AMIETE – CS (OLD SCHEME)

Code: AC10 **Subject: DISCRETE STRUCTURES**
Time: 3 Hours **Max. Marks: 100**

DECEMBER 2010

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: **(2×10)**

a. If A × B = {(3, 2),(3, 4), (5, 2), (5, 4)}, then

(A) A = {3, 5} B = {2, 4} **(B)** A = {4, 3} B = {5, 2}
(C) A = {4, 2} B = {3, 5} **(D)** A = {2, 4} B = {3, 5}

b. If R is a symmetric relation then

(A) $R \cap R^{-1} \neq \phi$ **(B)** $R \cap R^{-1} = \phi$
(C) $R \cup R^{-1} = \phi$ **(D)** $R \cup R^{-1} = R$

c. If f = {(1, 2), (3, 5), (4,1)} and g = {(2, 3), (5, 1), (1, 3)} then

(A) f*o*g = {(1, 3), (4, 5), (2, 2)} **(B)** f*o*g = {(2, 5), (5, 2), (1, 5)}
(C) f*o*g = {(1, 3), (3, 1), (4, 3)} **(D)** f*o*g = {(1, 5), (3, 4), (5, 2)}

d. If a∈ B (Boolean algebra) then a + a + a + a is
(A) 4a **(B)** 3a
(C) 2a **(D)** a

e. Conjunctive normal form of Boolean function f(x, y) = $x + x'y$ is

(A) x + y **(B)** $xy + xy' + x'y$
(C) $x' + y$ **(D)** none of these

f. Complete bipartite graph $K_{m,\,n}$ is Eularian if

(A) m is odd and n is even **(B)** both m and n are even
(C) both m and n are odd **(D)** m is even and n is odd

g. If p: " He is rich" and q: " he is unhappy" then choose the correct formula for the statement "He is poor or else he is both rich and unhappy"

(A) $\sim p \vee (p \wedge \sim q)$ **(B)** $p \wedge (p \wedge \sim q)$
(C) $\sim p \vee (p \wedge q)$ **(D)** $p \vee (p \leftrightarrow q)$

h. The number of solutions to the equation x + y – z = 7 such that x > 0, y > 2 and z >2 is

(A) 10 **(B)** 15
(C) 25 **(D)** 8

i. The number of internal (non pendant) vertices in a full binary tree of n-vertices is

(A) (n + 1) **(B)** (n + 1) / 2
(C) n(n + 1) / 2 **(D)** (n – 1) / 2

j. The language L(G) generated by the productions S → aS | a is

(A) L(G) = a^* **(B)** L(G) = (a, aaa, aaaaaa)
(C) L(G) = a^+ **(D)** L(G) = { }

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Define the principle of inclusion and exclusion. Determine the number of integers between 1 and 300 that are divisible by any of the integers 2, 3, 5 and 7. **(2+6)**

b. Define bijective mapping. Let f: R → R be defined as f(x) = 3x – 7. Show that (i) f is one-one and onto (ii) Find a formula for inverse f^{-1}: R → R. **(3+3+2)**

Q.3 a. (i) Determine the validity of the following argument:
"If wages increase, there will be inflation. The cost of living will not increase if there is no inflation. Wages will increase; therefore the cost of living will not increase". **(4)**
(ii) Show that $(p \wedge (\sim p \vee q)) \vee (q \wedge \sim (p \wedge q)) \equiv q$. **(4)**

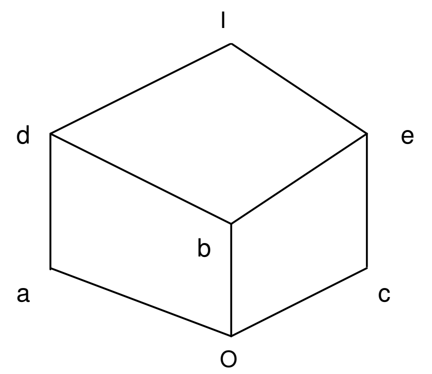
b. Show using the principle of mathematical induction that any positive integer greater than or equal to 2 is either prime or a product of prime. **(8)**

