ROLL NO. ______________

Code: AC10 **Subject: DISCRETE STRUCTU**

AMIETE – CS (OLD SCHEME)

Time: 3 Hours **OCTOBER 2012** **Max. Marks: 10**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Which of the following is an empty set?

(A) $\{x \mid x$ is a real number and $x^2+1 = 0\}$
(B) $\{x \mid x$ is a real number and $x^2-1 = 0\}$
(C) $\{x \mid x$ is a real number and $x = 2x+1\}$
(D) $\{x \mid x$ is a real number and $x^2 = 9\}$

b. The converse of the statement 'If 2+2=4, then I am not the queen of England' is

(A) If 2+2=4, then I am the queen of England.
(B) If I am the queen of England, then 2+2=4.
(C) If $2+2 \neq 4$, then I am not the queen of England
(D) If I am the queen of England, then $2+2 \neq 4$.

c. A bank password consists of 2 letters of the English alphabet, followed by 2 digits. How many different passwords are there?

(A) 6760 **(B)** 2600
(C) 260 **(D)** 67600

d. An undirected graph is called *connected* if

(A) there is a path between some two not necessarily distinct vertices of the graph.
(B) even if there is no path between some pair of vertices in the graph
(C) if there is a path between every pair of distinct vertices of the graph.
(D) if there is no path between every pair of distinct vertices of the graph

e. Let R and S be relations on a set A. If R and S are reflexive, choose the following that are also reflexive.

(A) $R - S$ **(B)** $R - A$
(C) $R \cup S$ **(D)** $S - R$

f. A relation is called a *partial order* if

(A) it is reflexive, symmetric and transitive
(B) it is reflexive, associative and abelian
(C) it is reflexive, antisymmetric and transitive
(D) it is irreflexive, not symmetric, and not-transmitted

g. Identity law in boolean attributes is

(A) $x+1=1$ **(B)** $x+0=x$
(C) $x.0=x$ **(D)** $xy=yx$

h. A rooted tree is a tree, in which

(A) more than one vertex has been designated as the root and every edge is directed towards the root.
(B) one vertex has been designated as the root and every edge is directed towards the root.
(C) one vertex has been designated as the root and every edge is directed away from the root.
(D) more than one vertex has been designated as the root and every edge is directed away from the roots.

i. If $G=(V,S,v_o,\to)$ is a phase structure grammar, the sets S and $V-S$ are called

(A) terminal and non-terminal symbols respectively
(B) non-terminal and terminal symbols respectively
(C) terminal and derivation symbols respectively
(D) derivation and terminal symbols respectively

j. If 30 dictionaries in a library contain a total of 61327 pages, then

(A) at most one of the dictionaries must have at most 2045 pages.
(B) at least one of the dictionaries must have at least 2045 pages.
(C) at least one of the dictionaries must have at most 2045 pages
(D) at most one of the dictionaries must have at least 2045 pages

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Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A survey of 520 television watchers produced the following information: 285 watch football, 195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, and 50 do not watch any of these three games.
(i) How many people in the survey watch all three games?
(ii) How many people watch exactly one of the games? **(6)**

b. Using mathematical induction, prove: $1^3 + 2^3 + 3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$. **(8)**

c. How many three letter words can be formed from letters in the set {*a, b, y, z*} if repeated letters are allowed. **(2)**

Q.3 a. Derive the explicit formula for the recursive relation:
$b_n = -3b_{n-1} - 2b_{n-2}, b_1 = -2, b_2 = 4$. **(6)**

b. Prove that if any 14 numbers from 1 to 25 are chosen, then one of them is a multiple of another number. **(5)**

c. What are the two methods used to represent graphs? Give an example for each. **(5)**

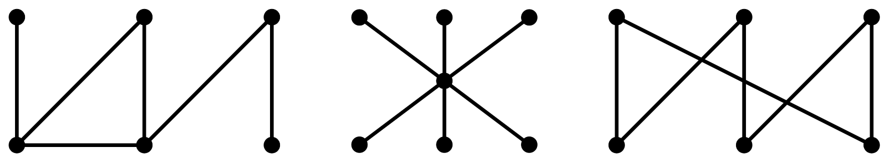
Q.4 a. Let *S* = {1, 2, 3, 4} and let *A* = *S*×*S*. Define the following relation *R* on *A*: $(a,b)R(a',b')$ if and only if $(a+b) = (a'+b')$. Show that *R* is an equivalence relation. **(8)**

b. Define transitive closure with an example. Write the Warshall's algorithm to compute the transitive closure of a directed graph. **(8)**

Q.5 a. Let *A* = {1, 2, 3, 4, 12}. The partial order of divisibility on *A* is defined such that if $a, b \in A$, then $a \le b$ if and only if a divides b. Draw the Hasse diagram of the poset $(A, \le)$. **(5)**

b. Simplify the following boolean expressions using Karnaugh maps
(i) $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$
(ii) $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$ **(8)**

c. Which of these graphs are trees? **(3)**



Q.6 a. Write the algorithm for Huffman coding. Encode the following symbols with the frequencies listed: A = 0.18, B = 0.10, C = 0.12, D = 0.15, E = 0.20, F = 0.25. **(10)**

b. Write the algorithms for In-order and Pre-order traversals. **(6)**

Q.7 a. Write the syntax diagrams for a decimal number and a digit. **(8)**

b. Write Prim's algorithm for computing a minimum spanning tree of a weigh graph. **(8)**

Q.8 a. Prove that 3 divides $n^3 + 2n$, whenever *n* is a positive integer. **(10)**

b. Give the recursive definition for the sequence $\{a_n\}, n = 1,2,3...$ and
(i) $a_n = 2n + 1$ (ii) $a_n = 4n - 2$. **(6)**

Q.9 a. Consider the set *A* = {1, 2, 3, 4, 5} and a relation *R* on *A*, where
$R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,2),(2,1),(2,3),(3,2),(1,3),(3,1),(4,5),(5,4)\}$
Prove that *R* is an equivalence relation and determine the partition induced by *R* on *A*. **(10)**

b. A box contains 6 red balls and 4 green balls. 4 balls are selected at random from the box. What is the probability that 2 of the selected balls will be red and 2 will be green? **(6)**