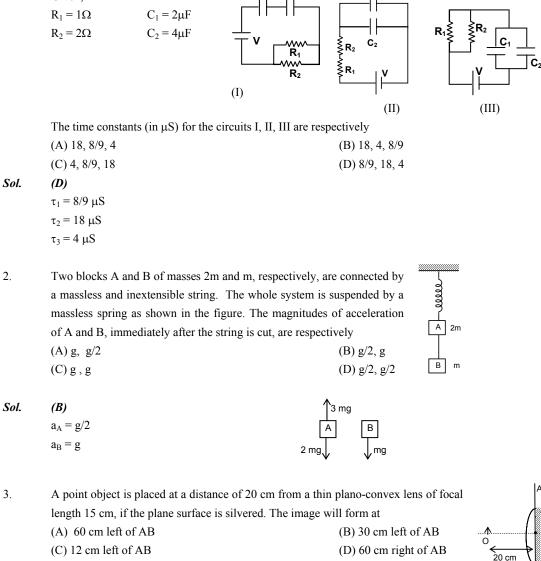
IIT-JEE2006-PH-1

Solutions to IITJEE-2006 **Physics**

Time: 2 hours

Note: The marking Scheme is (+3, -1) for question numbers 1 to 12, (+5, -1) for question numbers 13 to 20, (+5, -2) for question numbers 21 to 32 and (+6, 0) for question numbers 33 to 40.

```
1 Given,
```



Sol.

(C)

$$-\frac{1}{F} = \frac{2}{f_{\ell}} + \frac{1}{\infty} \Rightarrow F = -\frac{15}{2}$$

$$-\frac{2}{15} = \frac{1}{v} - \frac{1}{20}$$

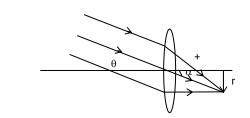
$$\Rightarrow v = -12 \text{ cm} \text{ i.e. 12 cm left of AB}$$

15 cm

IIT-JEE2006-PH-2

- 4. A biconvex lens of focal length f forms a circular image of sun of radius r in focal plane. Then (A) $\pi r^2 \propto f$ (B) $\pi r^2 \propto f^2$ (C) if lower half part is covered by black sheet, then area of the image is equal to $\pi r^2/2$ (D) if f is doubled, intensity will increase
- Sol. (B)

 $r = f \tan \alpha$ Hence, $\pi r^2 \propto f^2$



5. Given a sample of Radium-226 having half-life of 4 days. Find the probability, a nucleus disintegrates after 2 half lives.

(A) 1	(B)	1/2
(C) 1.5	(D)	3/4

Sol. (B)

Sol.

Disintegration of each nuclei is independent of any factor. Hence, each nuclei has same chance of disintegration.

6. Graph of position of image vs position of point object from a convex lens is shown. Then, focal length of the lens is
(A) 0.50 ± 0.05 cm
(B) 0.50 ± 0.10 cm
(C) 5.00 ± 0.05 cm

(D)
$$5.00 \pm 0.10$$
 cm

 $\Delta f = 0.15$

$$\begin{aligned} & (D) \\ & \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Longrightarrow f = 5 \text{ cm} \\ & f = \frac{uv}{u+v} \\ & \frac{\Delta f}{f} = \left|\frac{\Delta u}{u}\right| + \left|\frac{\Delta v}{v}\right| + \frac{|\Delta u| + |\Delta v|}{|u| + |v|} \end{aligned}$$

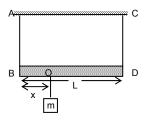
u cm -31 -30' -20 -10 0 (-9, +9)

The most appropriate answer is 5.00 ± 0.10 cm

(for f = 5 cm)

7. A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further, it is observed that the frequency of 1st harmonic (fundamental frequency) in AB is equal to 2nd harmonic frequency in CD. Then, length of BO is

