

Q2 (a) A battery has an internal resistance of 0.5Ω and open circuit voltage of $12V$. What is power lost in the battery and terminal voltage on full load if a resistance of 3Ω is connected across the terminals of the battery?

Answer

The internal resistance is series with load. Hence the current is

$$I = 12/3.5 = 3.43A$$

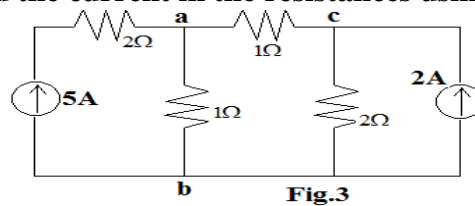
$$R_L = 3\Omega$$

$$\text{Terminal voltage} = IR_L = 3.43 \times 3 = 10.3v$$

$$R_{int} = 0.5\Omega$$

$$\text{Power loss} = I^2 R_{int} = (3.43)^2 \times 0.5 = 5.88 \text{ watt.}$$

Q2 (b) In the Fig.3, find the current in the resistances using node analysis.



Answer

At node a of fig3, using node analysis

$$S = \frac{V_a - V_b}{1} + \frac{V_a}{1}$$

$$\therefore 2V_a - V_b = 5 \text{ -----(1)}$$

Similarly, at node b, using node analysis

$$\frac{V_a - V_b}{1} + \frac{V_a}{2} + 5 + 2 = 0$$

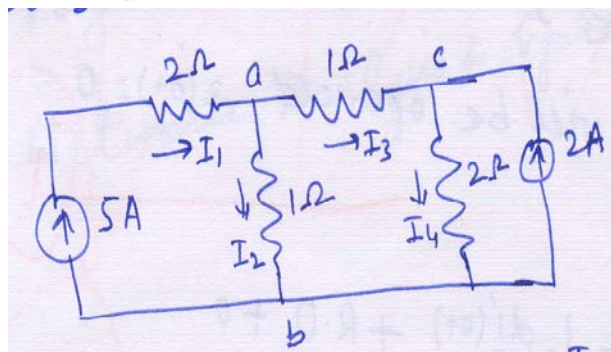
$$1.5V_b - V_a = -7$$

$$\text{or } 3V_b - 2V_a = -14 \text{ -----(2)}$$

Adding eq.(1) &(2), we get

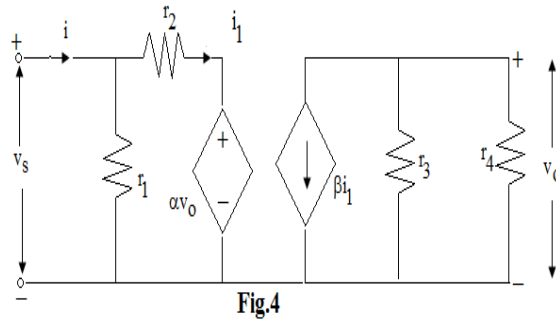
$$-V_b + 3V_b = 5 - 14 \rightarrow 2V_b = -9 \Rightarrow V_b = -4.5U$$

Substitute in eq.1, We get $V_a = 0.25V$



$$\begin{aligned} \therefore I_1 &= 5A \\ I_2 &= \frac{V_a - V_b}{1} = 4.75A \\ I_3 &= \frac{V_a}{1} = 0.25A \\ I_4 &= \frac{-V_b}{2} = -2.25A \end{aligned}$$

Q2 (c) Find v_o using Kirchoff's laws in the circuit as shown in Fig.4. Given that $r_1 = 1000\Omega$, $r_2 = 500\Omega$, $r_3 = 50\Omega$, $r_4 = 5\Omega$, $\alpha = 0.5, \beta = 2$ and $v_s = 10V$.



