ROLL NO. ___________________

Code: AC09/AT09 Subject: NUMERICAL COMPUTING

AMIETE – CS/IT (OLD SCHEME)

Time: 3 Hours **DECEMBER 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2 × 10)**

a. An approximate value of $\sqrt{2} = 1.414214$ is given by 1.414. The relative error in the approximation is given by

(A) .000151 **(B)** .000214
(C) $-.000151$ **(D)** $-.000214$

b. In bisection method, the minimum number of iterations required for converging to a root in the interval $(0,1)$ for permissible error 10^{-5} is given by

(A) 16 **(B)** 17
(C) 18 **(D)** 20

c. Partial Pivoting is normally used

(A) to avoid division by zero
(B) to reduce round off error
(C) to avoid division by zero and also to reduce round off error
(D) None of these

d. The matrix A has the same eigenvalue as

(A) A^{-1} **(B)** A^{T}
(C) Both A^{-1} and A^{T} **(D)** None of these

e. Given $f(2) = 4$ and $f(2.5) = 5.5$. The linear interpolating polynomial is given by

(A) $3x - 2$ **(B)** $2x - 3$
(C) $4x - 2$ **(D)** $4x - 3$

f. The following data is given

x :	−2	−1	0	1	2
f(x):	8	4	1	5	7

The least squares linear polynomial approximation to the above data is

(A) $5-0.1x$ (B) $5+0.1x$
(C) $5-x$ (D) $5+x$

g. In numerical differentiation formula

$$f''(x_{i+1}) = \frac{1}{h^2}\left[f(x_{i-1}) - 2f(x_i) + f(x_{i+1})\right]$$

The error is of order

(A) 1 (B) 2
(C) 3 (D) 4

h. "Error" is defined as

(A) Inherent Error – Relative Error
(B) Relative Error – Absolute Error
(C) True Value – Approximate Value
(D) None of these

i. The integral $I = \int_{-1}^{1} (1-x^2)^{3/2} \cos x \, dx$ is evaluated by the Gauss-Chebyshev two point formula. The value of I is given by

(A) 0.59709 (B) 3.14159
(C) 1.13200 (D) 1.08979

j. The initial value problem
$y' = x(y+x) - 2, y(0) = 2$
is solved by Euler's method. The approximate value of y(0.1), with h = 0.1 is given by

(A) 1.8 (B) 1.7
(C) 1.6 (D) 1.4

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The negative root of the smallest magnitude of the equation $f(x) = 3x^3 + 10x^2 + 10x + 7 = 0$ is to be obtained.
(i) Find an interval of unit length which contains this root.
(ii) Perform two iterations of the bisection method.
(iii) Taking the end points of the last interval as initial approximations, perform three iterations of secant method. **(8)**

b. How should the constant α be chosen to ensure the fastest possible convergence with the iteration formula.

$$x_{n+1} = \frac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$$

Obtain the value of α using Newton-Raphson method. **(8)**

Q.3 a. Solve the following system of equations by LU decomposition method with $u_{ii} = 1, i = 1,2,3$

$$x_1 + x_2 - x_3 = 2$$
$$2x_1 + 3x_2 + 5x_3 = -3$$
$$3x_1 + 2x_2 - 3x_3 = 6$$

(8)

b. Set up the Gauss-Seidel iteration scheme in matrix form to solve the following linear system of equations:

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

and obtain iterate three times starting with initial vector $x^{(0)} = 0$. Determine the rate of convergence of the method. **(8)**

Q.4 a. Using the Jacobi method find all the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$

(8)

b. Find the smallest eigenvalue in magnitude of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

using four iterations of the inverse power method. **(8)**

Q.5 a. Determine the maximum step size that can be used in the tabulation of $f(x) = e^x$ in [0, 1], so that the error in the quadratic interpolation will be less than 5×10^{-4}. **(8)**

b. Using divided differences, show that the data

x:	−3	−2	−1	1	2	3
$f(x)$:	18	12	8	6	8	12

represents a second degree polynomial. Hence determine the interpolating polynomial. **(8)**

Q.6 a. Obtain the least squares approximation of the form $f(x) = ax^b$ to the data **(8)**

x:	0.5	0.6	0.7	0.8	1.0
$f(x)$:	0.3136	0.4515	0.6146	0.8027	1.2542

b. A numerical differentiation formula for computing $f''(x_0)$ is given by,

$$f''(x_0) = \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

By use of repeated Richardson extrapolation, find $f''(0.6)$ from the following values: **(8)**

x	f(x)
0.2	1.420072
0.4	1.881243
0.5	2.128147
0.6	2.386761
0.7	2.657971
0.8	2.942897
1.0	3.559753

Q.7 a. Evaluate the integral $I = \int_0^{1/2} \frac{x}{\sin x} dx$, using Trapezoidal's rule with 2, 3, 5 points. Improve the results using Romberg integration. **(8)**

b. Evaluate the integral $I = \int_2^3 \frac{\cos 2x}{1 + \sin x} dx$, using Gauss-Legendre three point formula. **(8)**

Q.8 a. Use the fourth order Runge-Kutta method to find the numerical solution at $x = 0.6$ and $x = 0.8$ for the initial value problem $y' = \sqrt{x + y}, y(0.4) = 0.41$. **(8)**

b. Find an approximation to y(1.3) for the initial value problem $y' = -2xy^2, y(1) = 1$, using Taylor series method of second order with step size h = 0.1. compare with the exact solution $y = \frac{1}{x^2}$. **(8)**

Q.9 a. Derive the formula for the first derivative of y = f(x) of $O(h^2)$ using forward difference approximation. When $f(x) = \sin x$, estimate $f'\left(\frac{\pi}{4}\right)$ with $h = \frac{\pi}{12}$. **(6)**

b. Determine a, b and c such that the formula

$$\int_o^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$$

is exact for polynomials of as high order as possible and determine the order of the truncation error. **(10)**