Q.1 a. How many different 2-digit numbers can be made from the digits 0 to 9
(i) When repetition is allowed?
(ii) When repetition is not allowed?

Answer:

When repetition is allowed:
The tens place can be filled by 10 ways and the unit place can be filled by 10 ways. Therefore the total number of 2 digit numbers = 10*10=100
When repetition is not allowed:
The tens place can be filled by 10 ways and the unit place can be filled by 9 ways.
Therefore total number of 2 digit numbers=10*9=90

b. Let R be an equivalence relation on the set A={4, 5, 6, 7} defined by R= {(4,4), (5,5), (6,6), (7,7), (4,6), (6,4)}. Determine its equivalence classes.

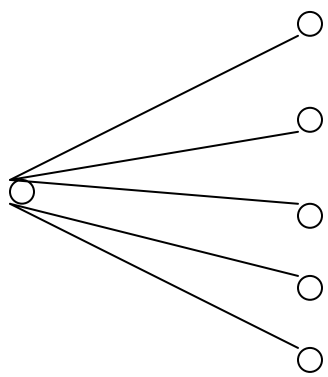
Answer:

Equivalence classes are [4]=[6]={4,6}
[5]=[5]
[7]=[7]

c. What is a complete bipartite graph? Draw the complete bipartite graph $K_{1,5}$.

Answer:

A graph g=(V,E) is called a complete bipartite graph if its vertices V can be partitioned into two subsets V_1 and V_2, such that each vertex of V_1 is connected to each vertex of $V_{2.}$ The number of edges in a complete bipartite graph is m*n as each of the m vertices is connected to each of the n vertices. It is denoted as $K_{m,n}$ and m≤n. $K_{1,5}$ is as below:



d. Prove that the argument p→q, q→r, r→s, ~s, pVt is valid without using truth table.

Answer:

(i) p→q (given)
(ii) q→r (given)
(iii) r→s, (given)
(iv) ~s, (given)
(v) pVt (given)

(vi)	p→r	(Syllogism using (i) and (ii))
(vii)	p→s	(Syllogism using (vi) and (iii))
(viii)	~p	(Modus tollens using (vii) and (iv))
(ix)	t	(Disjunctive Syllogism using (v) and (viii))

e. Simplify the logical expression $\overline{X}\,\overline{Y} + \overline{X}Z + YZ + \overline{Y}Z\overline{W}$

Answer:

The above expression can be written as

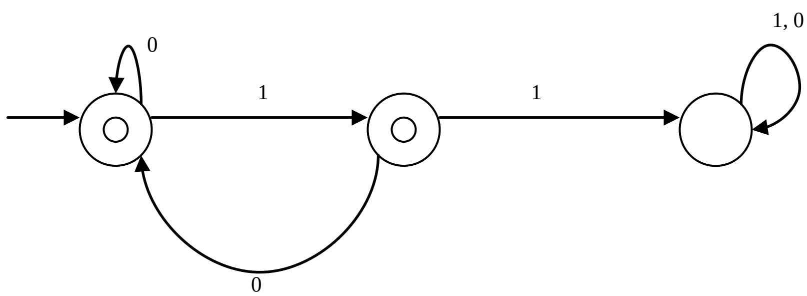
$$\overline{X}\,\overline{Y} + \overline{X}Z.1 + YZ + \overline{Y}Z\overline{W}$$
$$= \overline{X}\,\overline{Y} + \overline{X}Z(Y + \overline{Y}) + YZ + \overline{Y}Z\overline{W}$$
$$= \overline{X}\,\overline{Y} + \overline{X}ZY + \overline{X}Z\overline{Y} + YZ + \overline{Y}Z\overline{W}$$
$$= \overline{X}\,\overline{Y}(1 + Z) + YZ(\overline{X} + 1) + \overline{Y}Z\overline{W}$$
$$= \overline{X}\,\overline{Y} + YZ + \overline{Y}Z\overline{W}$$

f. State and prove the Euler formula to test the planarity of the graph.

Answer:

It is v-e+r=2, where, v is the number of vertices, e the edges and r the regions in the graph. It can be proved either by induction or using the method of spanning tree formation.

g. What kind of strings is not accepted by the following automaton? Explain how.



Answer:

All strings with two consecutive ones

Q.2 a. Write the negation of each of the following in good English sentence.

(i) Jack did not eat fat, but he did eat broccoli

(ii) The weather is bad and I will not go to work.

(iii) Mary lost her lamb or the wolf ate the lamb.

(iv) I will not win the game or I will not enter the contest.