Answer

Applying KUL at output Loop

$$V_o = -\beta I_1 (r_3 \parallel r_4) = \frac{\beta r_3 r_4}{r_3 + r_4} i \text{-----(1)}$$

$$\text{At input loop, } I_1 = \frac{V_s - \alpha V_o}{r_2} \text{----- (2)}$$

using (2) in eq.(1), we have

$$V_o = \frac{\beta r_3 r_4}{r_3 + r_4} \left(\frac{V_s - \alpha V_o}{r_2} \right)$$

or

$$V_o = \left(1 - \frac{\beta r_3 r_4 \alpha}{r_2 (r_3 + r_4)} \right) = - \frac{\beta r_3 r_4 V_s}{r_2 (r_3 + r_4)}$$

$$\therefore V_o = \frac{\left(- \frac{\beta r_3 r_4 \alpha}{r_2 (r_3 + r_4)} \right)}{\left(1 - \frac{\beta r_3 r_4 \alpha}{r_2 (r_3 + r_4)} \right)}$$

substituting the values of $r_1, r_2, r_3, r_4, \alpha, \beta$ and V_s , we get

$$V_o = -183.48mv$$

Q3 (a) For the circuit given in Fig.5, switch K is closed at $t = 0$. Find the i , $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

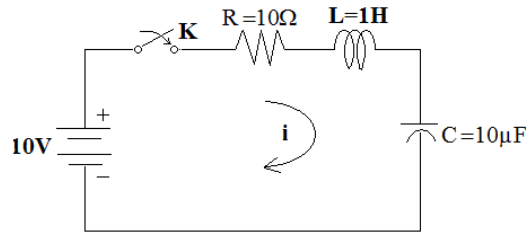


Fig.5

Answer

Apply KUL, we get

$$10 = L \frac{di}{dt} + Ri + \frac{1}{c} \int idt \dots \dots \dots (1)$$

for $t=0$, the circuit will be open click, $I(0^+) = 0$

$$\text{At } t=0, \frac{1}{c} \int idt = 0$$

$$10 = L \frac{di}{dt}(0^+) + R \cdot 0 + 0$$

\therefore eq (1) become

$$\therefore \frac{di(0^+)}{dt} = \frac{10}{L} = \frac{10}{1} = 10 \text{ amp / sec}$$

Differentiate eq.(1), we get

$$0 = L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{1}{c} i \dots \dots \dots (2)$$

Substitute values of di/dt and I at $t=0^+$

$$1. \frac{d^2i(0^+)}{dt^2} + 10 \cdot 10 + \frac{1}{10MF} \times 0 = 0$$

or

$$\frac{d^2i(0^+)}{dt^2} = -100 \text{ amp / sec}^2$$

Q3 (b) Find the general solution of the equation $2 \frac{di}{dt} + i(t) = 2i(t)$ with initial condition at $t = 0, i = 5(A)$

Answer

The given equation is

$$2 \frac{di(t)}{dt} + i(t) = 2i(t) \dots \dots \dots (1)$$

$$2 \frac{di(t)}{dt} + 2i(t) - 2i(t) = 0$$

$$\frac{di(t)}{dt} + \frac{1}{2}i(t) = 0 \dots \dots \dots (2)$$

The general solution of this equation is

$$I(t) = e^{-1/2t} \dots \dots \dots (3)$$

C = constant

Putting initial conditions, at $t=0, I=5A$

$$5 = e^0 C = C = 5$$

The solution is

$$i(t) = 5e^{-1/2t} \text{ for } t=0$$

Q4 (a) Using Laplace transform technique, find i_2 at $t = 0^+$ when switch k is closed at $t = 0$ in Fig.6.

Answer

$$I_1(8)(6+8) - I_2(8) = 10/8 \dots \dots \dots (5)$$

$$-I_1(8) + I_2(8)(6+8) = 0 \dots \dots \dots (6)$$

Substituting eqn 5 in 6 we get

$$I_2(8)(6+8)^2 - I_2(8) = 10/8$$

$$I_2(8)[(6+8)^2 - 1] = 10/8$$

$$I_2(8)[36 + 8^2 + 128 - 1] = 10/8$$

or

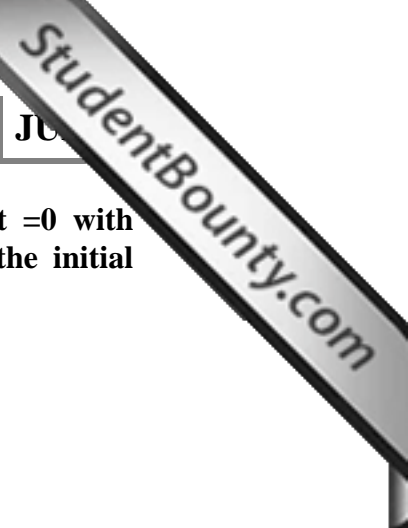
$$I_2(8) = 10/8(8^2 + 128 + 35) \dots \dots \dots (7)$$

