ROLL NO. ______________

Code: AE01/AC01/AT01 Subject: MATHEMATICS

AMIETE – ET/CS/IT (OLD SCHEME)

Time: 3 Hours **DECEMBER 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. The value of $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dx\, dy$ is

(A) -1 **(B)** 1
(C) 2 **(D)** -2

b. If $u = \sin^{-1}(x - y)$, x = 3t and y = $4t^3$ then the value of $\frac{du}{dt}$ is

(A) 1 **(B)** 0
(C) $\frac{3}{\sqrt{1-t^2}}$ **(D)** $-\frac{3}{\sqrt{1-t^2}}$

c. The value of $\lim\limits_{\substack{x\to 1 \\ y\to 2}} \frac{3x^2 y}{x^2 + y^2 + 5}$ is

(A) $\frac{3}{5}$ **(B)** $-\frac{3}{5}$
(C) $\frac{2}{5}$ **(D)** $-\frac{2}{5}$

d. If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{dy}{dt}\frac{dt}{dx}$ **(B)** $-\frac{dy}{dt}\frac{dt}{dx}$
(C) $\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ **(D)** $-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$

e. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is

(A) 3 **(B)** 4
(C) 2 **(D)** -2

f. Two matrices are said to be equal if

(A) both are of same order
(B) both are of same order and the position of corresponding elements are equal
(C) one should be identity matrix
(D) one should be null matrix

g. If $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ then the value of A^8 is

(A) $\begin{bmatrix} 625 & 0 \\ 0 & 625 \end{bmatrix}$ **(B)** $\begin{bmatrix} 600 & 0 \\ 0 & 600 \end{bmatrix}$
(C) $\begin{bmatrix} -625 & 0 \\ 0 & 625 \end{bmatrix}$ **(D)** Identity matrix

h. The solution of the differential equation $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{-2x} + e^{-3x}$ is

(A) $c_1e^x + c_2e^{2x} + c_3e^{3x} - \frac{1}{120}(2e^{-2x} + e^{-3x})$

(B) $c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x} - \frac{1}{120}(2e^{-2x} + e^{-3x})$

(C) $2e^x + 5e^{2x} + 7e^{3x} + \frac{1}{120}(2e^{-2x} + e^{-3x})$

(D) $e^x + e^{2x} + e^{3x} - \frac{1}{120}(2e^{2x} + e^{3x})$

i. The solution of $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ is

(A) $x + y = c$ **(B)** $x - y = c$
(C) $y \sin x + (\sin y + y)\, x = c$ **(D)** $2 \sin x + 3 \cos y = c$

j. The value of $\frac{d}{dx}\left(x^n J_n(x)\right)$ is

(A) $J_n(x)$ (B) $x^n J_{n-1}(x)$

(C) $J_{-n}(x)$ (D) $[J_n(x)]^2$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. If $u = x^2 - y^2$, $v = 2xy$ and $f(x, y) = \theta(u, v)$, then show that by substitution that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2\theta}{\partial u^2} + \frac{\partial^2\theta}{\partial v^2}\right)$ **(8)**

b. Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third may be maximum. **(8)**

Q.3 a. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dx \, dy \, dz$ **(8)**

b. If $u = \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4a^2 t}}$, then prove that $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$. **(8)**

Q.4 a. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ by using elementary –row transformation method. **(8)**

b. Determine two non-singular matrix P and Q such that PAQ is in normal form, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ **(8)**

Q.5 a. Find the characteristic equation of the matrix, $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. **(8)**

b. Test for consistency/ inconsistency and solve the following system of equations
$2x+3y+4z=11; \quad x+5y+7z=15; \quad 3x+11y+13z=25$ **(8)**

Q.6 a. Solve in series the equation $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-y=0$ **(8)**

b. Prove that $J_{-n}(x)=(-1)^n J_n(x)$ where n is positive integers. **(8)**

Q.7 a. Prove that $xP'_n(x)-P'_{n-1}(x)=nP_n(x)$ **(8)**

b. Express $f(x)=1-2x+x^2+5x^3$ in term of Legendre polynomials. **(8)**

Q.8 a. Solve the differential equation $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=x\,e^x\sin x$ **(8)**

b. Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2}-6\dfrac{dy}{dx}+9y=\dfrac{e^{3x}}{x^2}$. **(8)**

Q.9 a. Solve the differential equation $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ **(8)**

b. Solve $\dfrac{d^2y}{dx^2}+y=\operatorname{cosec} x$ **(8)**