Q.2 a. Define control system. When is a control system said to be robust?

Answer:

Control systems, combinations of components (electrical, mechanical, thermal, or hydraulic) that act together to maintain actual system performance close to a desired set of performance specifications**:**

A control system is said to be robust when

(i) It has low sensitivities

(ii) It is stable over a wide range of parameter variations; and

(iii) The performance stays within prescribed (but practical) limit bounds in presence of changes in the parameters of the controlled system and disturbance input

b. Define the following terms in respect of feedback control system:
(i) Feed forward element
(ii) Control signal
(iii) Feedback element
(iv) Actuating signal

Answer: Page Number 17-18 of Text Book

Q.3 a. Define the transfer function of a linear time-invariant system in terms of its differential equation model. What is the characteristic equation of the system?

Answer:

The input- relation of a linear time invariant system is described by the following nth-order differential output equation with constant real coefficients:

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots.. + a_1\frac{dy(t)}{dt} + a_0 y(t) = \frac{d^m u(t)}{dt^m} + b_{n-1}\frac{d^{m-1} u(t)}{dt^{m-1}} + \ldots.. + b_1\frac{du(t)}{dt} + b_0 u(t)$$

"Transfer function is defined as the ratio of Laplace transform of output variable to the input variable assuming all initial condition to be zero."i.e.

$$T(s)=Y(s)/U(s)=s^m+b_{n-1}s^{m-1}+ \ldots\ldots\ldots\ldots\ldots\ldots..+b_0/s^n+a_{n-1}s^{n-1}+\ldots\ldots.+a_0$$

The characteristic equation is given by

$$s^n + a_{n-1}s^{n-1} + \ldots\ldots + a_1 s + a_0 = 0$$

b. Obtain the Unit-step response of a unity-feedback whose open-loop transfer function is $G(s) = \frac{5(s+20)}{s(s+4.59)(s^2+3.14s+16.35)}$

Answer:

$$G(s) = \frac{5(s+20)}{s(s+4.59)(s^2+3.41s+16.35)}$$

$$M(s) = \frac{G(s)}{1+G(s)} = \frac{5(s+20)}{(s^2+4.59s)(s^2+3.41s+16.35)+5s+100}$$

$$= \frac{5(s+20)}{s^4+8s^3+32s^2+80.04s+100}$$

$$\frac{C(s)}{R(s)} = M(s) \qquad C(s) = 1/sM(s)$$

$$C(s) = \frac{5s+100}{s(s^4+8s^3+32s^2+80.04s+100)}$$

Applying inverse laplace transform

$$C(t) = 1+3/8e^{-t}\cos(3t) - 17/24\, e^{-t}\sin(3t) - 11/8e^{(-3t)}\cos(t) - 13/8\, e^{-3t}\sin(t)$$

Q.4 a. Explain the procedure to be followed when in the Routh's array all the elements of a row corresponding to S^4 are zeros.

Answer:

When all the elements of a row corresponding to s^4 are zeros, then write the auxiliary equation. This is an equation formed by the coefficients of the row just above the row having all zeros. Here it means make an equation from the row corresponding to s^5. Differentiate this equation and replace the entries in Routh's array by the coefficients of this differential equation. Then follow the usual procedure.

b. Write short note on compensation.

Answer:

- Compensation technique is used to make the unstable system stable by introducing the poles and or zeros at suitable place.
- In control system design, if the designed specifications do not satisfy the requirements of the system or leads to expensive and conflicting demands, then it is required to insert an additional component within the structure of feedback system. This adjustment is called compensation.
- Compensators are of three types:

- (a) Lag phase compensator (b) Lead phase compensator
  (c) Lead-lag compensator.
- Depending on the location the compensation is divided in following types:
  (a) Series or cascade compensation
  (b) Parallel or feedback compensation
  (c) Load compensation.
- Load compensations is provided to damp out oscillations in the system having mechanical output.
- Primary function of the lead compensator is to reshape the frequency response curve to provide sufficient phase lead angle to offset the excessive phase lag associated with the components of fixed systems.
- The primary function of lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin.
- The design of lead-lag compensator by the frequency response approach is based on the combination of the design techniques discussed under lead compensation and lag compensation.
- Tachnometer, rate gyro are the examples of rate feedback devices.
- Two different methods of designing control system with rate feedback are single parameter loci and multiple parameter loci.

c. Obtain the transfer function for RLC circuit shown in Fig.2 below.

