Q2 (a) Write the differential equation describing the dynamics of the system shown in Fig.1 and find $\frac{x_2(s)}{F(s)}$

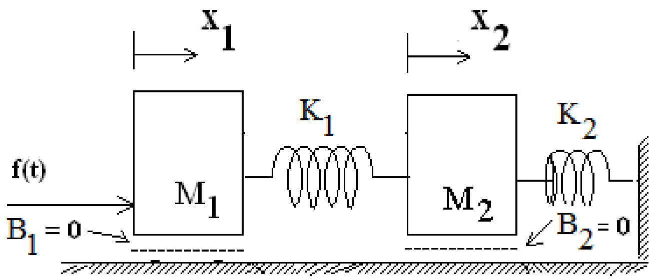
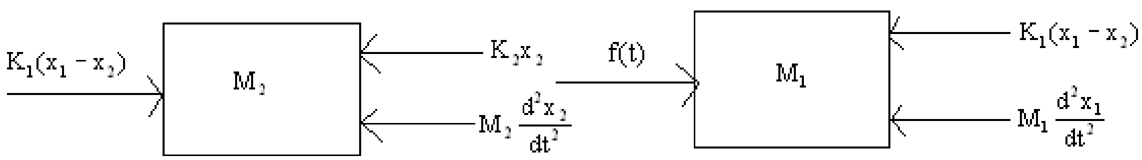


Fig.1

Answer

Free body diagrams for mass M_1 and M_2 are



For M_2 For M_1

$$F(t) = M_1\frac{d^2x_1}{dt^2} + K_1(x_1 - x_2) \_\_\_\_\_\_\_\_\_\_\_\_\_ (1)$$

$$K_1(x_1 - x_2) = K_2x_2 + M_2\frac{d^2x_1}{dt^2} \_\_\_\_\_\_\_\_\_\_\_\_\_ (2)$$

Take Laplace transform of eq. (1) & eq. (2) under initial conditions as zero.

$$F(s) = M_1s^2x_1(s) + K_1x_1(s) - K_2x_2(s) + M_2s^2x_2(s) \_\_\_\_\_\_\_\_\_\_ (3)$$

$$K_1x_1(s) - K_1x_2(s) = K_2x_2(s) + M_2s^2x_2(s) \_\_\_\_\_\_\_\_\_ (4)$$

Solving (3) & (4) we get $x_1(s) = \frac{x_2(s)}{K_1}(s^2M_2 + K_1 + K_2)$ ________ (5)

$$F(s) = \frac{x_2(s)}{K_1}(s^2M_2 + K_1 + K_2)(s^2M_1 + K_1x_2(s)$$

or $$\frac{x_2(s)}{f(s)} = \frac{K_1}{(s^2M_2 + K_1 + K_2)(K_1 + s^2M_1) - K_1^2}$$

Q2 (b) Obtain the F-I and F-V analogy of (a).

Answer Page Number 49 of Textbook

Q3 (a) In the Fig.4, identify the set of state variables and draw the signal flow graph of the circuit.

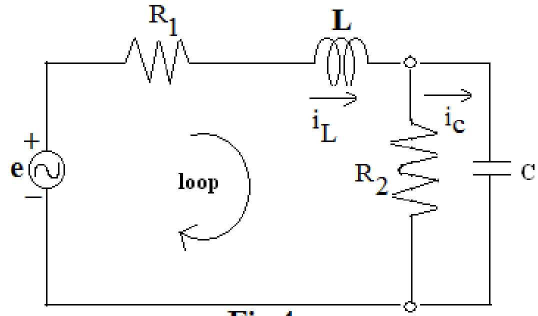


Fig.4

Also, determine transfer function from signal flow graph.

Answer

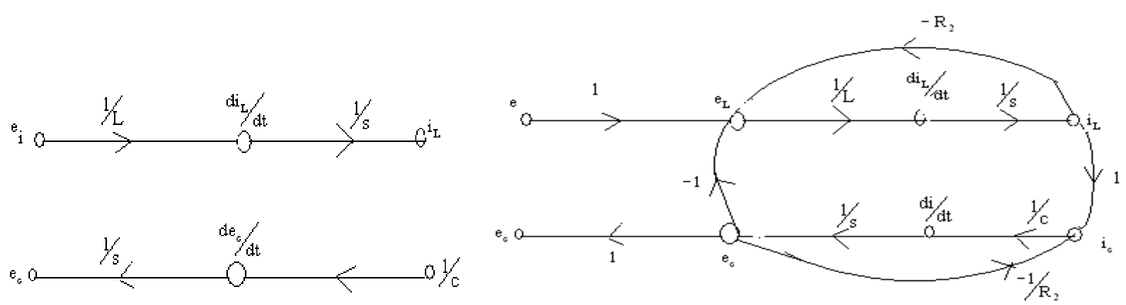
The Circuit of Fig.4 has two storage elements, so these shall be two state variables V_{iz} i_L and e_c.

