

Q.2 a. With neat block diagram explain the working of digital communication system.

Answer:

Digital Communication System

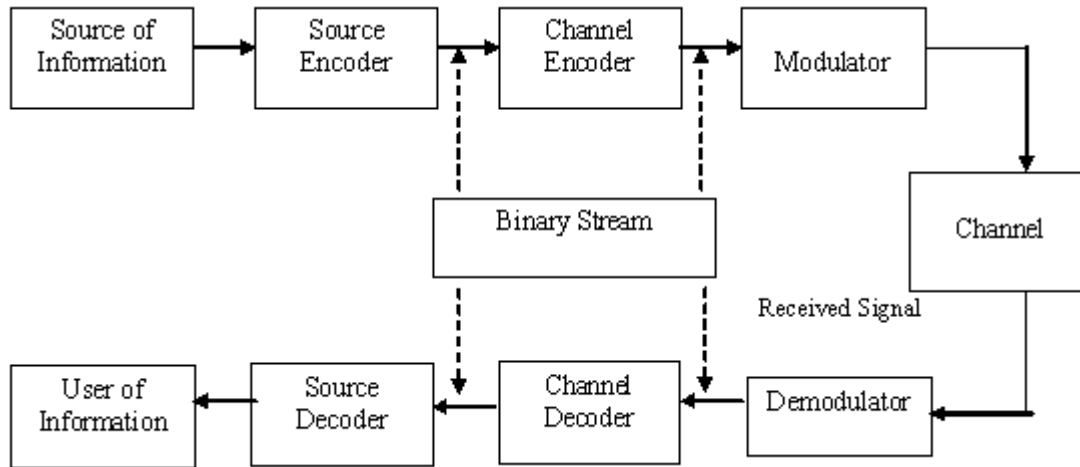


Fig 1.: Block Diagram of a Digital Communication System

SOURCE ENCODER / DECODER:

The Source encoder (or Source coder) converts the input i.e. symbol sequence into a binary sequence of 0's and 1's by assigning code words to the symbols in the input sequence.

At the receiver, the source decoder converts the binary output of the channel decoder into a symbol sequence.

Aim of the source coding is to remove the redundancy in the transmitting information, Ex: Huffman coding.

CHANNEL ENCODER / DECODER:

Error control is accomplished by the channel coding operation that consists of systematically adding extra bits to the output of the source coder. These extra bits do not convey any information but helps the receiver to detect and / or correct some of the errors in the information bearing bits.

Example: Block Coding, Convolution Coding

MODULATOR:

The Modulator converts the input bit stream into an electrical waveform suitable for transmission over the communication channel. Modulator can be effectively used to minimize the effects of channel noise, to match the frequency spectrum of transmitted signal with channel characteristics, to provide the capability to multiplex many signals.

DEMODULATOR:

The extraction of the message from the information bearing waveform produced by the modulation is accomplished by the demodulator. The output of the demodulator is bit stream. The important parameter is the method of demodulation.

CHANNEL:

The Channel provides the electrical connection between the source and destination. The different channels are: Pair of wires, Coaxial cable, Optical fibre, Radio channel, Satellite channel or combination of any of these.

b. Define the following terms:

- (i) Self information
- (ii) Entropy

Answer:

Self information: self information of a symbol S_k with probabilities P_k is

$$I[S_k] = \log[1/P_k]$$

Entropy: if source consists of M symbols $S_1, S_2, S_3, \dots, S_M$ with probabilities P_1, P_2, \dots, P_M respectively, then entropy

$$H[s] = \sum_{i=1}^M p_i \log(1/p_i)$$

Q.3 a. State and prove sampling theorem for low-pass signal.

Answer:

Statement:- "If a band –limited signal $g(t)$ contains no frequency components for $|f| > W$, then it is completely described by instantaneous values $g(kT_s)$ uniformly spaced in time with period $T_s \leq 1/2W$. If the sampling rate, f_s is equal to the Nyquist rate or greater ($f_s \geq 2W$), the signal $g(t)$ can be exactly reconstructed.

i.e $f_s \geq 2W$

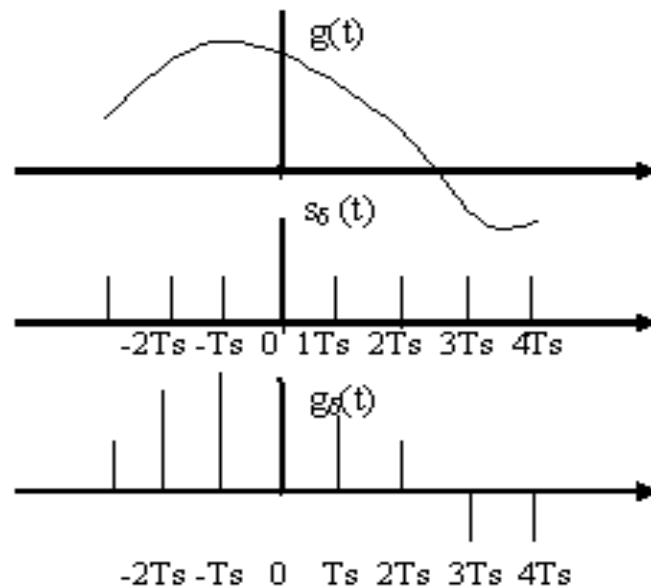


Fig : Sampling process

Proof:- Consider the signal $g(t)$ is sampled by using a train of impulses $s_\delta(t)$.