Q.4 a. Define composite relation. If R^{-1} and S^{-1} are the inverses of relation R and S respectively, then prove that $(S o R)^{-1} = R^{-1} o S^{-1}$. **(2+6)**

b. Let I be the set of integers and R be a binary relation defined on set I such that R = {(x, y)| x ≡ y (mod 3), x ϵ I, y ϵ I}.
(i) Show that R is an equivalence relation.
(ii) Find the quotient set (I/R) of I induced by R. **(4+4)**

Q.5 a. (i) Define Boolean algebra.
(ii) Let $(D_{63}, \leq)$ be a lattice of all positive divisors of 63 and $x \leq y$ means x divides y. Draw the Hasse diagram and prove or disprove the statement: $(D_{63}, \leq)$ is a Boolean algebra. **(4+4)**

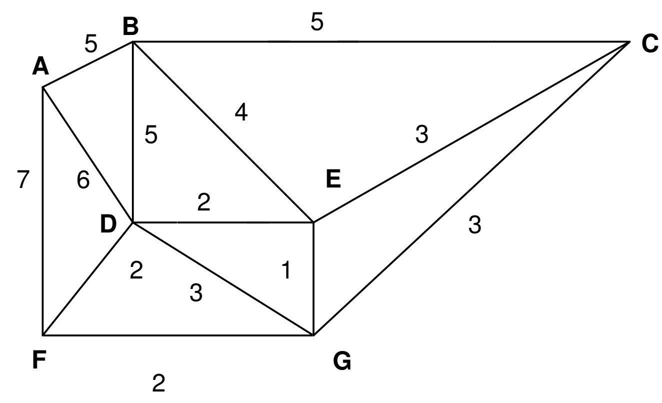
b. Consider the lattice L given below:



(i) Find all sub-lattices with 5-elements.
(ii) Find atoms.
(iii) Find complement of a and b if they exists.
(iv) Is L distributive? Complemented? **(2+2+2+2)**

Q.6 a. Prove or disprove:
(i) Every simple Euler graph with an even number of vertices has an even number of edges.
(ii) Peterson's graph is Hamiltonian. **(4+4)**

b. Apply Prim's algorithm to determine the minimal spanning tree in the given graph: **(8)**

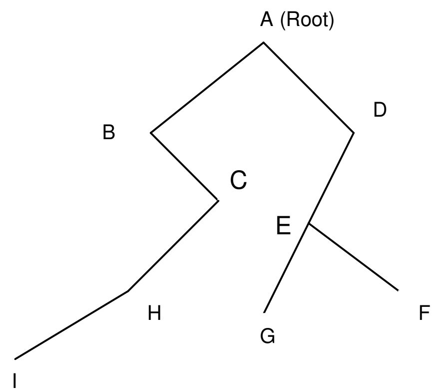


Q.7 a. In a shipment of 50 CDs 10 are defective. Determine
(i) In how many ways we can select 35 CDs.
(ii) In how many ways we can select 35 non-defective CDs.
(iii) In how many ways we can select 35 CDs containing exactly 5 defective CDs.

(iv) In how many ways we can select 35 CDs containing at least 5 defective CDs. **(2+2+2+2)**

b. Solve the difference equation $a_n - 6a_{n-1} + 9a_{n-2} = 3^n$, with the initial conditions $a_0 = 0$ and $a_1 = 1$. **(8)**

Q.8 a. Define binary tree. Write the pre-order, post-order and in-order traversal for the given tree. **(2+2+2+2)**



b. Following table gives the value of the function f(x, y, z). Find the corresponding function. Draw a simplified circuit diagram of the function. Also find the minterm normal form of f(x, y, z). **(8)**

x	y	z	f(x, y, z)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Q.9 a. Explain Chomsky's hierarchy. Give suitable example in each case. **(8)**

b. Design a deterministic finite state automaton that accepts all strings over {0, 1} starting with 01 and contains 110. **(8)**