrightarrow {}^{(c,d)}R_{(a,b)}.$$

R is symmetric as well.

***Transitivity**:*For any three pairs (a, b), (c, d) and (e, f) in N x N, if $^{(a,b)}R_{(c,d)}$ and $^{(c,d)}R_{(e,f)}$ then

$$a + d = b + c, \text{ and}$$

$$c + f = d + e \Leftrightarrow d + e = c + f$$

Subtracting second from first, we get,

$$a - e = b - f \Leftrightarrow a + f = b + e \Leftrightarrow {}^{(a,b)}R_{(e,f)}.$$

R is transitive also. Hence R is an **equivalence relation.**

Q.134 Minimize the Boolean expression (by algebraic method)

$F = \overline{A}C + \overline{A}B + A\overline{B}C + BC$ and then draw the circuit diagram using only NAND gate. **(8)**

Ans: The given expression can be minimized by algebraic method as follows:

$$\overline{A}C + \overline{A}B + A\overline{B}C + BC = \overline{A}C + \overline{A}B + A\overline{B}C + ABC + \overline{A}BC$$

$$= \overline{A}C + \overline{A}B + AC(\overline{B} + B) + \overline{A}BC \qquad [\textit{Distributive law}]$$

$$= \overline{A}C + \overline{A}B + AC + \overline{A}BC \qquad [\overline{B} + B = 1, \quad 1.A = A]$$

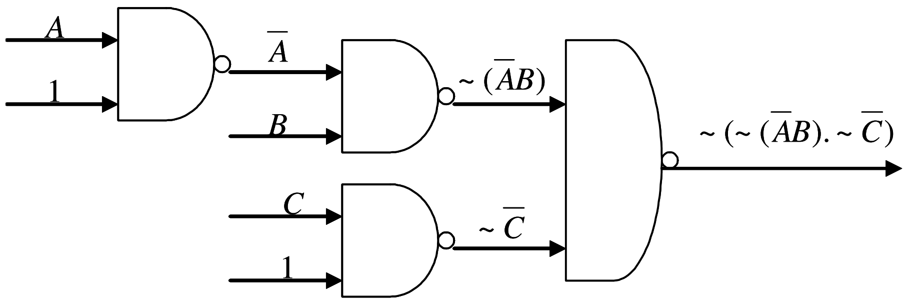
$$= (\overline{A} + A)C + \overline{A}BC + \overline{A}B \qquad [\textit{Distributive law}]$$

$$= C + \overline{A}BC + \overline{A}B \qquad [\overline{A} + A = 1, \quad 1.C = C]$$

$$= C + \overline{A}B \qquad [C + \overline{A}BC = C]$$

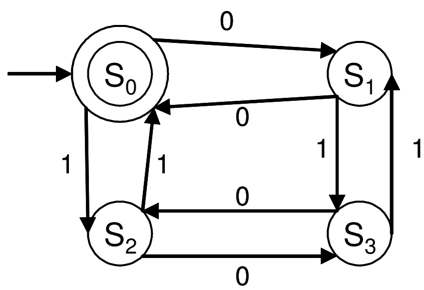
$$= \sim(\sim C \bullet \sim (\overline{A}B)) \qquad [\textit{Complement of complement}]$$

Now the circuit diagram using the NAND gate only is as below:



Q.135 Design an automaton which accepts only even numbers of 0s and even number of 1's. **(8)**

Ans: The required automata that accepts even number of 0's and even number of 1's is given below.



Q.136 Consider the context free grammar

$S \rightarrow SS+ \mid SS* \mid a$

(i) Show how the string aa + a* can be generated by this grammar.
(ii) Construct a parse tree for this string.
(iii) What language is generated by this grammar. **(8)**

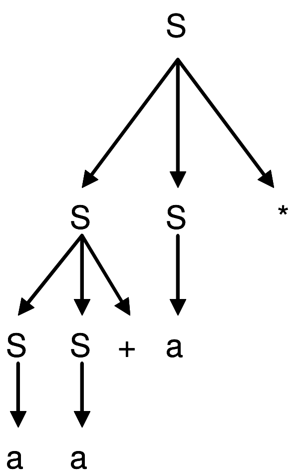
Ans: The production rule of the grammar can be numbered as

1. $S \rightarrow SS+$ 2. $S \rightarrow SS*$ 3. $S \rightarrow a$

(i)

$S \rightarrow SS*$	[Apply rule 2]
$\rightarrow Sa*$	[Apply rule 3]
$\rightarrow SS + a*$	[Apply rule 1]
$\rightarrow Sa + a*$	[Apply rule 3]
$\rightarrow aa + a*$	[Apply rule 3]

(ii) The parse tree for the above expression is drawn below.



(iii) Language generated by the grammar is L = {w | w = $a^n(+ \vee a \vee *)^n\ (+ \vee *)$}

Q.137 For any relation R in a set A, we can define the inverse relation R^{-1} by ${}^aR_b^{-1}$ iff ${}^aR_b$.
Prove that

(i) As a subset of A x A, $R^{-1} = \{(b,a)/(a,b) \in R\}$.

(ii) R is symmetric iff R = R^{-1}. **(8)**

Ans: As per the definition of the inverse of the relation (b, a) $\in R^{-1}$ iff (a, b) $\in$ R. Let (b, a) be any element of R^{-1} then (a, b) $\in$ R. This implies that a, b $\in$ A $\Rightarrow$ (b, a) $\in$ A x A and hence R^{-1} is subset of A x A i.e. $R^{-1} \subseteq$ A x A and R^{-1} = {(b, a) | (a, b) $\in$ R}
(ii) Let R is symmetric. For any pair (a, b) $\in R^{-1}$, (b, a) $\in$ R. since R is symmetric, (b, a) $\in$ R $\Rightarrow$ (b, a) $\in$ R. Thus, $R^{-1} \subseteq$ R. Conversely let (a, b) be any element in R, then by symmetry (b, a) $\in$ R and this $\Rightarrow$ (a, b) $\in R^{-1}$. i.e. (a, b) $\in$ R$\Rightarrow$ (a, b) $\in R^{-1}$. Thus, R$\subseteq R^{-1}$. Taken together it implies that R = R^{-1} .

Next let R = R^{-1} . By definition of inverse of relation then, for any pair (a, b) $\in$ R, we have (b, a) $\in$ R. Thus R is symmetric.

Q.138 Write Prim's Algorithm. **(6)**

Ans: Prim's algorithm to find a minimum spanning tree from a weighted graph in stepwise form is given below.

Let G = (V, E) be graph and S = (V_S, E_S) be the spanning tree to be found from G.

Step 1: Select a vertex v_1 of V and initialize

$V_S = \{v_1\}$ and
$E_S = \{\}$

Step 2: Select a nearest neighbor of v_i from V that is adjacent to some $v_j \in V_S$ and that edge (v_i, v_j) does not form a cycle with members edge of E_S. Set

$$V_S = V_S \cup \{v_i\} \text{ and}$$
$$E_S = E_S \cup \{(v_i, v_j)\}$$

Step 3: Repeat step2 until $|E_S| = |V| - 1$.

Q.139 Prove that A tree with *n* vertices has *(n – 1)* edges. **(6)**

Ans: From the definition of a tree a root has indegree zero and all other nodes have indegree one. There must be (n – 1) incoming arcs to the (n – 1) non-root nodes. If there is any other arc, this arc must be terminating at any of the nodes. If the node is root, then its indegree will become one and that is in contradiction with the fact that root always has indegree zero. If the end point of this extra edge is any non-root node then its indegree will be two, which is again a contradiction. Hence there cannot be more arcs. Therefore, a tree of n vertices will have exactly (n – 1) edges.

Q.140 Prove the De Morgan's Law by algebraic method

For every pair of elements x and y in A

(i) $\overline{(x+y)} = \bar{x} \cdot \bar{y}$

(ii) $\overline{(x \cdot y)} = \bar{x} + \bar{y}$ **(10)**

Ans:

(i) For any two variables a and b, whenever a .b' = 0 and a + b' = 1, we say that **a** and **b** are equal. Substituting **b** with (x + y)' and **a** with x'.y' in the LHS of the equation a .b' = 0, we get,

(x'.y'). ((x + y)')'
=(x'.y').(x + y) [Double Negation]
=((x'.y').x) + ((x'.y').y) [Distribution law]
=(x'.x.y') + (x'.y'.y) [Associative and commutative law]
=(0.y')+(x'.0) [Complementation]
=0 + 0 = 0

Similarly, substituting **b** with (x + y)' and **a** with x'.y' in the LHS of the equation a + b' = 1, we get

(x'.y') + ((x + y)')'
=(x'.y') + (x + y) [Double Negation]

$=((x'.y') + x) + ((x'.y') + y)$ [Distribution law]
$=((x'+x).(y'+ x)) + ((x'+y).(y'+ y))$ [Distribution law]
$=(1.(y'+ x)) + ((x'+y).1))$ [Complementation]
$=(y' + y) +(x' + x)$ [Associative and commutative law]
$=1 + 1$ [Complementation]
$=1$
Therefore $(x +y)' = x'.y'$

(ii) This can also be proved by substituting **b** with $(x.y)'$ and **a** with $x' + y'$ in the LHS of the equation $a .b' = 0$ and $a + b' = 1$.