Let $g_\delta(t)$ denote the ideally sampled signal, can be represented as

$$g_\delta(t) = g(t) \cdot s_\delta(t) \quad \text{----- 1}$$

where $s_\delta(t)$ – impulse train defined by

$$s_\delta(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT_s) \quad \text{----- 2}$$

Therefore
$$g_\delta(t) = g(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t - kT_s)$$

$$= \sum_{k=-\infty}^{+\infty} g(kT_s) \cdot \delta(t - kT_s) \quad \text{----- 3}$$

The Fourier transform of an impulse train is given by

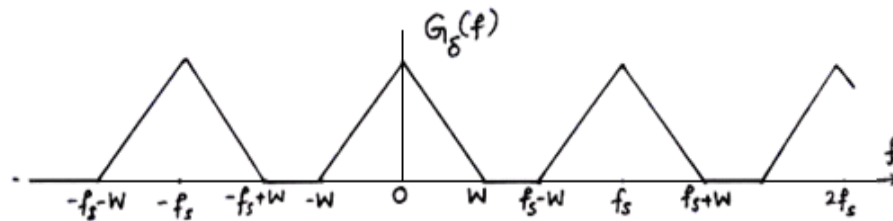
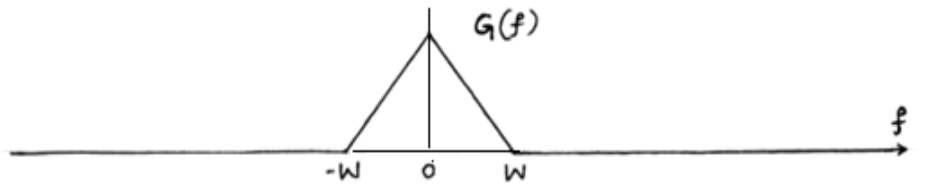
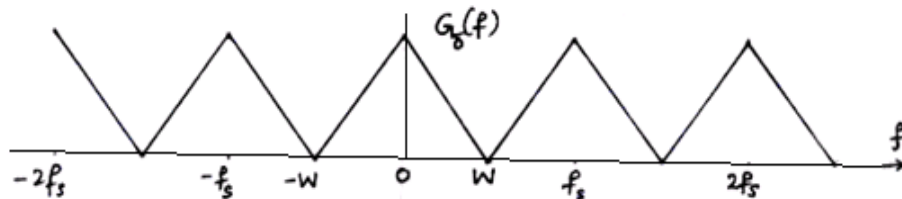
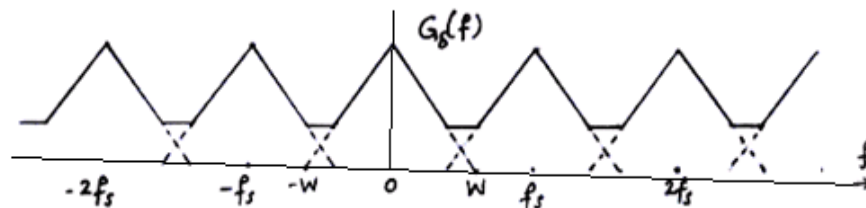
$$S_\delta(f) = F[s_\delta(t)] = f_s \sum_{n=-\infty}^{+\infty} \delta(f - nf_s) \quad \text{----- 4}$$

Applying F.T to equation 1 and using convolution in frequency domain property,

$$G_\delta(f) = G(f) * S_\delta(f)$$

Using equation 4,
$$G_\delta(f) = G(f) * f_s \sum_{n=-\infty}^{+\infty} \delta(f - nf_s)$$

$$G_\delta(f) = f_s \sum_{n=-\infty}^{+\infty} G(f - nf_s) \quad \text{----- 5}$$

Over Sampling ($f_s > 2W$)Nyquist Rate Sampling ($f_s = 2W$)Under Sampling ($f_s < 2W$)

The spectrum of sampled signal is periodic with period f_s . Therefore for proper reconstruction

$f_s \geq 2W$. The minimum sampling rate is called Nyquist rate. If the sampling frequency is less than Nyquist rate then aliasing

b. Briefly explain Quadrature Sampling of Band – Pass Signals.

Answer:

In this scheme, the band pass signal is split into two components, one is in-phase component and other is quadrature component. These two components will be low-pass signals and are sampled separately. This form of sampling is called quadrature sampling.

Let $g(t)$ be a band pass signal, of bandwidth ' $2W$ ' centered around the frequency, f_c , ($f_c > W$). The in-phase component, $g_I(t)$ is obtained by multiplying $g(t)$ with $\cos(2\pi f_c t)$

and then filtering out the high frequency components. Parallely a quadrature phase component is obtained by multiplying $g(t)$ with $\sin(2\pi fct)$ and then filtering out the high frequency components..

The band pass signal $g(t)$ can be expressed as,

$$g(t) = g_I(t) \cdot \cos(2\pi fct) - g_Q(t) \sin(2\pi fct)$$

The in-phase, $g_I(t)$ and quadrature phase $g_Q(t)$ signals are low-pass signals, having band limited to $(-W < f < W)$. Accordingly each component may be sampled at the rate of $2W$ samples per second.

