PHYSICS - 1999

PART - A

Directions : *Select the most appropriate alternative a, b, c & d in questions 1-25*

1. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is :

(A) same everywhere (B) lower in front side

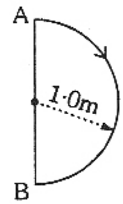
(C) lower in rear side (D) lower in upper side.

2. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300K is :

(A) $\sqrt{2/7}$ (B) $\sqrt{1/7}$

(C) $\sqrt{3}/5$ (D) $\sqrt{6}/5$

3. In 1·0S, a particle goes from point A to point B, moving in a semicircle (see figure). The magnitude of the average velocity is :



(A) 3·14 m/s (B) 2·0 m/s

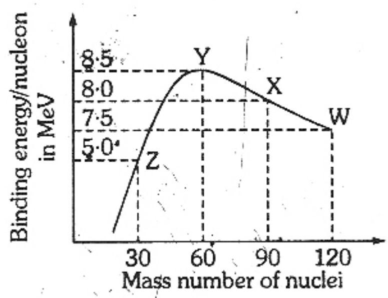
(C) 1·0 m/s (D) zero

4. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a :

(A) straight line (B) circle

(C) helix (D) cycloid

5. Binding energy per nucleon Vs mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is :



(A) $Y \rightarrow 2Z$

(B) $W \rightarrow X + Z$

(C) $W \rightarrow 2Y$

(D) $X \rightarrow Y + Z$

6. Order of magnitude of density of uranium nucleus is ($m_p = 1.67 \times 10^{-27}$ kg) :

(A) 10^{20} kg/m^3 (B) 10^{17} kg/m^3

(C) 10^{14} kg/m^3 (D) 10^{11} kg/m^3

7. Two identical circular loops of metal wire are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B :

(A) remains stastionery

(B) is attracted by the loop A

(C) is repelled by the loop A

(D) rotates about its CM, with CM fixed.

8. A spring of force constant K is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of :

(A) 2/3 K (B) 3/2 K

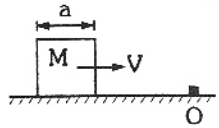
(C) 3 K (D) 6 K

9. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is :

(A) nitrogen (B) carbon

(C) boron (D) oxygen.

10. A cubical block of side a moving with velocity V on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :



(A) 3V/4a (B) 3V/2a

(C) $\sqrt{3}V/\sqrt{2}a$ (D) zero

11. Yellow light is used in a single slit diffraction experiment with slit width of 0·6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal :

(A) that the central maximum is narrower

(B) more number of fringes

(C) less number of fringes

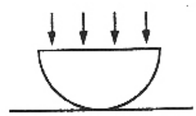
(D) no diffraction pattern

12. Two identical metal plates are given positive charges Q_1 and Q_2 ($< Q_1$) respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is :

(A) $(Q_1 + Q_2)/2C$ (B) $(Q_1 + Q_2)/C$

(C) $(Q_1 - Q_2)/C$ (D) $(Q_1 - Q_2)/2C$

13. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate as shown. The observed interference fringes from this combination shall be :



(A) straight

(B) circular

(C) equally spaced

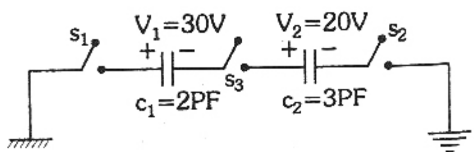
(D) having fringe spacing which increases as we go outwards.

14. A coil of inductance 8.4 mH and resistance 6Ω is connected to a 12V battery. The current in the coil is 1.0A at approximately the time :

(A) 500 s (B) 20 s

(C) 35 ms (D) 1 ms

15. For the circuit shown, which of the following statements is true :



(A) With S_1 closed,
$V_1 = 15V, V_2 = 20V$

(B) With S_3 closed,
$V_1 = V_2 = 25V$

(C) With S_1 and S_2 closed, $V_1 = V_2 = 0$

(D) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

16. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a :

(A) Convergent lens of focal length 3.5 R

(B) Convergent lens of focal length 3.0 R

(C) divergent lens of focal length 3.5 R

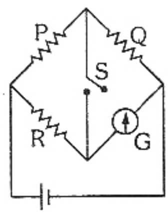
(D) divergent lens of focal length 3.0 R

17. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :

(A) 4 RT (B) 15 RT

(C) 9 RT (D) 11 RT

18. In the circuit shown $P \neq R$, the reading of galvonometer is same with switch S open or closed. Then :



(A) $I_R = I_G$

(B) $I_P = I_G$

(C) $I_Q = I_G$

(D) $I_Q = I_R$

19. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then :

(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$

(C) $\omega_A = \omega$ (D) $\omega_B = \omega$

20. In hydrogen spectrum the wavelength of H_α line is 656 nm; whereas in the spectrum of a distant galaxy H_α line wavelength is 706 nm. Estimated speed of galaxy with respect to earth is :

(A) 2×10^8 m/s (B) 2×10^7 m/s

(C) 2×10^6 m/s (D) 2×10^5 m/s

21. A particle free to move along the x-axis has potential energy given by $U(x) = K[1 - \exp(-x^2)]$ for $-\infty \leq x \leq +\infty$ where K is a positive constant of appropriate dimensions. Then :

(A) At points away from the origin, the particle is in unstable equilibrium

(B) For any finite non-zero value of x, there is a force directed away from the origin

(C) If its total mechanical energy is K/2, it has its minimum kinetic energy at the origin.