The signal flow graph is conflicted by KCL equation at node and KVL equation round the loop. There are

$$\iota_L = \frac{e_c}{R_2} + i_c \quad \text{or} \quad \iota_L = i_L \frac{e_c}{R_2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

and $e = R_1 i_L + e_L + e_c$ or $e_L = e - R_1 i_L - e_c$ ____________ (2)

The signal flow graph is drawn



(a) (b)

Signal flow graph

From signal flow graph, the two state variable equations be written as

$$\frac{di_L}{dt} = \frac{1}{L} e_L = \frac{1}{L}(-e_c - R_1 i_L + e) = \frac{-R_1}{L} i_L - \frac{1}{L} e_c + \frac{1}{L} e \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_ (3)}$$

and $\frac{de_c}{dt} = \frac{1}{C} i_L = \frac{1}{C}(i_L + \frac{e_c}{R_2}) = \frac{1}{C} i_L - \frac{1}{R_2 C} e_c$ ____________(4)

eq. (3) & (4) in the matrix from

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{di_c}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-R_1}{L} & \dfrac{1}{L} \\ \dfrac{1}{C} & \dfrac{1}{R_2 e} \end{bmatrix} \begin{bmatrix} i_L \\ e_c \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} e$$

Forward path $P_1 = \dfrac{1}{sL} \times \dfrac{1}{sC} = \dfrac{1}{s^2LC}, \Delta_1 = 1$

Single loop. $P_{11} - \dfrac{1}{sL}, P21 = -\dfrac{1}{sR_2C}, P_{31} = -\dfrac{1}{s^2LC}$

$$\Delta = 1 + \frac{R_1}{sL} + \frac{1}{sR_2C} + \frac{1}{s^2LC}$$

Hence $\dfrac{E_C(s)}{E(s)} = \dfrac{P_1}{\Delta} = \dfrac{\dfrac{1}{sR_2C}}{1 + \dfrac{1}{sR_2C} + \dfrac{1}{s^2LC}}$

$$= \frac{1}{1 + s\left(R_1C + \dfrac{L}{R_2}\right)} + s^2LC$$

Q3 (b) Find the overall transfer function of the system in Fig.5.

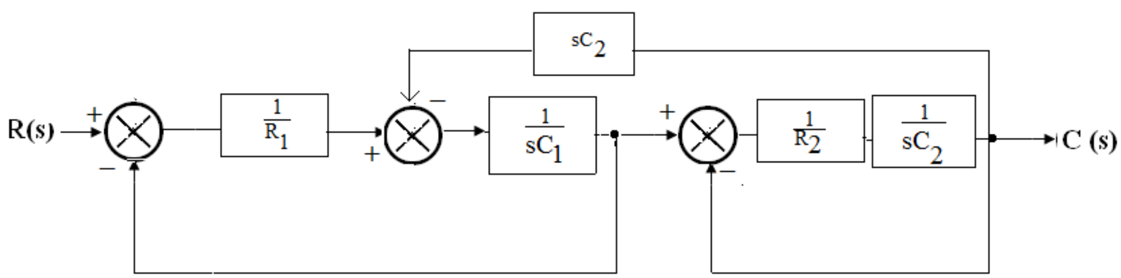
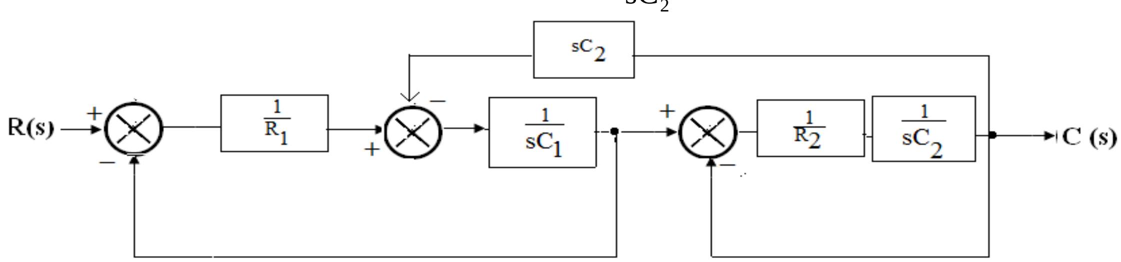