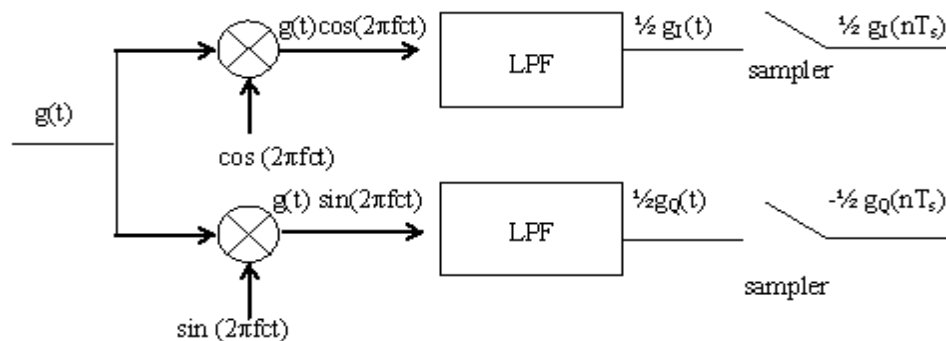


Fig : Generation of in-phase and quadrature phase samples

RECONSTRUCTION:

From the sampled signals $g_I(nTs)$ and $g_Q(nTs)$, the signals $g_I(t)$ and $g_Q(t)$ are obtained. To reconstruct the original band pass signal, multiply the signals $g_I(t)$ and $g_Q(t)$ by $\cos(2\pi fct)$ and $\sin(2\pi fct)$ respectively and then add the results.

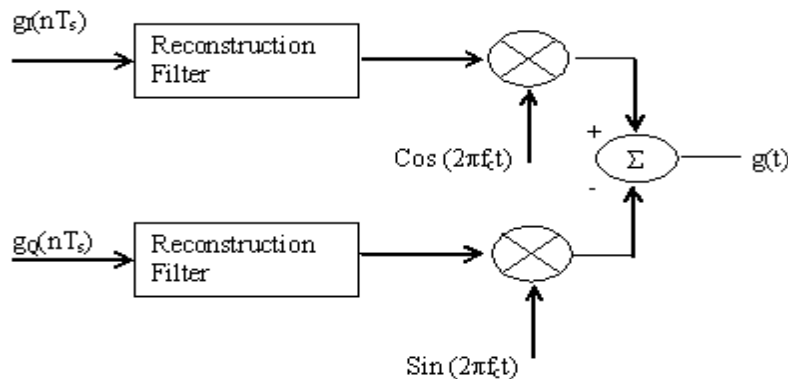


Fig : Reconstruction of Band-pass signal $g(t)$

- c. A message signal $m(t) = 1 + 2\sin 200\pi t + 4\sin 400\pi t$ is to be sampled at Nyquist rate of sampling. Find the sampling frequency.

Answer:

$$m(t) = 1 + 2\sin 200\pi t + 4\sin 400\pi t$$

$$f_1 = 0, f_2 = 100\text{Hz}, f_3 = 200\text{Hz}$$

$$F_{Nyq} = 2 \times F_{max} = 2 \times 200 = 400\text{Hz}$$

Q.4 a. With neat diagram explain the working of PCM system.

Answer:

Basic Blocks:

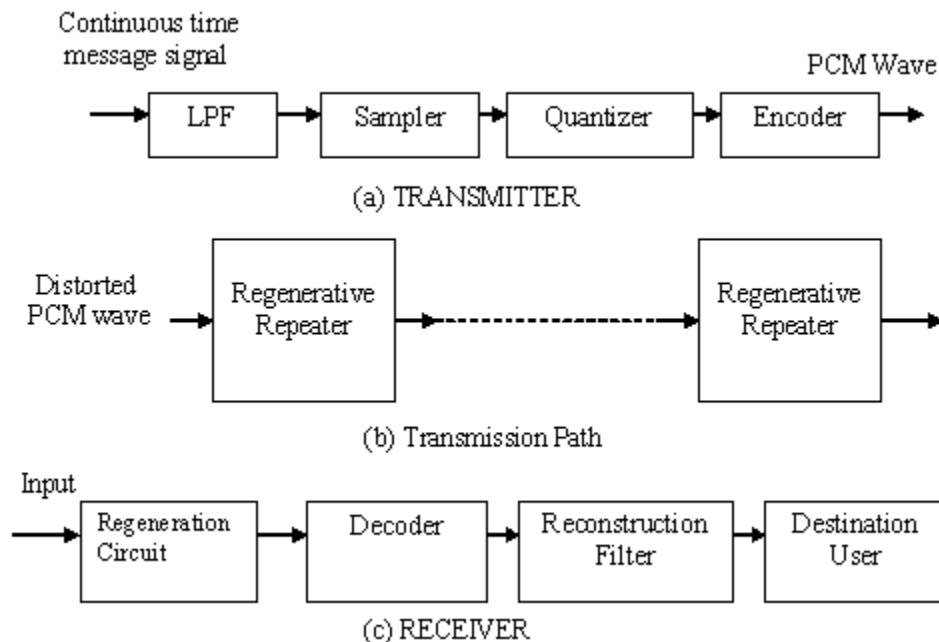
1. Anti aliasing Filter
2. Sampler
3. Quantizer
4. Encoder

An anti-aliasing filter is basically a filter used to ensure that the input signal to sampler is free from the unwanted frequency components.

For most of the applications these are low-pass filters. It removes the frequency components of the signal which are above the cutoff frequency of the filter. The cutoff frequency of the filter is chosen such it is very close to the highest frequency component of the signal.

Sampler unit samples the input signal and these samples are then fed to the Quantizer which outputs the quantized values for each of the samples. The quantizer output is fed to an encoder which generates the binary code for every sample. The quantizer and encoder together is called as analog to digital converter.