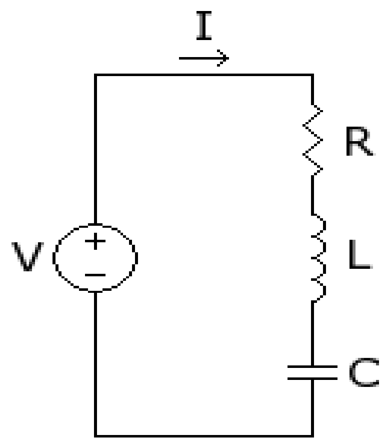


Fig.2

Answer:

Consider a RLC circuit. The governing differential equation is given by

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int idt=V.$$

But,

$$i=\frac{dq}{dt}.$$

Therefore, equation (3) reduces to

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V.$$

The Laplace transforms of the above equation yields

$$Ls^2Q(s)+RsQ(s)+\frac{1}{C}Q(s)=V(s),$$

$$\Rightarrow\frac{Q(s)}{V(s)}=\frac{1}{Ls^2+Rs+\frac{1}{C}}.$$

The above equation represents the transfer function of a RLC circuit.

Q.5 a. For the system whose signal flow graph is shown by Fig.3, find $\frac{Y(s)}{R(s)}$

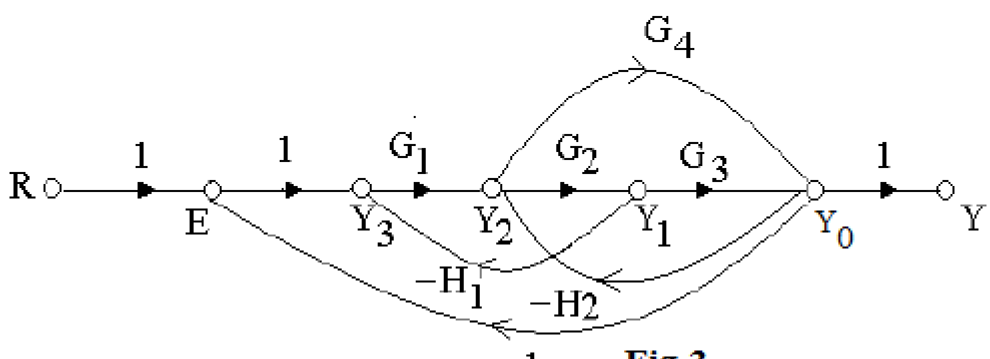


Fig.3

Answer:

$$P_1 = -G_1G_2G_3$$
$$P_2 = -G_1G_4$$

There are no non touching loops

$$\Delta = 1-(-G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_1G_4 - G_4H_2)$$

$$T = \frac{P_1\,\Delta_1 + P_2\,\Delta_2}{\Delta}$$

$$\text{Transfer function} = \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 + G_1G_4 + G_4H_2}$$

b. Determine the transfer function $\frac{C(s)}{R(s)}$ for the block diagram shown in Fig.4 by first drawing its signal flow graph and then using the Mason's gain formula.