using partial fraction, we get

$$I_2(8) = 2/7 + 10/84 + 1/6 + 5 \dots \dots \dots (8)$$

Taking inverse Laplace transform

$$i(t) = 2/7 + \frac{10}{84}e^{-7t} + \frac{1}{6}e^{-5t}$$



4 (b) A unit impulse voltage is applied to a series RC circuit at $t = 0$ with $R = 5\Omega$ and $C = 2F$. Find $i(t)$ using Laplace transform, assuming the initial charge stored in the capacitor is zero.

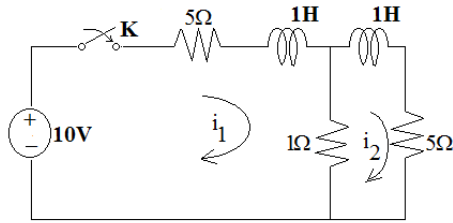


Fig.6

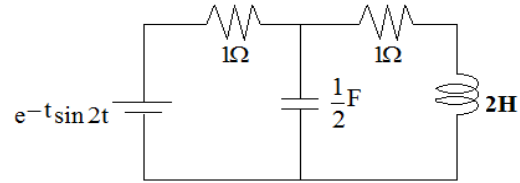


Fig.7

Answer

Apply KVL, to series RC circuit

$$\int_{-\infty}^t i(t) dt = g(t) \dots \dots \dots (1)$$

$$Ri(t) + 1/c \int_{-\infty}^0 i(t) dt = 1/c \int_0^t i(t) dt = \&t$$

RCCRT

Taking laplace transform, we get

$$RI(\&) + 1/c \frac{q(0+)}{\&} + 1/c \frac{I(\&)}{\&} = 1 \dots \dots \dots (3)$$

As initial charge store in c is zero $\therefore q(0+) = 0$

$$\therefore RI(\&) + 1/c \& I(\&) = 1$$

$$I(\&) = \frac{1}{R + \frac{1}{c \&}} = 1/R \cdot \frac{\&}{\& + \frac{1}{Rc}} = 1/5 \cdot \frac{\&}{\& + 1/10}$$

$$= 0.2 \& / \& + 0.1 = 0.2 \left[1 - \frac{0.1}{\& + 0.1} \right] \dots \dots \dots (4)$$

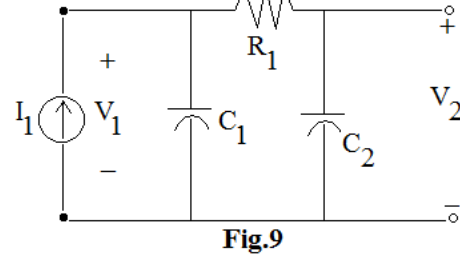
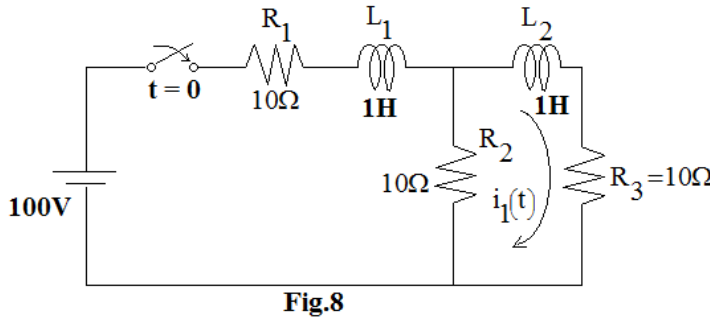
Taking inverselaplacetransform

$$i(t) = 0.2[8(t) - 0.1e^{-0.1t}]$$

Q5 (a) Determine Z(s) and I(s) for the network shown in Fig.7 using transform network.

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Q5 (b) Consider the network shown in Fig.8. Calculate $i_1(t)$ using Thevenin's theorem.



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Q6 (a) Compute the current gain $\alpha_{12}(s)$ and driving point impedance $Z_{12}(s)$ for the network shown in Fig.9 with $C_1 = 1F$, $R_1 = 1\Omega$ and $C_2 = 2F$.