In the transmission path the PCM waves are reconstructed using regenerative repeaters. At the receiver we first reconstruct the PCM wave. Decoder converts binary codes into discrete samples. Finally we use a Low pass filter to reconstruct the message from the sampled values.



- b. Obtain an expression for signal to quantization noise ratio for midtread type PCM system.

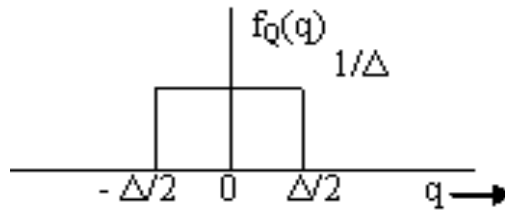
Answer:

Let Q = Random Variable denotes the Quantization error

q = Sampled value of Q

Assuming that the random variable Q is uniformly distributed over the possible range $(-\Delta/2 \text{ to } \Delta/2)$, as

$$f_Q(q) = \begin{cases} 1/\Delta & -\Delta/2 \leq q \leq \Delta/2 \\ 0 & \text{otherwise} \end{cases} \quad \text{----- (1)}$$



where $f_Q(q)$ = probability density function of the Quantization error. If the signal does not overload the Quantizer, then the mean of Quantization error is zero and its variance σ_Q^2

Therefore

$$\sigma_Q^2 = E\{Q^2\}$$

$$\sigma_Q^2 = \int_{-\infty}^{\infty} q^2 f_q(q) dq \quad \text{---- (2)}$$

$$\sigma_Q^2 = \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q^2 dq = \frac{\Delta^2}{12} \quad \text{--- (3)}$$

Thus the variance of the Quantization noise produced by a Uniform Quantizer, grows as the square of the step size. Equation (3) gives an expression for Quantization noise in PCM system.

Let σ_x^2 = Variance of the base band signal $x(t)$ at the input of Quantizer.

When the base band signal is reconstructed at the receiver output, we obtain original signal plus Quantization noise. Therefore output signal to Quantization noise ratio (SNR) is given by

$$(SNR)_o = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\sigma_x^2}{\sigma_Q^2} = \frac{\sigma_x^2}{\Delta^2/12} \quad \text{----- (4)}$$

Smaller the step size Δ , larger will be the SNR.

Let x = Quantizer input, sampled value of random variable X with mean X , variance σ_x^2 . The Quantizer is assumed to be uniform, symmetric and mid tread type.

x_{\max} = absolute value of the overload level of the Quantizer.

Δ = Step size

L = No. of Quantization level given by

$$L = \frac{2x_{\max}}{\Delta} + 1 \quad \text{----- (5)}$$

Let n = No. of bits used to represent each level.

In general $2^n = L$, but in the mid tread Quantizer, since the number of representation levels is odd,

$$L = 2^n - 1 \quad \text{----- (Mid tread only) ---- (6)}$$

From the equations (5) and (6),

$$2^n - 1 = \frac{2x_{\max}}{\Delta} + 1$$

$$\Delta = \frac{x_{\max}}{2^{n-1} - 1} \quad \text{---- (7)}$$

The ratio $\frac{x_{\max}}{\sigma_x}$ is called the loading factor. To avoid significant overload distortion, the amplitude of the Quantizer input x extend from $-4\sigma_x$ to $4\sigma_x$, which corresponds to loading factor of 4. Thus with $x_{\max} = 4\sigma_x$ we can write equation (7) as

$$\Delta = \frac{4\sigma_x}{2^{n-1} - 1} \quad \text{----- (8)}$$

$$(SNR)_O = \frac{\sigma_x^2}{\Delta^2 / 12} = \frac{3}{4} [2^{n-1} - 1]^2 \quad \text{----- (9)}$$

For larger value of n (typically $n > 6$), we may approximate the result as

$$(SNR)_O = \frac{3}{4} [2^{n-1} - 1]^2 \approx \frac{3}{16} (2^{2n}) \quad \text{----- (10)}$$

Hence expressing SNR in db

$$10 \log_{10} (SNR)_O = 6n - 7.2 \quad \text{----- (11)}$$

- c. A mid-riser type PCM system has number of quantization level of 1024. Find the number of bits required. If the minimum SNR required is 40db, find the number of bits required.

Answer:

$$1024 = 2^N$$

$$N = 10$$

$$SNR = 6 \cdot n + 1.76$$

$$40 = 6 \cdot n + 1.76$$

$$N = 7 \text{ bits [integer value]}$$

- Q.5** a. Explain the role of equalizer in digital communication system. Discuss the adaptive equalization technique used in communication system.

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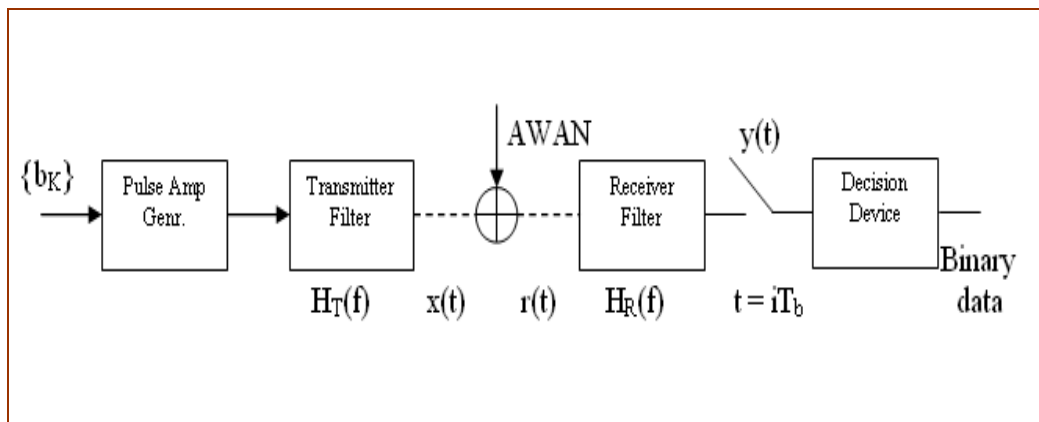
- b. What is Inter symbol Interference? Derive an expression for ISI in base band transmission.