(A) L/5	(B) 4L/5
(C) 3L/4	(D) L/4

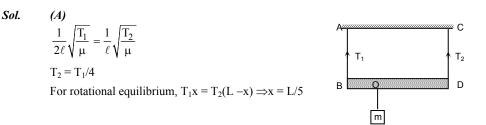


31

30

v cm

IIT-JEE2006-PH-3



8. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then

(A)
$$\frac{T_{A}}{T_{B}} = \left(\frac{r_{A}}{r_{B}}\right)^{3/2}$$
(B) $T_{A} > T_{B}$ (if $r_{A} > r_{B}$)
(C) $T_{A} > T_{B}$ (if $m_{A} > m_{B}$)
(D) $T_{A} = T_{B}$
(D) $T_{A} = T_{B}$
(E+3, -1]
(D)
 $\frac{Gm_{A}m_{B}}{(r_{A} + r_{B})^{2}} = \frac{m_{A}r_{A}4\pi^{2}}{T_{A}^{2}} = \frac{m_{B}r_{B}4\pi^{2}}{T_{B}^{2}}$
 $\Rightarrow m_{A}r_{A} = m_{B}r_{B}$
 $\therefore T_{A} = T_{B}$

9.

Sol.

A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

(A)
$$\frac{2R}{\sqrt{15}}$$
 (B) $R\sqrt{\frac{2}{15}}$
(C) $\frac{4R}{\sqrt{15}}$ (D) $\frac{R}{4}$ [+3, -1]

Sol.

$$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$$
$$r = \frac{2R}{\sqrt{15}}$$

10.

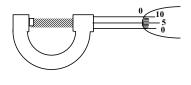
A student performs an experiment for determination of $g\left(=\frac{4\pi^2\ell}{T^2}\right)$, $\ell \approx 1$ m, and he commits an error of $\Delta\ell$.

For T he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1sec. For which of the following data, the measurement of g will be most accurate?

$\Delta \ell$	ΔT	n	Amplitude of oscillation
(A) 5 mm	0.2 sec	10	5 mm
(B) 5 mm	0.2 sec	20	5 mm
(C) 5 mm	0.1 sec	20	1 mm
(D) 1 mm	0.1 sec	50	1 mm
(D)			

Sol.

11. The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is
(A) 2.25 mm
(B) 2.20 mm
(C) 1.20 mm
(D) 1.25 mm



IIT-JEE2006-PH-4

Actual measurement

 $= 2 \times 0.5 \text{ mm} + 25 \times \frac{0.5}{50} - 0.05 \text{ mm}$

= 1 mm + 0.25 mm - 0.05 mm = 1.20 mm

Consider a cylindrical element as shown in the figure. Current flowing the through element is I and resistivity of material of the cylinder is ρ . Choose the correct option out the following.

- (A) Power loss in first half is four times the power loss in second half.
- (B) Voltage drop in first half is twice of voltage drop in second half.
- (C) Current density in both halves are equal.
- (D) Electric field in both halves is equal.

Sol.

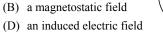
(B)

12

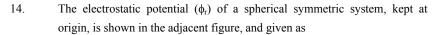
$$\frac{R_1}{R_2} = \frac{A_1}{A_2} = \frac{4}{1}$$
$$\frac{P_1}{P_2} = \frac{I^2 R_1}{I^2 R_2} = \frac{4}{1}$$
$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{4}{1}$$
$$\frac{J_1}{J_2} = \frac{1}{4}$$

More than One Choice may be correct.(+5, -1)

- 13. In the given diagram, a line of force of a particular force field is shown. Out of the following options, it can never represent
 - (A) an electrostatic field
 - (C) a gravitational field of a mass at rest



Sol. (A), (C)



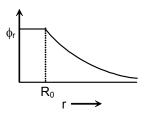
$$\begin{split} \phi_{r} &= \frac{q}{4\pi \in_{0} r} \qquad \left(r \geq R_{0} \right) \\ \phi_{r} &= \frac{q}{4\pi \in_{0} R_{0}} \qquad \left(r \leq R_{0} \right) \end{split}$$

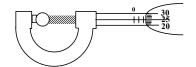
Which of the following option(s) is/are correct?