Q.141 Find the equivalent formula for $(P \to Q) \vee R$ that contains $\uparrow$ only. **(6)**

Ans: The binary connective corresponding to NAND is called the Sheffer stroke, and written with a vertical bar | or vertical arrow ↑. Since it is complete on its own, all other connectives can be expressed using only the stroke. For example,:

$P \to Q \equiv P\uparrow{\sim}Q$
$P \vee Q \equiv {\sim}P\uparrow{\sim}Q$
$P \,\&\, Q \equiv {\sim}(P\uparrow Q)$

Using the concept, the given formula: $(P \to Q) \vee R$, can be written using Sheffer stroke ↑, as

$(P \to Q) \vee R = (P\uparrow{\sim}Q) \vee R$
$= {\sim} (P\uparrow{\sim}Q) \uparrow{\sim} R$

Q.142 Find the number of ways in which 5 prizes can be distributed among 5 students such that
(i) Each student may get a prize.
(ii) There is no restriction to the number of prizes a student gets. **(8)**

Ans :
(i) Number of ways in which 5 prizes can be distributed among 5 students in such a way that each one may get a prize is 5*4*3*2*1 = 5! ways.
(ii) When no restriction on the number of prizes a student may get, then a student may get either 0 or 1 or 2 or…upto 5 prizes. It shows that a student have six possible ways in which it may get a prize. Therefore the number of ways in which prizes can be distributed is 6^5.

Q.143 What is equivalence relation? Prove that relation 'congruence modulo' ($\equiv$mod m) is an equivalence relation. **(8)**

Ans: A relation R defined on a nonempty set A is said to be an equivalence relation if R is Reflexive, Symmetric and Transitive on A.

Any integer **x** is said to 'congruence modulo **m**' another integer **y**, if both **x** and **y** yield the same remainder when divided by m. Let R be the relation 'congruence modulo **m**' over set of integers Z.

Reflexivity: Let $x \in Z$ be any integer, then $x \equiv_m x$ since both yield the same remainder w divided by m. Thus, $(x, x) \in R \ \forall \ x \in Z$. This proves that R is a reflexive relation.

Symmetry: Let x and y be any two integers and $(x, y) \in R$. This shows that $x \equiv_m y$ and hence $y \equiv_m x$. Thus, $(y, x) \in R$. Hence R is a symmetric relation also.

Transitivity: Let x, y and z be any three elements of Z such that (x, y) and $(y, z) \in R$. Thus, we have $x \equiv_3 y$ and $y \equiv_m z$. It implies that (x–y) and (y–z) are divisible by m. Therefore, $(x - y) + (y - z) = (x - z)$ is also divisible by m i.e. $x \equiv_m z$. Hence, (x, y) and $(y, z) \in R \Rightarrow (x, z) \in R$. So R is a transitive relation.

Therefore, R is **an equivalence relation**.

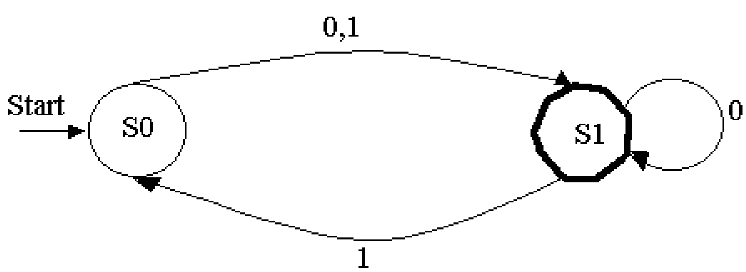
Q.144 Show that $x^n - y^n$ is divisible by (x - y) for all positive integral values of n. **(8)**

Ans: For n = 1, the result is obvious. Let it is true for n = k i.e. $(x^k - y^k)$ is divisible by (x – y). Now for n = k + 1, we have

$$(x^{k+1} - y^{k+1}) = (x - y)(x^k + x^{k-1}y + x^{k-2}y^2 + \ldots + y^k)$$

Since (x – y) is a factor in $(x^{k+1} - y^{k+1})$, it is divisible by (x – y).

Q.145 Specify the transition function of the DFA shown below and describe the language. **(8)**



Ans: The transition state table for the given DFA is:

State	0	1
S_0	S_1	S_1
S_1	S_1	S_0

The language accepted by this DFA is all strings w in (0, 1)* of the form

$$[(0\vee 1)0^{*}1]^{*}(0\vee 1)\,0^{*}$$

Q.146 Simplify the given expression

$$AB + (AC)' + AB'C(AB + C)$$ **(8)**

Ans: The given Boolean expression is AB + (AC)' + AB'C(AB + C). Let us use the algebraic method to simplify it.

AB + (AC)' + AB'C(AB + C)

= AB + (AC)' + AB'CAB +AB'C C [Distributive law]
= AB + (AC)' + AB'BC +AB'C [Associative &Absorption law]
= AB + (AC)' + AB'C [AB'BC = 0]
= ABC' + ABC + (AC)' + AB'C [AB = AB(C' + C)]
= ABC' + (AC)' + AC (B' + B) [Distributive law]
= ABC' + (AC)' + AC
= ABC' + 1
= 1

Q.147 A fair six sided die is tossed three times and the resulting sequence of numbers is recorded. What is the probability of the event E that either all three numbers are equal or none of them is a 4? **(8)**

Ans: Let us first count the possible number of ways of getting all three number equal or none of them is four. The ways in which we can get all three numbers equal is

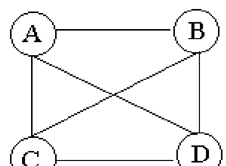
6x1x1 = 6

And the number of ways in which we can get numbers such that none of them is four is

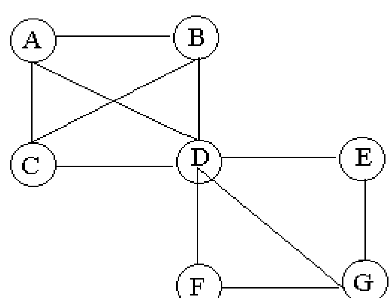
5x5x5 = 125

Thus the required probability is = $\frac{6+125}{6^3} = \frac{131}{216}$

Q.148 Define Euler Circuit and Euler Path. Which of the following graphs have an Euler circuit and Euler path. **(8)**

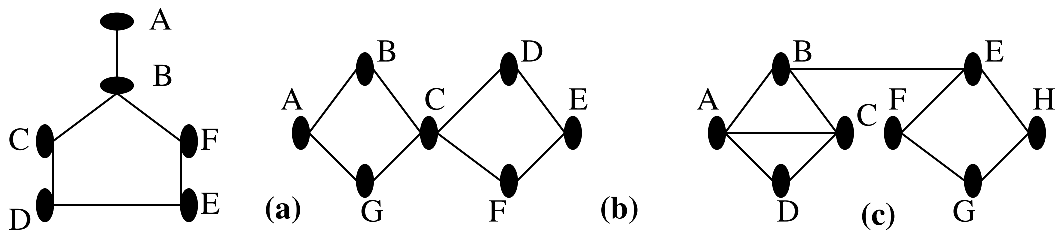


(i)



(ii)

Ans: In a graph G, a path is called an ***Euler path*** if it contains every edge of the graph exac once. An Euler path that is circuit is called an ***Euler circuit***. In the following, for example, (a) has an Euler path but no Euler circuit, (b) has both Euler circuit and Euler path whereas (c) has none.



A graph G, having more than two vertices of odd degrees, does not possess an Euler path. Since the given graph (i) and (ii) both contains more than two nodes of odd degree, neither (i) nor (ii) has any Euler path or Euler circuit.

Q.149 Prove that if a set A contains n elements, then P(A) contains 2^n elements. Also find the number of proper subsets of the set of letters UTTAR PRADESH. **(6+2=8)**

Ans: Let |A| =n. In an subset of the set A, either an element x is present or absent. Then in any subset an element can be selected in 2 ways and therefore there can be total of 2^n possible ways of selecting n element from the set A. Each such selection forms a subset of A. Therefore there can be 2^n possible subsets of A i.e. P(A) has 2^n elements.
The set containing letters from UTTARPRADESH is {U, T, A, R, P, D, E, S, H} and it has 9 elements. The number of proper subsets is equal to total number of subsets except the set itself and NULL set. Thus required number is = $2^9 - 2 = 510$.

Q.150 Prove that Prim's algorithm produces a minimum spanning tree of a connected weighted graph. **(8)**

Ans:
Let *G* be a connected, weighted graph. At every iteration of Prim's algorithm, an edge must be found that connects a vertex in a subgraph to a vertex outside the subgraph. Since *G* is connected, there will always be a path to every vertex. The output *T* of Prim's algorithm is a tree, because the edge and vertex added to *T* are connected. Let T_1 be a minimum spanning tree of G.

