Code: DE01 / DC01 **DECEMBER 2010** **Subject: MATHEMATICS - I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2x10)

a. The equation whose roots are double the roots of $x^2 - bx + c = 0$ is

(A) $4x^2 - 2bx + c = 0$ **(B)** $x^2 - 2bx + 4c = 0$
(C) $x^2 - 2bx + 2c = 0$ **(D)** $x^2 - 4bx + 2c = 0$

b. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$ is

(A) 0 **(B)** 1
(C) $\frac{1}{2}$ **(D)** $\frac{1}{4}$

c. If A(2, 1), B(4, 5) and C(K, -1) lie on a straight line, then value of k is

(A) 1 **(B)** 2
(C) 3 **(D)** 0

d. The equation of the straight line with slope 3 and x-intercept 2 is

(A) y=3x+2 **(B)** y=3x-2
(C) y=3x+6 **(D)** y=3x-6

e. If y = log (sec x + tan x), the value of $\frac{dy}{dx}$ is

(A) $\frac{1}{\sec x + \tan x}$ **(B)** $\sec x + \tan x$
(C) sec x **(D)** tan x

f. The value of $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$ is

(A) 0 **(B)** π

(C) $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

g. The area bounded by y = sin x, the x – axis between x = 0 and $x = \pi$ is

(A) 1 **(B)** 2

(C) 3 **(D)** 4

h. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 0$ is

(A) x+y = c **(B)** $x^2 + y^2 = c$

(C) xy = c **(D)** $\frac{x}{y} = c$

i. The value of $\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$ is

(A) x **(B)** $\frac{x}{2}$

(C) $\frac{x}{4}$ **(D)** 0

j. The value of $\frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$ is

(A) tan 5A **(B)** tan 3A

(C) tan 2A **(D)** tan A

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The sum of an infinite number of terms of a G.P. is 9 and sum of the squares of these terms is $\frac{81}{5}$. Find the G.P. **(8)**

b. If the r^{th} term to the expansion of $(1+x)^{20}$ has its coefficient equal to that of $(r+4)^{th}$ term. Find r. **(8)**

Q.3 a. If $A + B + C = \pi$, show that tan 2A + tan 2B + tan 2C = tan 2A.tan 2B.tan 2C **(8)**

b. If a, b, c be the sides opposite to the angles A, B, C for a triangle ABC, show that $\text{CosA} = \frac{b^2 + c^2 - a^2}{2bc}$. **(8)**

Q.4 a. Find the equation of a straight line when the length of perpendicular on it from the origin is given as 'p' and the inclination of this perpendicular to x-axis is given as α. **(8)**

b. Find the angle between the straight lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$. **(8)**

Q.5 a. Find the equation of the circle circumscribing the triangle formed by the lines x + y = 2, x – y = 0 and 3x – 4y = 6. **(8)**

b. Find the focus, vertex, latus rectum and directrix of the parabola $(y+3)^2 = 2(x+2)$ **(4×2 = 8)**

Q.6 a. If sin y = x sin (a+y), show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **(8)**

b. Show that the sum of the intercepts on axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant. **(8)**

Q.7 a. Find the local maximum and minimum values of the function
f(x) = (x-1) (x-2) (x-3) **(4+4)**

b. Evaluate $\int \sqrt{\left(\frac{2+x}{2-x}\right)} dx$ **(8)**

Q.8 a. Find the area bounded by the axis of x and the curve $y = 1 - x^2$ **(8)**

b. Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$ **(8)**

Q.9 Solve any **TWO** of the following differential equations:-

(i) $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

(ii) $x\frac{dy}{dx} + y = \log x$

(iii) $xdy - ydx = \sqrt{x^2 + y^2} dx$ **(8 + 8)**