**Subject: ENGINEERING MATHEMATICS - II**

**Time: 3 Hours** **Max. Marks: 10**

# DECEMBER 2010

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. Inverse transformation $w = \frac{1}{z}$, transforms the straight line $2ay + 3bx = 0$, into

**(A)** circle **(B)** an ellipse
**(C)** straight line through origin **(D)** straight line

b. The value of the integral $\int_C \frac{z+1}{z^3 - 2z^2} dz$, where C is the circle $|z| = 1$, is equal to

**(A)** $2\pi i$ **(B)** $-\frac{2}{3}\pi i$

**(C)** zero **(D)** $-\frac{3}{2}\pi i$

c. If r is the distance of a point (x, y, z) from the origin, the value of the expression $\hat{j} \times \text{grad}\frac{1}{r}$ equals

**(A)** $\left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}}\left(\hat{j}z - \hat{k}x\right)$ **(B)** $\left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}}\left(\hat{k}x - \hat{i}z\right)$

**(C)** zero **(D)** $\left(x^2 + y^2 + z^2\right)^{-\frac{3}{2}}\left(\hat{j}y - \hat{k}x\right)$

d. The value of the surface integral $\iint_S (yzdydz + zxdzdx + xydxdy)$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$, is

**(A)** $\frac{4\pi}{3}$ **(B)** $4\pi$
**(C)** zero **(D)** $12\pi$

e. One of the solutions to the auxiliary equations corresponding to the Lagrange's differential equation $\left(z^2 - 2yz - y^2\right)p + (xy + zx)q = xy - zx$ is

**(A)** $y^2 - 2yz - z^2 = C_1$ **(B)** $y^2 + 2yz - z^2 = C_1$
**(C)** $y^2 + 2yz + z^2 = C_1$ **(D)** $y^2 - 2yz + z^2 = C_1$

f. The fixed points of the transformation $W = \frac{2Z - 5}{Z + 4}$ are given by

**(A)** $\left(\frac{5}{2}, 0\right)$ **(B)** $(-4, 0)$
**(C)** $(-1 + 2i, -1 - 2i)$ **(D)** $\left(-1 + \sqrt{6}, -1 - \sqrt{6}\right)$

g. Which one of the following does not equal $\Delta f(x - 1)$?

**(A)** $\Delta E^{-1} f(x)$ **(B)** $E^{-1} \Delta f(x)$
**(C)** $\left(E^{-1} - 1\right) f(x)$ **(D)** $\left(1 - E^{-1}\right) f(x)$

h. $\Delta\left(\frac{1}{f(x)}\right)$ equals

**(A)** $\frac{\Delta f(x)}{f(x) f(x+1)}$ **(B)** $\frac{\Delta f(x+1)}{f(x+1) f(x)}$
**(C)** $\frac{-\Delta f(x)}{f(x) f(x+1)}$ **(D)** $-\frac{\Delta f(x+1)}{f(x) f(x+1)}$

i. A die is tossed thrice. Getting 1 or 6 on a toss is considered to be a success. The variance of the number of successes is

**(A)** $\frac{3}{2}$ **(B)** $\frac{1}{3}$
**(C)** $\frac{2}{3}$ **(D)** 1

j. If the diameter of an electric cable is assumed to be continuous random variable with probability density function $f(x) = kx(1 - x), 0 \le x \le 1$ then the value of k is

**(A)** 3 **(B)** 6
**(C)** $\frac{2}{3}$ **(D)** $\frac{3}{2}$

**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

**Q.2** a. If $f(z)=\begin{cases}\dfrac{x^2y(y-ix)}{x^4+y^2}, & \text{when } z\neq 0\\ \text{zero}, & \text{when } z=0\end{cases}$

prove that $\dfrac{f(z)-f(0)}{z}\to 0, \text{ as } z\to 0,$ along any radius vector but not as $z\to 0$ in any manner. **(8)**

b. Obtain the bilinear transformation which maps $z_1=i, z_2=1, z_3=-1$ onto $w_1=1, w_2=0, w_3=\infty$, respectively. Also, obtain its inverse transformation. **(8)**

**Q.3** a. Evaluate the integral $\int_C \dfrac{\sin\pi z^2}{(z-1)(z-2)}dz$ where C is the circle $|z|=3$. **(8)**

b. Find the Laurent's series of $f(z)=\dfrac{1}{z^2(1-z^2)}$ and determine the precise region of its convergence. **(8)**

**Q.4** a. Find f(r) such that $f(r)\vec{r}$ is both solenoidal and irrotational. **(8)**

b. Find the directional derivative of the function $\phi(x,y,z)=x^2-y^2+2z^2$ at the point P(2, 3, 4) in the direction of the line PQ where Q has coordinates (6, 0, 5). In what direction it will be maximum and what shall be its value? **(8)**

**Q.5** a. State Stoke's theorem. Evaluate the surface integral $\iint_S (\text{curl}\,\vec{F}\cdot n)ds$ by transforming it into line integral, S being that part of the surface of the paraboloid $z=1-x^2-y^2$, for which $z\geq 0$ and $\vec{F}=y\hat{i}+z\hat{j}+x\hat{k}$. **(8)**

b. Find $\iint_S \vec{F}\cdot\hat{x}ds$, where $\vec{F}=(2x+3z)\vec{i}-(3x+y)\vec{j}+(y+2z)\vec{k}$ and S is the surface of the sphere having centre $(3,-1,2)$ and radius 3. **(8)**

**Q.6** a. Solve the partial differential equation $u_x+u_y=2(x+y)u$, by the method of separation of variables. **(8)**

b. Apply Charpit's method to obtain the complete integral of the equation $p^2-q^2=x-y$. **(8)**

**Q.7** a. Using Newton's divided difference formula, calculate the value of f(6) from following data: **(8)**

| | | | | | |
|---|---|---|---|---|---|
| x | : | 1 | 2 | 7 | 8 |
| f(x) | : | 1 | 5 | 5 | 4 |

b. A train is moving at the speed (v) of 30 m/sec. Suddenly, brakes are applied. The speed of the train per second (after 't' seconds) is given by **(8)**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Time (t) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| Speed (v) | 30 | 24 | 19 | 15 | 13 | 11 | 10 | 8 | 7 | 5 |

Apply Simpson's three-eighth rule to determine the distance moved by the train in 45 seconds.

**Q.8** a. Find the mean and variance of the random variable whose density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{other wise} \end{cases}$$ **(8)**

b. A student takes examination in four subjects viz., English, Chemistry, Mathematics and Physics, which are abbreviated as E, C, M and P respectively. He estimates his chances of passing in E as $\frac{4}{5}$, in C as $\frac{3}{4}$, in M as $\frac{5}{6}$ and in P as $\frac{2}{3}$. To qualify, he must pass in E and at least in two other subjects. Find the probability that he qualifies. **(8)**

**Q.9** a. A factory producing cycle tyres, there is a small chance of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10. Using Poisson's distribution, calculate the approximate number of lots containing no defective and one defective tyre in a consignment of 10,000 lots (given $e^{-0.02} \simeq 0.9802$). **(8)**

b. In a normal distribution 31 % of the items are under 45 and 8 % over 64. Find the mean and standard deviation of the distribution (given for area 0.19, z = 0.496 and for the area 0.42, z = 1.405) **(8)**