

Subject: DIGITAL SIGNAL PROCESSING

Time: 3 Hours

DECEMBER 2010

Max. Marks: 100

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If the continuous signal  $x_c(t) = \cos(2\pi(1000)t)$  is as the input for an ideal C/D converter with the sampling period  $T=(1/3000)sec$ , the resulting discrete time signal  $x[n]$  is

- (A)  $\sin(2\pi n/3)$  (B)  $\cos(2\pi n/3)$   
(C)  $\sin(2\pi n/5)$  (D)  $\cos(2\pi n/5)$

b. In relation to the bilinear transform method of IIR design, which of the following apply?

- (A) The warping effect gets worse as we get closer to Nyquist frequency.  
(B) Pre-warping compensates for the distortion caused over the entire frequency response.  
(C) Pre-warping compensates for the distortion caused at only specified frequencies.  
(D) Frequency scaling eliminates the effects of warping in the frequency response.

c. The FIR filter structure is also referred to as a ----- structure.

- (A) transposed (B) cascade  
(C) transversal (D) parallel

d. As the length of the window increases in designing a FIR filter, width of the main lobe.....

- (A) does not change (B) increases  
(C) is zero (D) decreases

e. The maximum length of the sequence resulting from the linear convolution of a sequence of length L with a sequence of length P is

- (A)  $L - P + 1$  (B)  $L + P - 1$   
(C)  $L - P - 1$  (D)  $L + P + 1$

- f. Given two finite length sequences,  $x_1[n] = \{1, 2, 1, 1, 2, 1, 1, 2\}$  and  $x_2[n] = \{1, 3, 2\}$ , and  $x_3[n]$  is the 8-point circular convolution of the two sequences. The value of  $x_3[2]$  is
- (A) 9 (B) 10  
(C) 11 (D) 8
- g. In an N-point FFT algorithm, ----- complex storage registers are required to store the coefficients.
- (A)  $N - 2$  (B)  $N^2$   
(C)  $N/2$  (D)  $2N$
- h. In DFT analysis of sinusoidal signals, the degree of leakage depends on the relative ----- of the main lobe and the side lobes of the window function.
- (A) phase (B) width  
(C) angle (D) amplitude
- i. When a rectangular window sequence is used in estimating the power density spectrum, the estimator is called the
- (A) Spectrogram (B) Periodogram  
(C) Averaged Periodogram (D) Modified Periodogram
- j. The first four outputs of an 8-point, radix-2 FET are  $X[0] = 27$ ,  $X[1] = -4 + 3j$ ,  $X[2] = 4 + j$ ,  $X[3] = 0 - 5j$ . Which of the following statements are true?
- (A)  $X[7] = 0 + 5j$  (B)  $X[7] = -4 - j$   
(C) dc value is 27 (D)  $X[7] = 5$

**Answer any FIVE Questions out of EIGHT Questions.  
Each question carries 16 marks.**

**Q.2** a. Explain with suitable diagrams the effect in the frequency domain of sampling in the time domain and derive the Nyquist sampling theorem. (8)

b. Consider a system for discrete time processing of continuous signals, with the discrete time system an ideal LPF with cutoff frequency  $\pi/8$  radians/sec. If  $x_c(t)$  is bandlimited to 5 kHz, what is the maximum value of  $T$  that will avoid aliasing in a C/D converter? If  $1/T = 10$  kHz, what will be the cutoff frequency of the effective continuous-time filter? Repeat for  $1/T = 20$  kHz. (8)

**Q.3** a. Describe the following systems (i) Minimum phase systems (ii) All pass systems. (8)

- b. Consider the LTI system with input and output related through the difference equation  $y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n]$ . Find the system function  $H(z)$  and discuss the characteristics of the system for different ROCs. Indicate the ROCs in the pole-zero plots. (8)
- Q.4** a. Consider the system function  $H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$ . Draw the following structures for the system. (i) Direct form I (ii) Direct form II (iii) Cascade (iv) Parallel. (10)
- b. Consider the causal LTI system with system function  $H(z) = 1 - \frac{1}{3}z^{-1} + \frac{1}{6}z^{-2} + z^{-3}$ . Draw the Direct form and transposed Direct form representation of this system. (6)
- Q.5** a. Explain the bilinear transformation algorithm for designing IIR filters. (6)
- b. Design a first order digital lowpass filter with a 3dB cutoff frequency of  $0.25\pi$  by applying the bilinear transformation to the analog Butterworth filter. (10)
- Q.6** a. Consider the following two 4-point sequences  $x[n] = \cos\left(\frac{\pi n}{2}\right)$  and  $h[n] = 2^n$ . (i) Calculate the 4-point DFT  $X[k]$ . (ii) Calculate the 4-point DFT  $H[k]$ . (iii) Calculate the 4-point circular convolution of  $x[n]$  with  $h[n]$ . (10)
- b. Discuss overlap add method for performing linear convolution of large length signal. (6)
- Q.7** a. Describe Goertzel algorithm for computing DFT and compare the complexity with direct DFT algorithm. (8)
- b. Compute the 8-point DFT of the sequence  $x[n] = \cos\left(\frac{\pi n}{2}\right)$  using the decimation in time FFT algorithm (8)
- Q.8** a. What is a spectrogram? Explain time-dependent Fourier analysis of speech signals. (8)
- b. Consider a bandlimited continuous time signal  $x_c(t)$  such that  $X_c(j\Omega) = 0$  for  $|\Omega| \geq 2\pi(2500)$ . Assume that the antialiasing filter is ideal and the sampling rate for the C/D converter is  $1/T = 5000$  samples/second. If the DFT samples  $X[k]$  are to be equivalent to the samples of  $X_c(j\Omega)$  that are at most 10 Hz apart, what is the minimum value required for the DFT size? Calculate the equivalent continuous time frequency spacing. If it is determined that  $X[11] = 2000(1+j)$ , what is  $X[501]$ ? Find also the corresponding spectrum values in continuous time domain. (8)

- Q.9** a. Explain with an example how bandpass signals are represented using Fourier transform.
- b. Given  $X_R(e^{j\omega}) = \frac{1 - \alpha \cos \omega}{1 - 2\alpha \cos \omega + \alpha^2}$ ,  $|\alpha| < 1$  with  $\alpha$  real, find the sequence  $x[n]$  and the corresponding  $X[z]$ . (8)