Answer:

When channel bandwidth is close to signal bandwidth, i.e. if we transmit digital data which demands more bandwidth which exceeds channel bandwidth, spreading will occur and cause signal pulses to overlap. This overlapping is called **Inter Symbol Interference**.

PAM signal transmitted is given by

$$x(t) = \sum_{K=-\infty}^{\infty} a_K V(t - KT_b) \quad \text{----- (1)}$$



$V(t)$ is basic pulse, normalized so that $V(0) = 1$,

$x(t)$ represents realization of random process $X(t)$ and a_k is sample value of random variable a_k which depends on type of line codes.

The receiving filter output

$$y(t) = \mu \sum_{K=-\infty}^{\infty} a_K P(t - KT_b) \quad \text{----- (2)}$$

The output pulse $\mu P(t)$ is obtained because input signal $a_k \cdot V(t)$ is passed through series of systems with transfer functions $H_T(f)$, $H_C(f)$, $H_R(f)$

Therefore $\mu P(f) = V(f) \cdot H_T(f) \cdot H_C(f) \cdot H_R(f) \quad \text{----- (3)}$

$$P(f) \Longleftrightarrow p(t) \quad \text{and} \quad V(f) \Longleftrightarrow v(t)$$

The receiving filter output $y(t)$ is sampled at $t_i = iT_b$. where 'i' takes intervals $i = \pm 1, \pm 2, \dots$

$$y(iT_b) = \mu \sum_{K=-\infty}^{\infty} a_K P(iT_b - KT_b)$$

$$y(iT_b) = \mu a_i P(0) + \mu \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} a_K P(iT_b - KT_b) \quad \text{----- (4)}$$

$K = i \qquad K \neq i$

In equation(4) first term μa_i represents the output due to i^{th} transmitted bit. Second term represents residual effect of all other transmitted bits that are obtained while decoding i^{th} bit. This unwanted residual effect indicates ISI.

In absence of ISI desired output would have $y(t_i) = \mu a_i$

- c. A source outputs data at the rate of 50,000 bits/sec. The transmitter uses binary PAM with raised cosine pulse in shaping of optimum pulse width. Determine the bandwidth of the transmitted waveform. Given:

- | | |
|----------------------|----------------------|
| (i) $\alpha = 0$ | (ii) $\alpha = 0.25$ |
| (iii) $\alpha = 0.5$ | (iv) $\alpha = 0.75$ |
| (v) $\alpha = 1$ | |

Answer:

$$B = B_0(1+\alpha) \quad B_0 = R_b/2$$

- i). Bandwidth = $25,000(1 + 0) = 25 \text{ kHz}$
 ii). Bandwidth = $25,000(1 + 0.25) = 31.25 \text{ kHz}$
 iii). Bandwidth = $25,000(1 + 0.5) = 37.5 \text{ kHz}$
 iv). Bandwidth = $25,000(1 + 0.75) = 43.75 \text{ kHz}$
 v). Bandwidth = $25,000(1 + 1) = 50 \text{ kHz}$

- Q.6** a. Explain the working of BPSK system and obtain an expression for probability of error in BPSK system.

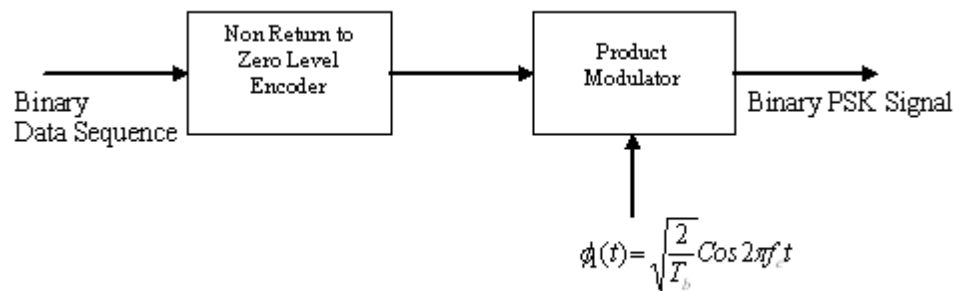
Answer:

In a Coherent binary PSK system the pair of signals $S_1(t)$ and $S_2(t)$ are used to represent binary symbol '1' and '0' respectively.

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \quad \text{-----} \quad \text{for Symbol '1'}$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \quad \text{-----} \quad \text{for Symbol '0'}$$

Where E_b = Average energy transmitted per bit $E_b = \frac{E_{b0} + E_{b1}}{2}$



Fig(a) Block diagram of BPSK transmitter

In the case of PSK, there is only one basic function of Unit energy which is given by

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \quad 0 \leq t \leq T_b$$

Therefore the transmitted signals are given by

$$S_1(t) = \sqrt{E_b} \phi_1(t) \quad 0 \leq t \leq T_b \quad \text{for Symbol 1}$$

$$S_2(t) = -\sqrt{E_b} \phi_1(t) \quad 0 \leq t \leq T_b \quad \text{for Symbol 0}$$

A Coherent BPSK is characterized by having a signal space that is one dimensional (N=1) with two message points (M=2)

$$S_{11} = \int_0^{T_b} S_1(t) \phi_1(t) dt = +\sqrt{E_b}$$

$$S_{21} = \int_0^{T_b} S_2(t) \phi_1(t) dt = -\sqrt{E_b}$$

The message point corresponding to $S_1(t)$ is located at $S_{11} = +\sqrt{E_b}$ and $S_2(t)$ is located at $S_{21} = -\sqrt{E_b}$.