Answer

In the current divider network of fig 9, we have

$$I_1(s) = I_{C_1}(s) + I_{R_1}(s) = V_1(s)[Y_{C_1}^s + Y_{R_1}^s] \dots \dots \dots (1)$$

$$\text{since } I_{R_1}(s) = \frac{Y_{R_1}(s)}{V_1(s)}$$

$$\therefore I_{R_1}(s) = \frac{Y_{R_1}(s)}{Y_{C_1}^s + Y_{R_1}^s} I_1(s) \dots \dots \dots (2)$$

now

$$L_{12}(s) = \frac{I_2(s)}{I_1(s)} = \frac{y_2(s)}{y_{C_1}(s) + y_{R_1}(s)} \dots \dots \dots (3)$$

also

$$y_{R_1}(s) = \frac{1}{R_1} / (\frac{1}{s} + y_{R_1 C_1}) \text{ and } y_{C_1} = c_1 / s$$

substituting in eq.(3)

$$L_{12}(s) = \frac{1}{R_1 C_1} \cdot \frac{1}{s + (c_1 + c_2) / R_1 C_1 C_2} \dots \dots \dots (4)$$

now

$$V_2(s) = \frac{1}{C_2} \cdot I_2(s) \text{ in fig 9.}$$

$$Z_{12}(s) = \frac{u_2(s)}{I_1(s)} = \frac{1}{R_1 C_1 C_2} \cdot \frac{1}{s + (1 + c_2) / R_1 C_1 C_2} \dots \dots \dots (5)$$

Substituting the value of C_1, C_2 & R_1 in eq(4) & (5),...we get

$$L_{12}(s) = \frac{1}{s + 1.5}$$

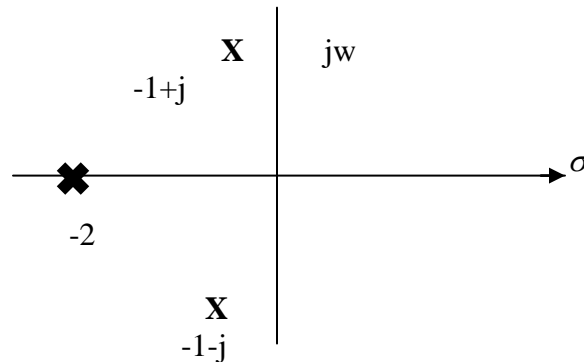
$$\text{and } Z_{12}(s) = \frac{0.5}{s + 1.5}$$

Q6 (b) A network function is given by $H(s) = \frac{2s}{(s+2)(s^2+2s+2)}$. Obtain pole-zero diagram.

Answer

$$\therefore H(s) = \frac{2s}{(s+2)(s^2+2s+2)} \dots\dots\dots(1)$$

The poles are located at $s = -2, (-1+j), (-1-j)$ and zero is at $s = 0$



Q6 (c) Check the positive realness of the function $F(s) = \frac{s^2 + 10s + 4}{s + 2}$.

Answer

Apply the test of positive realness

- (i) Since all the coefficient of polynomials in the numerator and denominator are the hence $f(s)$ is real if s is real.
- (ii) The poles of the function lies on left half of the s -plane

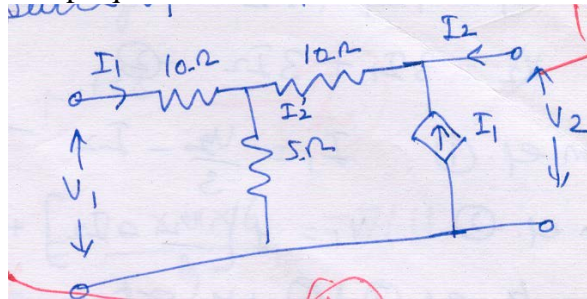
$$\begin{aligned} \text{(iii) } \operatorname{Re}[F(jw)] &= \operatorname{Re} \left[\frac{-w^2 + 10jw + 4}{jw + 2} \right] \left[\frac{-jw + 2}{-jw + 2} \right] \\ \operatorname{Re} &= \left[\frac{-2w^2 - 20jw + 8 + jw^3 + 10w^2 - 4jw}{w^2 + 4} \right] \\ &= \operatorname{Re} \left[\frac{8w^2 + 16jw + jw^3 + 8}{w^2 + 4} \right] = \left[\frac{(8w^2 + 8) + j(w^3 + 16w)}{w^2 + 4} \right] \\ &= \frac{8w^2 + 8}{w^2 + 4} \end{aligned}$$

Since for all values of w , $\operatorname{Re} [(f(jw))] \geq 0$
Therefore $f(s)$ is positive real function.