(D) For small displacements from x = 0, the motion is simple hormonic

22. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles λ_1/λ_2 is :

(A) m_1/m_2 (B) m_2/m_1

(C) 1.0 (D) $\sqrt{m_2}/\sqrt{m_1}$

23. A circular loop of radius R, carrying current I, lies in x-y plane with its centre at the origin. The total magnetic flux through x-y plane is :

(A) directly proportional to I (B) directly proportional to R

(C) directly proportional to R^2 (D) zero

24. Which of the following is a correct statement :

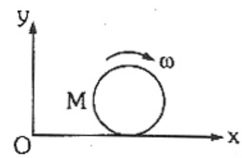
(A) Beta rays are same as cathode rays

(B) Gamma rays are high energy neutrons

(C) Alpha particles are singly ionized helium atoms

(D) Protons and neutrons have exactly the same mass.

25. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :



(A) $\left(\frac{1}{2}\right) MR^2\omega$ (B) $MR^2\omega$

(C) $\left(\frac{3}{2}\right) MR^2\omega$ (D) $2\ MR^2\omega$

Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answers. All correct answers must be marked to get any credit in these questions.*

26. The coordinates of a particle moving in a plane are given by x (t) = a cos(pt) and y (t) = b sin (pt) where a, b (< a) and p are positive constants of appropriate dimensions. Then :

(A) the path of the particle is an ellipse

(B) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$

(C) the acceleration of the particle is always directed towards a focus

(D) the distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a.

27. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially both of them have the same number of atoms. Then :

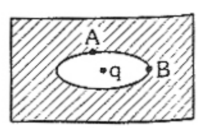
(A) X and Y have the same decay rate initially

(B) X and Y decay at the same rate always

(C) Y will decay at a faster rate than X

(D) X will decay at faster rate than Y

28. An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then :



(A) electric field near A in the cavity = electric field near B in the cavity

(B) charge density at A = charge density at B

(C) potential at A = potential at B

(D) total electric field flux through the surface of the cavity is q/ϵ_0.

29. Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by 45°, then :

(A) the resultant amplitude is $(1 + \sqrt{2})\, a$

(B) the phase of the resultant motion relative to the first is 90°

(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) the resulting motion is not simple harmonic

30. As a wave propagates :

(A) the wave intensity remains constant for a plane wave

(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times.

31. A bimetallic strip is formed out of two identical strips – one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is :

(A) proportional to ΔT

(B) inversely proportional to ΔT

(C) proportional to $|\alpha_B - \alpha_C|$

(D) inversely proportional to $|\alpha_B - \alpha_C|$

32. When a potential difference is applied across, the current passing through :

(A) an insulater at 0 K is zero

(B) a semiconductor at 0 K is zero

(C) a metal at 0 K is finite

(D) a p-n diode at 300 K is finite if it is reverse biased.

33. $Y(x, t) = \dfrac{0 \cdot 8}{[(4x + 5t)^2 + 5]}$ represents a moving pulse where x and y are in metres and t in second. Then :

(A) pulse is moving in positive x direction

(B) in 2s it will travel a distance of 2.5 m

(C) its maximum displacement is 0.16 m

(D) it is a symmetric pulse

34. In a wave motion $y = a \sin (Kx - \omega t)$, y can represent :

(A) electric field

(B) magnetic field

(C) displacement

(D) pressure.

35. Standing waves can be produced :

(A) on a string clamped at both ends

(B) on a string clamped at one end and free at the other

(C) when incident wave gets reflected from a wall

(D) when two identical waves with a phase difference of π are moving in the same direction.

ANSWERS

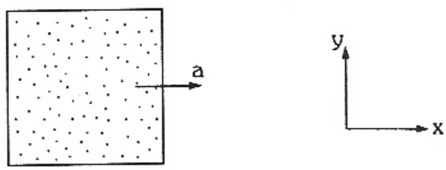
1. B,	**2.** C,	**3.** B,	**4.** A,	**5.** C,	**6.** B,
7. C,	**8.** B,	**9.** B,	**10.** A,	**11.** D,	**12.** D,
13. A,	**14.** D,	**15.** D,	**16.** A,	**17.** D,	**18.** A,
19. C,	**20.** B,	**21.** D,	**22.** C,	**23.** D,	**24.** A,
25. C,	**26.** A, B, C,	**27.** C,	**28.** C, D,	**29.** A, C,	**30.** A, C, D,
31. B, D,	**32.** A, B, D,	**33.** B, C, D,	**34.** A, B, C, D,		**35.** A, B, C.

SOLUTIONS

1. (B)

If a fluid (gas or liquid) is accelerated in positive x-direction, then pressure decreases in positive x-direction. Change in pressure has following differential equation—

$$\frac{dP}{dx} = -\rho a$$



where ρ is the density of the fluid. Therefore, pressure is lower in front side.

2. (C)

Speed of sound in an ideal gas is given by

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \quad V \propto \sqrt{\frac{\gamma}{M}} \qquad \text{[T is same for both the gases]}$$

$$\therefore \quad \frac{V_{N_2}}{V_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{H_e}} \cdot \frac{M_{H_e}}{M_{N_2}}}$$

$$= \sqrt{\frac{(7/5)}{(5/3)}\left(\frac{4}{28}\right)}$$

$$= \sqrt{3}/5$$

$\gamma_{N_2} = 7/5$ (Diatomic)

$\gamma_{He} = 5/3$ (Monoatomic)

3. (B)

$$|\text{ average velocity }| = \left|\frac{\text{Displacement}}{\text{time}}\right| = \frac{AB}{\text{time}} = \frac{2}{1} = 2\text{m/s}$$

4. (A)

The charged particle will be accelerated parallel (if it is a positive charge) or antiparallel (if it is a negative charge) to the electric field, i.e., the charged particle will move parallel or antiparallel to electric and magnetic field. Therefore net magnetic force on it will be zero and its path will be a straight line.