If $T_1=T$ then *T* is a minimum spanning tree. Otherwise, let *e* be the first edge added during the construction of *T* that is not in T_1, and *V* be the set of vertices connected by the edges added before *e*. Then one endpoint of *e* is in *V* and the other is not. Since T_1 is a spanning tree of *G*, there is a path in T_1 joining the two endpoints. As one travels along the path, one must encounter an edge *f* joining a vertex in *V* to one that is not in *V*. Now, at the iteration when *e* was added to *T*, *f* could also have been added and it would be added instead of *e* if its weight was less than *e*. Since *f* was not added, we conclude that

$$w(f) \geq w(e).$$

Let T_2 be the graph obtained by removing f and adding e from T_1. It is easy to show that T_2 connected, has the same number of edges as T_1, and the total weights of its edges is not larger than that of T_1, therefore it is also a minimum spanning tree of G and it contains e and all the edges added before it during the construction of V. Repeat the steps above and we will eventually obtain a minimum spanning tree of G that is identical to T. This shows T is a minimum spanning tree.

Q.151 Prove that, the complement of every element in a Boolean algebra B is unique. **(8)**

Ans: ***Proof:*** Let I and 0 are the unit and zero elements of B respectively. Let b and c be two complements of an element $a \in B$. Then from the definition, we have

$a \wedge b = 0 = a \wedge c$ and
$a \vee b = I = a \vee c$

We can write $b = b \vee 0 = b \vee (a \wedge c)$
$= (b \vee a) \wedge (b \vee c)$ [since lattice is distributive]
$= I \wedge (b \vee c)$
$= (b \vee c)$

Similarly, $c = c \vee 0 = c \vee (a \wedge b)$
$= (c \vee a) \wedge (c \vee b)$ [since lattice is distributive]
$= I \wedge (b \vee c)$ [since $\vee$ is a commutative operation]
$= (b \vee c)$

The above two results show that $b = c$.

Q.152 Prove that $A - (B \cap C) = (A - B) \cup (A - C)$. **(8)**

Ans: In order to prove this let x be any element of $A - (B \cap C)$, then

$x \in A - (B \cap C) \Leftrightarrow x \in A$ and $x \notin (B \cap C)$
$\Leftrightarrow x \in A$ and $[x \notin B$ or $x \notin C]$
$\Leftrightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$
$\Leftrightarrow (x \in A - B)$ or $(x \in A - C)$
$\Leftrightarrow x \in (A - B) \cup (A - C)$

This implies that

$A - (B \cap C) \subseteq (A - B) \cup (A - C)$ and
$(A - B) \cup (A - C) \subseteq A - (B \cap C)$

Thus $A - (B \cap C) = (A - B) \cup (A - C)$

Q.153 Test the validity of argument:

"If it rains tomorrow, I will carry my umbrella, if its cloth is mended. It will rain tomorrow and the cloth will not be mended. Therefore I will not carry my umbrella" **(8)**

Ans: Let p : 'It will rain tomorrow' ; q : 'I will carry my umbrella'; r **:** 'Cloth of umbrella mended' Now the given statement "If it rain tomorrow, I will carry my umbrella, if its cloth is mended" can be written as $(p \wedge r) \rightarrow q$. "It will rain tomorrow" is written as p and " cloth will not be mended" is written $\neg r$. Thus, we have

$(p \wedge r) \rightarrow q$
p and
$\neg r$ together $\Rightarrow (p \wedge r) \rightarrow q$, p , $\neg r$
$\Rightarrow [\neg(p \wedge r) \vee q]$, p , $\neg r$ $\quad [(\neg p \vee \neg r) \vee q), p \Rightarrow (r \rightarrow q)]$
$\Rightarrow (r \rightarrow q)$, $\neg r$ $\quad [(r \rightarrow q), \neg r \Rightarrow \neg q \rightarrow \neg r, \neg r \Rightarrow \neg q]$
$\Rightarrow \neg q$

That is "I will not carry my umbrella" is valid.

Q.154 A valid identifier in the programming language FORTAN consists of a string of one to six alphanumeric characters (the 36 characters A,B,….,Z,0,1,…9) beginning with a letter. How many valid FORTRAN identifiers are there? **(8)**

Ans: There are 26 letters and 10 digits a total of 36 characters. It is restricted that an identifier can be of 1 to 6 character long only, and first character must be a letter. Thus, Total number of identifiers of length 1 = 26
Total number of identifier of length 2 = 26*36
Total number of identifier of length 3 = $26*36^2$
Total number of identifier of length 4 = $26*36^3$
Total number of identifier of length 5 = $26*36^4$
Total number of identifier of length 6 = $26*36^5$
Thus number of valid identifiers = $26*[1 + 36 + 36^2 + 36^3 + 36^4 + 36^5]$ = 1617038306.

Q.155 Let R be a relation on a set A. Then prove that R^∞ is the transitive closure of R. **(8)**

Ans: Transitive closure of a relation R is the smallest transitive relation S that contains R. Transitive closure of R is denoted by R^∞. R^∞ is defined as the union of different powers of R i.e. $R^\infty = \bigcup_{i \in w} R^i$, where are distinct powers of R such that none of them are repeated and W = {x : R^x is distinct power of R}. It is to prove that the relation R^∞ is transitive and contains *R*. Furthermore, any transitive relation containing *R* must also contain R^∞.

Let *A* be any set of elements. Suppose $\exists$ S transitive relations such that $R \subseteq S$ and $R^\infty \not\subseteq S$. So, $\exists$ $(a, b) \notin S$ and $(a, b) \in R^\infty$. It means that particular $(a, b) \notin R$.

Now, by definition of R^∞, we know that $\exists\, n \in W$ such that $(a, b) \in R^n$.

Then, $\forall i, i < n \Rightarrow e_i \in A$ and, there is a path from *a* to *b* like this: $\mathbf{aRe_1 R...Re_{(n-1)} Rb}$.

But, by transitivity of S on R, $\forall i \in W$, $i < n \Rightarrow (a, e_i) \in S$, …. $(a, e_{(n-1)}) \in S$ and $(e_{(n-1)}, b) \in S$. By transitivity of S, we get $(a, b) \in S$. **A Contradiction of $(a, b) \notin S$.**

Therefore, $\forall (a, b) \in A\times A$, $(a, b) \in R^{\infty} \Rightarrow (a, b) \in S$. This means that $R^{\infty} \subseteq S$, for any transitive S containing R. So, R^{∞} is the **smallest** transitive relation containing R.

Q.156 Show that $x^n - y^n$ is divisible by (x - y) for all positive integral values of n. **(8)**

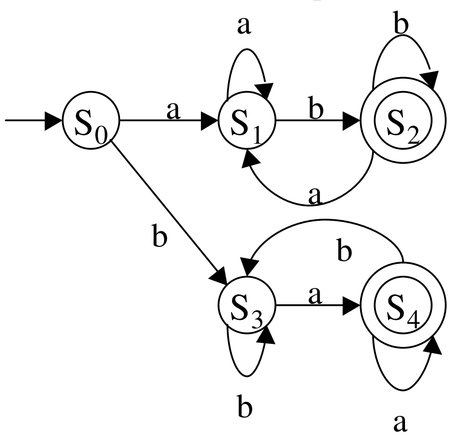
Ans: For n = 1, the result is obvious. Let it is true for n = k i.e. $(x^k - y^k)$ is divisible by (x – y). Now for n = k + 1, we have

$(x^{k+1} - y^{k+1}) = (x - y)((x^k + x^{k-1}y + x^{k-2}y^2 + \ldots + y^k)$

Since (x – y) is a factor in $(x^{k+1} - y^{k+1})$, it is divisible by (x – y).

Q.157 Build a Fine Automaton that accept all words that have different first and last letters (i.e. if the word begins with an "a" to be accepted it must end with "b" and vice versa.) **(8)**

Ans: The Finite state automata for the specified string is as below.



Q.158 How many relations are possible from a set A of 'm' elements to another set B of 'n' elements? **(8)**

Ans: A relation R from a set A to another set B is defined as any subset of A x B. *If* $|A| = m$ *and* $|B| = n$, *then* $|A \times B| = mn$ *and so number of possible subsets of* $A \times B = 2^{mn}$. Each subset is a relation from A to B. Thus there are 2^{mn} different types of relations from A to B.

Q.159 What is Partially Ordered Set? Let $S = \{a, b, c\}$ and $A = P(S)$. Draw the Hasse diagram of the poset A with the partial order $\subseteq$ (set inclusion). **(8)**

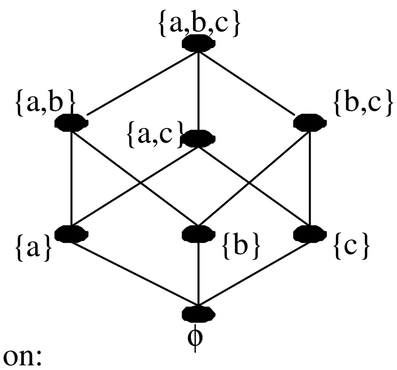
Ans: Let R be a relation defined on a non-empty set A. The mathematical structure (A, R) is set to be a *Partial order set* or **poset** if the relation R is a partial order relation on A.