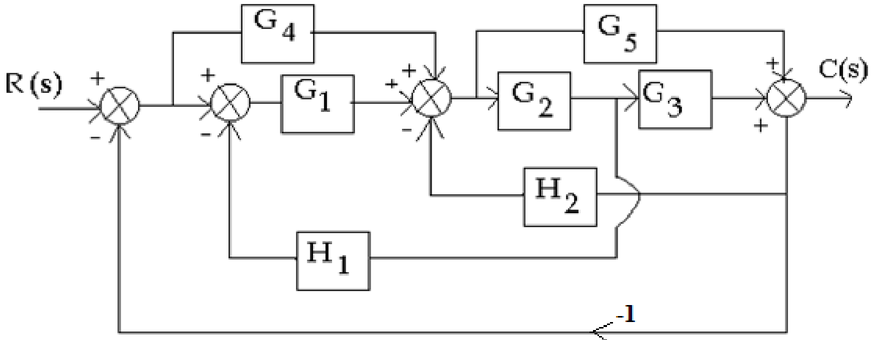
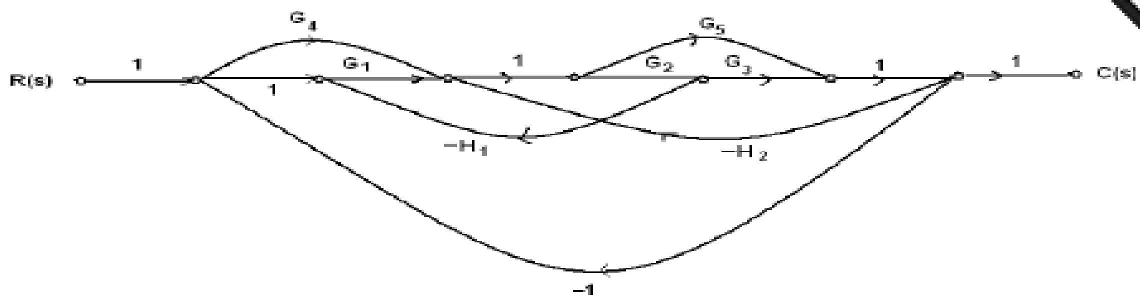


Fig.4

Answer:

SIGNAL FLOW GRAPH OF THE BLOCK DIAGRAM:



Mason's gain formula

Overall system gain is given by

$$T = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

P_k – gain of k[th] forward path

Δ = det of the graph
= 1 – sum of loop gains of all individual loops + (sum of gain products of all possible combinations of two non-touching loops) –(sum of gain products of all possible combinations of three non touching loops)+ ......
Δ_k = the value of Δ for that part of the graph not touching the kth forward path.

There are four path gains

$$P_1 = G_1G_2G_3$$
$$P_2 = G_4G_2G_3$$
$$P_3 = G_1G_5$$
$$P_4 = G_4G_5$$

Individual loop gains are

$$P_{11} = -G_1G_2H_1$$
$$P_{21} = -G_5H_2$$
$$P_{31} = -G_2G_3H_2$$
$$P_{41} = -G_4G_5$$
$$P_{51} = -G_1G_2G_3$$
$$P_{61} = -G_4G_2G_3$$
$$P_{71} = -G_1G_5$$

There are no non touching loops

$$\Delta = 1-(-G_1G_2H_1-G_5H_2-G_2G_3H_2-G_4G_5-G_1G_2G_3-G_4G_2G_3-G_1G_5)$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_4G_2G_3 + G_1G_5 + G_4G_5}{1+G_1G_2H_1+G_5H_2+G_2G_3H_2+G_4G_5+G_1G_2G_3+G_4G_2G_3+G_1G_5}$$

Q.6 a. The transfer functions for a single-loop non-unity-feedback control system are given as $G(s)=\frac{1}{s^2+s+2}$ and $H(s)=\frac{1}{s+1}$. Find the steady-state errors due to a unit-step input, a unit-ramp input and a parabolic input.