5. (C)

Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleons) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (C), this happens.

Given $W \to 2Y$

Binding energy of reactants $= 120 \times 7.5 = 900$ MeV

and binding energy of products $= 2(60 \times 8.5) = 1020 \text{ MeV} > 900$ MeV

6. (B)

Radius of a nucleus is given by

$$R = R_0 A^{1/3} \quad (\text{where } R_0 = 1.25 \times 10^{-15} \text{ m})$$
$$= 1.25\, A^{1/3} \times 10^{-15} \text{ m}$$

Here A is the mass number

and mass of the Uranium nucleus will be

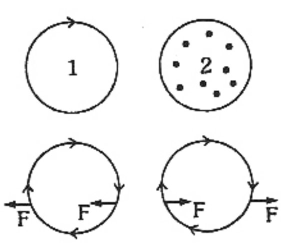
$$m \approx A m_p \quad m_p = \text{mass of proton}$$
$$= A(1.67 \times 10^{-27} \text{ Kg})$$

$$\therefore \quad \text{Density } \rho = \frac{\text{mass}}{\text{volume}}$$
$$= \frac{m}{\frac{4}{3}\pi R^3} = \frac{A(1.67 \times 10^{-27} \text{ kg})}{A(1.25 \times 10^{-15} \text{ m})^3}$$

or $\rho \approx 2.0 \times 10^{17} \text{ Kg/m}^3$

7. (C)

For understanding, let us assume that the two loops are lying in the plane of paper as shown. The current in loop 1 will produce • magnetic field in loop 2. Therefore, increase in current in loop 1 will produce an induced current in loop 2 which produces ⊗ magnetic field passing through it i.e. induced current in loop 2 will also be clockwise as shown alongside.

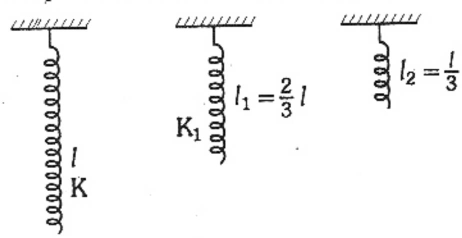


⊙ Perpendicular to paper outwards

⊗ Perpendicular to paper inwards

The loops will now repel each other as the currents at the nearest and farthest points of the two loops flow in the opposite directions.

8. (B)



$$l_1 = 2l_2$$
$$l_1 = \frac{2}{3} l$$

$$\text{Force constant } K \propto \frac{1}{\text{length of spring}}$$

$$K_1 = \frac{3}{2} K$$

9. **(B)**

Atomic number of Neon is 10.

By the emission of two α-particles, atomic number will be reduced by 4. Therefore, atomic number of the unknown element will be

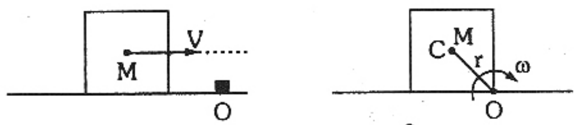
$$Z = 10 - 4$$
$$= 6$$

Similarly mass number of the unknown element will be

$$A = 22 - 2 \times 4$$
$$= 14$$

∴ Unknown nucleus is carbon (A = 14, Z = 6)

10. **(A)**



$$r = \sqrt{2}\,\frac{a}{2} \quad \text{or} \quad r^2 = \frac{a^2}{2}$$

Net torque about O is zero.

Therefore, angular momentum (L) about point O will be conserved

or $$L_i = L_f$$

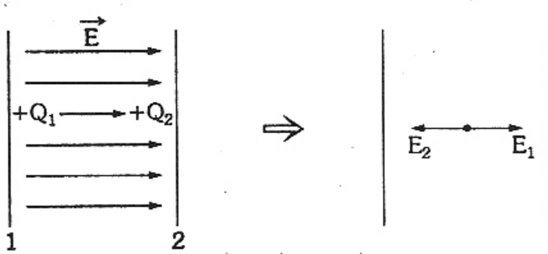
$$MV\left(\frac{a}{2}\right) = I_0\,\omega$$
$$= (I_{com} + Mr^2)\,\omega$$
$$= \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega$$
$$= \frac{2}{3}Ma^2\,\omega$$

∴ $$\omega = \frac{3V}{4a}$$

11. **(D)**

Diffraction is obtained when the slit width is of the order of wavelength of light (or any electromagnatic wave) used. Here wavelength of X-rays (1-100 Å)< < slit width (0.6 mm). Therefore no diffraction pattern will be observed.

12. **(D)**



Electric field within the plates $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$

or
$$E = E_1 - E_2$$
$$= \frac{Q_1}{2A \epsilon_0} - \frac{Q_2}{2A \epsilon_0}$$
$$E = \frac{Q_1 - Q_2}{2A \epsilon_0}$$

∴ Potential difference between the plates

$$V_A - V_B = E \cdot d = \left(\frac{Q_1 - Q_2}{2A \epsilon_0}\right) d$$
$$= \frac{Q_1 - Q_2}{2\left(\frac{A \epsilon_0}{d}\right)}$$
$$= \frac{Q_1 - Q_2}{2C}$$

13. (A)

Locus of equal path difference are the lines running parallel to the axis of the cylinder. Hence straight fringes are obtained.