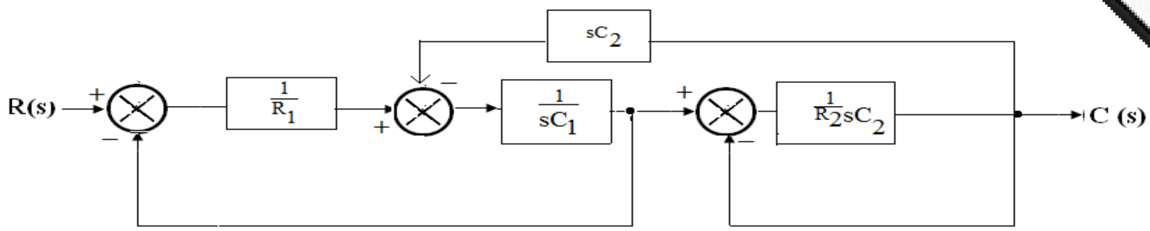
Fig.5

Answer

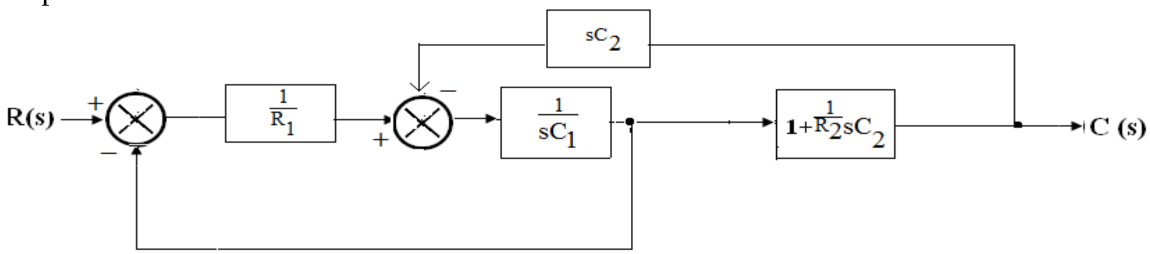
Step1: Shift the pick off point beyond the block $\dfrac{1}{sC_2}$



