Subject: SIGNALS AND SYSTEMS

Time: 3 Hours **Max. Marks: 100**

DECEMBER 2010

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. System function $H(z)$ for the system described by the difference equation $4y(n) = 3x(n) + 2x(n-1) - y(n-1)$ is

(A) $\frac{3+2z^{-1}}{4+z^{-1}}$ **(B)** $\frac{1+4z^{-1}}{2+3z^{-1}}$

(C) $\frac{4+3z^{-1}}{1+2z^{-1}}$ **(D)** $\frac{2+z^{-1}}{3+4z^{-1}}$

b. A system has an input-output relation given by $y = ax + b$. The system is linear if

(A) a and b are arbitrary **(B)** $b = 0$
(C) $a = 0$ **(D)** $b < 0$

c. A system is characterized by the equation $y(t) = 10\,x(t) + 5$ is

(A) Stable, time-invariant **(B)** Unstable, time-invariant
(C) Stable, time-variant **(D)** Unstable, time-variant

d. Inverse Z-transform of $X(z) = \frac{2z}{(z-2)^2}$ is

(A) $4u(n)$ **(B)** $2^n u(n)$

(C) $8u(n)$ **(D)** $n2^n u(n)$

e. The impulse response of a discrete-time system is given by $h(n) = \frac{1}{2}(\delta[n] + \delta[n-1])$. The magnitude response can be expressed as

(A) $|\cos(\Omega/2)|$ **(B)** $\cos(\Omega/2)$
(C) $|\sin(\Omega/2)|$ **(D)** $\sin(\Omega/2)$

f. A series RL (R = 1 ohm, L = 1 H) circuit, is energized with a voltage v
$\cos t\, u(t)$ with initial current $i(0) = 2$ A. The natural response for the curre
in the circuit is

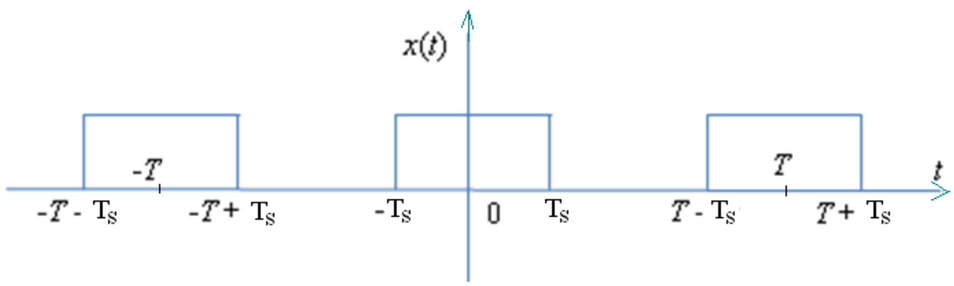
(A) $\frac{1}{2}\cos t$ **(B)** $\frac{1}{2}\cos t + \frac{1}{2}\sin t$

(C) $\frac{1}{2}\sin t$ **(D)** $\frac{3}{2}e^{-t}$

g. The zero-frequency component in the Fourier series representation of the square wave shown in Fig.1 is

(A) $\frac{1}{2}T_s / T$ **(B)** $\frac{1}{2}T / T_s$

(C) $2T_s / T$ **(D)** $|z| = 0$



h. Inverse DTFT of $\delta(\Omega)$, $-\pi < \Omega \le \pi$ is

(A) $u(n)$ **(B)** $\frac{1}{2\pi}$

(C) $\delta(n)$ **(D)** 2π

i. Fourier transform of the function $x(t) = \begin{cases} 1, & -T \le t \le T \\ 0, & |t| > T \end{cases}$ is

(A) $\frac{2}{\omega}\sin(\omega T)$ **(B)** $\frac{1}{\omega}\sin(\omega T)$

(C) $\frac{\omega}{2}\sin(\omega T)$ **(D)** $\omega \sin(\omega T)$

j. Fourier transform of a periodic unit impulse train of period τ is an impulse train of period and magnitude, respectively,

(A) $\frac{\pi}{\tau}, \frac{2\pi}{T}$ **(B)** $\frac{2\pi}{\tau}, \frac{2\pi}{T}$

(C) $\frac{\pi}{\tau}, \frac{\pi}{T}$ **(D)** $\frac{2\pi}{\tau}, \frac{\pi}{T}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Determine the Laplace transform of the signal $v(t) = 0.5(\sin t)(\sin 1000t)u(t)$ **(6)**

b. Determine the voltage $v(t)$ for $t \geq 0$ for the circuit shown in Fig.2 when $e(t) = 1 + \sin t$. Use Laplace transform method. Assume no initial charge on the capacitor.