Any relation R defined on a non-empty set A is said to be a Partial Order Relation**,** if R is

- Reflexive on A i.e., xRx $\forall$ x$\in$ A

- Anti-symmetric on A i.e., xRy and yRx $\Rightarrow$ x = y and
- Transitive on A i.e., xRy and yRz $\Rightarrow$ xRz for x, y, z $\in$ A.

A partial order relation is denoted by the symbol '$\leq$'. A general notation for a poset is (A, $\leq$), where A is any non-empty set and '$\leq$' is any partial order relation defined on the set A. The Hasse diagram for the poset (P(S), $\subseteq$) is as below. The poset has 8 elements – 8 possible subsets of S. Null set is the minimum element and S itself is the maximal element..



Q.160 Write short note on:

(i) Pigeonhole Principle
(ii) Graph Isomorphism
(iii) Tree Traversal
(iv) Types of Grammar **($4 \times 4 = 16$)**

Ans:

(i) "If **five** men (pigeonholes) are married to **six** women (pigeons) then at least one men has more than one wife."

Or

"If **five** men (pigeons) are married to **four** women (pigeonholes) then at least one woman has more than one husband"

Or

"If we have **six** pigeons in **five** pigeonholes then at least one pigeonhole would have more than one pigeons"

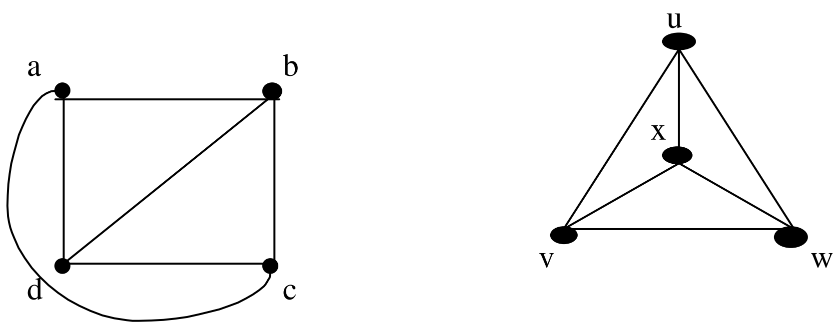
Arguments of this kind are used in solution to many mathematical problems and quite a few beautiful theorems have been proven with their help. All these arguments share the name "**Pigeonhole Principle**". In general

If **n** objects (pigeons) are placed in **m** places (pigeonholes) for m < n, then one of the places (pigeonholes) must contains at least

$$\left\lfloor \frac{n-1}{m} \right\rfloor + 1$$

objects (pigeons).This so called '**pigeonhole principle'** is very simple. Its application, however, is very intelligence intensive. It needs skill, to identify the pigeons, pigeonholes and their respective numbers. Once you have these figures, you have almost found the solution to the problem.

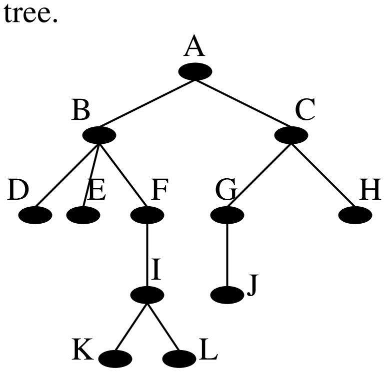
(ii) Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic graphs iff $\exists$ func f: $V_1 \rightarrow V_2$ and g: $E_1 \rightarrow E_2$ such that both f and g are bijection and the incidences are preserve i.e. if there is an edge **e** between nodes u and v of V_1 then g(**e**) should be an edge between f(u) and f(v) of V_2. The following two graphs are isomorphic graphs.



Testing of graph isomorphism, in most of the time, is quite simple contrary to the theoretical difficulties involved. It is practically not possible to check for all possible functions f: $V_1 \rightarrow V_2$. For example, if there are 20 vertices in a graph then there are 20! such functions. The easiest way is to look for the fixed parameters like number of nodes, number of edges and ***degree spectrum*** of graphs. A degree spectrum is defined as the list of degrees of all nodes in a graph. Two graphs cannot be isomorphic if any of these parameters are not matching. However, it cannot be said that two graphs will always be isomorphic if these parameters are matching.

(iii) Traversal of tree means tree searching for a purpose. The purpose may be for searching or sorting of the items contained in a tree. A tree may contain an item at its node as a label. Traversing a tree is a recursive procedure. To implement this, a tree is considered to have three components: root, left subtree and right subtree. These three components can be arranged in six different ways: (*left, root, right), (root, left, right), (left, right, root), (right, left, root), (right, root, left) and (root, right, left).* The first three are used whereas the last three combinations are of no use as it alters the positions of a node in a positional tree.

There are other two important types of tree traversal: Breadth First Search (BFS) and Depth First Search (DFS). These traversal methods we can use for both tree and graph. Consider the following labeled tree.



In the BFS traversal a tree is traversed in the order of level of nodes. At a particular level nodes are traversed from left to right. First visit the root node. Then go to the first level nodes and visit all nodes from left to right. Then go to the second level and visit all nodes from left to right and so on until all nodes have been visited. The algorithm is stated as "*visit all successors of a visited node before visiting any successors of any of those successors.* The sequence of node is **A B C D E F G H I J K L**. A DFS traversal of a tree is similar to the preorder traversal of a tree.

(iv) Any language is suitable for communication provided the syntax and semantic of the language is known to the participating sides. It is made possible by forcing a standard on the way to make sentences from words of that language. This standard is forced through a set of rules. This set of rules is called **grammar** of the language. A grammar G = (N, Σ, P, S) is said to be of

Type 0: if there is no restriction on the production rules i.e., in $\alpha \to \beta$, where $\alpha, \beta \in (N \cup \Sigma)^*$. This type of grammar is also called an ***unrestricted grammar***.

Type 1: if in every production $\alpha \to \beta$ of P, $\alpha, \beta \in (N \cup \Sigma)^*$ and $|\alpha| \le |\beta|$. Here $|\alpha|$ and $|\beta|$ represent number of symbols in string α and β respectively. This type of grammar is also called a ***context sensitive*** grammar (or ***CSG***).

Type 2: if in every production $\alpha \to \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \Sigma)^*$. Here α is a single non-terminal symbol. This type of grammar is also called a ***context free*** grammar (or ***CFG***).

Type 3: if in every production $\alpha \to \beta$ of P, $\alpha \in N$ and $\beta \in (N \cup \Sigma)^*$. Here α is a single non-terminal symbol and β may consist of at the most one non-terminal symbol and one or more terminal symbols. The non-terminal symbol appearing in β must be the extreme right symbol. This type of grammar is also called a ***right linear*** grammar or ***regular*** grammar (or ***RG***).

Q.161 Shirts numbered consecutively from 1 to 20 are worn by 20 members of a bowling league. When any three of these members are chosen to be a team, the league proposes to use the sum of their shirt numbers as the code number for the team. Show that if any eight of the 20 are selected, then from these eight one may form at least two different teams having the same code number. **(8)**

Ans: When a team consisting of three persons is selected and number inscribed on the shirt is added, the possible minimum number is (1+ 2 + 3 =) 6 and the maximum number is (18 + 19 + 20 =) 57. Thus a team of three can have a code number from the possible range of 52 codes from 6 to 57 both inclusive. Now after selecting 8 from 20 members, any three out of 8 can be selected in ${}^{8}C_{3} = 56$.
Now using the Pigeon Hole principle, let 56 pigeons are placed into 52 holes marked with codes between 6 and 57, then there are at least two teams will be in the same hole, implying that these two teams will have the same code number.

Q.162 In a group of athletic teams in a certain institute, 21 are in the basket ball team, 26 in the hockey team, 29 in the foot ball team. If 14 play hockey and basketball, 12 play foot ball and basket ball, 15 play hockey and foot ball, 8 play all the three games.

(i) How many players are there in all?

(ii) How many play only foot ball? **(4+4)**

Ans: Let A, B and C is set of players who play Basket ball, Hockey and Foot ball respectively. Thus

$|A| = 21$, $|B| = 26$, $|C| = 29$,

$|A\cap B| = 14$, $|A\cap C| = 12$, $|B\cap C| = 15$, and

$|A\cap B\cap C| = 8$.