⟶ *Circular rings (also called Newton's rings) are observed in interference pattern when a plano-convex lens of large focal length is placed with its convex surface in contact with a plane glass plate because locus of equal path difference in this case is a circle.*

14. (D)

The current-time (i–t) equation in L-R circuit is given by [Growth of current in L–R circuit]

$$i = i_0 (1 - e^{-t/\tau_L}) \qquad \ldots(1)$$

where $i_0 = \frac{V}{R} = \frac{12}{6} = 2\ A$

and $\tau_L = \frac{L}{R} = \frac{8.4 \times 10^{-3}}{6} = 1.4 \times 10^{-3}\ s$

and $i = 1A$ (given)

$t = ?$

Substituting these values in equation (1), we get

$$t = 0.97 \times 10^{-3}\ s$$

or $t = 0.97$ ms

$t \approx 1$ ms

15. (D)

When S_3 is closed, due to attraction with opposite charge, no flow of charge takes place through S_3. Therefore, potential difference across capacitor plates remains unchanged or $V_1 = 30$ V and $V_2 = 20$ V.

Alternate Solution

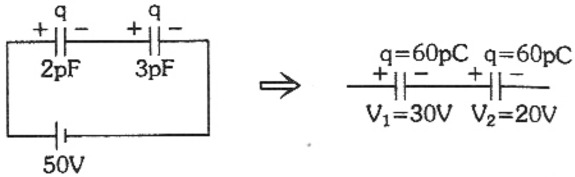
Charges on the capacitors are—

$$q_1 = (30)(2) = 60\ pC \text{ and}$$

$$q_2 = (20)(3) = 60 \text{ pC}$$

or $$q_1 = q_2 = q \text{ (say)}$$

The situation is similar as the two capacitors in series are first charged with a battery of emf 50V and then disconnceted.



$\therefore$ When S_3 is closed, $V_1 = 30$ V and $V_2 = 20$ V

16. (A)

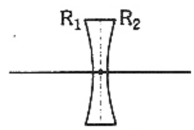
$$R_1 = -R,\ R_2 = +R,\ \mu_g = 1.5 \text{ and } \mu_m = 1.75$$

$\therefore$ $$\frac{1}{f} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Substituting the values, we have

$$\frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(\frac{1}{-R} - \frac{1}{R}\right)$$

$$= \frac{1}{3.5\,R}$$



$\therefore$ $$f = +3.5\,R$$

Therefore, in the medium it will behave like a convergent lens of focal length 3.5 R. It can be understood as, $\mu_m > \mu_g$, the lens will change its behaviour.

17. (D)

Internal energy of n moles of an ideal gas at temperature T is given by—

$$U = n\left(\frac{f}{2}RT\right)$$

where f = degrees of freedom.
= 5 for O_2 and 3 for Ar

Hence

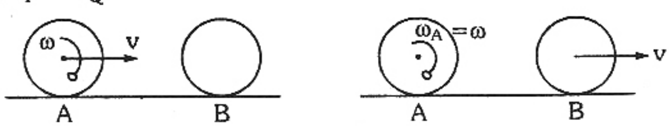
$$U = U_{O_2} + U_{Ar}$$

$$= 2\left(\frac{5}{2}RT\right) + 4\left(\frac{3}{2}RT\right) = 11\,RT$$

18. (A)

As there is no change in the reading of galvanometer with switch S open or closed. It implies that bridge is balanced. Current through S is zero and $I_R = I_G,\ I_P = I_Q$.

19. (C)



Before collision   After collision

Since it is head on elastic collision between two identical balls, they will exchange their linear velocities i.e. A comes to rest and B starts moving with linear velocity v. As there is no friction anywhere, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged. Therefore $\omega_A = \omega$ and $\omega_B = 0$.

20. (B)

Since the wavelength (λ) is increasing, we can say that the galaxy is receding. Doppler effect can be given by—

$$\lambda' = \lambda\sqrt{\frac{c+v}{c-v}} \qquad \ldots(1)$$

or $$706 = 656\,\frac{\sqrt{c+v}}{c-v}$$

or $$\frac{c+v}{c-v} = \left(\frac{706}{656}\right)^2 = 1.16$$

$\therefore$ $$c + v = 1.16\,c - 1.16\,v$$

$\therefore$ $$v = \frac{0.16\,c}{2.16}$$

$$= \frac{0.16 \times 3.0 \times 10^8}{2.16}\ \text{m/s}$$

$$= 0.22 \times 10^8\ \text{m/s}$$

$$v \approx 2.2 \times 10^7\ \text{m/s}$$

If we take the approximation then equation (1) can be written as—

$$\Delta\lambda = \lambda\left(\frac{v}{c}\right) \qquad \ldots(2)$$

From here $$v = \left(\frac{\Delta\lambda}{\lambda}\right).c$$

$$= \left(\frac{706 - 656}{656}\right)(3 \times 10^8)\ \text{m/s}$$

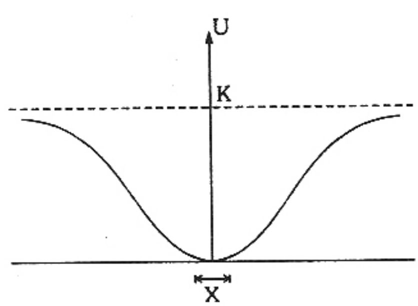
$$v = 0.23 \times 10^8\ \text{m/s}$$

which is almost equal to the previous answer. So we may use equation (2) also.

21. (D)

$$U(x) = K(1 - e^{-x^2})$$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore U versus x graph will be as shown



From the graph it is clear that at origin—

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is also zero because $F = \frac{-dU}{dx} = -(\text{slope of } U - x \text{ graph}) = 0$.

Therefore, origin is the stable

equilibrium position. Hence particle will oscillate simple harmonically about $x = 0$ for small displacements. Therefore, correct option is (D).

(A), (B) and (C) options are wrong due to following reasons.