Answer:

i. Jack did eat fat or he did not eat broccoli.
ii. The weather is not bad or I will go to work.
iii. Mary did not lose her lamb and the wolf did not eat the lamb.
iv. I will win the game and I will enter the contest.

b. Prove that $^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$

Answer:

RHS $^{n}C_{r-1} + {}^{n}C_r$

$$= \frac{\angle n}{\angle r-1 \angle n-r+1} + \frac{\angle n}{\angle r \angle n-r}$$

$$= \frac{\angle n}{\angle r-1 (n-r+1) \angle n-r} + \frac{\angle n}{r \angle r-1 \angle n-r}$$

$$= \frac{\angle n}{\angle r-1 \angle n-r}\left[\frac{1}{n-r+1} + \frac{1}{r}\right]$$

$$= \frac{\angle n}{\angle r-1 \angle n-r}\left[\frac{r+n-r+1}{r(n-r+1)}\right]$$

$$= \frac{\angle n (n+1)}{r \angle r-1 \angle n-r (n-r+1)}$$

$$= \frac{\angle n+1}{\angle r \angle n-r+1}$$

$= {}^{n+1}C_r$

LHS

Q.3 a. On a set S = {1,2,3,4,5}, find the equivalence relation on S, which generate the partition { {1,2}, {3}, {4,5} }. Draw the graph of the relation.

Answer:

R={(1,1),(1,2),(2,1),(2,2),(3,3),(4,4),(4,5),(5,4),(5,5)}

b. How many different sub-committees can be formed each containing three women from an available set of 20 women and four men from an available set of 30 men?

Answer:

Here order does not matter so we are merely counting the number of possible subsets. Three women can be chosen from 20 in $^{20}C_3$ ways and 4 men can be chosen from 30 in $^{30}C_4$ ways. Together it is $^{20}C_3 \times {}^{30}C_4$ ways i.e. 31,241,700 ways.

Q.4 a. State and prove the condition to find out if a given graph is an Euler graph.

Answer:

If the degree of all the vertices of a graph is even, then the graph is an Euler graph. To prove the condition, we can trace the whole graph starting from any vertex. When we enter a new vertex, it contributes one to the degree of the vertex. While exiting the vertex, it contributes again one to the degree. Thus the degree becomes even. If again, we visit the same vertex, we need to exit from the edge, which has not been traversed. Thus always, the degree is contributed twice. As the degree increases only in multiple of two, the degree will be even. It happens with the start vertex also.

b. Define Boolean algebra. Prove that the power set of any set forma a Boolean algebra.

Answer:

Boolean Algebra is defined as a five tuple <S, {0,1}, ~, +,*>, Where S is the set, {0,1} are the elements and rest three are the operations defined on the set. On the elements of the Power set of any set all the operations can be applied. Thus it is a Boolean algebra.

Q.5 a. What is a Hasse diagram? Draw the Hasse diagrams of the following sets under the partial ordering relation "divides" and indicate those which are totally ordered.

(i) {2, 6, 24}
(ii) {1,2,3,6,12}
(iii) {3,9,27,54}

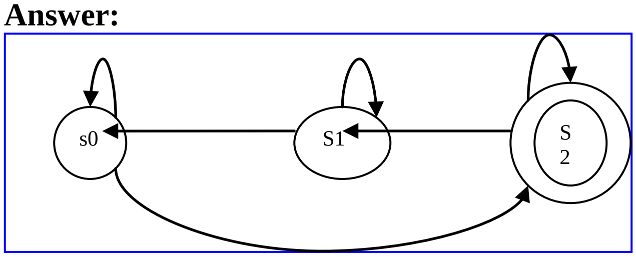
Answer:

Hasse diagram is the representation of any relation. With this the properties of any relation can easily be inferred. Depending on the relation, the elements are positioned inform of the vertices. Relation puts the edge between the vertices.

b. Construct the finite automaton for the state transition table given below.

	a	b
start s_0	s_0	s_2
s_1	s_0	s_1
s_{2*}	s_2	s_1

Answer:



Q.6 a. Prove that if (A, ≤) has a least element, then (A, ≤) has a unique least element.

Answer:

Suppose a and b are the least elements of (A, ≤). Then a ≤ b and b ≤ a. Since ≤ is antisymmetric, a=b.

Q.7 a. Write the Preorder, Inorder and Postorder tree traversal algorithm.

Answer:

Preorder

Visit the root
Search the left subtree, if it exists
Search the right subtree, if it exists.

Inorder

Search the left subtree, if it exists
Visit the root
Search the right subtree, if it exists.

Postorder

Search the left subtree, if it exists
Search the right subtree, if it exists.
Visit the root

b. Write down the Warshall's algorithm for the connectivity amongst the vertices of the graph.

Answer:

Refer page 159, Discrete Mathematical Structures, Kolman, Busby, Ross

Text Books

1. J.L. Hein, Discrete Structures, Logic and Computability, Jones and Bartlett Publishers, 3rd Edition

2. R.L. Graham, D.E. Knuth, O. Patashnik, Concrete Mathematics: A Foundation for Computer Science, Pearson Education, 1994