ROLL NO. ____________________

Code: AC10 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (OLD SCHEME)

Time: 3 Hours **DECEMBER 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Let $A = \{1, 2, 3, 4, 5\}$. Which of the following sets is not equal to A?

(A) {1, 2, 3, 4, 5} **(B)** {x | x is a real number and $x^2 \le 25$}
(C) {5, 4, 3, 2, 1} **(D)** {2, 3, 4}

b. The contrapositive of the statement ``If I am not President, then I will walk to work'' is

(A) If I do not walk to work, then I am President.
(B) If I am President, then I will not walk to work
(C) If I walk to work, then I am not President.
(D) If I walk to work, then I am President.

c. If no letter or digit can be repeated, how many license plates having 2 letters followed by 4 digits can be manufactured?

(A) 6,760,000 **(B)** 3,276,000
(C) 3,72600 **(D)** 327600

d. How many edges are there in a graph with 10 vertices, each of degree 6?

(A) 60 **(B)** 30
(C) 20 **(D)** 100

e. To make $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,1), (3,2)\}$ a binary equivalence relation on the set $A = \{1, 2, 3, 4, 5\}$, choose the set that one needs to add to R.

(A) $\{(4,4), (5,5)\}$ **(B)** $\{(5,5), (1,3)\}$
(C) $\{(4,4), (5,5), (1,3)\}$ **(D)** $\{(4,4), (1,2,3)\}$

f. Consider a poset (A, R), where A is a finite set and R is a relation on the set A. Then, the greatest element of A is

(A) any $a \in A$ such that $xRA, \forall x \in A$
(B) any $a \in A$ such that $aRx, \forall x \in A$
(C) the same as the upperbound of a subset B of A
(D) the same as the lowerbound of a subset B of A

g. Domination law in boolean attributes is

(A) $x.0 = 0$ **(B)** $x + x = x$
(C) $x.1 = x$ **(D)** $xy = yx$

h. A rooted tree is a full *m*-ary tree, if

(A) every external vertex has at most *m* children.
(B) every internal vertex has at most *m* children.
(C) every external vertex has exactly*m* children.
(D) every internal vertex has exactly*m* children.

i. A phase structure grammar *G*is defined to be a

(A) 3 tuple $(V, S, \rightarrow)$, $V \subseteq S$ and $\rightarrow$ is a finite relation on V^*
(B) 4 tuple $(V, S, v_o, \rightarrow)$, V is a finite set, $S \subseteq V, v_0 \in V - S, \rightarrow$ is a finite relation on V^*
(C) 5 tuple $(V, S, v_o, \rightarrow, \leftarrow)$, V is a finite set, $S \subseteq V, v_o \in S - V, \rightarrow$ and $\leftarrow$ are finite relations on V^*
(D) 4 tuple $(V, S, v_o, \rightarrow)$, V is an infinite set, $V \subseteq S, v_0 \in V - S, \rightarrow$ is a finite relation on V^*

j. The explicit version of the recursive relation $a_n = 2 \times 5. a_{n-1}, a_1 = 4$, is

(A) $a_n = 4 \times 10^n$ **(B)** $a_n = 10^n$
(C) $a_n = 4^n$ **(D)** $a_n = (4 \times 10)^n$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. A survey has been taken on the methods of commuter travel. Each respondent was asked to check BUS, TRAIN or AUTOMOBILE as a major method of traveling to work. More than one answer was permitted. The results were reported as follows: BUS: 30 people; TRAIN: 35 people; AUTOMOBILE: 100 people; BUS and TRAIN: 15 people; BUS and AUTOMOBILE: 15 people; TRAIN and AUTOMOBILE: 20 people; and all the three methods, 5 people. How many people completed a survey form? **(6)**

