ROLL NO. ___________________

Code: AC65 **Subject: DISCRETE STRUCT**

AMIETE – CS (NEW SCHEME)

Time: 3 Hours **JUNE 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Let A be a finite set of size n. The number of elements in the power set of A×A is

(A) 2^{2^n} **(B)** 2^{n^2}
(C) $(2^n)^2$ **(D)** $(2^2)^n$

b. Two events A and B are mutually exclusive events if

(A) $A \cap B = \phi$ **(B)** $A \cap B = A - B$
(C) $A \cup B = S$ (sample space) **(D)** None of these

c. The contra-positive of $P \rightarrow \sim Q$ is

(A) $\sim P \rightarrow \sim Q$ **(B)** $P \rightarrow Q$
(C) $\sim P \rightarrow Q$ **(D)** $P \rightarrow \sim Q$

d. Find the negation of "There exists a dog that is 25 years old".

(A) Some dog is not 25 years old **(B)** All dog is 25 years old
(C) Every dog is 25 years old **(D)** Every dog is not 25 years old

e. Which of the following is correct?

(A) $(\forall X)(\exists Y)\{X+Y = 100\}$ **(B)** $(\exists Y)(\forall X)\{X+Y = 100\}$
(C) $(\forall X)(\forall Y)\{X+Y = 100\}$ **(D)** None of these

f. A relation R is reflexive on a set A if

(A) Domain (R) $\subseteq$ Range (R) **(B)** Domain (R) $\subset$ Range (R)
(C) Domain (R) = Range (R) $\neq$ A **(D)** Domain (R) = Range (R) = A

g. Relation $a_n = 3a_{n-1} + n$ is

(A) homogeneous recurrence relation
(B) non homogeneous recurrence relation
(C) generating function
(D) None of these

h. The value of the parameter α, for which the function $f(x) = 1 + \alpha x$, $\alpha \neq 0$ is th inverse of itself, is

(A) –2 **(B)** –1
(C) 1 **(D)** 2

i. Which of the following is not true?

(A) Every group of prime order is cyclic
(B) Every subgroup of a cyclic group is cyclic
(C) Every proper subgroup of an infinite cyclic group is infinite
(D) Every cyclic group may not be abelian

j. Which of the following is not a ring with respect to addition and multiplication?

(A) Set of all natural numbers
(B) Set of all integers
(C) Set of all rational numbers
(D) Set of all $n \times n$ matrices with their elements as real numbers

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Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If A, B and C are any sets, prove that $A \cap (B - C) = (A \cap B) - C$ **(5)**

b. Write all the subsets of the set $S = \{\phi, \{\phi\}\}$ where ϕ is null set. **(5)**

c. A problem is given to four students A, B, C, D whose chances of solving it are $1/2, 1/3, 1/4, 1/5$ respectively. Find the probability that the problem is solved. **(6)**

Q.3 a. Simplify $[\sim (P \vee Q) \wedge R] \vee [R \wedge (Q \vee P)]$. **(5)**

b. Prove that $[(P \vee \sim Q) \rightarrow P]$ and $(P \vee Q)$ are equivalent. **(5)**

c. Prove that $P \rightarrow (Q \rightarrow R) = (P \wedge Q) \rightarrow R)$ **(6)**

Q.4 a. Write the predicate calculus (using quantifiers) representation of the following sentence:
"Some baby boys are more mischievous than all baby girls".
Use B(X): X is a baby boy, G(X): X is a baby girl, N(X, Y): X is mischievous than Y. **(7)**

b. Consider the following knowledge base:
$\forall X \forall Y \text{ cat}(X) \wedge \text{fish}(Y) \Rightarrow \text{likes\_to\_eat}(X, Y)$
$\forall X \text{ calico}(X) \Rightarrow \text{cat}(X)$
$\forall X \text{ tuna}(X) \Rightarrow \text{fish}(X)$
Convert these into Horn clauses (convert into conjunction of disjuncts by removing all quantifiers and implications). **(9)**

Q.5 a. Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$. **(8)**

b. Prove the following by mathematical induction:
$(1 + x)^n > 1 + nx$ for natural number $n \geq 2$, $x > -1$ and $x \neq 0$. **(8)**

Q.6 a. Let A denotes the set of ordered pair of all natural numbers (N×N). Relation R is defined as follows:
for $(a, b), (c, d) \in$ A, (a, b) R (c, d) iff $(a + d) = (b + c)$.
Prove that the relation R is equivalence relation. **(8)**

b. If $(L, \leq)$ is a lattice, then prove that the dual of the poset $(L, \leq)$ is also a lattice. **(8)**

Q.7 a. Let A : $\{a, b, c, d\}$, B = $\{p, q, r, s\}$ denotes two sets. Identify which of the following is (are) a function/ not a function from A to B? Give reasons for each.
(i) $\{(a, p), (b, q), (c, r)\}$
(ii) $\{(a, p), (b, q), (c, s), (d, r)\}$
(iii) $\{(a, p), (b, s), (b, r), (c, q)\}$
(iv) $\{(a, p), (b, r), (d, r), (c, q)\}$ **(8)**

b. Let f : R → R be a mapping defined as $f(x) = ax + b$; $a, b \in$ R, $a \neq 0$. Is f invertible? Justify. **(8)**

Q.8 a. Prove that fourth roots of unity form a cyclic group. Find the order of each element of the group. **(8)**

b. Let G be a multiplicative group and $ba = a^p b^q$ where $a, b \in$ G and $p, q \in Z^+$. Prove that order of $a^p b^{q-2}$ is equal to order of $a b^{-1}$. **(8)**

Q.9 a. Show that the set of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers is a ring with respect to addition and multiplication. **(8)**

b. Write a note on Generator matrix of an encoding function E and parity-check matrix associated with it. **(8)**