Answer:

$$G(s) = \frac{1}{s^2 + s+2} \qquad H(s) = \frac{1}{(s+1)}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} \qquad e_{ss} = \lim_{s\to 0} sE(s)$$

for unit step input

$$e_{ss} = \lim_{s\to 0} \frac{s*1/s}{1 + (1/s^2+s+2)(1/s+1)}$$

$$= \frac{1}{1 +(½)} = 2/3$$

for unit ramp input

$$e_{ss} = \lim_{s\to 0} \frac{s*1/ s^2}{1 + G(s)H(s)} = \infty$$

for unit parabolic input

$$e_{ss} = \lim_{s\to 0} \frac{s*1/s^3}{1 + G(s)H(s)} = \infty$$

b. Define sensitivity. Discuss the sensitivity of transfer function with different parameters.

Answer:

Changes in Parameter Values

Definition of Sensitivity

The term that is used to represent the relative change in the total transfer function from input to output for a given relative change in some transfer function or parameter in the system is sensitivity. The sensitivity of a system is a transfer function, which is denoted by S(s). To explicitly represent the sensitivity of transfer function A(s) with respect to transfer function B(s), the notation S^A_B is used

Assume that a closed-loop system has a forward transfer function G(s) and a feedback transfer function H(s). The sensitivity of the closed-loop transfer function T(s) to changes in the forward transfer function G(s) is

$$S_G^T \triangleq \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{\partial T(s)}{\partial G(s)} \cdot \frac{G(s)}{T(s)} = \frac{\partial}{\partial G(s)}\left[\frac{G(s)}{1+G(s)H(s)}\right] \cdot \frac{G(s)}{\left[\frac{G(s)}{1+G(s)H(s)}\right]} \quad (5)$$

$$S_G^T = \left[\frac{(1+G(s)H(s))(1) - G(s)H(s)}{(1+G(s)H(s))^2}\right] \cdot [1+G(s)H(s)] \quad (6)$$

$$S_G^T = \frac{1}{1+G(s)H(s)} = S(s) \quad (7)$$

Small values for $|S_G^T|$ mean that changes in the forward transfer function $G(s)$ produce only small changes in the closed-loop transfer function $T(s)$. This is accomplished when $|G(s)H(s)| \gg 1$. Note that in an open-loop configuration, $H(s) = 0$, and $S_G^T = 1$. This means that the total transfer function from $R(s)$ to $Y(s)$ changes by the same amount as $G(s)$. This makes sense since in an open-loop system, $G(s)$ is the total transfer function from $R(s)$ to $Y(s)$. The $G(s)$ that is used in (7) is the nominal (unperturbed) transfer function. Note that for an open-loop system, $H(s) = 0$, and $S_G^T = 1$, which indicates that there is a one-to-one correspondence between changes in $G(s)$ and changes in the transfer function between $R_{OL}(s)$ and $Y(s)$.

The sensitivity of the closed-loop transfer function $T(s)$ to changes in the feedback transfer function $H(s)$ is

$$S_H^T \triangleq \frac{\partial T(s)/T(s)}{\partial H(s)/H(s)} = \frac{\partial T(s)}{\partial H(s)} \cdot \frac{H(s)}{T(s)} = \frac{\partial}{\partial H(s)}\left[\frac{G(s)}{1+G(s)H(s)}\right] \cdot \frac{H(s)}{\left[\frac{G(s)}{1+G(s)H(s)}\right]} \quad (8)$$

$$S_H^T = \left[\frac{(1+G(s)H(s))(0) - G(s)G(s)}{(1+G(s)H(s))^2}\right] \cdot \frac{[1+G(s)H(s)]\,H(s)}{G(s)} \quad (9)$$

$$S_H^T = \frac{-G(s)H(s)}{1+G(s)H(s)} \quad (10)$$

If we have $|G(s)H(s)| \gg 1$ in order to have $|S_G^T| \ll 1$, the sensitivity to changes in the feedback transfer function is $|S_H^T| \approx 1$. Thus, there is a trade-off between sensitivity reduction to forward-path perturbations and sensitivity reduction to feedback-path perturbations. The negative sign in the numerator of (10) indicates a reversal in the direction of change between $H(s)$ and $T(s)$. Note that with $H(s) = 1$, we have the following relationship between the sensitivity $S(s)$ (which equals S_G^T) and the closed-loop transfer function $T(s)$.

$$S(s) = \frac{1}{1+G(s)}, \qquad T(s) = \frac{G(s)}{1+G(s)} \quad \Rightarrow \quad S(s) + T(s) = 1 \quad (11)$$

which clearly shows that there is a trade-off between these two functions; they cannot both be made small in magnitude at the same time.

