ROLL NO. ______________

Code: AE01/AC01/AT01 **Subject: MATH[illegible]S-I**

AMIETE – ET/CS/IT (OLD SCHEME)

Time: 3 Hours **OCTOBER 2012** **Max. Marks: 10[illegible]**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2 × 10)**

a. The value of $\lim\limits_{(x,y)\to(\infty,2)} \dfrac{xy+4}{\left(x^2+2y^2\right)}$ is

(A) 0 **(B)** 1
(C) limit does not exist **(D)** -1

b. If $u = x^y$ then the value of $\dfrac{\partial u}{\partial x}$ is equal to

(A) 0 **(B)** yx^{y-1}
(C) xy^{x-1} **(D)** $x^y \log(x)$

c. If $z = \sin^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then the value of $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is

(A) z/2 **(B)** 2z
(C) tan(z)/2 **(D)** sin(z)/2

d. The value of integral $\int\limits_{-1}^{1}\int\limits_{0}^{z}\int\limits_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$ is equal to

(A) 1 **(B)** 0
(C) -1 **(D)** None of these

e. The differential equation of the coaxial circles of the system $x^2 + y^2 + 2ax + c^2 = 0$ Where c is a constant and a is a variable is given by

(A) $2xy\dfrac{dy}{dx} = c^2 - x^2 + y^2$ **(B)** $x^2\left(1 + (\dfrac{dy}{dx})^2\right) = y^2$

(C) $c^2\left(1 + (\dfrac{dy}{dx})^2\right) = x^2$ **(D)** $(x^2 + y^2)\left(1 + (\dfrac{dy}{dx})^2\right) = c^2$

f. The solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$ satisfying the initial conditions y(0) = 1, y(π/4) = 2 is

(A) y = 2cos2x + sin2x **(B)** y = cos2x + 2 sin2x
(C) y = cos2x + sin2x **(D)** y = 2cos2x + 2 sin2x

g. If $A\begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}, where\ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the A is equal to

(A) $\begin{pmatrix} 2 & 0 \\ -1/2 & -1/2 \end{pmatrix}$ **(B)** $\begin{pmatrix} 0 & 1 \\ -1/2 & -1/2 \end{pmatrix}$
(C) $\begin{pmatrix} 2 & -1 \\ -1/2 & -1/2 \end{pmatrix}$ **(D)** $\begin{pmatrix} 2 & 1 \\ -1/2 & -1/2 \end{pmatrix}$

h. The matrix A is idempotent if

(A) $A^2 + A = 0$ **(B)** $A^2 = A$
(C) $A^2 - A = I$ **(D)** None of these

i. The value of $\int_{-1}^{1} P_0(x)dx$ is equal to

(A) 1 **(B)** 2
(C) -1 **(D)** 0

j. The value of the integral $J_{-2}(x)$ is equal to

(A) $-J_2(x)$ **(B)** $-J_{-2}(x)$
(C) $J_2(x)$ **(D)** $J_{-1}(x)$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. For the function $f(x,y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ show that $f_{xy}(0,0) \neq f_{yx}(0,0)$. **(8)**

b. Find the maxima and minima of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. **(8)**

Q.3 a. If $f(x,y)=\tan^{-1}(y/x)$, find an approximate value of f(1.1,0.9) using the Taylor's series quadratic approximation. **(8)**

b. Evaluate the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \left(x^2+y^2\right) dx\, dy$ by changing to polar coordinates. **(8)**

Q.4 a. Find the solution of the differential equation $(2x+y-3)dy=(x+2y-3)dx$ **(6)**

b. Solve the differential equation $\sec x \sec^2 y \dfrac{dy}{dx}=e^x-\sec x \tan x \tan y$. **(6)**

c. Show that the functions 1, sinx, cosx are linearly independent. **(4)**

Q.5 a. Using method of variation of parameters, solve $y''-2y'=e^x \sin x$. **(8)**

b. Solve $\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+3y=2xe^{3x}+3e^x\cos 2x$. **(8)**

Q.6 a. If $A=\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, show that AA* is a Hermitian matrix, where A* is the conjugate transpose of A. **(8)**

b. Examine the following vectors for linear dependence and find the relation if it exists, $X_1=(1,2,4), X_2=(2,-1,3), X_3=(0,1,2), X_4=(-3,7,2)$. **(8)**

Q.7 a. Examine, whether the matrix A is diagonalizable. $A=\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. If, so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix. **(8)**

b. Investigate the values of μ and λ so that the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ has
(i) no solutions
(ii) a unique solution and
(iii) an infinite number of solutions. **(8)**

Q.8 a. Find the power series solution of the equation $y''+(x-1)^2y'-4(x-1)y=0$, about the point $x_0=1$ **(11)**

b. Prove that $P_n'(1) = \frac{1}{2}n(n+1)$.

Q.9 a. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. **(8)**

b. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials. **(8)**