Q7 (a) Determine the Z-parameter of the network shown in Fig.10.

Answer

With open circuiting the output port C-D and applying a voltage place V_1 at input parts A-B, The loop equations are



$$V_1 = I_1(10 + 5) - 5I_2^1 \text{ -----(1)}$$

or

$$V_1 = 15I_1 - 5I_2^1$$

$I_2 = 0$, being output open circuit

$$I_2^1 = I_1$$

eq....(1) becomes $V_1 = 15I_1 + 5I_1 = 20I_1$

$$\therefore \frac{V_1}{I_1} \Big|_{I_2 = 0} = Z_{11} = 20\Omega \text{ -----(2)}$$

$$\text{and } I_2^1 = -I_1 = \frac{-V_1}{20} \text{ A -----(3)}$$

Again $V_2 = (I_1 - I_2^1)5 - 10I_2^1 = (I_1 + I_1)5 + 10I_1 \therefore I_1 = -I_2^1 = 20I_1$

$$\therefore \frac{V_2}{I_1} \Big|_{I_2 = 0} = Z_{21} = 20\Omega \text{ -----(4)}$$

Again V_2 is applied at output part C.D. The input is open, $I_1 = 0$

Here $I_2 = I_2^1$

$$\text{and } V_2 = I_2^1(10 + 5) = 15I_2^1 = 15I_2$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1 = 0} = 15\Omega \text{ -----(5)}$$

$$\text{Also } V_1 = I_2^1 \times 5 = 5I_2$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1 = 0} = 5\Omega \text{ -----(6)}$$

$$\therefore Z_{11} = 20 - \Omega, Z_{12} = 5\Omega$$

$$\therefore Z_{21} = 20 - \Omega, Z_{22} = 15\Omega$$

Q7 (b) The Z-parameter of a circuit are given by $\begin{bmatrix} 4 & 1 \\ 3 & 3 \end{bmatrix}$. Obtain the transmission line ABCD parameters.

Answer

The Z Parameters equation is given by

$$V_1 = 4I_1 + I_2 \text{ -----(1)}$$

$$V_2 = 3I_1 + 3I_2 \text{ -----(2)}$$

$$\text{From eq..(1)} I_1 = \frac{V_2}{3} - I_2 \text{ -----(3)}$$

$$\text{From eq..(2)} V_1 = 4 \left[\frac{V_2}{3} - I_2 \right] I_2 = \frac{4}{3}V_2 + 3(-I_2) \text{ -----(4)}$$

Recreate eq... (3)& (4) we get

$$\begin{aligned} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} &= \begin{bmatrix} 4/3 & 3 \\ 1/3 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \\ &= \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4/3 & 3 \\ 1/3 & 1 \end{bmatrix} \end{aligned}$$

Q8 (a) Obtain the driving point impedance of the given network across A-B shown in Fig.11 using Transform network.

Answer

Transforming the given network into domain

$$Z_{AB}(s) = Z_{L_1}(s) + Z_{ep(s)}$$

$$\begin{aligned} Z_{CD}(s) &= \left[\left(\frac{s+1}{s} \right) 11 \frac{1}{s} \right] \\ &= \frac{s(s+1/s)}{s+s+1/s} = \frac{s(s+1/s)}{2s+1/s} = \frac{s(s^2+1)}{2s^2+1} \end{aligned}$$

Also,

$$Z_{L_1}(s) = s$$

$$\therefore Z_{AB}(s) = \Delta + \frac{s(s^2+1)}{2\Delta^2+1} = \frac{2s^3+s+s^3+s}{2s^2+1} = \frac{3s^2+2s}{2s^2+1}$$

$$\therefore Z_{AB}(s) = \frac{s(3s^2+2)}{2s^2+1}$$

Q8 (b) The driving point impedance of an LC network is

$$Z(s) = \frac{10(s^2+4)(s^2+16)}{s(s^2+9)}. \text{ Obtain Foster form of network.}$$

Answer

Two poles exist at $W=0$ and at $W=\infty$.