Now for part (i), we have to find $|A\cup B\cup C|$. This can be computed using the formula

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$
$$= 21 + 26 + 29 - 14 - 12 - 15 + 8 = 84 - 41 = 43.$$

For part (ii) we have to find number of players playing only football. This is

$$= |C| - |A\cap C| - |B\cap C| + |A\cap B\cap C| = 29 - 12 - 15 + 8 = 10.$$

Q.163 Let A = {a,b,c,d,e} and R={(a,a), (a,b), (b,c), (c,e), (c,d), (d,e)}

Compute (i) R^2 and (ii) R^∞ **(8)**

Ans: The given relation can be represented by 5x5 Boolean matrix as below:

$$R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \textit{The square of } R \textit{ is given by } R^2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii) R^∞ is called transitive closure of relation R and is obtained by finding the successive powers of R. the process terminates when two consecutive powers of R become equal i.e. $R^n = R^{n+1}$. Let us find the successive powers of R.

$$R^3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R^4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R^5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since $R^4 = R^5$, The process is stopped and R^∞ is obtained by adding all distinct ma obtained so far including R. Thus $R^\infty = R + R^2 + R^3 + R^4$ i.e.

$$R^\infty = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

When translated into set of ordered pairs, it is

{(a, a), (a, b), (a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, d), (c, e), (d, e)}

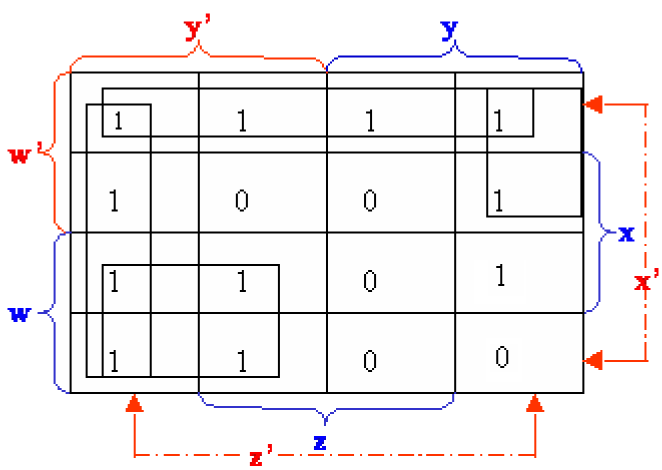
Q.164 Simplify the Boolean function:

$F(w,x,y,z) = \sum (0, 1, 2, 3, 4, 6, 8, 9, 12, 13, 14)$ **(8)**

Ans:

$$f(w, x, y, z) = \sum(0, 1, 2, 3, 4, 6, 8, 9, 12, 13, 14)$$

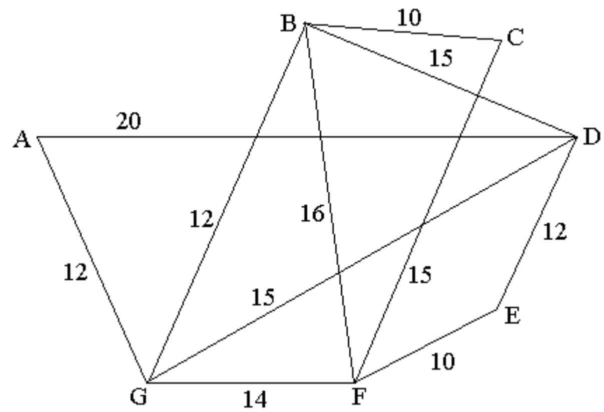
The function is of four Boolean variables. The K-Map for this function is given as in figure below. Minterms of four variables having decimal equivalent 0, 1, 2, 3, 4, 6, 8, 9, 12, 13 and 14 produce **1** as output for the function, so squares corresponding to minterms 0000, 0001, 0010, 0011, 01000, 0110, 1000, 1001, 1100, 1101 and 1110 contain an entry 1. The remaining squares contain 0 as an entry.



There is five quads: three marked with closed rectangles and one wrapped around w' and z of four corners. Therefore the given function can be expressed in simplified form as

f(w, x, y, z) = **y'z' + w'x' + wy' + w'z' + xz'**

Q.165 Use Kruskal's algorithm to find a minimal spanning tree for the following graph. **(10)**



Ans: Let G = (V, E) be graph and S = (V_S, E_S) be the spanning tree to be found from G. Let |V| = n and E = {e_1, e_2, e_3, …, e_m}. The stepwise Kruskal's Algorithm to find minimum spanning tree is outlined below.

Step 1: Select an edge e_1 from E such that e_1 has least weight. Replace

$E = E - \{e_1\}$ and
$E_S = \{e_1\}$

Step 2: Select an edge e_i from E such that e_i has least weight and that it does not form a cycle with members of E_S. Set

$E = E - \{e_i\}$ and
$E_S = E_S \cup \{e_i\}$

Step 3: Repeat step2 until $|E_S| = |V| - 1$.

In the given graph there are 7 nodes so 6 minimum weight edges are to be selected so that they do not form a cycle. There are 2 edges of weight 10, 3 of 12 and one of 14. They do not form cycle within themselves. Thus, edges in spanning tree are:{(F, E), (B, C), (E, D), (B, G), (A, G), (F, G)}.

Q.166 How do you find the second minimum spanning tree of a graph? Find the second minimum spanning tree of the above graph. **(6)**

Ans: The second minimum spanning tree is obtained by replacing the maximum weight e the first minimum spanning tree with next highest weight edge in the graph. In the given gra the second minimum spanning tree is obtained by replacing (F, G) with any one of the edge (C, F), (B, D), (D, G). Thus one of the second minimum spanning tree is {(F, E), (B, C), (E, D), (B, G), (A, G), (C, F)}.

Q.167 Prove that for every pair of elements x and y in A (using algebraic method).

(i) $(x+y)' = x'.y'$

(ii) $(x.y)' = x'+y'$ **(10)**

Ans:

(i) For any two variables a and b, whenever $a .b' = 0$ and $a + b' = 1$, we say that **a** and **b** are equal. Substituting **b** with $(x + y)'$ and **a** with $x'.y'$ in the LHS of the equation $a .b' = 0$, we get,

$(x'.y'). ((x + y)')'$

$=(x'.y').(x + y)$ [Double Negation]

$=((x'.y').x) + ((x'.y').y)$ [Distribution law]

$=(x'.x.y') + (x'.y'.y)$ [Associative and commutative law]

$=(0.y')+(x'.0)$ [Complementation]

$=0 + 0 = 0$

Similarly, substituting **b** with $(x + y)'$ and **a** with $x'.y'$ in the LHS of the equation $a + b' = 1$, we get

$(x'.y') + ((x + y)')'$

$=(x'.y') + (x + y)$ [Double Negation]

$=((x'.y') + x) + ((x'.y') + y)$ [Distribution law]

$=((x'+x).(y'+ x)) + ((x'+y).(y'+ y))$ [Distribution law]

$=(1.(y'+ x)) + ((x'+y).1))$ [Complementation]

$=(y' + y) +(x' + x)$ [Associative and commutative law]

$=1 + 1$ [Complementation]

$=1$

Therefore $(x +y)' = x'.y'$

(ii) This can also be proved by substituting **b** with $(x.y)'$ and **a** with $x' + y'$ in the LHS of the equation $a .b' = 0$ and $a + b' = 1$.

Q.168 Prove that $(p \leftrightarrow q) \leftrightarrow r = p \leftrightarrow (q \leftrightarrow r)$ **(6)**

Ans: This can also be proved using algebraic method. Here one other way using the truth table is provided. Since the given expression is a tautology, it is proved that LHS is RHS.

p	q	r	p↔q	(p↔q)↔r	q↔r	p↔(q↔r)	(p↔q)↔r = p↔(q↔r)
0	0	0	1	0	1	0	1
0	0	1	1	1	0	1	1
0	1	0	0	1	0	1	1
0	1	1	0	0	1	0	1
1	0	0	0	1	1	1	1
1	0	1	0	0	0	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

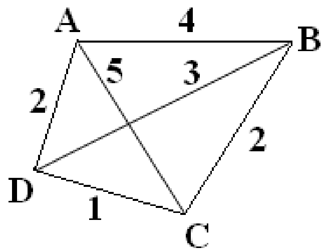
Q.169 Find the number of ways in which 5 prizes can be distributed among 5 students such that
(i) Each student may get a prize
(ii) There is no restriction to the number of prizes a student gets. **(8)**

Ans :

(i) Number of ways in which 5 prizes can be distributed among 5 students in such a way that each one may get a prize is 5*4*3*2*1 = 5! ways.

(ii) When no restriction on the number of prizes a student may get, then a student may get either 0 or 1 or 2 or…upto 5 prizes. It shows that a student have six possible ways in which it may get a prize. Therefore the number of ways in which prizes can be distributed is 6^5.

Q.170 For the given graph, find the minimal spanning tree using Prim's algorithm. **(8)**



Ans: Let G = (V, E) be graph and S = (V_S, E_S,) be the spanning tree to be found from G. The Prim's algorithm is given as below:

Step 1: Select a vertex v_1 from V and initialize

$V_S = \{v_1\}$ and
$E_S = \{\}$

Step 2: Select a nearest neighbor of v_i from V that is adjacent to some $v_j \in V_S$ and that edge (v_i, v_j) does not form a cycle with members edge of E_S. Set

$V_S = V_S \cup \{v_i\}$ and
$E_S = E_S \cup \{(v_i, v_j)\}$
Step 3: Repeat step2 until $|E_S| = |V| - 1$.

