

Code: AC65

Subject: DISCRETE STRUCTURES

AMIETE – CS

Time: 3 Hours

JUNE 2013

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

- a. If $A \times B = B \times A$, (where A and B are general matrices) then
- | | |
|----------------|--------------|
| (A) $A = \phi$ | (B) $A = B'$ |
| (C) $B = A$ | (D) $A' = B$ |
- b. A partial order relation is transitive, reflexive and
- | | |
|-------------------|-----------------|
| (A) antisymmetric | (B) bisymmetric |
| (C) antireflexive | (D) asymmetric |
- c. $p \rightarrow q$ is logically equivalent to
- | | |
|----------------------------|----------------------------|
| (A) $\sim q \rightarrow p$ | (B) $\sim p \rightarrow q$ |
| (C) $\sim p \wedge q$ | (D) $\sim p \vee q$ |
- d. A recursive definition for the sequence $a_n = n(n+2)$ for $n \geq 1$ is
- | | |
|------------------------------|--------------------------------|
| (A) $a_{n+1} = 2 \times a_n$ | (B) $a_{n+1} = a_{n-1} + 2$ |
| (C) $a_{n+1} = a_n + 2n + 3$ | (D) $a_{n+1} = a_n + (2n + 1)$ |
- e. The number of distinct relations on a set of 3 elements is:
- | | |
|--------|---------|
| (A) 8 | (B) 9 |
| (C) 18 | (D) 512 |
- f. $(P \rightarrow q) \wedge (P \rightarrow R)$ is
- | | |
|----------------------------------|---------------------------------------|
| (A) $P \rightarrow (Q \vee R)$ | (B) $P \rightarrow (q \rightarrow R)$ |
| (C) $P \rightarrow (q \wedge R)$ | (D) $P \rightarrow (R \rightarrow q)$ |
- g. Let $N = \{1, 2, 3, \dots\}$ be ordered by divisibility, which of the following subset is totally ordered
- | | |
|----------------|-----------------|
| (A) (2, 6, 24) | (B) (3, 5, 15) |
| (C) (2, 9, 16) | (D) (4, 15, 30) |

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- h. Three unbiased coins are tossed. What is the probability of getting at most two heads?
- (A) $3/4$ (B) $1/4$
(C) $3/8$ (D) $7/8$
- i. Context free languages are closed under
- (A) union, intersection (B) intersection, complement
(C) union, kleene star (D) complement, kleene star
- j. If any 6 numbers are chosen from 1 to 10, then two of them have their sum equal to
- (A) 6 (B) 10
(C) 15 (D) 11

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

- Q.2** a. Prove that $A - (B \cap C) = (A - B) \cup (A - C)$ (5)
- b. A die is tossed thrice. Find the probability of getting an odd number at least once. (5)
- c. A speaks truth in 80% of the cases and B speaks truth in 60% of the cases. Find the probability of the cases of which they are likely to contradict each other in stating the same fact. (6)
- Q.3** a. Prove the following logical equivalences without using truth tables:
- (i) $P \vee [P \wedge (P \vee Q)] \Leftrightarrow P$
(ii) $[P \vee Q \vee (\neg P \wedge \neg Q \wedge R)] \Leftrightarrow P \vee Q \vee R$
(iii) $[(\neg P \vee \neg Q) \rightarrow (P \wedge Q \wedge R)] \Leftrightarrow P \wedge Q$
(iv) $[(P \rightarrow q) \wedge (r \rightarrow q)] \Leftrightarrow [(P \vee r) \rightarrow q]$ (8)
- b. What are tautologies and contradictions? Prove that, for any propositions P, Q, R the following compound propositions are tautologies:
- (i) $[(P \rightarrow Q) \wedge (P \rightarrow R)] \rightarrow (P \rightarrow R)$
(ii) $[P \rightarrow (Q \rightarrow R)] \rightarrow [(P \rightarrow Q) \rightarrow (P \rightarrow R)]$ (8)
- Q.4** a. Prove that $q \vee s$ is a valid conclusion from the premises $P \rightarrow q, r \rightarrow s, P \vee r$. (8)
- b. Test the validity of the following argument:
If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
 \therefore I must have watched TV in the evenings. (8)

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- Q.5** a. Find an explicit definition of the sequence defined recursively by
 $a_1 = 7, a_n = 2a_{n-1} + 1$ for $n \geq 2$ (8)
- b. Prove by induction
 $1.1! + 2.2! + \dots + n.n! = (n+1)! - 1$ (8)
- Q.6** a. If R is a relation defined by iff $a + d = b + c$, show that R is an equivalence relation. (8)
- b. For any relation R in a set A , we can define the inverse relation R^{-1} by aR_b^{-1} iff aR_b
 Prove that
 (i) As a subset of $A \times A$, $R^{-1} = \{(b, a) / (a, b) \in R\}$
 (ii) R is symmetric iff $R = R^{-1}$ (8)
- Q.7** a. Prove that the inverse of a linear function is also linear and the two slopes are reciprocals of each other. (8)
- b. A toy manufacturer has a new product to sell. The number of units to be sold, x , is a function of the price p : $n(p) = 30 - 25p$. The revenue earned is a function of the number of units sold: $r(x) = 1000 - (1/4)x^2$. Find the function for revenue in terms of price, p . (8)
- Q.8** a. Let G be a cyclic group of order 6. How many of its elements generate G ? (8)
- b. State and prove Lagrange's theorem. Explain any two direct consequences of Lagrange's theorem. (8)
- Q.9** a. Let R be a ring with a unity element 1. Show that the set R^* of units in R is a group under Multiplication. (8)
- b. Give steps and an example to generate a parity check matrix. (8)