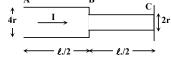
- (A) For spherical region $r \le R_0$, total electrostatic energy stored is zero.
- (B) Within $r = 2R_0$, total charge is q.
- (C) There will be no charge anywhere except at $r = R_0$.
- (D) Electric field is discontinuous at $r = R_0$.

Sol. (A), (B), (C), (D)

The potential shown is for charged spherical conductor.







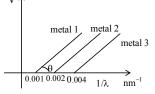
IIT-JEE2006-PH-5

15. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ . The coefficient of friction between the cylinder and incline is μ . Then (A) frictional force is always μ mg cos θ (B) friction is a dissipative force (C) by decreasing θ , frictional force decreases (D) friction opposes translation and supports rotation. Sol. (C), (D) Function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM 16. (B) if A = -B; C = 2B, amplitude = $B\sqrt{2}$ (A) for any value of A, B and C (except C = 0) (D) if A = B; C = 2B, amplitude = |B|(C) if A = B; C = 0(A), (B), (D) Sol. $x = \frac{A}{2}(1 - \cos 2\omega t) + \frac{B}{2}(1 + \cos 2\omega t) + \frac{C}{2}\sin 2\omega t$ For A = 0, B = 0 $x = \frac{C}{2} \sin 2\omega t$ For A = -B and C = 2B $x = B \cos 2\omega t + B \sin 2\omega t$ Amplitude = $|B\sqrt{2}|$ For A = B; C = 0x = A, Hence this is not correct option For A = B, C = 2B $x = B + B \sin 2\omega t$ It is also represents SHM.

- 17. In a dark room with ambient temperature T_0 , a black body is kept at a temperature T. Keeping the temperature of the black body constant (at T), sunrays are allowed to fall on the black body through a hole in the roof of the dark room. Assuming that there is no change in the ambient temperature of the room, which of the following statement(s) is/are correct?
 - (A) The quantity of radiation absorbed by the black body in unit time will increase.
 - (B) Since emissivity = absorptivity, hence the quantity of radiation emitted by black body in unit time will increase.
 - (C) Black body radiates more energy in unit time in the visible spectrum.
 - (D) The reflected energy in unit time by the black body remains same.

Sol. (A), (B), (C), (D)

18. The graph between 1/λ and stopping potential (V) of three metals having work functions φ₁, φ₂ and φ₃ in an experiment of photo-electric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct? [Here λ is the wavelength of the incident ray].



- (A) Ratio of work functions φ₁: φ₂: φ₃ = 1 : 2 : 4
 (B) Ratio of work functions φ₁: φ₂: φ₃ = 4 : 2 : 1
- (C) $\tan \theta$ is directly proportional to hc/e, where h is Planck's constant and c is the speed of light.
- (D) The violet colour light can eject photoelectrons from metals 2 and 3.

IIT-JEE2006-PH-6

Sol. (4), (C) $\frac{hc}{\lambda} - \phi = eV$ $V = \frac{hc}{e\lambda} - \frac{\phi}{c}$ For plate 1: plate 2 plate 3 $\frac{\phi_1}{hc} = 0.001 \quad \frac{\phi_2}{hc} = 0.002 \qquad \frac{\phi_3}{hc} = 0.004$ $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$ For plate 2, threshold wavelength $\lambda = \frac{hc}{\phi_2} = \frac{hc}{0.002 hc} = \frac{1000}{2} = 500 \text{ nm}$ For plate 3, threshold wavelength $\lambda = \frac{hc}{\phi_3} = \frac{hc}{0.004 hc} = \frac{1000}{4} = 250 \text{ nm}$

> Since violet colour light λ is 400 nm, so $\lambda_{violet} < \lambda_{threshold}$ for plate 2 So, violet colour light will eject photo-electrons from plate 2 and not from plate 3.

19. An infinite current carrying wire passes through point O and in perpendicular to the plane containing a current carrying loop ABCD as shown in the figure. Choose the correct option (s).(A) Net force on the loop is zero.

(A) Net loree on the loop is zero.

(B) Net torque on the loop is zero.