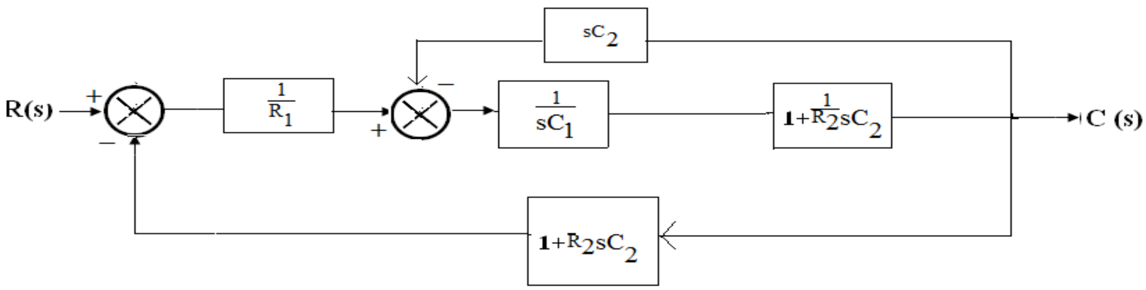
Step2: Two blocks are in cascade



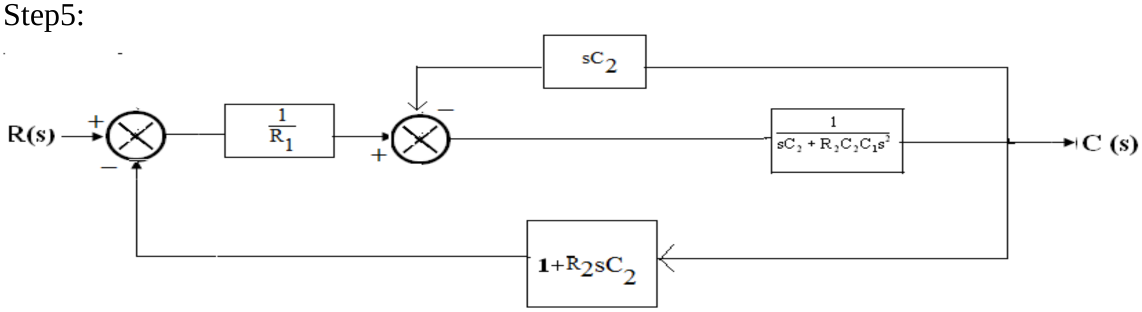
Step3:



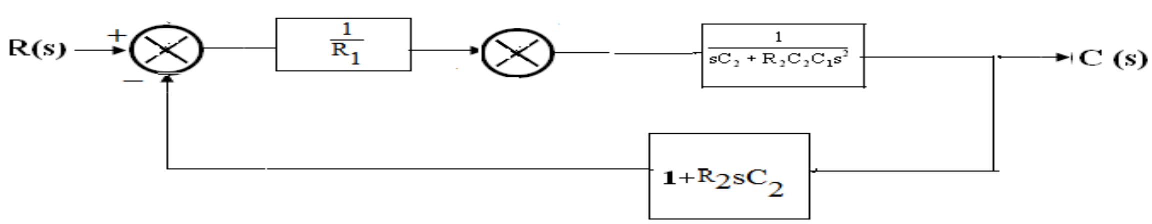
Step4:



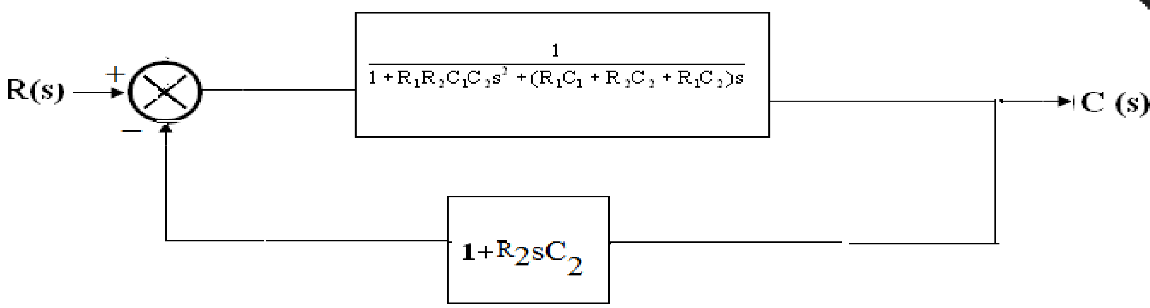
Step5:



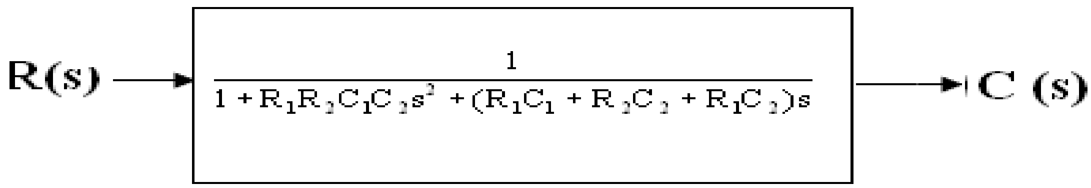
Step6:



Step7:



Step8:



$$\frac{C(s)}{R(s)} = \frac{1}{1 + R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s}$$

Q4 (a) Explain how the parameter variation is reduced by the use of feedback.

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Q4 (b) What are different controller components? Explain in brief.

Answer Page Number 134 of Textbook

Q5 (a) A second order system with ξ = 0.5 and ω_n = 6 rad /sec is subjected to a unit step input. Determine the rise time, peak time, settling time and peak overshoot.