Using the algorithm and starting from node **A**, we get the following edges (A, C), (C, D), (D, B) for minimum spanning tree.

Q.171 Show that $n^3 + 2n$ is divisible by 3. **(8)**

Ans: Let us prove this using mathematical induction. For n =1, the result is obvious as $n^3 + 2n = 3$ and it is divisible by 3.

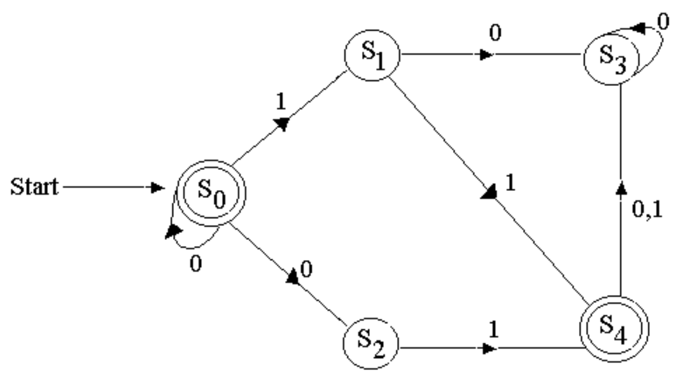
Let us assume that it is true for n = k, i.e. $(k^3 + 2k)$ is divisible by 3. We now show that it is true for n = k + 1 i.e. $((k+1)^3 + 2(k+1))$ is divisible by 3.

$(k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + 2k + 2$

$= (k^3 + 2k) + 3(k^2 + k + 1)$

Since both term are individually divisible by 3, sum is also divisible by 3.

Q.172 Find the state table for the NFA with the state diagram given below. Find the language recognized by it. **(8)**



Ans: The transition state table for the given NFA is:

State	0	1
S_0	{ S_0, S_2}	{ S_1}
S_1	{ S_3}	{ S_4}
S_2	{ }	{ S_4}
S_3	{ S_3, S_4}	{ S_4}
S_4	{ }	{ }

The language accepted by this NFA is all strings w except '10' in (0, 1)* that contains at the most two 1's.

Q.173 Find the number of ways in which we can put *n* distinct objects into two identical boxes so that no box remains empty. **(8)**

Ans: This problem of counting can be solved using generating function concept, Let x and y be the number of count of objects in two boxes such that

$x + y = n$ where $1 \le x \le n - 1$ & $1 \le y \le n - 1$

Therefore, the number of ways in which n objects can be put into two boxes under the condition, is equal to the coefficient of x^n in the expression

$$(x + x^2 + x^3 + \ldots + x^{n-1})\,(x + x^2 + x^3 + \ldots + x^{n-1})$$

i.e. in $\left[\dfrac{x(1-x^{n-1})}{1-x}\right]^2$. This is equal to the coefficient of x^{n-2} in $(1-x)^{-2}$ i.e. ${}^{2+n-2-1}C_{n-2} = n - 1$.

Q.174 Let L, be distributive Lattice, for any a,b,c $\in$ L, then show that

$a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ imply $b = c$ **(8)**

Ans: In any distributive lattice L, for any three elements a, b, c of L, we can write

$b = b\vee (a \wedge b)$
$= (b\vee a) \wedge (b\vee b)$ [Distributive law]
$= (a\vee b) \wedge b$ [Commutative and Idempotent law]
$= (a\vee c) \wedge b$ [Given that $(a\vee b) = (a\vee c)$]

$= (a \wedge b) \vee (c \wedge b)$ [Distributive law]
$= (a \wedge c) \vee (c \wedge b)$ [Given that $(a \wedge b) = (a \wedge c)$]
$= (c \wedge a) \vee (c \wedge b)$ [Commutative law]
$= c \wedge (a \vee b)$ [Distributive law]
$= c \wedge (a \vee c)$ [Given that $(a \vee b) = (a \vee c)$]
$= c$

Q.175 Which of the following sets are equal?

$S_1 = \{1, 2, 2, 3\}$, $S_2 = \{x \mid x^2 - 2x + 1 = 0\}$,
$S_3 = \{1, 2, 3\}$, $S_4 = \{x \mid x^3 - 6x^2 + 11x - 6 = 0\}$, **(8)**

Ans: The elements of S_2 and S_4 is given as below:

$S_2 = \{1, 1\}$ and $S_4 = \{1, 2, 3\}$

Two sets are said to be equal if both contain same elements and the multiplicity of every element is same in both the sets. This implies that S_3 and S_4 are equal. Though S_1 has same elements as in S_3 and S_4, it not equal because multiplicity of 2 in S_1 is 2 whereas that in S_3 and S_4 is 1.

Q.176 A graph G has 21 Edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in G. **(8)**

Ans: It is given that graph G has 21 edges, thus total degree of graph is 42. it is also given that three vertices are of degree 4 and other vertices are degree 3. Let number of vertices of degree 3 is y. Then

$3y + 4x3 = 42$
$\Rightarrow y = (42 - 12) / 3 = 10.$

Thus total number of vertices in the graph G is 10 + 3 = 13.

Q.177 Write the negation of each of the following in good English sentence.

(i) Jack did not eat fat, but he did eat broccoli.
(ii) The weather is bad and I will not go to work.
(iii) Mary lost her lamb or the wolf ate the lamb.
(iv) I will not win the game or I will not enter the contest. **(2 x 4)**

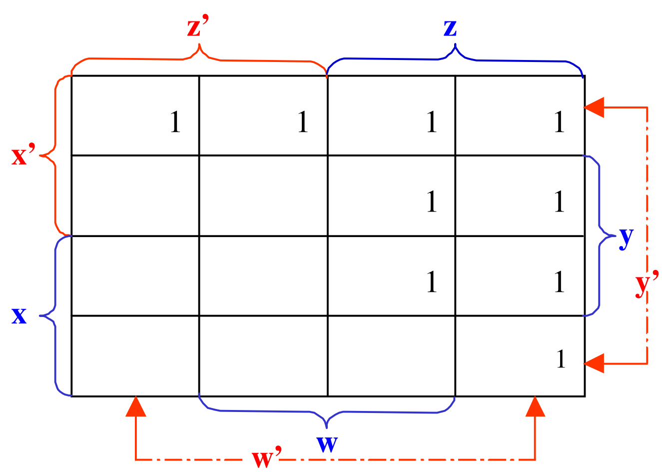
Ans: The negation of statement is written using concept of propositional rules. For example the negation of "p AND q" is "not p OR not q"; the negation of " p implies q" is "not q implies not q".

(i) If Jack did not eat broccoli then he did ate fat.
(ii) The weather is not bad or I will go to work.
(iii) Mary did not loss her lamb and the wolf did not eat the lamb.
(iv) I will win the game and I will enter the contest.

Q.178 Simplify the logical expression $\overline{X}\,\overline{Y}+\overline{X}\,Z+Y\,Z+\overline{Y}\,Z\,\overline{W}$. **(8)**

Ans: The K-Map for the given Boolean expression is given by the following diagram. The optimized expression then obtained is given below.

x'y' + zw' + yz



Q.179 How many different sub-committees can be formed each containing three women from an available set of 20 women and four men from an available set of 30 men? **(8)**

Ans: The number of subcommittee that can be formed is equal to the number of ways the members of committee can be selected from the given group of people. The number of ways in which three women members from 20 women and four men members from 30 men is

$$^{20}C_3 * {}^{30}C_4 = 31241700$$

Q.180 A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls will be red and two will be green? **(8)**

Ans: Total number of ways in which 4 balls can be selected out of 10 is $^{10}C_4 = 210$. The number of ways in which 2 red balls out of six and 2 green balls can be selected is $^{6}C_2 * {}^{4}C_2 = 90$ Therefore the probability of selecting 2 red and 2 green balls is $\frac{90}{210} = \frac{3}{7}$

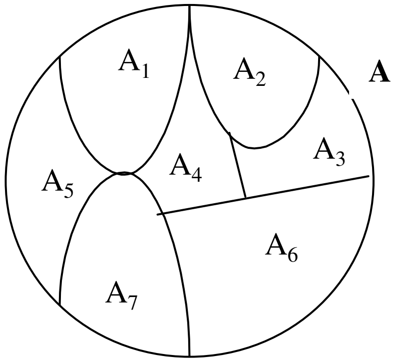
Q.181 What is a partition on a set? Let A = {1,2,3,4,5} and a relation R defined on A is R={(1,1), (1,2), (2,1), (2,2), (3,3), (4,4), (4,5), (5,4), (5,5)}. Obtain the cells of the partition. **(8)**

Ans: A *partition* of a non-empty set A is defined as a collection of subsets A_i's of A such that

* Union of all such subsets A_i is equal to A i.e. $\cup A_i = A$

* All such subsets are mutually disjoint i.e. $A_i \cap A_j = \phi$.for $i \neq j$ and

* All subsets A_i's are non-empty i.e. $| A_i | \neq \phi$.



