Subject: FINITE AUTOMATA & FORMULA LANGUAGES

Time: 3 Hours **Max. Marks: 100**

DECEMBER 2010

NOTE: There are 9 Questions in all.
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Given an arbitrary non-deterministic finite automaton (NFA) with N states, the maximum number of states in an equivalent minimized DFA is at least

(A) N^2 **(B)** 2N
(C) 2^N **(D)** N!

b. Consider a DFA over {a, b} accepting all strings with even number of a's and even number of b's. What is the minimum number of states that the DFA will have?

(A) 2 **(B)** 4
(C) 5 **(D)** 6

c. What is the language of the grammar with the following production rules?

$S \rightarrow ASb \mid c$
$A \rightarrow a$

(A) $\{a^n c b^n \mid n \geq 1\}$
(B) $\{xcb \mid x \in \{a\}^*\}$
(C) $\{acy \mid y \in \{b\}^*\}$
(D) All of the answers above are incorrect

d. Context-free languages are closed under:

(A) Union, intersection **(B)** Union, Kleene closure
(C) Intersection, complement **(D)** Complement, Kleene Closure

e. Let L(P) be the set of all languages accepted by a PDA P by final state and L(E) the set of all languages accepted by a PDA E by empty stack. Which of the following is true?

(A) $L(P) = L(E)$ **(B)** $L(P) \neq L(E)$
(C) $L(P) \subseteq L(E)$ **(D)** $L(E) \subseteq L(P)$

f. If we want a Deterministic Push Down Automata to accept a language by empty stack, then the strings of the language should have

(A) Substring property
(B) Identity property
(C) Prefix property
(D) Suffix property

g. Which of the following regular expression over {0,1} denotes the set of all strings not containing 100 as a substring

(A) (1+0)*
(B) 0*1010*
(C) (10+1)* 0*
(D) 0*1*01*

h. The simplification of grammar needs to follow the order (i) Eliminate useless symbol (ii) eliminate unit products (iii) eliminate ε-productions

(A) (i), (ii) and (iii)
(B) (ii), (i) and (iii)
(C) (iii), (ii) and (i)
(D) (iii), (i) and (ii)

i. The language $[0^n \ 1^n \ 2^n \mid 1 \le n \le 106]$ is

(A) Regular
(B) context-free
(C) Recursive
(D) Recursively enumerable

j. Counter Machine uses

(A) Finite Automata.
(B) Push Down automata.
(C) Turing Machine.
(D) all of the above.

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Define the terms – alphabet, string and language and provide one example for each. **(6)**

b. Consider the language: {w | w consists of an equal number of 0's and 1's}. Provide the following:
(i) Alphabet of this language
(ii) Any three different strings of length 6 belonging to the language.
(iii) Does any palindrome string belong to this language? Justify with an example. **(6)**

c. Using the method of structural inductions, prove that every expression has an equal number of left and right parentheses. **(4)**

Q.3 a. Define Deterministic Finite Automata. Find DFA that accepts all strings from {a, b} that does not contain either aa or bb. **(6)**

b. Consider the following ε-NFA.

State	δ			
	ε	A	b	c
→ p	{q, r}	Φ	{q}	{r}
q	Φ	{p}	{r}	{p, q}
* r	Φ	Φ	Φ	Φ

(i) Compute ε-closure of each state
(ii) Give all strings of length three or less accepted by the automata
(iii) Convert the automata to a DFA. **(10)**

Q.4 a. Find regular expressions to represent **(6)**

(i) All strings over {a, b} that contain both aa and bab as substrings
(ii) L = {w | w has odd number of 1's followed by even number of 0's}

b. Convert the DFA defined by the transition table given below to regular expression using state elimination method. **(5)**

State	δ	
	a	b
→ q0	{q1}	{q0}
q1	{q1}	{q2}
q2	{q3}	{q2}
* q3	{q3}	{q3}

c. Construct an ε-NFA for the language L = 0* + 1* + 2*. **(5)**

Q.5 a. Prove that the language $\{a^n b^n \mid n >= 1\}$ is not regular. **(5)**

b. Is there another equivalent for the DFA given in below transition table? Justify your answer. **(5)**

State	δ	
	0	1
→ * q0	{q1}	{q2}
* q1	{q1}	{q2}
q2	{q0}	{q2}

c. Define Context Free Grammar (CFG). Design a CFG to accept palindrome strings over 0's and 1's. **(6)**

Q.6 a. Show that the grammar G = (S, {a, b}, {S → SbS | a}, S) is ambiguous. **(5)**

b. Construct a PDA to accept strings containing equal number of a's and b's. Show the moves of the PDA for the input string 'abbaab'. **(6)**

c. Prove that if $L = L(P_F)$ for some PDA $P_F = (Q, \Sigma, \Gamma, \delta_F, q_0, Z_0, F)$, then there is a PDA P_N such that $L = N(P_N)$. **(5)**

Q.7 a. Convert the grammar with following productions to Chomsky Normal Form.
P={S → ASB | ε, A → aAS | a, B →SbS | A | bb} **(8)**

b. State and prove pumping lemma for Context Free Languages. Show that $L = \{a^i b^i c^i \mid i \geq 1\}$ is not CFL. **(8)**

Q.8 a. With a proper diagram, briefly explain the working of a Turing Machine. Formally define the language accepted by a Turing Machine. **(8)**

b. Design a Turing Machine to accept the strings over {0, 1} with equal number of 0's and 1's. Show the moves of the Turing Machine for the input string 0011. **(8)**

Q.9 a. Define the following languages. Also show pictorially the relationship between them.
(i) Recursively Enumerable
(ii) Recursive, and
(iii) Non-Recursively Enumerable **(8)**

b. Define Post's Correspondence Problem (PCP). Obtain a solution for the following instance of PCB: **(8)**

	List A	List B
1	110	110110
2	0011	00
3	0110	110