Q.7 Construct root locus and comment on the stability of a unity-feedback control system having the open-loop transfer function $G(s) = \frac{10}{s(s-1)(2s+3)}$

Answer:

$$G(s) = \frac{10}{s(s-1)(2s+3)}$$

Poles s = 0, 1, -1.5

$$\text{Angle of asymptotes} = \frac{\pm 180\,(2q+1)}{(n-m)}$$

± 60 , ± 180

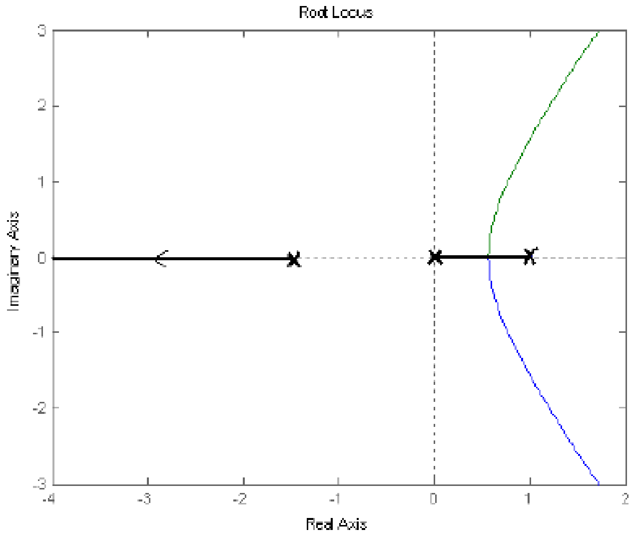
$$\text{centroid} = \frac{1 - 1.5}{2} = -0.25$$

$$\frac{C(s)}{R(s)} = \frac{k}{s(s-1)(2s+3)+k}$$

$(s^2-s)(2s+3)+k=0$

$k = -(2s^3+3s^2-2s^2-3s) = -(2s^3+s^2-3s)$

$dk/ds = -(6s^2+2s-3) = 0 \rightarrow s = 0.5598$



ROOT LOCUS FIGURE

From the root locus, we find that for any gain the system has right half poles. So the closed loop system is always unstable.

Q.8 a. Explain the properties of polar plots.

Answer: Page Number 252 of Text Book

b. Use the Nyquist criterion to determine the range of values of K>0 for the stability of the system in Fig.5.

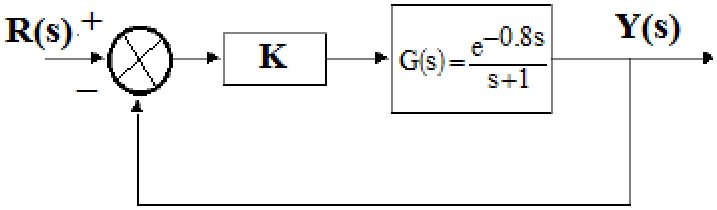


Fig.5

Answer:

$$G(jw) = \frac{e^{-0.8jw}}{jw+1}$$

$$= \frac{1}{1+w^2}\left[\cos(0.8w) - w\sin(0.8w) - j(\sin(0.8w) + w\cos(0.8w))\right]$$

The imaginary part is equal to zero if $(\sin(0.8w) + w\cos(0.8w)) = 0$; this gives w value as -tan (0.8w); solving this equation for smallest positive value of w, we get ,w = 2.4482