Answer

Given that $\xi = 0.5$ and w_n =6 rad/sec

Rise time $t_r = \frac{\pi - ten^{-1}\sqrt{1-\xi^2}}{w_n\sqrt{1-\xi^2}}$ = 0.403 sec

Peak time $t_p = \frac{\pi}{w_n\sqrt{1-\xi^2}}$ =0.605 sec

Settling time $t_s = \frac{4}{\xi w_n}$ = 1.00 sec

Maximum/Peak overshoot = $M_p = e - \frac{\pi\xi}{\sqrt{1-\xi^2}} \times 100 = 1.63\%$

Q5 (b) The transfer function of a unity feedback system is G(s) = $\frac{10}{s(s+1)}$**.**

Find the dynamic error coefficient and steady state error to the input r(t) = $P_0 + P_1 t + P_2 t^2$

Answer

$$G(s) = \frac{10}{s(s+1)}, H(s) = 1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = \frac{s+s^2}{10+s+s^2} \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

$= 0.1s + 0.09\, s^2 - 0.019\, s^3$…..

$\therefore$ E(s) = 0.1s R(s) + 0.09 s^2 R(s) – 0.019s^3R(s)………..

Take inverse Laplace

e(t) = 0.1 r(t) + 0.09 r(t) – 0.019 r(t) ________ (2)

Now

$r(t) = P_0 + P_1 t + P_2\, t^2$

$r(t) = P_1 + 2\, P_2\, t$

$r(t) = 2P_2$

$r(t) = 0$

eq. (2) becomes

$e(t) = 0.1(P_1+2P_2 t)+0.18\, P_2$

The steady state error is

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} 0.1\,(P_1+2P_2 t) + 0.18\, P_2$$

The dynamic error coefficients from eq. (2)

$$K_1 = \frac{1}{01} = 10,\; K_2 = \frac{1}{0.09} = 11.1,\; K_3 = \frac{1}{-0.019} = -52.63$$

Q5 (c) A unity negative feedback control system has open loop transfer function is G(s) = $\frac{K(s+1)(s+2)}{(s+0.1)(s-1)}$ **using Routh stability criterion, determine the range of values of K for which the closed loop system has 0,1 or 2 poles in the right – half of S plane.**

Answer

The characteristics equation is 1 + G(s) = 0

(s + 0.1)(s – 1) + K(s + 1)(s +2) = 0

or $(1+K)s^2 + (3K - 0.9)s + (2K - 0.1) = 0$

Apply Routt criterion

s^2	1+K	2K-0.1
s^1	3K-0.9	
s^0	2K – 0.1	

(i) No Pole in right half S plane

K +1 > 0 or K > -1
3K – 0.9 > 0 K > 0.3
2K – 0.1 > 0 K > 0.05

(ii) 1 Pole in right half s Plane (= No sign change in first column terms)
-1 < K < 0.05

(iii) 2 Poles in right half s plane = (two change in sign in first column terms)
0.05 < K < 0.3

Q6 (a) The open loop transfer function of feedback system is $\frac{K}{s(s+4)(s^2+4s+20)}$**. Draw root locus for this system.**

Answer
Step1: plot poles & zeros

Poles are at s=0,s=-4, s^2 +4s+20=0

S=-2 $\pm$ y4

Step2: Segment between s=0 and s=-4 is the part of roots focus.

Step3: No of root loci N=p=4

Step4: Centroid of asymptote

Step5: Angle of asymptote

K=0	$\varphi_1 = 45^o$
K=1	$\varphi_2 = 135^o$
K=2	$\varphi_2 = 225^o$
K=3	$\varphi_4 = 315^o$

