Subject: ENGINEERING MATHEMATICS - I

Time: 3 Hours **DECEMBER 2010** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. For which value of 'b' the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$ is 2

(A) 1 **(B)** 2
(C) 3 **(D)** 0

b. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(A) 0 **(B)** $\sqrt{2}$
(C) 1 **(D)** $\sqrt{3}/2$

c. For what values of x, the matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular

(A) 0 **(B)** 3
(C) 0, 3 **(D)** 0, -3

d. Regula-Falsi method requires _______________ initial approximations to the root.

(A) 1 **(B)** 2
(C) 3 **(D)** None of the above

e. The necessary and sufficient condition for the differential equation Mdx+N
0 to be exact is

(A) $\frac{\partial M}{\partial y}+\frac{\partial N}{\partial x}=0$ (B) $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=0$

(C) $\frac{\partial M}{\partial y}\times\frac{\partial N}{\partial x}=1$ (D) None of the above

f. Two function $y_m(x)$ and $y_n(x)$ defined on some interval $a \le x \le b$, are said to be orthogonal on this interval w.r.t. the weight function $p(x) > 0$ if

(A) $\int_a^b p(x) y_m(x) y_n(x) dx = 0$ for $m \ne n$

(B) $\int_a^b p(x) \frac{y_m(x)}{y_n(x)} dx = 0$ for $m \ne n$

(C) $\int_a^b p(x) \frac{y_n(x)}{y_m(x)} dx = 0$ for $m \ne n$

(D) $\int_a^b p(x) y_m(x) y_n(x) dx = 1$ for $m \ne n$

g. The value of $\int_0^\infty \sqrt{x} e^{-\sqrt[3]{x}} dx$

(A) $\frac{16}{315}\sqrt{\pi}$ (B) π

(C) 1 (D) $\frac{315}{16}\sqrt{\pi}$

h. The value of $e^{\frac{1}{2}x(t-1/t)}$ is

(A) $\sum_{n=-\infty}^{\infty} t^n J_n(x)$ (B) $\sum_{n=0}^{\infty} t^n J_n(x)$

(C) $\sum_{n=-\infty}^{0} t^n J_{n-1}(x)$ (D) $\sum_{n=0}^{\infty} t^n J_{n-1}(x)$

i. Find the complementary function of $(D-2)^2 = 8\left(e^{2x} + \sin 2x + x^2\right)$

(A) $\left(C_1 + C_2 e^{2x}\right)x$ (B) $\left(C_1 + C_2 x\right)e^{2x}$

(C) $\left(C_1 x + C_2 x^2\right)e^{2x}$ (D) $\left(C_1 x + C_2 e^x\right)\cdot 2x$

j. For a system of $(m \times n)$ equations, if the rank of the coefficient matrix in r that of augmented matrix in r', then the equations will be consistent and there will be infinite number of solutions if:
(where m is the number of equation and n is the number of unknown)

(A) $r \neq r'$ **(B)** $r = r' = n$
(C) $r = r' < n$ **(D)** $r \neq r' > n$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $z = (x+y) + (x+y)\phi\left(\frac{y}{x}\right)$, then prove that

$$x\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y \partial x}\right) = y\left(\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y}\right)$$ **(8)**

b. Find the equation of the tangent plane and normal to the surface $x^2 + 2y^2 + 3z^2 = 12$ at $(1,2,-1)$ **(8)**

Q.3 a. Change the order of integration of $\int_0^\infty \int_0^\infty e^{-xy} \sin nx \, dx dy$

Show that $\int_0^\infty \frac{\sin nx}{x} dx = \frac{\pi}{2}$ **(8)**

b. Find the position of the centre of gravity of a semicircular lamina of radius 'a' if its density varies as the square of the distance from the diameter. **(8)**

Q.4 a. Find the values of λ for which the equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$
$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$
$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

are consistent, and find the ratios of x:y:z when λ has the smallest of these values. What happens when λ has the greatest of these values. **(8)**

b. Find the eigen values and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(8)**

Q.5 a. Using Newton-Raphson method, derive formulas to find

(i) $1/N$ (ii) $N^{1/q}, N > 0, q$ integer.

Hence find $1/18, (18)^{1/3}$ to four decimals. Use suitable initial approximation. **(8)**

b. Apply Runge-Kutta method (fourth order) to find an approximate value of y when x = 0.2 given that $\frac{dy}{dx} = x + y^2$ and y = 1 when x = 0. **(8)**

Q.6 a. Solve $xy\left(1 + xy^2\right)\frac{dy}{dx} = 1$. **(8)**

b. Solve $\frac{dy}{dx} = \frac{2y - x - 4}{y - 3x + 3}$ **(8)**

Q.7 a. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **(8)**

b. Solve $[D^2 + 5D + 6][y] = e^x$ **(8)**

Q.8 a. Prove that $\int_0^1 xJ_n(\alpha_x)J_n(\beta_x)dx = \begin{cases} 0 & \alpha \neq \beta \\ \frac{1}{2}[J_{n+1}(\alpha)]^2, & \alpha = \beta \end{cases}$

Where α and β are the roots of $J_n(x)$. Also discuss the orthogonality relation of Bessel function. **(8)**

b. Prove that $J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0(x)$, where $J_n(x)$ in the Bessel function of first kind. **(8)**

Q.9 a. Show that $\Gamma 2n = \frac{2^{2n-1}}{\sqrt{\pi}} \Gamma\left(n + \frac{1}{2}\right) \Gamma n$ and $\Gamma \tfrac{1}{4} \Gamma \tfrac{3}{4} = \pi\sqrt{2}$ **(8)**

b. Evaluate $\int_0^\infty e^{-ax} x^{m-1} \sin bx \, dx$ in terms of Gamma function. **(8)**