(A) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e. slope of U–X graph should be zero and from the graph we can see that slope is zero at $x = 0$ and $x = \pm \infty$. Now among these equilibriums stable equilibrium position is that where U is minimum (Here $x = 0$). Unstable equilibrium position is that where U is maximum (Here none).
Neutral equilibrium position is that where U is constant (Here $x = \pm \infty$). Therefore, option (A) is wrong.

(B) For any finite non-zero value of x, force is directed towards the origin, because origin is in stable equilibrium position. Therefore, option (B) is incorrect.

(C) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, option (C) is also wrong.

22. (C)

From law of conservation of momentum,

$$P_1 = P_2 \quad \text{(in opposite directions)}$$

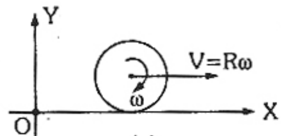
Now de-Broglie wavelength is given by

$$\lambda = \frac{h}{P} \quad h = \text{Planck's constant}$$

Since momentum (P) of both the particles is equal,

therefore $\lambda_1 = \lambda_2$

or $\lambda_1/\lambda_2 = 1$



(a)

23. (D)

Total magnetic flux passing through whole of the X-Y plane will be zero, because magnetic lines form a closed loop. So as many lines will move in –Z direction same will return to + Z direction from the X–Y plane.

24. (A)

Both the beta rays and the cathode rays are made up of electrons. So only option (A) is correct.

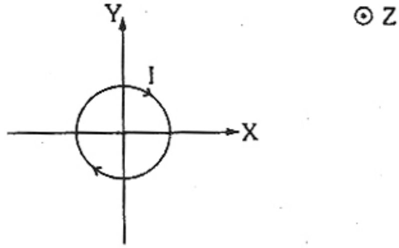
(B) Gamma rays are electromagnetic waves.

(C) Alpha particles are doubly ionized helium atoms, and

(D) Protons and Neutrons have approximately the same mass.

Therefore (B), (C) and (D) are wrong options.

25. (C)



From the theorem—

$$\vec{L}_0 = \vec{L}_{com} + M(\vec{r} \times \vec{V}) \quad \ldots(1)$$

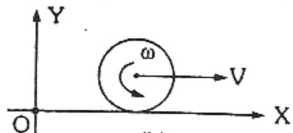
We may write

Angular momentum about O = Angular momentum about COM + Angular momentum of COM about origin

$\therefore \quad L_0 = I\omega + MRV$

$$= \frac{1}{2} MR^2 \omega + MR(R\omega)$$

$$= \frac{3}{2} MR^2 \omega$$



(b)

Note that in this case both the terms in equation (1) i.e. $\vec{L}_{com}$ and $M(\vec{r} \times \vec{V})$ have the same direction ⊗. That is why we have used $L_0 = I\omega + MRV$. We will use $L_0 = I\omega \sim MRV$ if they are in opposite directions as shown in figure (b).

26. (A, B, C)

$$x = a \cos pt \Rightarrow \cos(pt) = \frac{x}{a} \quad \ldots(1)$$

$$y = b \sin pt \Rightarrow \sin(pt) = \frac{y}{b} \quad \ldots(2)$$

Squaring and adding (1) and (2), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Therefore, path of the particle is in ellipse. Hence option (A) is correct.

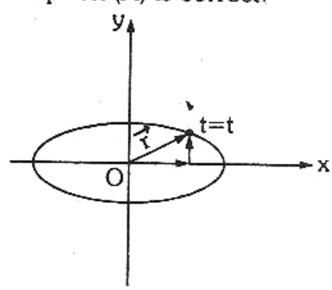
From the given equations we can find—

$$\frac{dx}{dt} = v_x = -a\,p \sin pt$$

$$\frac{d^2x}{dt^2} = a_x = -ap^2 \cos pt$$

$$\frac{dy}{dt} = v_y = bp \cos pt \text{ and}$$

$$\frac{d^2y}{dt^2} = a_y = -bp^2 \sin pt$$



At t = t, position of the particle

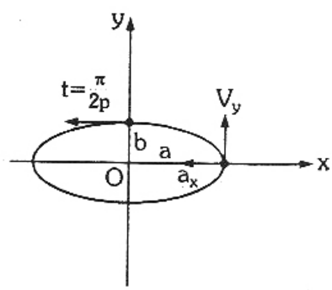
$$\vec{r}(t) = x\hat{i} + y\hat{j}$$

$$= a \cos pt\ \hat{i} + b \sin pt\ \hat{j}$$

At time $t = \pi/2p$ or $pt = \pi/2$

a_x and v_y become zero (because $\cos \pi/2 = 0$) only v_x and a_y are left, or we can say that velocity is along negative x-axis and acceleration along y-axis.

Hence at $t = \pi/2p$, velocity and acceleration of the particle are normal to each other. So option (B) is also correct.



and acceleration of the particle is

$$\vec{a}(t) = a_x \hat{i} + a_y \hat{j}$$

$$= -p^2 [a \cos pt\hat{i} + b \sin pt\ \hat{j}]$$

$$= -p^2 [x\hat{i} + y\hat{j}] = -p^2\, \vec{r}(t)$$

Therefore acceleration of the particle is always directed towards origin.

Hence option (C) is also correct.

$t = 0$

$y = 0 = v_x = a_y$

$x = a$

$v_y = bp$ and

$a_x = -ap^2$

At t = 0, particle is at (a,0) and at t = p/2p, particle is at (0, b). Therefore, the distance covered is one-fourth of the elliptical path not a. Hence option (D) is wrong.

27. (C)

$$(t_{1/2})_x = (t_{mean})_y$$

or
$$\frac{0.693}{\lambda_x} = \frac{1}{\lambda_y}$$

$\therefore$
$$\lambda_x = 0.693\,\lambda_y$$
$$\lambda_x < \lambda_y$$

or Rate of decay = λN

Initially number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, y will decay at a faster rate than x.