$G(jw)\big|_{w=0} = 1+j0$; $G(jw)\big|_{w=\infty} = 0$ and $G(jw)\big|_{w=2.4482} = -0.378+j0$

The polar plot spiral into the $w \to \infty$ point at the origin

The critical value of K is obtained by letting G(j2.4482) equal to -1.This gives K=2.65

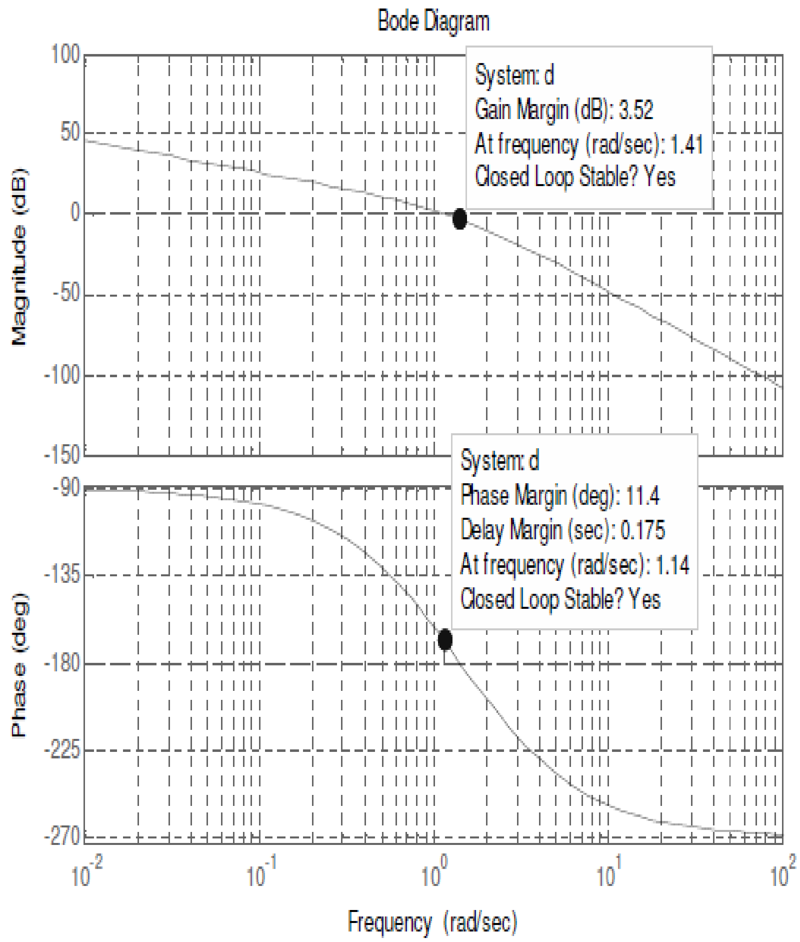
The closed loop system is stable for K < 2.65.

Q.9 A unity-feedback system has open-loop transfer function $G(s) = \dfrac{4}{s(s+1)(s+2)}$

(i) Using Bode plots of G(jw), determine the phase margin of the system.

(ii) How should the gain be adjusted so that phase margin is 50°?

Answer:



(i). From the bode plot we get PM = 11.4 deg

(ii). For the phase margin of 50 deg , we require that mag(G(jw)H(jw)) =1 and ang(G(jw)H(jw)) = -130 deg , for some value of w ,from the phase curve of G(jw)H(jw) ,we find that ang(G(jw)H(jw)) = -130 deg at w = 0.5. The magnitude of G(jw)H(jw) at this frequency is approximately 3.5 .The gain must be reduced by a factor of 3.5 to achieve a phase margin of 50 deg .

TEXT BOOK

Feedback and Control Systems (Schaum's Outlines), Joseph J DiStefano III, Allen R.Stubberud and Ivan J. Williams, 2nd Edition, 2007, Tata McGraw-Hill Publishing Company Ltd.