See the figure shown above. Let A be a non-empty set and A_1, A_2, A_3, A_4, A_5, A_6 and A_7 be subsets of A. It is obvious from the figure that

$$(i) \quad \bigcup_{i=1}^{7} A_i = A$$

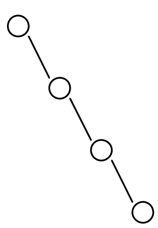
$$(ii) \quad A_i \cap A_j = \Phi \quad for\ i \neq j$$

$$(iii) \quad | A_i | \neq 0$$

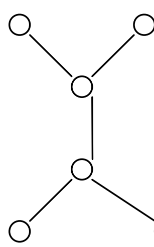
A *partition* is represented by P. Each subset of A in P is called ***block*** or ***cell***. A partition of a set is also called ***quotient Set.***

The Cells of partitions are {1, 2}, {3} and {4,5}.

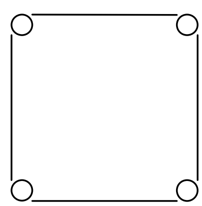
Q.182 What is a lattice? Which of the following graphs are lattice and why? **(8)**



(a)



(b)



(c)

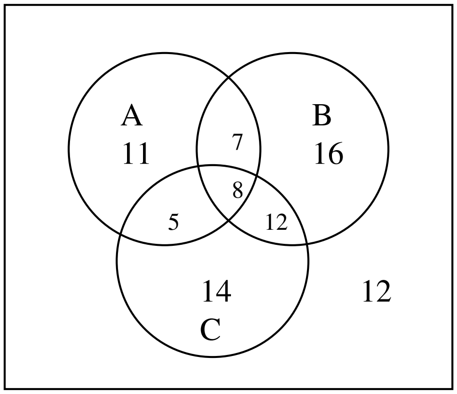
Ans: Let (L, ≤) be a poset. If every subset {x, y} containing any two elements of L, has a **glb** (Infimum) and a **lub** (Supremum), then the poset (L, ≤) is called a **lattice**. A **glb** ({x, y}) is represented by x∧y and it is called ***meet*** of x and y. Similarly, **lub** ({x, y}) is represented by x∨y and it is called ***join*** of x and y. Therefore, **lattice** is a mathematical structure equipped with two binary operations **meet** and **join.**

In the given examples, (a) is a lattice because every pair of elements has a meet and join in the set under the relation represented by the graph. Graph in (b) does not represent a lattice because bottom two elements have no meet and top two elements have no join.

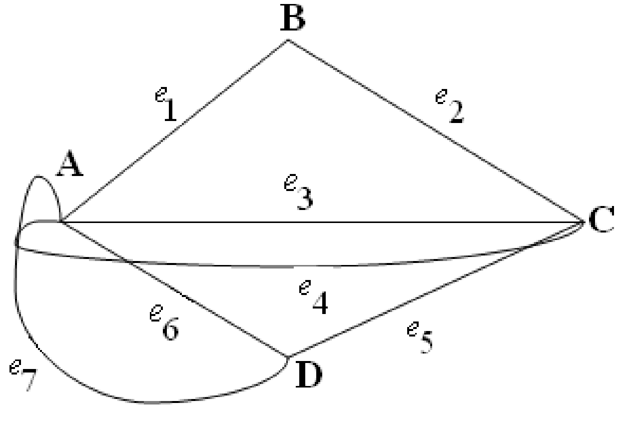
In the case of (c), the relation represented is not even anti symmetric because two bottoms and two top level elements are at the same level and represented as related to each other (symmetric) without being the same element (equal).

Q.183 In a survey of 85 people it is found that 31 like to drink milk 43 like coffee and 39 like tea. Also 13 like both milk and tea, 15 like milk and coffee, 20 like tea and coffee and 12 like none of the three drinks. Find the number of people who like all the three drinks. Display the answer using Venn Diagram. **(8)**

Ans: Let A, B and C is set of people who like to drink milk, take coffee and take tea respectively. Thus $|A| = 31$, $|B| = 43$, $|C| = 39$, $|A\cap B| = 15$, $|A\cap C| = 13$, $|B\cap C| = 20$, $|A\cup B\cup C| = 85 - 12 = 73$. Therefore, $|A\cap B\cap C| = 73 + (15 + 13 + 20) - (31 + 43 + 39) = 8$.



Q.184 Obtain the incidence and the adjacency matrix of the graph given below. **(8)**



Ans: The adjacency matrix M_A and incidence matrix M_I for the graph is given as shown belo

The adjacency matrix is of order 4 x 4 because there are 4 nodes in the graph. Since the graph is undirected, we have used numbers to represent number of arc between the node I and node J at the (I, J) the location in the matrix. For example '0' stand for no edge between them, '2' stands for two nodes between them and so on..

The incidence matrix is Boolean matrix of order N x E where N is the number of nodes and E is the number of edges in the graph. An edge joins two nodes. The presence of '1'in the matrix represents the nodes for that edge.

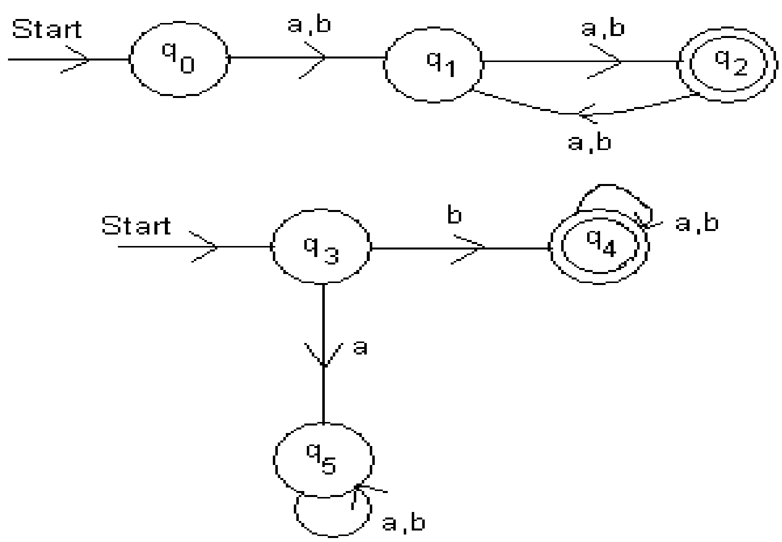
$$M_A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix} \qquad M_I = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Q.185 What are the possible ways to combine two automatons? Explain with an example. **(8)**

Ans:
A finite automaton is drawn for a regular language. A regular language is closed under the operation of Union, Concatenation, Negation, Kleene Star, Reverse, Intersection, Set difference and Homomorphism. If L_1 and L_2 are two regular languages having M_1 and M_2 corresponding automata then M_1 and M_2 can be combined for the above operations. The General Approach is outlined below.

- Build automata (dfas or nfas) for each of the languages involved.
- Show how to combine the automata to create a new automaton that recognizes the desired language.
- Since the language is represented by an nfa or dfa, conclude that the language is regular.

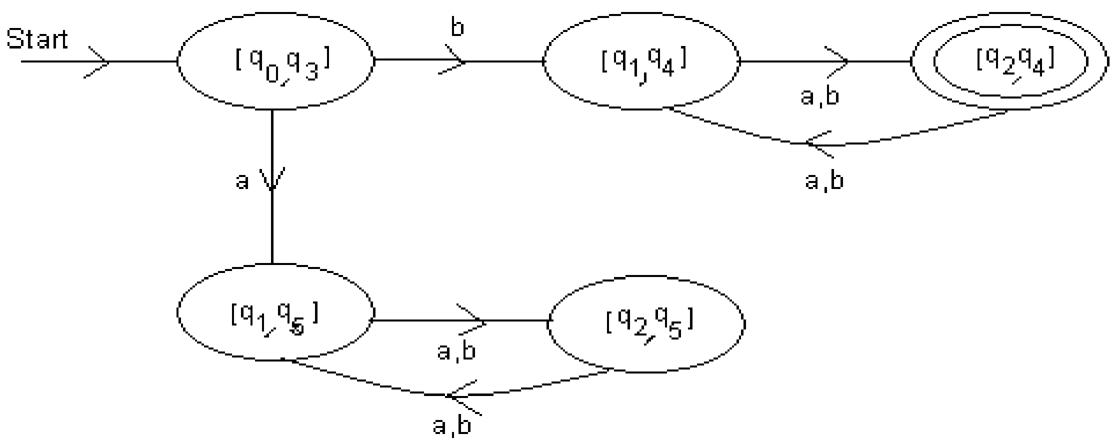
In order to demonstrate the working method, let us take intersection of languages as an example. Let us construct DFA for L = L1 $\cap$L2 over alphabet {a, b}.Where
L1 = All strings of even length.
L2 = All strings starting with b.
.