28. (C, D)

Under electrostatic condition, all points lying on the conductor are in same potential. Therefore, potential at A = potential at B. Hence option (C) is correct. From Gauss theorem, total flux through the surface of the cavity will be q/ϵ_0.

→ *Instead of an elliptical cavity, if it would had been a spherical cavity then options (A) and (B) were also correct.*

29. (A, C)

From superposition principle—

$$y = y_1 + y_2 + y_3$$
$$= a \sin \omega t + a \sin (\omega t + 45°) + a \sin (\omega t + 90°)$$
$$= a \{\sin \omega t + \sin (\omega t + 90°)\} + a \sin (\omega t + 45°)$$
$$= 2a \sin (\omega t + 45°) \cos 45° + a \sin (\omega t + 45°)$$
$$= (\sqrt{2} + 1)\, a \sin (\omega t + 45°)$$
$$= A \sin (\omega t + 45°)$$

Therefore, resultant motion is simple harmonic of amplitude

$$A = (\sqrt{2} + 1)\, a$$

and which differ in phase by 45° relative to the first.

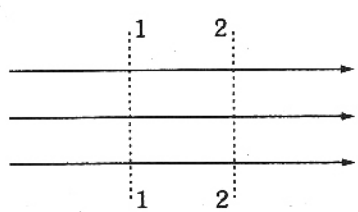
Energy in SHM $\propto$ (amplitude)^2 $\qquad E = \frac{1}{2} m A^2 \omega^2$

$$\therefore \quad \frac{E_{resultant}}{E_{single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2} + 1)^2 = (3 + 2\sqrt{2})$$

$$\therefore \quad E_{resultant} = (3 + 2\sqrt{2})\, E_{single}$$

30. (A, C, D)

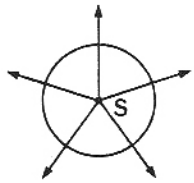
For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.

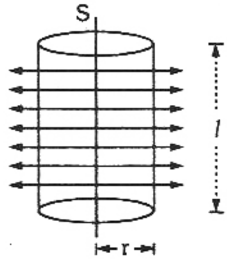


$I_{11} = I_{22}$

But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2} \quad \text{or} \quad I \propto \frac{1}{r^2}$$





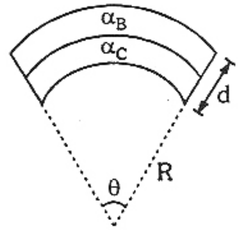
→ *For a line source* $I \propto \frac{1}{r}$

because $I = \frac{P}{\pi r l}$

31. (B, D)

Let l_0 be the initial length of each strip before heating.
Length after heating will be—

$$l_B = l_0 (1 + \alpha_B \Delta T) = (R + d)\,\theta \text{ and}$$
$$l_C = l_0 (1 + \alpha_C \Delta T) = R\,\theta$$



$$\therefore \quad \frac{R + d}{R} = \left(\frac{1 + \alpha_B \Delta T}{1 + \alpha_C \Delta T}\right)$$

$$\therefore \quad 1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\,\Delta T \qquad \text{[From binomial expansion]}$$

$$R = \frac{d}{(\alpha_B - \alpha_C)\,\Delta T}$$

or $\quad R \propto \frac{1}{\Delta T}$ and

$$\propto \frac{1}{|\alpha_B - \alpha_C|}$$

32. (A, B, D)

At 0 K, a semiconductor becomes a perfect insulator. Therefore at 0 K, if some potential difference is applied across an insulator or a semiconductor, current is zero. But a conductor will become a superconducter at 0 K. Therefore, current will be infinite. In reverse biasing at 300 K through a p-n junction diode, a small finite current flows due to minority charge carriers.

33. (B, C, D)

The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in figure (a)

$y(0, 0) = \frac{0.8}{5} = 0.16$ m

From the figure it is clear that $y_{max} = 0.16$ m

Pulse will be symmetric (Symmetry is checked about y_{max}) if

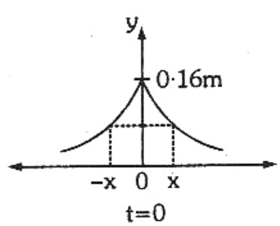
$$\text{At } t = 0;\ y(x) = y(-x)$$

From the given equation

$$\left.\begin{aligned} y(x) &= \frac{0.8}{16x^2 + 5} \\ \text{and} \quad y(-x) &= \frac{0.8}{16x^2 + 5} \end{aligned}\right\} \text{ at } t = 0$$

or $\quad y(x) = y(-x)$

Therefore pulse is symmetric.

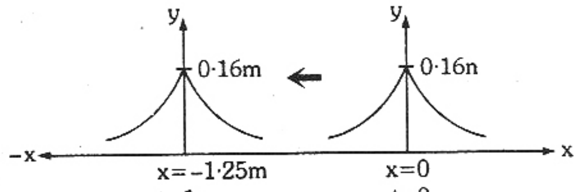


(a)

Speed of pulse →

At $t = 1s$ and $x = -1.25$ m

value of y is again 0.16 m. i.e. pulse has travelled a distance of 1.25 m in 1 second in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 seconds. The above statement can be better understood from figure (b).



(b)

Alternate method

If equation of a wave pulse is

$$y = f(ax \pm bt)$$

the speed of wave is $\frac{b}{a}$ in negative x direction for $y = f(ax + bt)$ and positive x direction for $y = f(ax - bt)$. Comparing this from given equation we can find that speed of wave is $5/4 = 1.25$ m/s and it is travelling in negative x-direction.