To generate a binary PSK signal we have to represent the input binary sequence in polar form with symbol '1' and '0' represented by constant amplitude levels of $+\sqrt{E_b}$ & $-\sqrt{E_b}$ respectively. This signal transmission encoding is performed by a NRZ level encoder. The resulting binary wave [in polar form] and a sinusoidal carrier $\phi_1(t)$ [whose frequency $f_c = \frac{n_c}{T_b}$] are applied to a product modulator. The desired BPSK wave

is obtained at the modulator output.

To detect the original binary sequence of 1's and 0's we apply the noisy PSK signal $x(t)$ to a Correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$ as shown in fig (b). The correlator output x_1 is compared with a threshold of zero volt.

If $x_1 > 0$, the receiver decides in favour of symbol 1.

If $x_1 < 0$, the receiver decides in favour of symbol 0.

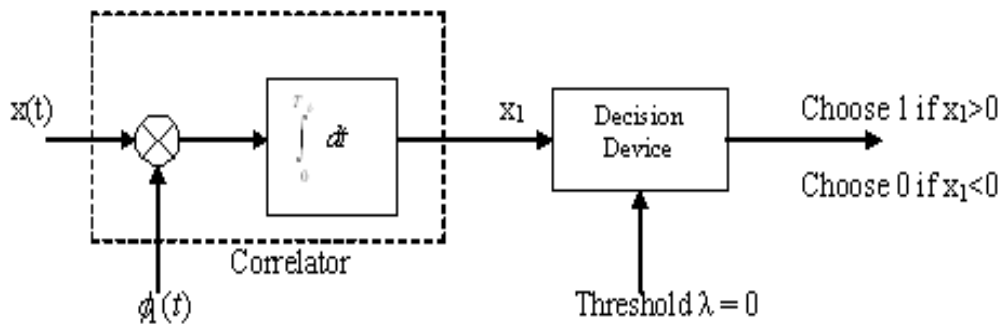


Fig (b) Coherent binary PSK receiver

Bit Error rate Calculation [BER Calculation] :-

In BPSK system the basic function is given by

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \quad 0 \leq t \leq T_b$$

The signals $S_1(t)$ and $S_2(t)$ are given by

$$S_1(t) = \sqrt{E_b} \phi_1(t) \quad 0 \leq t \leq T_b \quad \text{for Symbol 1}$$

$$S_2(t) = -\sqrt{E_b} \phi_1(t) \quad 0 \leq t \leq T_b \quad \text{for Symbol 0}$$

The signal space representation is as shown in fig

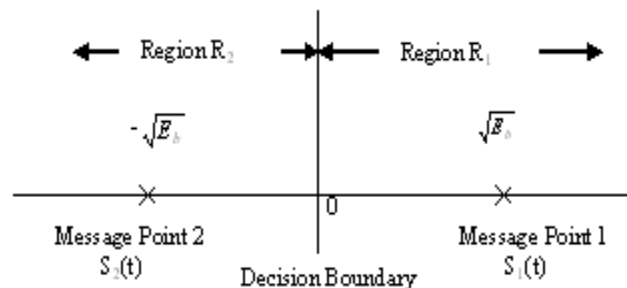


Fig Signal Space Representation of BPSK

The observation vector x_1 is related to the received signal $x(t)$ by

$$x_1 = \int_0^T x(t) \phi_1(t) dt$$

If the observation element falls in the region R_1 , a decision will be made in favour of symbol '1'. If it falls in region R_2 a decision will be made in favour of symbol '0'.

The error is of two types

1) $P_e(0/1)$ i.e. transmitted as '1' but received as '0' and

2) $P_e(1/0)$ i.e. transmitted as '0' but received as '1'.

Error of 1st kind is given by

$$P_e(1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty \exp\left[-\frac{(x_1 - \mu)^2}{2\sigma^2}\right] dx_1 \quad \text{Assuming Gaussian Distribution}$$

Where μ = mean value = $-\sqrt{E_b}$ for the transmission of symbol '0'

σ^2 = Variance = $\frac{N_0}{2}$ for additive white Gaussian noise.

Threshold Value $\lambda = 0$. [Indicates lower limit in integration]

Therefore the above equation becomes

$$P_{e0} = P_e(1/0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{(x_1 + \sqrt{E_b})^2}{N_0}\right] dx_1$$

$$\text{Put } Z = \frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}}$$

$$P_{e0} = P_e(1/0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^\infty \exp[-Z^2] dz$$

$$P_e(1/0) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

$$\text{Similarly, } P_e(0/1) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

The total probability of error $P_e = P_e(1/0)P_e(0) + P_e(0/1)P_e(1)$ assuming probability of 1's and 0's are equal.

$$P_e = \frac{1}{2} [P_e(1/0) + P_e(0/1)]$$

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}$$

- b. Binary data is transmitted at a rate of 106 bits/sec over a microwave link having a bandwidth of 3 MHz. Assume that the noise power spectral density at the receiver input is $\eta/2 = 10^{-10}$ watt/Hz. Find the average carrier power required at the receiver input for coherent PSK and DPSK signalling schemes to maintain $P_e \leq 10^{-4}$.

