ROLL NO. ______________

Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEMATICS

AMIETE – ET/CS/IT (NEW SCHEME)

Time: 3 Hours **DECEMBER 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $u = x^y$ then the value of $\frac{\partial u}{\partial x}$ is equal to

(**A**) 0 (**B**) yx^{y-1}
(**C**) xy^{x-1} (**D**) $x^y \log(x)$

b. The value of integral $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ is equal to

(**A**) $\frac{3}{4}$ (**B**) $\frac{3}{8}$
(**C**) $\frac{3}{5}$ (**D**) $\frac{3}{7}$

c. If two eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the third eigen value is

(**A**) 0 (**B**) 1
(**C**) -1 (**D**) 2

d. In solving simultaneous equations by Gauss-Jordan Method, the coefficient matrix is reduced to ___________ matrix.

(**A**) Identity (**B**) Diagonal
(**C**) Null (**D**) None of these

e. The differential equation $\left(x+x^8+ay^2\right)dx+\left(y^8-y+bxy\right)dy=0$ is exact if

(A) b = 2a **(B)** b = a
(C) a = 2b **(D)** a = -b

f. The square matrix 'A' is called orthogonal if

(A) $A=A^2$ **(B)** $A'=A^{-1}$
(C) $A=A^{-1}$ **(D)** $AA^{-1}=I$

g. The Bessel's equation of order 0 is given as

(A) $xy''+y'x+xy=0$ **(B)** $y''+y'x+xy=0$
(C) $xy''+y'+xy=0$ **(D)** $xy''+y'x+y=0$

h. The value of integral $\int_0^2\int_1^3\int_1^2 xy^2z\,dx\,dy\,dz$ is equal to

(A) 22 **(B)** 26
(C) 5 **(D)** 25

i. If λ is an eigen value of a non-singular matrix A then the eigen value of A^{-1} is

(A) $1/\lambda$ **(B)** λ
(C) $-\lambda$ **(D)** $-1/\lambda$

j. The value of the integral $\int x^2J_1(x)dx$ is

(A) $x^2J_1(x)+c$ **(B)** $x^2J_{-1}(x)+c$
(C) $x^2J_2(x)+c$ **(D)** $x^2J_{-2}(x)+c$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $x+y=2e^{\theta}\cos\phi$ and $x-y=2ie^{\theta}\sin\phi$, show that

$$\frac{\partial^2u}{\partial\theta^2}+\frac{\partial^2u}{\partial\phi^2}=4xy\frac{\partial^2u}{\partial x\partial y}$$ **(8)**

b. If $u = a^3x^2 + b^3y^2 + c^3z^2$ where $x^{-1} + y^{-1} + z^{-1} = 1$, show that the stationary value of u is given by $x = \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ **(8)**

Q.3 a. Evaluate the integral $\iint_R \sqrt{\left(x^2+y^2\right)}\,dxdy$ by changing to polar coordinates, R is the region in the x-y plane bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. **(8)**

b. Evaluate the integral $\iiint_T z\,dxdydz$, where T is region bounded by the cone $x^2 \tan^2\alpha + y^2 \tan^2\beta = z^2$ and the planes z=0 to z=h in the first octant. **(8)**

Q.4 a. Investigate the values of λ for which the following equations are consistent

$$(\lambda-1)x + (3\lambda+1)y + 2\lambda z = 0,$$
$$(\lambda-1)x + (4\lambda-2)y + (\lambda+3)z = 0,$$
$$2x + (3\lambda+1)y + 3(\lambda-1)z = 0$$

hence find the ratios of x:y:z when λ has the smallest of these values. **(8)**

b. Find the eigen value and eigen vector of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. **(8)**

Q.5 a. Find the solution of the differential equation (y-x+1)dy – (y+x+2) dx = 0. **(8)**

b. Solve the differential equation $\cot 3x \frac{dy}{dx} - 3y = \cos 3x + \sin 3x,\ 0 < x < \pi/2$. **(8)**

Q.6 a. Find the general solution of the equation $y'' - 4y' + 13y = 18e^{2x}\sin 3x$. **(8)**

b. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + y = \log x \frac{\sin(\log x)+1}{x}$. **(8)**

Q.7 a. Find the power series solution about the point $x_0 = 2$ of the equation $y'' + (x-1)y' + y = 0$. **(10)**

b. Prove that $\int_0^1 \frac{x^2}{\sqrt{\left(1-x^4\right)}}dx \times \int_0^1 \frac{dx}{\sqrt{\left(1+x^4\right)}} = \frac{\pi}{4\sqrt{2}}$ **(6)**

Q.8 a. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. **(8)**

b. Express $f(x) = x^4 + 2x^3 - 6x^2 + 5x - 3$ in terms of Legendre Polynomial. **(8)**

Q.9 a. Solve by Gauss-Seidel method, the following system of equations: **(8)**

$28x + 4y - z = 32;$

$x + 3y + 10z = 24;$

$2x + 17y + 4z = 35.$

b. Using Runge-Kutta method of fourth order, solve for y(0.1), y(0.2) given that $\frac{dy}{dx} = xy + y^2, y(0) = 1.$ **(8)**