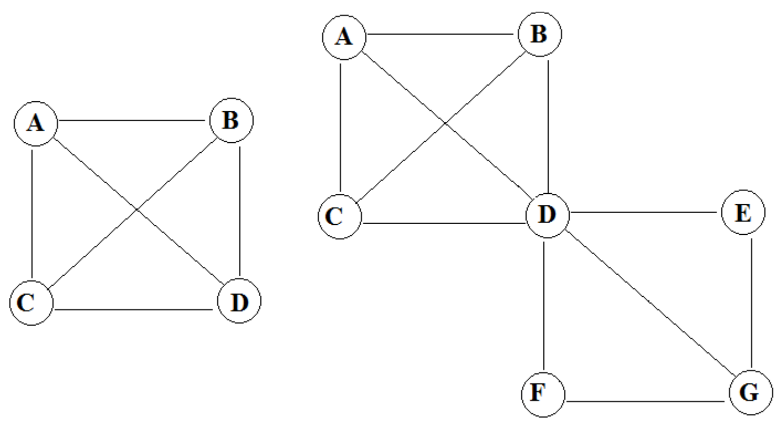
b. The Harmonic numbers $H_j,\ j = 1,2,3,\cdots$ is defined by $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{j}$. Use, mathematical induction to show that $H_{2^n} \geq 1 + n/2$. **(8)**

c. A label identifier for a computer program consists of one letter followed by 3 digits. If repetitions are allowed, how many distinct label identifiers are possible? **(2)**

Q.3 a. Derive the explicit formula for the recursive relation: $a_n = 4a_{n-1} + 5a_{n-2}, a_1 = 2, a_2 = 6$. **(5)**

b. Show that if any 8 positive integers are chosen, two of them will have the same remainder when divided by 7. **(4)**

c. Define Euler circuit and Euler path. Which of the following graphs have an Euler circuit and Euler path: **(7)**



Q.4 a. Draw a digraph for each of the following relations:

(i) Let $A = \{a, b, c, d\}$ and let $R = \{(a,b),(b,d),(a,d),(d,a),(b,a),(c,c)\}$

(ii) Let $A = \{1,2,3,4,5,6,7,8\}$ and let ${}^{x}R_{Y}$, wherever Y is divisible by x

(iii) Determine which of the relations are reflexive, transitive, symmetric and antisymmetric. **(3+3+2)**

b. Prove that a graph G is a tree iff G has no cycles and $|E| = |V| - 1$. **(8)**

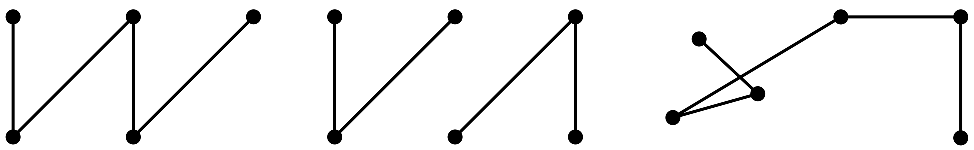
Q.5 a. Let S = {a, b, c} and A is the power set of S. Draw the Hasse diagram of the poset A with the partial order $\subseteq$ (set inclusion). **(5)**

b. Simplify the following boolean expressions using Karnaugh maps.

(i) $xy\bar{z} \mid x\bar{y}\bar{z} \mid \bar{x}yz \mid \bar{x}\bar{y}\bar{z}$

(ii) $x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$ **(8)**

c. Which of these graphs are trees? **(3)**



Q.6 a. Write the algorithm for Huffman coding. Encode the following symbols with the frequencies listed: A = 0.25, B = 0.20, C = 0.15, D = 0.12, E = 0.10, F = 0.18. **(10)**

b. Write the algorithms for In-order and Post-order traversals. **(6)**

Q.7 a. Write the syntax diagrams for a letter and an identifier. **(8)**

b. Write Kruskal's algorithm for computing a minimum spanning tree of a weighted graph. **(8)**

Q.8 a. Prove that 2 divides $n^2 + n$, whenever n is a positive integer. **(10)**

b. Give the recursive definition for the sequence $\{a_n\}, n = 1,2,3....$ and

(i) $a_n = 2n + 1$ and

(ii) $a_n = n^2$. **(6)**

Q.9 a. Design an automaton which accepts only even number of 0's and even number of 1's. **(10)**

b. Choose 4 cards at random from a standard 52-card deck. What is the probability that 4 kings will be chosen? **(6)**