ROLL NO. ________________

Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEM... I

AMIETE – ET/CS/IT

Time: 3 Hours **JUNE 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \\ 1 & 2 & 4 & 4 \\ 1 & 2 & 3 & 5 \end{vmatrix}$ is

(A) 0 **(B)** 1
(C) 2 **(D)** 3

b. The rank of the matrix $\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$ is

(A) 4 **(B)** 3
(C) 2 **(D)** 1

c. If the curves $f(x, y) = 0$ and $\phi(x, y) = 0$ touch each other, then at the point of contact,

(A) $\frac{\partial f}{\partial x}\frac{\partial \phi}{\partial y} = \frac{\partial f}{\partial y}\frac{\partial \phi}{\partial x}$ **(B)** $\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial \phi}{\partial x}\frac{\partial \phi}{\partial y}$
(C) $\frac{\partial f}{\partial x}\frac{\partial \phi}{\partial x} = \frac{\partial f}{\partial y}\frac{\partial \phi}{\partial y}$ **(D)** None of these

d. The value of the integral $\int_0^1 \int_0^{\sqrt{(1-y^2)}} x^3 y \, dxdy$ is

(A) $\frac{1}{6}$ **(B)** $\frac{1}{8}$
(C) $\frac{1}{12}$ **(D)** $\frac{1}{24}$

e. Using Newton-Raphson method, a recurrence formula for finding $\sqrt{N}$ is

(A) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ **(B)** $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{Nx_n}\right)$
(C) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{N}{x_n}\right)$ **(D)** $x_{n+1} = \frac{1}{2}\left(x_n - \frac{1}{Nx_n}\right)$

f. A family of straight lines passing through the origin is represented by

(A) ydx + xdy = 0 **(B)** ydx – xdy = 0
(C) xdx + ydy = 0 **(D)** xdx – ydy = 0

g. Particular integral of the differential equation $\frac{d^2y}{dx^2} + x^2 y = \cos(nx + \alpha)$

(A) $\frac{x}{2n}\cos(nx + \alpha)$ **(B)** $-2nx\cos(nx + \alpha)$
(C) $\frac{x}{2n}\sin(nx + \alpha)$ **(D)** $-2nx\sin(nx + \alpha)$

h. $\beta\left(\frac{1}{2}, \frac{3}{2}\right)$ is equal to

(A) $\sqrt{\pi}$ **(B)** π
(C) $\frac{\sqrt{\pi}}{2}$ **(D)** $\frac{\pi}{2}$

i. The value of $J^2_{\frac{1}{2}}(x) + J^2_{-\frac{1}{2}}(x)$ is

(A) $\frac{2}{\pi x}$ **(B)** $\frac{\pi x}{2}$
(C) $\frac{2x}{\pi}$ **(D)** $\frac{x}{2\pi}$

j. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then

(A) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} - z\frac{\partial u}{\partial z} = 0$ **(B)** $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} - z\frac{\partial u}{\partial z} = 0$

(C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(D)** None of these

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Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $x^x y^y z^z = c$, show that at x=y=z, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ **(8)**

b. Expand $f(x, y) = \tan^{-1}(xy)$ in powers of $(x-1)$ and $(y-1)$ upto second degree terms. **(8)**

Q.3 a. Change the order of integration and then evaluate it $\int_0^\infty \int_0^x x e^{\frac{-x^2}{y}} dy dx$ **(8)**

b. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ **(8)**

Q.4 a. Solve the equation $\begin{vmatrix} x+1 & 2x+1 & 3x+1 \\ 2x & 4x+3 & 6x+3 \\ 4x+1 & 6x+4 & 8x+4 \end{vmatrix} = 0$ **(8)**

b. Find the values of λ for which the equations $(2-\lambda)x + 2y + 3 = 0$, $2x + (4-\lambda)y + 7 = 0$, $2x + 5y + (6-\lambda) = 0$ are consistent and find the values of x and y corresponding to each of these values of λ. **(8)**

Q.5 a. Use Regula-Falsi method to compute the real root of $xe^x = 2$ correct to three decimal places. **(8)**

b. Use Runge-Kutta method of order four to find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$. Take h = 0.2. **(8)**

Q.6 a. Solve the equation $\frac{dy}{dx} = -\left(\frac{x + y\cos x}{1 + \sin x}\right)$ **(8)**

b. Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2\lambda y + C = 0$, λ being the parameter. **(8)**

Q.7 a. Solve the differential equation $\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$ **(8)**

b. Use method of variation of parameters to solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ **(8)**

Q.8 a. Show that

(i) $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$

(ii) $\beta(m, n+1) + \beta(m+1, n) = \beta(m, n)$ **(4+4)**

b. Solve in series the equation $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$ **(8)**

Q.9 a. Show that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$ **(8)**

b. Show that $\int_{-1}^{1} \left(1 - x^2\right)\; P'_m(x)\; P'_n(x)\, dx = 0$ **(8)**