Taking partial fraction of $2(s)$ we get

$$2(s) = \frac{A}{s} + \frac{B}{s + j3} + \frac{B^*}{s - j3} + Hs$$

$$A = \frac{10(s^2 + 4)(s^2 + 16)}{s(s - j3)} \Big|_{s=0} = \frac{10 \times 4 \times 16}{9} = 71.11$$

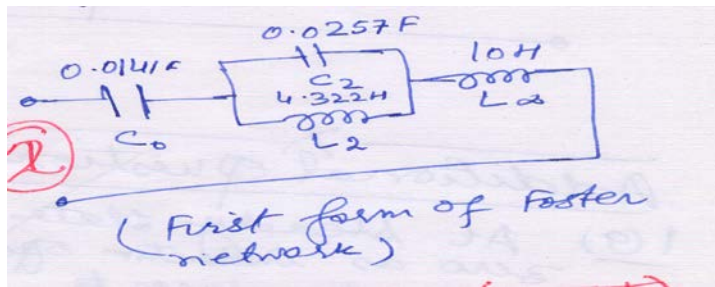
$$B = \frac{10(s^2 + 4)(s^2 + 16)}{s(s - j3)} \Big|_{s=-j3} = \frac{(10(-j3)^2 + 4)((-j3)^2 + 16)}{(-j3)(-j3 - j3)} = \frac{350}{18} = 19.45$$

$$C_0 = \frac{1}{A} = \frac{1}{71.11} = 0.0141F$$

$$L_\infty = H = 10H$$

$$C_2 = \frac{1}{2}B = \frac{1}{2 \times 19.45} = 0.0257F$$

$$L_2 = 2B/w^2n = \frac{2 \times 19.45}{32} = 4.322H.$$



Q9 (a) What are the error criteria in any approximation problem in network theory? Derive amplitude approximation for maximally flat low pass filter approximation.

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Q9 (b) Synthesize the voltage ratio $\frac{V_2}{V_1} = \frac{s^2 + 1}{s^2 + 2s + 1}$ as a constant resistance bridged-T network terminated in a 1Ω resistor.

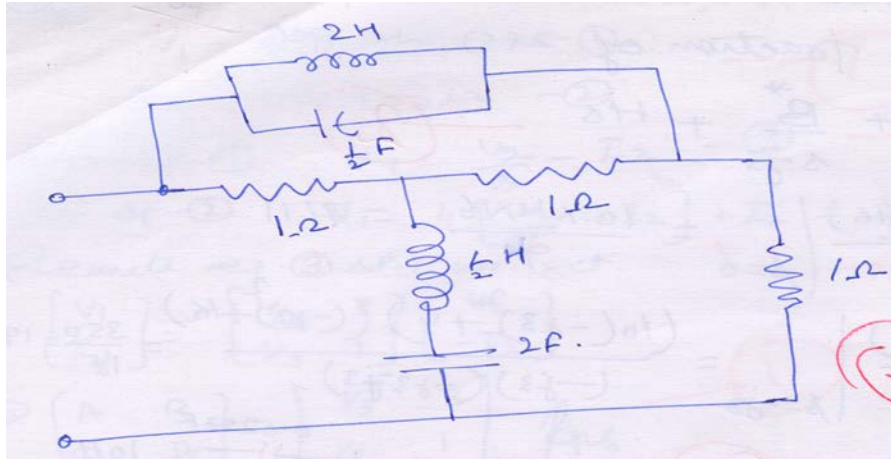
Answer

$$\frac{V_2}{V_1} = \frac{s^2 + 1}{s^2 + 2s + 1} \text{-----(1)}$$

$$Z_a = \frac{2s}{s^2 + 1} \text{-----(2)}$$

$$Z_b = \frac{s^2 + 1}{2s} \text{-----(3)}$$

Z_a , as parallel L_c to network circuit and Z_b as series L_c network circuit. The network is



Text Books

1. Network Analysis, M.E. Van Valkenberg, 3rd Edition, Prentice-Hall India, EEE 2006.
2. Network Analysis and Synthesis, Franklin F Kuo, 2nd Edition, Wiley India Student Edition 2006.