**Code: AC09/AT09** **Subject: NUMERICAL COMPUTING**
**Time: 3 Hours** **Max. Marks: 100**

**DECEMBER 2010**

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after half an hour of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following: (2x10)**

a. The true value of a number is 6. It is approximated as 5.9994. Then, the relative error in the approximation is given by

**(A)** 0.0001 **(B)** 0.0006
**(C)** 0.0011 **(D)** –0.0006

b. The equation $x^4 + x - 6 = 0$ is given. Two approximations to the root are 1.2 and 1.4. The next approximation to the root obtained by the secant method is

**(A)** 1.527 **(B)** 1.477
**(C)** 1.6 **(D)** 1.567

c. Attempt is made to solve the system of equations $\begin{bmatrix} 1 & 5 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 18 \end{bmatrix}$, by Gauss-Jacobi iterative method. Then, the

**(A)** iteration diverges
**(B)** rate of convergence is 0.698
**(C)** rate of convergence is 0.898
**(D)** rate of convergence is 1.07

d. The matrix $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ is to be reduced to the diagonal form to find its eigen values, by the Jacobi method. The angle of the *first* orthogonal rotation is given by

**(A)** $\pi/3$. **(B)** $\pi/2$.
**(C)** $\tan^{-1}(\sqrt{5})$. **(D)** $-\pi/4$.

e. Let $P(x) = 2x^3 + 3x^2 - 5x + 6$. Then, $\nabla^3 P(x)$, where $\nabla$ is the back difference operator, with $h = 1$, is given by

**(A)** 24 **(B)** 18
**(C)** 16 **(D)** 12.

f. A numerical differentiation formula for finding $f'(x)$ is written as

$$f'(x) = [f(x+h) + af(x) - f(x-h)]/(2h).$$

Then, the value of $a$ is

**(A)** 2 **(B)** −2
**(C)** 1 **(D)** 0

g. The integral $I = \int_{-1}^{1} \frac{dx}{9x^2+1}$ is evaluated by the Gauss-Legendre two point formula. The value of $I$ is given by

**(A)** $\sqrt{3}$ **(B)** 1.0
**(C)** 0.5 **(D)** 1/3

h. The initial value problem $y' = 3x + 2y^2$, $y(1) = 2$, is given. The approximation to $y(1.1)$ by using the Euler method is given by ( $h = 0.1$)

**(A)** 3.1 **(B)** 2.7
**(C)** 2.9 **(D)** 3.5

i. The following data is given.

| $x$ | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|
| $f(x)$ | 2 | 5 | 8 | 10 |

The least squares linear polynomial approximation to the above data is

**(A)** $27x + 0.5$ **(B)** $27x - 0.5$
**(C)** $25x + 1.5$ **(D)** $25x - 1.5$

j. The divided difference $f[x_0, x_1, x_2]$ is equal to

**(A)** $\nabla^2 f_2$ **(B)** $\nabla^2 f_2 / h^2$
**(C)** $\nabla^2 f_2 /(2h^2)$ **(D)** $\nabla^3 f_2 /(6h^2)$

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**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

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**Q.2** a. The polynomial equation $3x^3 + 8x^2 + 8x + 5 = 0$ is given **(8)**
(i) Locate the negative root of smallest magnitude, in an interval of length 1.
(ii) Using this interval, find another interval in which the root lies by performing two iterations of bisection method.
(iii) Taking any two approximations in the interval obtained in(ii), perform one iteration of the secant method to find an approximation to the root.

b. Determine the values $a$ and $b$ so that the order of the meth[...] $x_{n+1} = ax_n + \frac{bN}{x_n^2}$, for computing $N^{1/3}, N > 0$, becomes as high as possible. What is the order of the method? **(8)**

**Q.3** a. Find the solution of the system of equations **(8)**

$$\begin{bmatrix} 5 & 1 & 0 \\ 1 & 5 & 2 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 4 \\ 8.5 \end{bmatrix}$$

using two iterations of the Gauss-Jacobi iteration method. Assume the initial approximations as $(-1.6, 0.45, 1.55)$. Obtain the iteration matrix and hence find the rate of convergence.

b. Obtain the Choleski decomposition of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 7 & -3 \\ -1 & -3 & 2 \end{bmatrix}$.