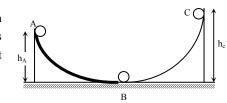
(C) As seen from O, the loop rotates clockwise.

(D) As seen from O, the loop rotates anticlockwise

Sol. (A), (C)

Magnetic force on wire BC would be perpendicular to the plane of the loop along the outward direction and on wire DA the magnetic force would be along the inward normal, so net force on the wire loop is zero and torque on the loop would be along the clockwise sense as seen from O.

20. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A , K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then (A) $h_A > h_C$; $K_B > K_C$

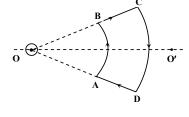


(D)
$$h_A < h_C$$
; $K_B > K_C$

Sol.

(C) $h_A = h_C$; $K_B = K_C$ (A), (B), (D) $E_A = mgh_A + K_A$ $E_B = K_B$ $E_c = mgh_C + K_C$ Using conservation of energy $E_A = E_B = E_C$ $K_B > K_C$ $K_B > K_A$ $Mg(h_A - h_C) + (K_A - K_C) = 0$ $\Rightarrow h_A - h_C = \frac{K_C - K_A}{Mg}$

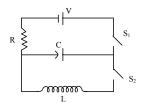
(B) $h_A > h_C$; $K_C > K_A$



IIT-JEE2006-PH-7

*Comprehension -I

The capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing switch S_1 while keeping switch S_2 open. The capacitor can be connected in series with an inductor 'L' by closing switch S_2 and opening S_1 .



- 21. Initially, the capacitor was uncharged. Now, switch S_1 is closed and S_2 is kept open. If time constant of this circuit is τ , then
 - (A) after time interval $\tau,$ charge on the capacitor is CV/2
 - (B) after time interval 2τ , charge on the capacitor is $CV(1-e^{-2})$
 - (C) the work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged.
 - (D) after time interval 2τ , charge on the capacitor is $CV(1-e^{-1})$

Sol. (B)

$$\begin{split} Q &= Q_0(1-e^{-t/\tau})\\ Q &= CV(1-e^{-t/\tau}) \text{ after time interval } 2\tau. \end{split}$$

22. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then,

(A) at t = 0, energy stored in the circuit is purely in the form of magnetic energy

- (B) at any time t > 0, current in the circuit is in the same direction
- (C) at t > 0, there is no exchange of energy between the inductor and capacitor

(D) at any time t > 0, instantaneous current in the circuit may $V_{\sqrt{\frac{C}{L}}}$

$$q = Q_0 \cos \omega t$$

$$i = -\frac{dq}{dt} = Q_0 \omega \sin \omega t$$

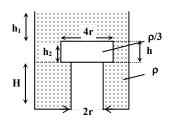
$$\Rightarrow i_{kax} = C\omega V = V \sqrt{\frac{C}{L}}$$

23. If the total charge stored in the LC circuit is Q₀, then for $t \ge 0$ (A) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} + \frac{t}{\sqrt{LC}}\right)$ (B) the charge on the capacitor is $Q = Q_0 \cos\left(\frac{\pi}{2} - \frac{t}{\sqrt{LC}}\right)$ (C) the charge on the capacitor is $Q = -LC \frac{d^2Q}{dt^2}$ (D) the charge on the capacitor is $Q = -\frac{1}{\sqrt{LC}} \frac{d^2Q}{dt^2}$ Sol. (C)

IIT-JEE2006-PH-8

Comprehension-II

A wooden cylinder of diameter 4r, height h and density $\rho/3$ is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H.



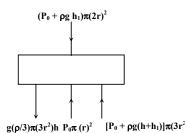
If level of liquid starts decreasing slowly when the level of liquid is at a height h₁ above the cylinder, the block just starts moving up. Then, value of h₁ is
(A) 2h/3
(B) 5h/4

(A)	2h/3	(B)	5h/4
(C)	5h/3	(D)	5h/2

Sol.