34. (A, B, C, D)

In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

→ *In general, y is any general physical quantity which is made to oscillate at one place and these oscillations are propagated to other places also.*

35. (A, B, C)

Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.

CHEMISTRY - 1999

PART - A

Directions : *Select the most appropriate alternative A, B, C or D in questions 1–25.*

1. The electrons, identified by quantum numbers n and l, (i) $n = 4, l = 1$, (ii) $n = 4, l = 0$, (iii) $n = 3, l = 2$, and (iv) $n = 3, l = 1$ can be placed in order of increasing energy, from the lowest to highest, as :

(A) (iv) < (ii) < (iii) < (i)
(B) (ii) < (iv) < (i) < (iii)
(C) (i) < (iii) < (ii) < (iv)
(D) (iii) < (i) < (iv) < (ii)

2. The number of neutrons accompanying the formation of $^{139}_{54}Xe$ and $^{94}_{38}Sr$ from the absorption of a slow neutron by $^{235}_{92}U$, followed by nuclear fission is :

(A) 0
(B) 2
(C) 1
(D) 3

3. The correct order of increasing C—O bond length of CO, CO_3^{2}, CO_2 is :

(A) $CO_3^{2-} < CO_2 < CO$
(B) $CO_2 < CO_3^{2-} < CO$
(C) $CO < CO_3^{2-} < CO_2$
(D) $CO < CO_2 < CO_3^{2-}$

4. A gas will approach ideal behaviour at :

(A) low temperature and low pressure
(B) low temperature and high pressure
(C) high temperature and low pressure
(D) high temperature and high pressure

5. The normality of 0.3 M phosphorus acid (H_3PO_3) is :

(A) 0.1
(B) 0.9
(C) 0.3
(D) 0.6

6. The coordination number of a metal crystallizing in a hexagonal close-packed structure is :

(A) 12
(B) 4
(C) 8
(D) 6

7. A gas X at 1 atm is bubbled through a solution containing a mixture of 1 M Y^- and 1 M Z^- at 25°C. If the reduction potential of Z > Y > X, then :

(A) Y will oxidize X and not Z
(B) Y will oxidize Z and not X
(C) Y will oxidize both X and Z
(D) Y will reduce both X and Z.

8. The pH of 0.1 M solution of the following salts increases in the order :

(A) $NaCl < NH_4Cl < NaCN < HCl$
(B) $HCl < NH_4Cl < NaCl < NaCN$
(C) $NaCN < NH_4Cl < NaCl < HCl$
(D) $HCl < NaCl < NaCN < NH_4Cl$

9. For the chemical reaction $3X(g) + Y(g) \rightleftharpoons X_3Y(g)$, the amount of X_3Y at equilibrium is affected by :

(A) temperature and pressure

(B) temperature only

(C) pressure only

(D) temperature, pressure and catalyst

10. In the dichromate dianion :

(A) 4 Cr—O bonds are equivalent

(B) 6 Cr—O bonds are equivalent

(C) all Cr—O bonds are equivalent

(D) all Cr—O bonds are nonequivalent

11. One mole of calcium phosphide on reaction with excess water gives :

(A) one mole of phosphine

(B) two moles of phosphoric acid

(C) two moles of phosphine

(D) one mole of phosphorus pentoxide

12. The oxidation number of sulphur in S_8, S_2F_2, H_2S respectively, are :

(A) 0, + 1 and − 2 (B) + 2, + 1 and − 2

(C) 0, + 1 and + 2 (D) − 2, + 1 and − 2

13. On heating ammonium dichromate, the gas evolved is :

(A) oxygen (B) ammonia

(C) nitrous oxide (D) nitrogen

14. In the commerical electrochemical process for aluminium extraction, the electrolyte used is :

(A) $Al(OH)_3$ in NaOH solution

(B) an aqueous solution of $Al_2(SO_4)_3$

(C) a molten mixture of Al_2O_3 and Na_3AlF_6

(D) a molten mixture of AlO(OH) and $Al(OH)_3$

15. The geometry of H_2S and its dipole moment are :

(A) angular and non-zero (B) angular and zero

(C) linear and non-zero (D) linear and zero

16. The geometry of $Ni(CO)_4$ and $Ni(PPh_3)_2Cl_2$ are :

(A) both square planar

(B) tetrahedral and square planar, respectively

(C) both tetrahedral

(D) square planar and tetrahedral, respectively

17. In compounds of type ECl_3, where E = B, P, As or Bi, the angles Cl—E—Cl for different E are in the order :

(A) $B > P = As = Bi$ (B) $B > P > As > Bi$

(C) $B < P = As = Bi$ (D) $B < P < As < Bi$

18. In the compound $CH_2=CH—CH_2—CH_2—C\equiv CH$, the C_2-C_3 bond is of the type :

(A) $sp-sp^2$ (B) sp^3-sp^3

(C) $sp-sp^3$ (D) sp^2-sp^3

19. When propionic acid is treated with aqueous sodium bicarbonate, CO_2 is liberated. The 'C' of CO_2 comes from :

(A) methyl group (B) carboxylic acid group

(C) methylene group (D) bicarbonate

20. The enol form of acetone, after treatment with D_2O, gives :

(A) $CH_3—C(OD)=CH_2$ (B) $CD_3—C(=O)—CD_3$

(C) $CH_2=C(OH)—CH_2D$ (D) $CD_3=C(OD)—CD_3$

21. A positive carbylamine test is given by :

(A) N, N–dimethylaniline (B) 2, 4-dimethylaniline

(C) N-methyl-o-methylaniline (D) *p*-methylbenzylamine

22. The optically active tartaric acid is named as D – (+) – tartaric acid because it has a positive :

(A) optical rotation and is derived from D-glucose

(B) pH in organic solvent

(C) optical rotation and is derived from D – (+) – glyceraldehyde

(D) optical rotation only when substituted by deuterium

23. A solution of (+) –2–chloro–2–phenylethane in toluene racemises slowly in the presence of small amount of $SbCl_5$, due to the formation of :