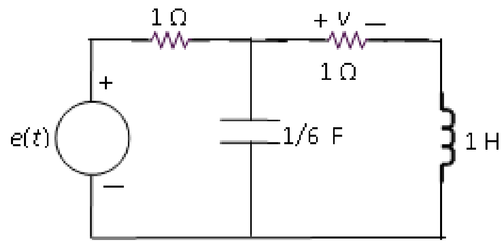


Fig.2 **(10)**

Q.3 a. Determine the impulse response h(t) for the system characterized by the differential equation $\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} + 2y(t) = x(t)$ **(6)**

b. (i) State the Sampling theorem **(4)**

(ii) Determine the condition on the sampling interval so that $x(t) = \sin(10\pi t)/\pi t$ can be uniquely represented by the discrete-time sequence. **(6)**

Q.4 a. The impulse response of a linear time-invariant system is $h(t) = u(t)$. Determine the output of the system if the input $x(t) = e^{-at}u(t)$, $a > 0$ by convolution. Show all the steps graphically (rough sketch) also. (No graph paper to be used. **(9)**

b. Sketch the odd part of the signal shown in Fig.3 **(7)**

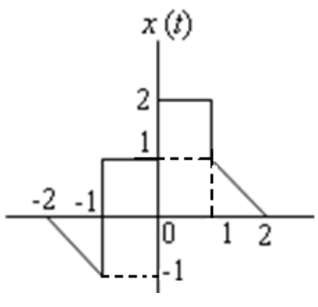


Fig.3

Q.5 a. Consider a discrete-time LTI system described by $y[n] - \frac{1}{2}y[n-1] = x[n] + \frac{1}{2}x[n-1]$

(i) Determine the frequency response $H(e^{j\omega})$ of the system.
(ii) Find the impulse response h[n] of the system.
(iii) Determine its response y[n] to the input $x[n] = \cos\frac{\pi}{2}n$. **(9)**

b. Consider the periodic function defined over one period T is **(7)**

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

(i) Sketch the waveform $x(t)$.
(ii) Which type of symmetry does the function exhibit?
(iii) Determine the Fourier series coefficients
(iv) Plot the magnitude spectra of the function when $T = 4T_1$

Q.6 a. Explain the linearity and time-shifting properties of the *z*-transform. **(10)**

b. Find the Z-transform and the region of convergence of the sequence
$x(n) = b^{|n|}$ **(6)**

Q.7 a. Let x(t) be a signal with Fourier transform $X(j\omega)$. Derive the following properties
(i) Parseval's relation (ii) Integration property **(8)**

b. Determine the Fourier transform of the function $f(t) = e^{-at}\cos(\omega t + \theta)$ **(8)**

Q.8 a. Verify the convolution theorem for DTFT. **(8)**

b. Determine the discrete Fourier series representation for each of the following sequences:
(i) $x[n] = \cos\frac{\pi}{4}n$ (ii) $x[n] = \cos\frac{\pi}{3}n + \sin\frac{\pi}{4}n$ **(8)**

Q.9 a. Two random variables *X* and *Y* have the joint probability density function

$$P_{XY}(x, y) = \begin{cases} Ae^{-(2X+Y)}, & x, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of A **(2)**
(ii) Compute $P_X(x)$ and $P_Y(y)$. **(8)**

b. If X and Y are independent random variable having normal distributions with parameters $(\mu_1, \sigma_1^{\ 2})$ and $(\mu_2, \sigma_2^{\ 2})$, respectively. Find the distribution of $X + Y$. **(6)**