

DipIETE – ET/CS (NEW SCHEME) – Code: DE55 / DC55**Subject: ENGINEERING MATHEMATICS - II**

Time: 3 Hours

DECEMBER 2011

Max. Marks: 100

NOTE: There are 9 Questions in all.

- Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.
- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of the limit $\lim_{x \rightarrow 0} \left(\frac{2^x - 1}{(1+x)^{1/2} - 1} \right)$ is equal to

- (A) $2 \log 2$ (B) $\log 2$
(C) 0 (D) 1

b. The value of definite integral $\int_0^{\pi} \theta \sin^3 \theta \cos \theta d\theta$ is equal to

- (A) $\frac{3\pi}{32}$ (B) $-\frac{3\pi}{32}$
(C) $\frac{\pi}{32}$ (D) $-\frac{\pi}{32}$

c. The solution of $xdy - ydx = \sqrt{x^2 + y^2} dx$ is

- (A) $y - \sqrt{x^2 + y^2} = cx^2$ (B) $x - y^2 e^{-y} = cy^2$
(C) $y + \sqrt{x^2 + y^2} = cx^2$ (D) None of these.

d. z is a complex number with $|z|=1$, $\arg(z) = 3\pi/4$ the value of z is

- (A) $(1+i)/\sqrt{2}$ (B) $(-1+i)/\sqrt{2}$
(C) $(1-i)/\sqrt{2}$ (D) $(-1-i)/\sqrt{2}$

e. How many seconds a clock would lose per day if the length of its pendulum were increased in the ratio 900 : 901

- (A) 48 (B) 45
(C) 40 (D) 44

f. Laplace transform of $te^{at} \sin(at)$, $t > 0$ is

- (A) $\frac{(s-a)}{(s-a)^2 + a^2}$ (B) $\frac{a(s-a)}{(s-a)^2 + a^2}$
(C) $\frac{2a(s-a)}{[(s-a)^2 + a^2]^2}$ (D) $\frac{(s-a)^2}{(s-a)^2 + a^2}$

g. $L^{-1}\left(\tan^{-1}\frac{1}{s}\right)$ is

- (A) $\frac{\cos t}{t}$ (B) $\frac{\sin t}{t}$
(C) $1 + \cos t$ (D) $1 - \cos t$

h. If $f(x) = \cos x$, $(-\pi, \pi)$ then the value of b_n is

- (A) $-\pi$ (B) 0
(C) π (D) 2π

i. The volume of the parallelepiped whose three coterminus edges are given by $\vec{a} = -\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$ is

- (A) 2 (B) 4
(C) -2 (D) -4

j. If the admittance and current of a circuit are given by the complex numbers $7+i$, $1-i$ respectively, then the voltage of the circuit is

- (A) $\frac{4}{25} + i\frac{4}{25}$ (B) $-\frac{4}{25} - i\frac{4}{25}$
(C) $\frac{4}{25} - i\frac{4}{25}$ (D) $-\frac{4}{25} + i\frac{4}{25}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Evaluate $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x}$ (8)

b. If $f(x)$ is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$,
 $h(x) = [f(x)]^2 + [g(x)]^2$, then find the value of $h(10)$ if $h(5) = 11$. (8)

Q.3 a. Find the volume of the solid generated by the revolution of the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about x-axis. (8)

b. If $U_n = \int_0^{\pi/2} x(\sin^n x) dx (n > 1)$ then prove that $U_n = \frac{n-1}{n} U_{n-2} + \frac{1}{n^2}$.

Deduce that $U_5 = \frac{149}{225}$ (8)

Q.4 a. Separate $\tan^{-1}(a + ib)$ into real and imaginary parts. (8)

b. If n is a positive integer, prove that $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1}$ where $i = \sqrt{-1}$ (8)

Q.5 a. Find the moment about a line through the origin having direction of $2\hat{i} + 2\hat{j} + \hat{k}$ due to a 30kg force acting at a point $(-4, 2, 5)$ in the direction of $12\hat{i} - 4\hat{j} - 3\hat{k}$. (8)

b. If $|\vec{A} + \vec{B}| = 60$, $|\vec{A} - \vec{B}| = 40$, $|\vec{B}| = 46$, find $|\vec{A}|$ (8)

Q.6 a. Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1$, $x(\pi/2) = -1$ (8)

b. Solve $x \sin x \frac{dy}{dx} + (x \cos x + \sin x) y = \sin x$ (8)

Q.7 a. Find the Fourier series of the function $f(x) = \begin{cases} 0, & -2 < x < -1 \\ 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ (8)

b. Given that $f(x) = x + x^2$ for $-\pi < x < \pi$ find the Fourier expansion of $f(x)$.

$$\text{Deduce that } \frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots \quad (8)$$

Q.8 a. Find the Laplace transform of $\frac{1 - \cos t}{t^2}$ (8)

b. Find the Laplace transform of the function $f(t) = \begin{cases} \sin wt & \text{for } 0 < t < \frac{\pi}{w} \\ 0 & \text{for } \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases}$ (8)

Q.9 a. Evaluate $L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right)$ (8)

b. Find $L^{-1}\left[\frac{3s - 8}{s^2 - 4s + 20}\right]$ (8)