Answer:

The probability of error for the PSK scheme is

$$(P_e)_{PSK} = Q\left(\sqrt{2S_{av}T_b/\eta}\right) \leq 10^{-4},$$

thus

$$\sqrt{2S_{av}T_b/\eta} \geq 3.75$$

$$(S_{av}) \geq (3.75)^2 (10^{-10})(10^6) = 1.48 \text{ dBm}$$

For the DPSK scheme

$$(P_e)_{DPSK} = \frac{1}{2} \exp\left[-(A^2 T_b / 2\eta)\right] \leq 10^{-4},$$

Hence,

$$S_{av} T_b / \eta \geq 8.517$$

$$(S_{av})_{DPSK} \geq 2.3.3 \text{ dBm}$$

This example illustrates that the DPSK signaling scheme requires about 1 dB more power than the coherent PSK scheme when the error probability is of the order of 10^{-4} .

Q.7 a. What is a matched filter? Derive the condition for maximum output of a matched filter.

Answer:

Matched filter is an optimum filter which will maximize output SNR to minimize the probability of error.

Let

$x(t)$ = input signal to the matched filter

$h(t)$ = impulse response of the matched filter

$w(t)$ = white noise with power spectral density $N_o/2$

$\phi(t)$ = known signal

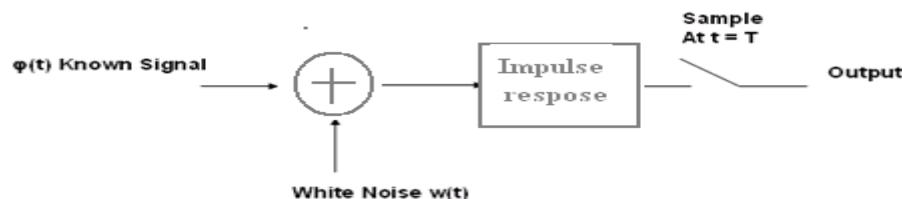
Input to the matched filter is given by

$$x(t) = \phi(t) + w(t) \quad 0 \leq t \leq T$$

since the filter is linear, the resulting output $y(t)$ is given by

$$y(t) = \phi_0(t) + n(t)$$

where $\phi_0(t)$ and $n(t)$ are produced by the signal and noise components of the input $x(t)$.



The signal to noise ratio at the output of the matched filter at $t = T$ is

$$(SNR)_0 = \frac{|\phi_0(T)|^2}{E[n^2(t)]} \dots\dots\dots (1)$$

aim is to find the condition which maximize the SNR

let

$$\phi_0(t) \leftrightarrow \phi_0(f)$$

$$h(t) \leftrightarrow H(f)$$

are the Fourier transform pairs, hence the output signal $\phi_0(t)$ is given by

$$\phi_0(t) = \int_{-\infty}^{\infty} H(f)\Phi(f)\exp(j2\pi ft)df$$

output at $t = T$ is

$$|\phi_0(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)\Phi(f)\exp(j2\pi fT)df \right|^2 \dots\dots\dots (2)$$

For the receiver input noise with psd (power spectral density) $N_0/2$ the receiver output noise psd is given by

$$S_N(f) = \frac{N_0}{2} |H(f)|^2 \dots\dots\dots (3)$$

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)df$$

and the noise power is given by

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df \dots\dots\dots (4)$$

substituting the values of eqns (2) & (4) in (1) we get

$$(SNR)_0 = \frac{\left| \int_{-\infty}^{\infty} H(f)\Phi(f)\exp(j2\pi fT)df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df} \dots\dots\dots (5)$$

using Schwarz's inequality

$$\left| \int_{-\infty}^{\infty} X_1(f)X_2(f)df \right|^2 \leq \int_{-\infty}^{\infty} |X_1(f)|^2 df \int_{-\infty}^{\infty} |X_2(f)|^2 df \dots\dots\dots (6)$$

Eqn (6) is equal when $X_1(f) = kX_2^*(f)$

let $X_1(f) = H(f)$

& $X_2(f) = \Phi(f)\exp(j2\pi fT)$

under equality condition

$$H(f) = K \Phi^*(f) \exp(-j2\pi fT) \dots \dots \dots (7)$$

Thus substituting in (6) we get the value

$$\left| \int_{-\infty}^{\infty} H(f) \Phi(f) \exp(j2\pi fT) df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |\Phi(f)|^2 df$$

substituting in eqn (5) and simplifying

$$(SNR)_0 \leq \frac{2}{N_0} \int_{-\infty}^{\infty} |\Phi(f)|^2 df$$

Using **Rayleigh's energy theorem**

$$\int_{-\infty}^{\infty} |\phi(t)|^2 dt = \int_{-\infty}^{\infty} |\Phi(f)|^2 df = E, \quad \text{energy of the signal}$$

$$(SNR)_{0,\max} = \frac{2E}{N_0} \dots \dots \dots (8)$$

Under maximum SNR condition, the transfer function is given by (k=1), eqn (7)

$$H_{opt}(f) = \Phi^*(f) \exp(-j2\pi fT)$$

The impulse response in time domain is given by

$$\begin{aligned} h_{opt}(t) &= \int_{-\infty}^{\infty} \Phi^*(f) \exp[-j2\pi fT] \exp(j2\pi ft) df \\ &= \phi(T-t) \end{aligned}$$

Thus the impulse response is folded and shifted version of the input signal $\phi(t)$

- b. Write short note on detection of signals with unknown phase in noise.

