MATHEMATICS

(Three hours)

*Candidates are allowed additional 15 minutes for **only** reading the paper.*
They must NOT start writing during this time)

Section A - *Answer* **Question 1** *(compulsory) and* **five** *other questions.*

Section B & Section C - *Answer* **two** *questions from* ***either*** *Section B* ***or*** *Section C.*

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The number of marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and squared paper are provided.

Slide rule may be used.

SECTION A

Find x and y, if $x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. [3]

Find the equation of the straight line through origin and passing through the point of intersection of lines $x - 2y + 3 = 0$ and $3x + 5y + 6 = 0$. [3]

Find the equations of the normal to the ellipse $5x^2 + 3y^2 = 137$ at the point where the abscissa is 2. [3]

If $y = \frac{1 - \cos x}{1 - \cos x}$, find $\frac{dy}{dx}$. [3]

Evaluate $\int \frac{x^2}{(x^2 - 4)}\, dx$. [3]

Find the equation of tangents to the hyperbola $3x^2 - y^2 = 3$ which are perpendicular to the line $x - 3y = 2$. [3]

In a single throw of two dice, find the probability of getting a total of at most 9. [3]

This Paper consists of 7 printed pages and 1 blank page.

Turn over

(viii) If the standard deviation of the numbers 2, 3, 11 and x is 3½, find the value of x.

(ix) Find the value of x and y, given that $(x + iy)(2 - 3i) = 4 + i$.

(x) Solve the differential equation:

$$(x + 1)\frac{dy}{dx} - y = e^{3x}(x + 1)^2.$$

Question 2

(a) Prove that:

$$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2)$$

(b) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

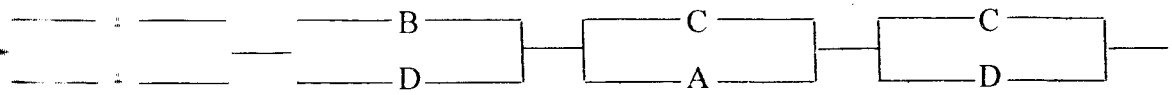
Question 3

(a) (i) Show that the second degree equation $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ represents a pair of straight lines.

(ii) Find the equation of the individual lines and their point of intersection.

[5]

Write down the Boolean expression corresponding to the switching circuit given below:-



Simplify the expression and construct the switching circuit for the simplified expression.

[5]

Solve for x : $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$.

[5]

Find $\frac{dy}{dx}$ if $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$.

[5]

Use Lagrange's mean value theorem to determine a point P on the curve $y = \sqrt{x-2}$ defined in the interval [2, 3] where the tangent is parallel to the chord joining the end points on the curve.

[5]

An open box with a square base is to be made out of a given quantity of cardboard whose area is c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

[5]

Evaluate $\int_0^5 f(x)\,dx$, *where* $f(x)$ *is defined by*

$$f(x) = \begin{cases} \sin x \,; if\ 0 \le x \le \frac{\pi}{2} \\ 1 \,; \frac{\pi}{2} \le x \le 5 \\ e^{x-5} \,; 5 \le x \le 9 \end{cases}$$

 sed by the curve and the line x = 2.

Question 7

The data for marks in Physics and History obtained by ten students are given below:-

Marks in Physics	15	12	8	8	7	7	7	6	5	3
Marks in History	10	25	17	11	13	17	20	13	9	15

Using this data:

(a) Compute Karl Pearson's coefficient of correlation between the marks in Physics and History obtained by the 10 students.

(b) (i) Find the line of regression in which Physics is taken as the independent variable.

(ii) A candidate had scored 10 marks in Physics but was absent from the History test. Estimate his probable score for the latter test.

Question 8

(a) There are 3 urns A, B and C. Urn A contains 4 red balls and 3 black balls. Urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and 1 black ball?

(b) The probability that a teacher will give an unannounced test during any class meeting is $\frac{1}{5}$. If a student is absent twice, find the probability that the student will miss at least one test.

Question 9

(a) If the ratio $\frac{z-i}{z-1}$ is purely imaginary, prove that the point z lies on the circle whose centre is the point $\frac{1}{2}(1+i)$ and radius is $\frac{1}{\sqrt{2}}$.

(b) Solve : $(x^2 + y^2)\ dx - 2xy\ dy = 0$, given that $y = 0$, when $x = 1$.

SECTION B

4. Find the coordinates of the point where the line joining the points (1, -2, 3) and (2, -1, 5) cuts the plane $x - 2y + 3z = 19$. Hence, find the distance of this point from the point (5, 4, 1). [5]

5. If A (-1,4,-3) is one end of a diameter AB of the sphere $x^2 + y^2 + z^2 - 2y + 2z - 15 = 0$ then find the coordinates of the other end point B. [5]

6. A small industrial concern used three raw materials A, B and C in its manufacturing process. The prices of the materials was as shown below:- [5]

Commodity	Price in Rs. in the year 1995	Price in Rs. in the year 2005
A	4	5
B	60	57
C	36	42

Using 1995 as the base year, calculate a simple aggregate price index for 2005.

7. Recorded monthly sales figures of a particular brand of T.V. for 18 months commencing January 1, 2005 are as follows:- [5]

YEAR	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
2005	18	16	23	27	28	19	31	29	35	27	28	24
2006	24	28	29	30	29	22						

Calculate six monthly moving averages and display these and the original figures on the same graph using the same axes for both.

Turn over

Question 12

(a) Find $\vec{a} \cdot \vec{b}$ if $|\vec{a}|=2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$.

(b) Given $\vec{a}=i-2j+k$, $\vec{b} = 2i+j+k$ and $\vec{c} = i+2j-k$
Find: $\vec{a} \times (\vec{b} \times \vec{c})$.

SECTION C

Question 13

(a) The banker's gain on a certain bill due 6 months hence is Rs. 100, the rate of interest being 10% per annum. Find the face value of the bill.

(b) Mr. Aggarwal buys a house at Rs. 30,00,000 for which he agrees to make equal payments at the end of each year for 10 years. If money is worth 10% p.a., find the amount of each instalment. [Take $(1.1)^{-10} = 0.3855$]

Question 14

(a) A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work atmost for 18 hours and 15 hours per day respectively. He earns a profit of Rs.30 per trunk on the first type of trunk and Rs.25 per trunk on the second type. Formulate a Linear Programming Problem to find out how many trunks of each type he must make each day to maximise his profit.

(b) The average cost function associated with producing and marketing x units of an item is given by $AC = 2x - 11 + \frac{50}{x}$. Find:

(i) The total cost function and marginal cost function.

(ii) The range of values of the output x, for which AC is increasing.

[redacted] 5

[5]

(i) Eight coins are thrown simultaneously.

Show that the probability of getting at least 6 heads is $\frac{37}{256}$.

What is the probability of getting at least 3 heads?

(ii) A class consists of 50 students out of which there are 10 girls. In the class 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be a rank holder, what is the probability that the student selected is a girl?

of interest

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