Step6: Break point characteristic is $1 + G(s)H(s) = 0$

$$s^4 + 8s^3 + 36s^2 + 80s + k = 0$$

or

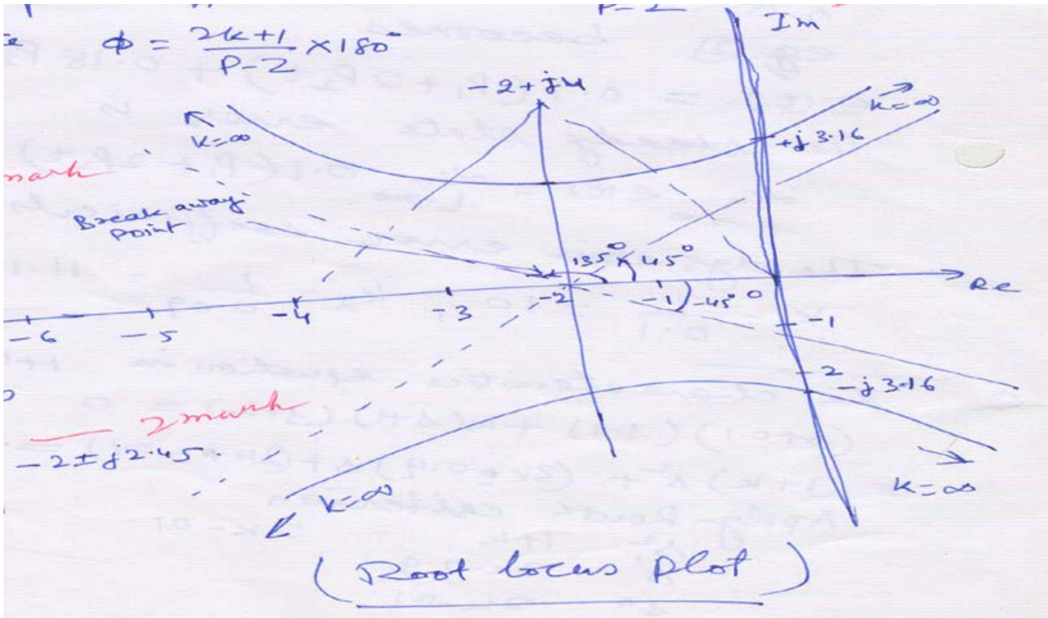
$$k = -(s^4 + 8s^3 + 36s^2 + 80s)$$

$$\frac{dk}{ds} = -(4B^3 + 24s^2 + 72s + 80) = 0$$

$\therefore$ *Break* even point s = -2s

two complex break even points are $-2 \pm j2.45$

step7 : point of intersection



s^4	1	36	K
s^3	8	80	
s^2	26	K	
s^1	80-0.307k		
s^0	k		

Maximum/Peak overshoot $= M_P = e^{-\frac{\pi \in}{\sqrt{1-\in 2}}} \times 100 = 16.3\%$

Q6 (b) Explain the sensitivity of the roots of the characteristics equation.

Answer

$a(s) = \frac{10}{s(s+1)}, H(s) = 1$

$\frac{E(s)}{R(s)} = \frac{1}{1 + a(s)H(s)} = \frac{s + s^2}{10 + s + s^2}$.............(1)

$= 0.1s + 0.09s^2 .0.019s^3$...............

$\therefore E(s) = 0.1sR(s) + 0.09s^2 R(s) - 0.019s^3 R(s)$......

Take inverse laplace

e(t) = 0.1r(t) + 0.09r(t) - 0.019r(t)..........(2)

Now

$r(t) = P_0 + P_1 t + P_2 t^2$

$r(t) = P_1 + 2P_2 t$

$r(t) = 2P_2$

$r(t) = 0$

$\therefore$ *eg*...(2)*becomes*

$e(t) = 0.1(P_1 + 2P_2 t) + 0.18P_2$

The steady state error is

$\underset{t \to \infty}{Lim}\, e(t) = \underset{t \to \infty}{Lim}\, 0.1(P_1 + 2P_2 t) + 0.18P_2$

The dysnanaic error coefficients ep(2)

$k_1 = \frac{1}{0.1} = 10, k_2 = \frac{1}{0.09} = 11.1, k_3 = \frac{1}{-0.019} = -52.63$

The characteristics equation is 1+G(s) =0
(s+0.1)(s-1)+k(s+1) (s+2) =0
or
(1+k) s^2 + (3k-0.9) s+(2k-0.1)=0

Apply Routh criterion

s^2	1+k	2k-0.1
s^1	3k-0.9	
s^0	2k-0.1	

For stability k>0
80-0.307k>0 or k<260
at k=260, the auxiliary efn A(s)=26 s^2 +k
26 s^2 +260=0-→s= $\pm$ 3.16j

Step8: Angle of departure
$\phi d = 180^o - (117^o + 90^o + 63^o) = -90^o$

Q7 (a) Why logarithmic scale is used for Bode plot ? Sketch the Bode plot for the transfer function H(s) = $\frac{1000}{(1+0.1s)(1+0.001s)}$ determine (i) Phase margin (ii) Gain margin.

