Code: AE35/AC35/AT35 **Subject: MATHEMAT**
Time: 3 Hours **DECEMBER 2010** **Max. Marks:**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The function $f(z)=|z|^2$ satisfy which condition:

(A) Differentiable everywhere
(B) Differentiable at z = 0 and nowhere else
(C) Not differentiable
(D) None of the above

b. Using Cauchy's integral formula, the value of $\int_C \frac{dz}{z(z-\pi i)}$, where C is $|z+3i|=1$

(A) 1 **(B)** -1
(C) 0 **(D)** 2

c. The Taylor series which represents the function $\frac{z^2-1}{(z+2)(z+3)}$ in the region $|z|<2$ is :

(A) $1+\sum_{1}^{\infty}\left[\frac{3}{2^{n+1}}-\frac{8}{3^{n+1}}\right]z^n$ **(B)** $1+\sum_{0}^{\infty}(-1)^n\left[\frac{3}{2^{n+1}}+\frac{8}{3^{n+1}}\right]z^n$

(C) $1+\sum_{1}^{\infty}\left[\frac{3}{2^{n+1}}+\frac{8}{3^{n+1}}\right]z^n$ **(D)** $1+\sum_{0}^{\infty}(-1)^n\left[\frac{3}{2^{n+1}}-\frac{8}{3^{n+1}}\right]z^n$

d. If $\vec{r}=\sin t\,\hat{i}+\cos t\,\hat{j}+t\hat{k}$, then the value of $\left|\frac{d^2\vec{r}}{dt^2}\right|$ is :

(A) $\sqrt{2}$ **(B)** 1
(C) 3 **(D)** $\sqrt{3}$

e. The curl of grad of scalar field F is

(A) 0 **(B)** -1
(C) $\nabla^2 F$ **(D)** $\nabla(\nabla F)$

f. If $\vec{f} = 3xy\hat{i} - y^2\hat{j}$, then $\int_C \vec{f} \cdot d\vec{r}$ where C is the curve in xy plane, $y = 2x^2$ from $(0,0)$ to $(1,2)$:

(A) $-\frac{7}{6}$ **(B)** $\frac{7}{6}$
(C) $\frac{7}{3}$ **(D)** $-\frac{7}{3}$

g. A variate X has the probability distribution

x:	-3	6	9
P(X=x):	$1/6$	$1/2$	$1/3$

Then the value of E(X), is :

(A) $\frac{2}{11}$ **(B)** $\frac{11}{2}$
(C) $\frac{93}{2}$ **(D)** $\frac{5}{2}$

h. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than '2' will get a bad reaction.

(A) 0.31 **(B)** 0.30
(C) 0.32 **(D)** 0.33

i. If $f(x,y,z) = x^2y + y^2x + z^2$; find ∇f at the point (1, 1, 1)

(A) $3\hat{i} + 3\hat{j} + 2\hat{k}$ **(B)** $3\hat{i} - 3\hat{j} + 2\hat{k}$
(C) $3\hat{i} + 3\hat{j} - 2\hat{k}$ **(D)** $3\hat{i} - 3\hat{j} - 2\hat{k}$

j. The type of singularity of the function $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$ is

(A) isolated essential singularity **(B)** non-isolated essential singularity
(C) double pole **(D)** simple pole

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that $u = \frac{1}{2}\log\left(x^2 + y^2\right)$ is harmonic and find its harmonic conjugate. **(8)**

b. Find the image of closed half disk $|z| \le 1, I_m(z) \ge 0$ under the bi

transformation $w = \dfrac{z}{z+1}$.

Q.3 a. Find the value of the integral $\int_0^{1+i} \left(x - y + ix^2\right) dz$

(i) along the straight line from z = 0 to z = 1+i
(ii) along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1+i **(8)**

b. Expand log(1+z) in a Taylor's series about z = 0 and determine the region of convergence for the resulting series. **(8)**

Q.4 a. Find the nature and location of the singularities of the function $f(z) = \dfrac{1}{z\left(e^z - 1\right)}$. Prove that f(z) can be expanded in the form

$\dfrac{1}{z^2} - \dfrac{1}{2z} + a_0 + a_2 z^2 + a_4 z^4 + \ldots\ldots\ldots..$

Where $0 < |z| < 2\pi$ and find the values of a_0 and a_2. **(8)**

b. Evaluate the integral $I = \int_0^{2a} e^{\cos\theta} \cos(\sin\theta) d\theta$. **(8)**

Q.5 a. A particle move along the curves $x = 3t^2, y = t^2 - 2t, z = t^3$. Find its velocity and acceleration at t = 1 in the direction of vector $\vec{a} = \hat{i} + \hat{j} - \hat{k}$. **(8)**

b. Find $\vec{f} \times (\nabla \times \vec{g})$ at the point $(1,-1,2)$ if $\vec{f} = xz^2\hat{i} + 2y\hat{j} - 3xz\hat{k}, \vec{g} = 3xz\hat{i} + 2yz\hat{j} - z^2\hat{k}$ **(8)**

Q.6 a. Verify Green's theorem in the plane for $\oint \left(xy + y^2\right)dx + x^2 dy$, where C is the closed curve of the region bounded by y = x and y = x^2. **(8)**

b. Evaluate by Stoke's theorem $\oint (\sin z dx - \cos x dy + \sin y dz)$ where C is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$. **(8)**

Q.7 a. Using the method of separation of variables, solve $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$. **(8)**

b. A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in a position given by $y = y_0 \sin^3\left(\dfrac{\pi x}{e}\right)$. If it is released from rest from this position, find the displacement y(x, t) **(8)**

Q.8 a. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditi
$u(x,0) = 3\sin n\pi x, u(0,t) = 0$ and $u(1,t) = 0$, where $0 < x < 1$, $t > 0$. **(8)**

b. A die is tossed thrice. A success is 'getting 1 or 6' on a toss. Find the mean and variance of the number of success. **(8)**

Q.9 a. In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts. **(8)**

b. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for
(i) more than 2150 hours
(ii) less than 1950 hours and
(iii) more than 1920 hours and but less than 2160 hours. **(8)**