(C)

$$\begin{split} [P_0+\rho g h_1]\pi(4r^2) + \frac{\rho}{3}\pi 4r^2hg \ = [P_0+\rho g(h_1+h_2)]\pi(3r^2) + P_0\pi r^2 \\ h_1 = 5h/3 \end{split}$$

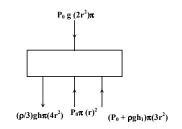


25. Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h₂ in figure) for which the cylinder remains in original position without application of force is

(A)	h/3	(B)	4h/9
(C)	2h/3	(D)	h

Sol. (B)

$$\begin{split} P_0\pi(4r^2) + \frac{\rho}{3}\pi 4r^2hg \ &= (P_0+\rho gh_2)\pi(3r^2) + P_0\pi r^2 \\ h_1 = 4h/9 \end{split}$$



26. If height h_2 of water level is further decreased, then

- (A) cylinder will not move up and remains at its original position.
- (B) for $h_2 = h/3$, cylinder again starts moving up
- (C) for $h_2 = h/4$, cylinder again starts moving up
- (D) for $h_2 = h/5$ cylinder again starts moving up
- Sol.

(A)

For $h_2 < 4h/9$ cylinder does not moves up

27. Two waves $y_1 = A \cos(0.5 \pi x - 100 \pi t)$

and $y_2 = A \cos(0.46 \pi x - 92 \pi t)$ are travelling in a pipe placed along x-axis. Find the number of times intensity is maximum in time interval of 1 sec.

(A) 4 (B) 6 (C) 8 (D) 10

IIT-JEE2006-PH-9

Sol.	(A)	
200	$ \mathbf{f}_1 - \mathbf{f}_2 = 4 \text{ s}^{-1}$	
28.	Find wave velocity of louder sound	
	(A) 100 m/s	(B) 192 m/s
	(C) 200 m/s	(D) 96 m/s
Sol.	(C)	
	$v_1 = v_2 = 200 \text{ m/s}$	
29.	Find the number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 sec	
	(A) 100	(B) 46
	(C) 192	(D) 96
Sol.	(D)	
	$y_1 + y_2 = A \cos 100\pi t + A \cos 92\pi t = 0$	
	$\cos 100\pi t = -\cos 92\pi t$	

$$100\pi t = (2n + 1)\pi - 92\pi t$$

$$t = \frac{(2n + 1)}{192}$$

$$\Delta t = t_{n+1} - t_n = \frac{2}{192}$$

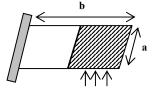
Questions 30-32 could not be retrieved due to large length of comprehension.

33. There is a rectangular plate of mass M kg of dimensions ($a \times b$). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v?

It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m; a = 1m; g = 10 m/s². Torque about hinge side

$$a \times \frac{b}{2}n(2 \text{ mv}) \times \frac{3b}{4} = Mg\frac{b}{2}$$

 $v = \frac{2}{3}\frac{Mg}{abnm} = \frac{2}{3} \times \frac{M \times 10}{2 \times 100 \times 0.01} = 10 \text{ m/s}$



34. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Then, find the final temperature of the mixture.

 $Given, \ L_{fusion} = 80 \ cal/g = 336 \ J/g, \ L_{vaporization} = 540 \ cal/g = 2268 \ J/g, \\ S_{ice} = 2100 \ J/kg \ K = 0.5 \ cal/gK \ and \ S_{water} = 4200 \ J/kg \ K = 1 \ cal/gK$

Sol. $\sum \Delta Q = 0$

Sol.

Heat lost by steam to convert into 0°C water $H_L = 0.05 \times 540 + 0.05 \times 10 \times 1$

= 27 + 5 = 32 kcal

Heat required by ice to change into 0°C water

$$H_g = 0.45 \times \frac{1}{2} \times 20 + 0.45 \times 80 = 4.5 + 36.00 = 40.5$$
 kcal

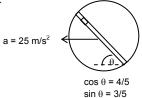
Thus, final temperature of mixture is 0°C.