Answer

put s=jw

$$H(jw) = \frac{1000}{(1+jo.1w)(1+jo.01w)}$$

Starting point is

$20\log 10\, k = 20\log_{10}1000 = 60db$

Corner frequency $w_1 = \frac{1}{01} = 10 red/\sec$

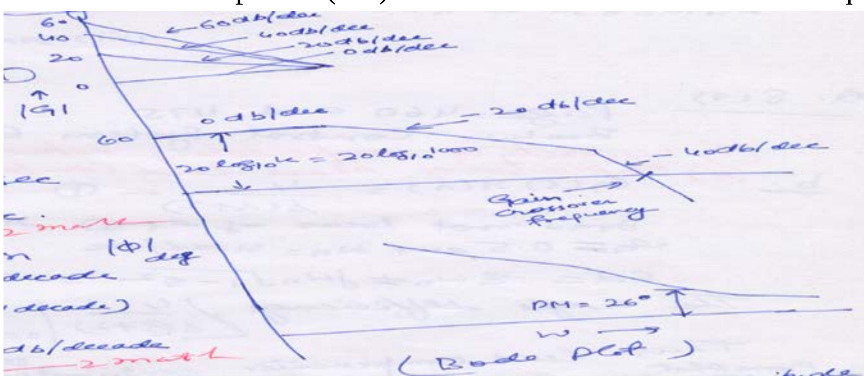
$W_2 = \frac{1}{0.001} = 1000 red/\sec$

Magnitude plot

(i) Make starting point 60db on y axis & drown a line stop of adb/decade

(ii) Draw a line with stop (0-20=-20 db/decade) from 1st corner frequency w_1

(iii) Draw a line of stop –20+(-20)=-40 db/decade from 2nd corner frequency w_2



Phase plot

W	-Arg(1+j0.1w)	-arg(1+j0.001w	Resulant
50	-76.6	-2.86	-81.46
100	-84.2	-5.7	-90
150	-86.2	-8.5	-94
200	87.13	-11.3	-98
500	88.85	-26.56	-115.4
800	-88.85	-38.65	-127.93
1000	-89.28	-450	-134.42
2000	--89.72	-63.43	-153.15
5000	-89.88	-71.56	-161.36
8000	-89.92	-78.69	-168.57
		-82.87	-172.79

Phase Margin:-

→ Throw point of integration of magnitude curve with 0 db draw a line on phase curve. This line into phase curve at -154^o

$\therefore$ Phase margin $154^0 - (-180^0) = +26^0$

→Gain margin $= \infty$

Since phase margin is $+26^0 c$ and gain margin is ∞, the system is inherently stable .

Q7 (b) The forward path transfer function of a unity feedback control system is G(s) = $\frac{100}{(s+6.54)}$ find the (i) resonance peak (ii) resonance frequency and (iii) bandwidth.

Answer

G(s) = $\frac{100}{s(s+6.54)}, H(s) = 1$

$$\frac{c(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{100}{s(s+6.54)}}{1+\frac{100}{s(s+6.54)}} = \frac{100}{s^2+6.54s+100}$$

Compare with $\frac{\omega^2 n}{s^2+2Ew_n s+w^2 n}$

$w^2 n = 100 \Rightarrow w_n = 10 rad/\sec$

(i) Re*sonent* frequncy

$w_r = w_n\sqrt{1-2E^2} = 8.86 rad/\sec$

(ii) Re*sonant* peak $= \frac{1}{2E\sqrt{1-E^2}} = 1.618$

$(iii) Bandwidth = w_n\sqrt{1-2E^2+(2-4E^2+4E^4)1/2}$

$= 14.34 rad/\sec$

Q8 (a) What is the necessity of compensating network? Explain phase lead compensator and give its comparison with phase lag compensator.

Answer Page Number 460 and 475 of TextBook

Q8 (b) Design a lead compensator for the system shown in Fig. 6. Given that ω_n = 4 rad / sec and ξ = 0.5 for compensated system.

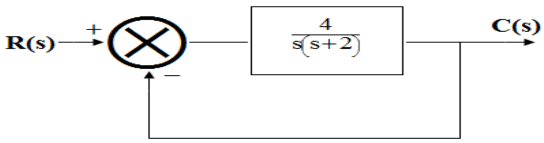


Fig.6

Answer

$$G(s)H(s) = \frac{4}{s(s+2)} \text{...........(1)}$$

Draw root focus of equation (1) it is shown in fig x.