DFA M_1(top) and M_2(bottom) for languages L_1 and L_2 are shown above. They are respectively defined as $M_1 = (\{q_0, q_1, q_2\}, \{a, b\}, \delta_1, q_0, \{q_2\})$ where δ_1 is state transition as per the diagram. $M_2 = (\{q_3, q_4, q_5\}, \{a, b\}, \delta_2, q_3, \{q_4\})$ where δ_2 is state transition function. So M = {[q0,q3], [q1,q4],q1,q5],[q2,q4],[q2,q5],}, {a,b}, δ, [q0,q3], {q2,q4]}) where δ is state transition as shown below.

Transition diagram for M is shown below.

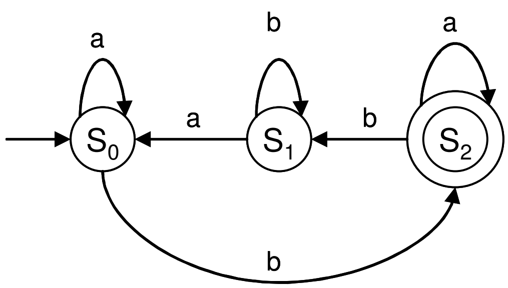
δ	a	b
$[q_0, q_3]$	$[q_1, q_5]$	$[q_1, q_4]$
$[q_1, q_5]$	$[q_2, q_5]$	$[q_2, q_5]$
$[q_1, q_4]$	$[q_2, q_4]$	$[q_2, q_4]$
$[q_2, q_5]$	$[q_1, q_5]$	$[q_1, q_5]$
$[q_2, q_4]$	$[q_1, q_4]$	$[q_1, q_4]$



Q.186 Construct the finite automaton for the state transition table given below. **(8)**

	a	b
start s_0	s_0	s_2
s_1	s_0	s_1
s_{2*}	s_2	s_1

Ans: The finite automata is shown below. The initial state is marked with arrow sign and the final state within double circle.



Q.187 Prove that if $(A, \leq)$ has a least element, then $(A, \leq)$ has a unique least element. **(8)**

Ans: Let (A, ≤) be a poset. Suppose the poset A has two least elements x and y. Since x is the least element, it implies that x ≤ y. Using the same argument, we can say that y ≤ x, since y is supposed to be another least element of the same poset. ≤ is an anti-symmetric relation, so x ≤ y and y ≤ x ⇒ x = y. Thus, there is at most one least element in any poset.

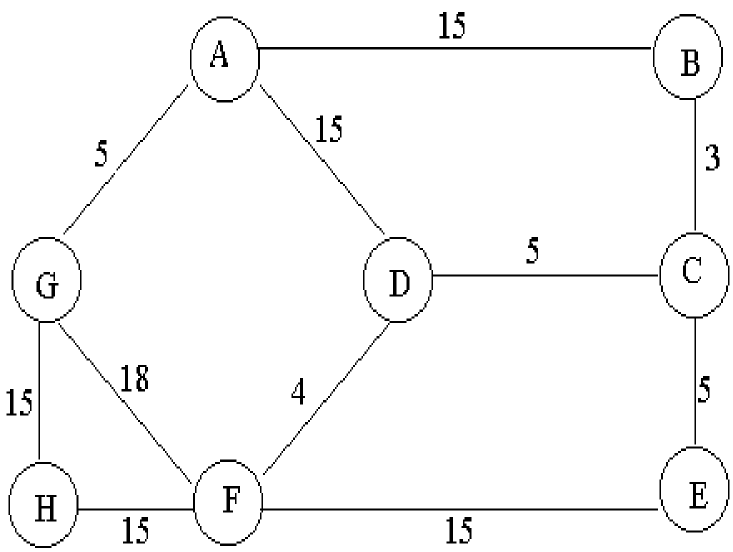
Q.188 Show that in a Boolean algebra, for any a and b, $(a \wedge b) \vee (a \wedge b') = a$. **(8)**

Ans: This can be proved either using the distributive property of join over meet (or of meet over join) or using a truth table for the expression. Let us use the truth table.

a	b	$a \wedge b$	$a \wedge b'$	$(a \wedge b) \vee (a \wedge b')$
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

From the truth table, it is obvious that entries for the expression and that for ‘a’ are the same and hence the result.

Q.189 What is minimum spanning tree? Determine a railway network of minimal cost for the cities in the following graph using Kruskal's algorithm.
(10)



Ans: A minimum spanning tree in a connected weighted graph is spanning tree that has the smallest possible sum of weights of its edges.

We collect the edges in sorted order as:

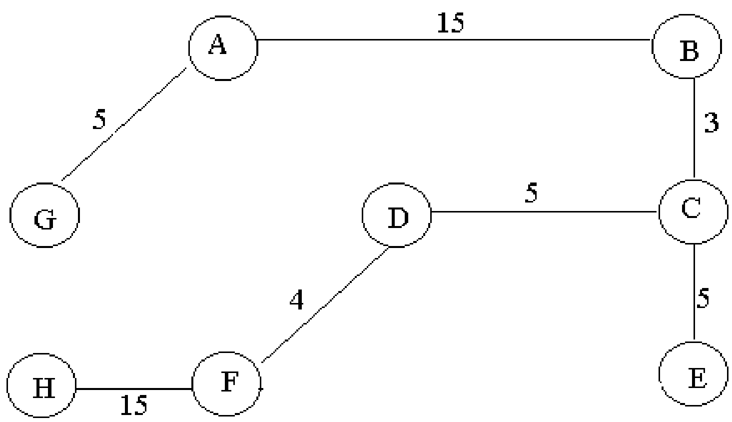
Edge	Cost
(B,C)	3
(D,F)	4
(A,G)	5
(C,D)	5
(C,E)	5
(A,B)	15
(A,D)	15
(F,H)	15
(G,H)	15
(E,F)	15
(F,G)	18

Choose the edges (B,C),(D,F),(A,G),(C,D),(C,E).

Then we have option we may choose only one of (A,B) and (A,D), because selection of makes a circuit. Suppose we choose (A,B).

Likewise we may choose only one of (G,H) and (F,H).Suppose we choose (F,H).

We have a spanning tree as :



Q.190 Draw a diagraph for each of the following relation

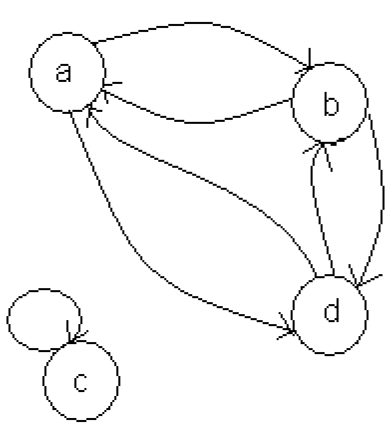
(i) Let $A = \{a, b, c, d\}$ and let $R = \{(a, b), (b, d), (a, d), (d, a), (d, b), (b, a), (c, c)\}$

(ii) Let $A = \{1,2,3,4,5,6,7,8\}$ and Let ${}^{x}R_{y}$, wherever y is divisible by x.

(iii) Determine which of the relations are reflexive, which are transitive, which are symmetric and which are anti symmetric. **(4, 4, 2)**

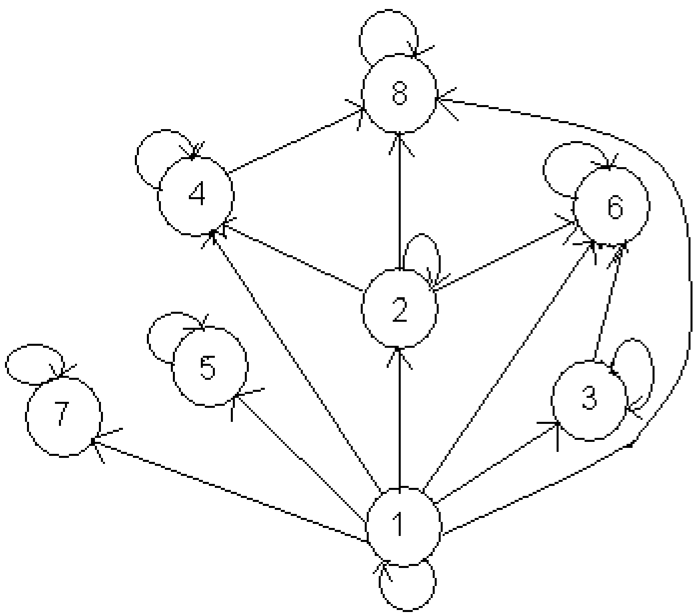
Ans: The diagraphs for the defined relation is shown below:

(i)



(ii) The R={ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(1,8),(2,2),(2,4),(2,6),(2,8),(3,3), (3,6),(4,4),(4,8),(5,5),(6,6),(7,7),(8,8) }

(iii)



Relation (ii) is reflexive, (i) is not.
Symmetry: Relation (i) is symmetrc whereas (ii) is not.
Transitivity: Relation (ii) is trasitive whereas (i) is not
Antisymmetry: Relation (ii) is anti-symmetrc whereas (i) is not