Hence, find $A^{-1}$. **(8)**

**Q.4** a. Transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, to tridiagonal form using Given's method. Use exact arithmetic. Using Sturm's sequence, obtain two of the eigen values of $A$. **(8)**

b. Find the smallest eigen value of the matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$, using five iterations of the inverse power method. Assume, the corresponding eigen vector as $[-1, 0.8]^T$. **(8)**

**Q.5** a. Write the Newton's backward difference interpolation polynomial for a given data $(x_i, f_i)$, $i = 0, 1, 2, \ldots, n$. Hence, show that **(8)**

$$f'(x_n) = \frac{1}{h}[\nabla f_n + \frac{1}{2}\nabla^2 f_n + \frac{1}{3}\nabla^3 f_n + \frac{1}{4}\nabla^4 f_n + \ldots]$$

Hence, determine $f'(5)$, where $f(x)$ is given in the following table of values.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 13 | 35 | 75 | 139 |

b. A numerical differentiation formula for computing $f''(x_0)$ is given by **(8)**

$$f''(x_0) = \frac{1}{12h^2}[-f(x_0 - 2h) + 16f(x_0 - h) - 30f(x_0) + 16f(x_0 + h) - f(x_0 + 2h)]$$

Using Taylor series expansions, find the order of the formula and the leading term of the error. Hence, compute $f''(2)$ from the following table of values.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 1.00000 | 2.71828 | 7.38906 | 20.08554 | 54.59815 |

**Q.6** a. Find an interpolating polynomial that fits the data

| $x$ | 0 | 0.5 | 1.5 | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 2.875 | 15.625 | 31 | 91 |

Hence, interpolate at $x = 1$.

b. Fit the least squares quadratic polynomial approximation $f(x) = a + bx + cx^2$, for the data

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| $f(x)$ | $-1.0$ | $-0.592$ | $-0.136$ | 0.416 | 1.112 | 2.0 |

**(8)**

**Q.7** a. Compute $I = \int_0^1 \frac{x^3}{\sqrt{x(1-x)}}dx,$ using Gauss-Chebychev two point formula.

**(8)**

b. A generalized trapezoidal rule can be written as **(8)**

$$\int_0^h f(x)dx = \frac{h}{2}[f(0) + f(h)] + ph^2[f'(0) - f'(h)]$$

where $p$ is a constant. Find the value of $p$ such that the method is of as high order as possible. Deduce the composite rule for evaluating the integral $\int_0^{nh} f(x)dx.$

**Q.8** a. Find an approximation to $y(0.1)$ and $y(0.2)$ using a second order Runge-Kutta method for the initial value problem **(7)**

$$y' = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1, \quad \text{with} \quad h = 0.1$$

b. Use the Taylor series method of fourth order to approximate $y(0.2)$ and $y(0.4)$ for the initial value problem $y' = x + 5y,\ y(0) = 1,$ with $h = 0.2$ **(9)**

**Q.9** a. The following data for a function is given. **(6)**

| x | 1 | 2 | 4 |
|---|---|---|---|
| f(x) | 3 | 11 | 69 |

Interpolate the value of $f(3)$ by Lagrange interpolation. Estimate the error in interpolating this value.

b. The system of equations $x^2y + y^3 = 10, xy^2 - x^2 = 3,$ has a solution near $x =$ 0.8, $y = 2.2$. Perform one iteration of the Newton's method to obtain this solution.

**(6)**

c. If $\delta$ and $\nabla$ are the central and backward difference operators respectively, show that $\delta = \nabla(1-\nabla)^{-1/2}$. **(4)**