R=0.5, and $w_n = 4rad/\sec$

Sd =E $w_n \pm jwn\sqrt{1-E^2} = -2 \pm j3.46$

The angle deficiency $\angle \frac{4}{s(s+2)} |_{s=-2\pm 3.46 f} = -210^o$

Or

$180^0 - (90+120) = 30^0$

Thus load compensator contribute $\phi = 30^o$ *atthispo* int

From plot Zero at s=-2.96$\therefore = 2.96$ $\frac{1}{LT} = 5.5$

$\alpha = 0.538$

Pole at s=-5.5 T=0.337

The open loop COMPENSATED transfer function of compensated system is

$$Gc(s)G(s) = k_c \frac{s+2.96}{s+5.5} . \frac{4}{s(s+2)} = \frac{k^1(s+2.96)}{s(s+2)(s+5.5)} \text{..................(2)}$$

$$k^1 = \frac{k^1(s+2.96)}{s(s+2)(s+5.5)} |_{sd=-2\pm 3.46 j} = 1 = k^1 = 18.7 \ \mathrm{k_c} = \frac{18.7}{4} = 4.675$$

$$Ked = 4.675 \times 0.538 = 2.52.$$

$$\textit{transfer} \text{ function of load compensation} = 2.52.\frac{1+0.337\mathrm{s}}{1+0.182s}$$

or

$$Gc(s) = 4.675\frac{s+2.96}{s+5.5}$$

open loop compensated trsfer function of compensated system

$$\mathrm{Gc(s)G(s)} = \frac{18.7(s+2.96)}{s(s+2)(s+5.5)}$$

$$\textit{the} \text{ velocity error constent } \mathrm{k_v} = \lim_{s\to 0} sGc(s)G(s) = \lim_{s\to 0} \frac{s18.5(s+2.96)}{s(s+2)(s+5.5)}$$

$$= k_v = 5.02\sec^{-1}$$

Q9 (a) A system with state model is $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u}$

Where u (t) is unit step occurring at t = 0 and $x^T(0) = [1 \quad 0]$**. Obtain the time response of the system and compute state transition matrix.**

Answer

$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

State transitions matrix $\phi(t) = 1 + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3$

Substituting values of a, we get

$$e^{At} = \begin{bmatrix} 1 + t + 0.5t^2 + ... & 0 \\ t + t^2 + ... & 1 + t + 0.5t^2 + ... \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

Time response of the system is

$$x(t) = \begin{matrix} \phi(t) = \left[x_0 + \int_0^t \phi(-t)Budt \right] \\ \phi(-t)Bu = \begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{-t} \\ e^{-t}91 - t) \end{bmatrix} \end{matrix}$$

$$x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & -e^{-t} \\ t & e^{-t} \end{bmatrix} = \begin{bmatrix} 2e^t - 1 \\ 2e^t \end{bmatrix}$$

Q9 (b) Test the following system for controllability and observability.

$$\dot{x} = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix} \mathbf{u} \quad \textbf{and} \quad \mathbf{y} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \mathbf{x}.$$

Answer

$$A = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$[A \quad B] = \begin{bmatrix} 2 & -3+1 \\ 2 & -1+1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 0 \\ 2 & 1 \end{bmatrix}$$

$$[A^2B] = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 0 & 3 \\ 2 & 1 \end{bmatrix}$$

$Q_c = [B \quad AB \quad A^2B]$

$$= \begin{bmatrix} 012 & -2-27 \\ 002 & 003 \\ 212 & 121 \end{bmatrix}$$ Qc has Rank=3, thus system is controllable

Test for Observability:-

$$A = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 0 & -4 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}, A^T = \begin{bmatrix} -3 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & -4 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}, (A^T)^2 C^T = \begin{bmatrix} 0 & 11 \\ 0 & -4 \\ 1 & 1 \end{bmatrix}$$

$$Qo = [C^T : A^T C^T : (A^T)^2 C^T] = \begin{bmatrix} 010 & -40 & 11 \\ 010 & 10 & -4 \\ 101 & 21 & -1 \end{bmatrix}$$

$$\text{Check for Rank} = \begin{vmatrix} -4 & 0 & 11 \\ 1 & 0 & -4 \\ 2 & 1 & -1 \end{vmatrix} = -5 \text{ is not equal to } 0$$

Rank of Qo =3, Thus System is completely Observable

Text Book

Control Systems Engineering, I.J. Nagrath and M. Gopal, Fifth Edition (2007 New Age International Pvt. Ltd.