IIT-JEE2006-PH-10

35. In hydrogen-like atom (z = 11), nth line of Lyman series has wavelength λ equal to the de-Broglie's wavelength of electron in the level from which it originated. What is the value of n?

$$\begin{aligned} \frac{1}{\lambda} &= Rz^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\ \frac{1}{\lambda} &= R(11)^2 \left(\frac{1}{1} - \frac{1}{n^2} \right) \\ \lambda &= \frac{h}{\rho} = \frac{h}{mv} \\ \lambda &= \frac{hr}{mvr} = \frac{rh2\pi}{nh} = \frac{2\pi r}{n} \\ \lambda &= \frac{2\pi r}{n} = \frac{\pi (0.529 \times 10^{-10})n^2}{(n)(11)} \\ \therefore &\frac{1}{\lambda} = \frac{11}{2\pi (0.529 \times 10^{-10})n} = \frac{11}{(2\pi)(0.529 \times 10^{-10})n} = 1.1 \times 10^7 (11)^2 \left(1 - \frac{1}{n^2} \right) \\ &= \frac{1}{(2\pi)(0.529 \times 10^{-10})(1.1 \times 10^2)(11)} = n - \frac{1}{n} \\ n - \frac{1}{n} = 25 \\ n^2 - 1 = 25 n \\ n^2 - 25 n - 1 = 0 \\ n = 25 \\ \text{Hence answer = 24} \end{aligned}$$

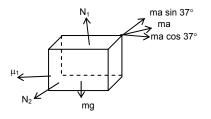
36. A circular disc with a groove along its diameter is placed horizontally. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove in contact is $\mu = 2/5$. The disc has an acceleration of 25 m/s². Find the acceleration of the block with respect to disc.



Sol.
$$N_1 = mg$$

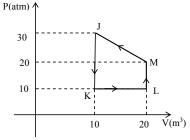
 $N_2 = m a \sin 37^\circ$
 $a_{bd} = \frac{ma \cos 37^\circ - \mu N_2 - \mu N_1}{m} = 10 m/s^2$.

Sol.



37. Heat given to process is positive, match the following option of column I with the corresponding option of column II

Column I	Column II
(A) JK	(P) $\Delta W > 0$
(B) KL	(Q) $\Delta Q < 0$
(C) LM	(R) $\Delta W < 0$
(D) MJ	(S) $\Delta Q > 0$



Sol. $(A) \rightarrow (Q), (B) \rightarrow (P), (S), (C) \rightarrow (S), (D) \rightarrow (Q), (R)$

IIT-JEE2006-PH-11

38. Match the following Columns

	Column I	Column II
(A)	Nuclear fusion	(P) Converts some matter into energy
(B)	Nuclear fission	(Q) Generally possible for nuclei with low atomic number
(C)	β-decay	(R) Generally possible for nuclei with higher atomic number
(D)	Exothermic nuclear reaction	(S) Essentially proceeds by weak nuclear forces

Sol. $(A) \rightarrow (P), (Q), (B) \rightarrow (P), (R), (C) \rightarrow (S), (P), (D) \rightarrow (P), (Q), (R)$

39 Match the following Columns

	Column I	Column II
(A)	Dielectric ring uniformly charged	(P) Time independent electrostatic field out of system
(B)	Dielectric ring uniformly charged rotating	(Q) Magnetic field
	with angular velocity ω	
(C)	Constant current in ring i ₀	(R) Induced electric field
(D)	$i = i_0 \cos \omega t$	(S) Magnetic moment

Sol. $(A) \rightarrow (P), (B) \rightarrow (Q), (S), (C) \rightarrow (Q), (S), (D) \rightarrow (Q), (R), (S)$

40. A simple telescope used to view distant objects has eyepiece and objective lens of focal lengths f_e and f_0 , respectively. Then

Column I	Column II
(A) Intensity of light received by lens	(P) Radius of aperture (R)
(B) Angular magnification	(Q) Dispersion of lens
(C) Length of telescope	(R) focal length f_0 , f_e
(D) Sharpness of image	(S) spherical aberration

Sol. $(A) \rightarrow (P), (B) \rightarrow (R), (C) \rightarrow (R), (D) \rightarrow (P), (Q), (S)$