Answer: Page Number 96/3.9 of Text Book I

- Q.8** a. Explain in detail the working of Direct – Sequence Spread Spectrum with coherent binary Phase shift Keying.

Answer:

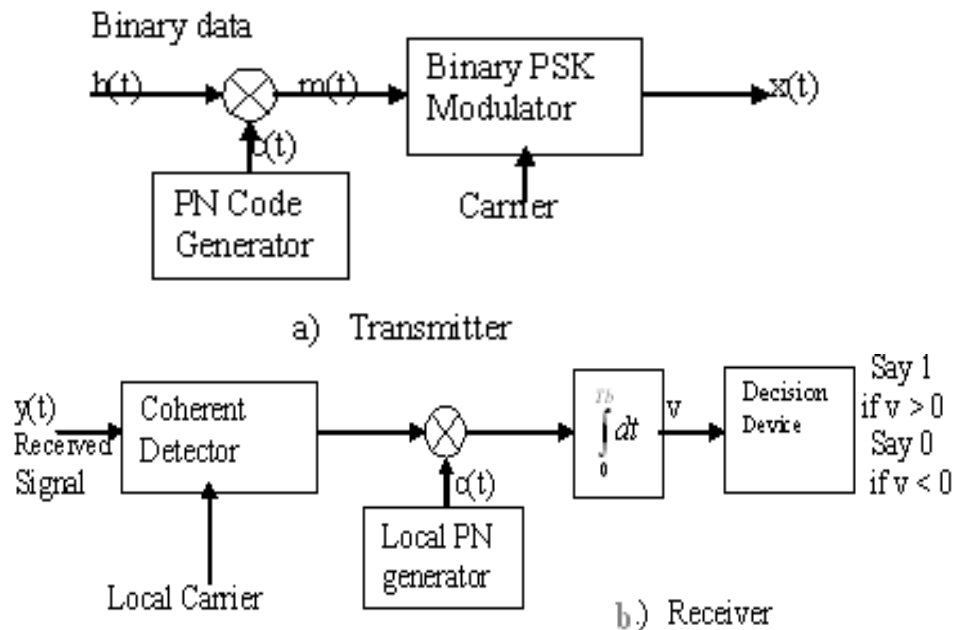


Fig: model of direct – sequence spread binary PSK system (alternative form is also available)

To provide band pass transmission, the base band data sequence is multiplied by a Carrier by means of shift keying. Normally binary phase shift keying (PSK) is used because of its advantages.

The transmitter first converts the incoming binary data sequence $\{b_k\}$ into an NRZ waveform $b(t)$, which is followed by two stages of modulation.

The first stage consists of a multiplier with data signal $b(t)$ and the PN signal $c(t)$ as inputs. The output of multiplier is $m(t)$ is a wideband signal. Thus a narrow – band data sequence is transformed into a noise like wide band signal.

The second stage consists of a binary Phase Shift Keying (PSK) modulator. Which converts base band signal $m(t)$ into band pass signal $x(t)$. The transmitted signal $x(t)$ is thus a direct – sequence spread binary PSK signal. The phase modulation $\theta(t)$ of $x(t)$ has one of the two values '0' and ' π ' (180°) depending upon the polarity of the message signal $b(t)$ and PN signal $c(t)$ at time t .

Polarity of PN & Polarity of PN signal both +, + or - - Phase '0'

Polarity of PN & Polarity of PN signal both +, - or - + Phase ' π '

The receiver consists of two stages of demodulation.

In the first stage the received signal $y(t)$ and a locally generated carrier are applied to a coherent detector (a product modulator followed by a low pass filter), Which converts band pass signal into base band signal.

The second stage of demodulation performs Spectrum desreading by multiplying the output of low-pass filter by a locally generated replica of the PN signal $c(t)$, followed by integration over a bit interval T_b and finally a decision device is used to get binary sequence.

- b. Mention the properties of PN sequence.

Answer:

Properties of PN Sequence

Randomness of PN sequence is tested by following properties

1. Balance property
2. Run length property
3. Autocorrelation property

1. Balance property

In each Period of the sequence , number of binary ones differ from binary zeros by at most one digit .

2. Run length property

Among the runs of ones and zeros in each period, it is desirable that about one half the runs of each type are of length 1, one- fourth are of length 2 and one-eighth are of length 3 and so-on.

3. Auto correlation property

Auto correlation function of a maximal length sequence is periodic and binary valued.

Autocorrelation sequence of binary sequence in polar format is given by

$$R_c(k) = \frac{1}{N} \sum_{n=1}^N c_n c_{n-k}$$

Where N is length or period of the sequence and k is the lag of the autocorrelation

- c. A direct sequence spread binary phase shift keying system uses a feedback shift register of length 19 for the generation of PN sequence. Calculate the processing gain of the system.

Answer:

Given length of shift register = $m = 19$

Therefore length of PN sequence $N = 2^m - 1$
 $= 2^{19} - 1$

Processing gain $PG = T_b/T_c = N$
in db $= 10 \log_{10} N = 10 \log_{10}(2^{19})$
 $= 57 \text{db}$

Q.9

Write short note on:

- (i) Light wave transmission link
- (ii) Digital Radio

Answer: Page Number 225, 350 of Text Book

TEXTBOOK

Digital Communication' by Siman Haykin