(A) carbanion (B) carbene

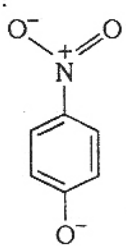
(C) free-radical (D) carbocation

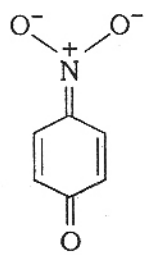
24. The product(s) obtained *via* oxymercuration ($HgSO_4 + H_2SO_4$) of 1-butyne would be :

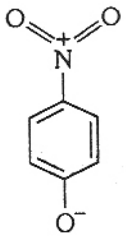
(A) $CH_3—CH_2—C(=O)—CH_3$ (B) $CH_3—CH_2—CH_2—CHO$

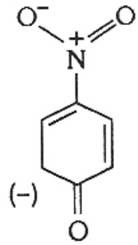
(C) $CH_3—CH_2—CHO + HCHO$ (D) $CH_3—CH_2—COOH + HCOOH$

25. The most unlikely representation of resonance structures of *p*-nitrophenoxide ion is :

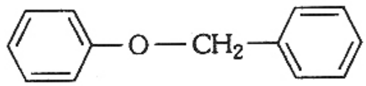
(A) 

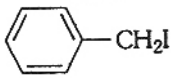
(B) 

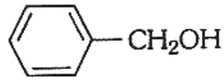
(C) 

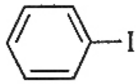
(D) 

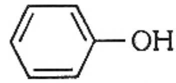
Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answer. All correct answers must be marked to get any credit in these questions.*

26. The ether  when treated with HI produces :

(A) 

(B) 

(C) 

(D) 

27. Toluene, when treated with Br_2/Fe, gives p-bromotoluene as the major product because the CH_3 group :

(A) is *para* directing

(B) is *meta* directing

(C) activates the ring by hyperconjugation

(D) deactivates the ring

28. The following statement(s) is (are) correct :

(A) A plot of log K_p *versus* $1/T$ is linear

(B) A plot of log [X] *versus* time is linear for a first order reaction, $X \rightarrow P$

(C) A plot of log p *versus* $1/T$ is linear at constant volume

(D) A plot of p *versus* $1/V$ is linear at constant temperature

29. The following is (are) endothermic reaction(s) :

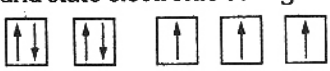
(A) Combustion of methane

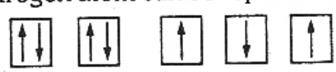
(B) Decomposition of water

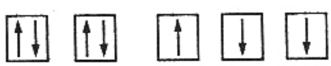
(C) Dehydrogenation of ethane to ethylene

(D) Conversion of graphite to diamond.

30. Ground state electronic configuration of nitrogen atom can be represented by :

(A) 

(B) 

(C) 

(D)

31. In the depression of freezing point experiment, it is found that the :

(A) vapour pressure of the solution is less than that of pure solvent

(B) vapour pressure of the solution is more than that of pure solvent

(C) only solute molecules solidify at the freezing point
(D) only solvent molecules solidify at the freezing point

32. Ionic radii of :
(A) $Ti^{4+} < Mn^{7+}$
(B) $^{35}Cl^- < {}^{37}Cl^-$
(C) $K^+ > Cl^-$
(D) $P^{3+} > P^{5+}$

33. Ammonia, on reaction with hypochlorite anion, can form :
(A) NO
(B) NH_4Cl
(C) N_2H_4
(D) HNO_2

34. A buffer solution can be prepared from a mixture of :
(A) sodium acetate and acetic acid in water
(B) sodium acetate and hydrochloric acid in water
(C) ammonia and ammonium chloride in water
(D) ammonia and sodium hydroxide in water

35. An aromatic molecule will :
(A) have $4n\ \pi$ electrons
(B) have $(4n + 2)\ \pi$ electrons
(C) be planar
(D) be cyclic

ANSWERS

1. (A)	**2.** (D)	**3.** (D)	**4.** (C)	**5.** (D)	**6.** (A)
7. (A)	**8.** (B)	**9.** (A)	**10.** (B)	**11.** (C)	**12.** (A)
13. (D)	**14.** (C)	**15.** (A)	**16.** (C)	**17.** (B)	**18.** (D)
19. (D)	**20.** (B)	**21.** (B), (D)	**22.** (C)	**23.** (D)	**24.** (A)
25. (C)	**26.** (A), (D)	**27.** (A), (D)	**28.** (A), (B), (D)	**29.** (B), (C) (D)	**30.** (A), (D)
31. (A), (D)	**32.** (D)	**33.** (C)	**34.** (A)	**35.** (B), (C) & (D)	

SOLUTIONS

Reason of Correctness

1. On the basis of $(n + l)$ Rule
(i) Value of $(n + l) = 4 + 1 = 5$
(ii) Value of $(n + l) = 4 + 0 = 4$
(iii) Value of $(n + l) = 3 + 2 = 5$
(iv) Value of $(n + l) = 3 + 1 = 4$

In these $(n + l)$ is lower for (ii) & (iv) but equal both, so in these n is minimum for (iv).
Hence energy order = (iv) < (ii)
Similaring in (i) & (iii)
(iii) < (i)

Hence correct order of energy (iv) < (ii) < (iii) < (i) **Ans. (A)**

2. $_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{139} + {}_{38}Sr^{94} + 3\,{}_0n^1$ **Ans. (D)**

3. Bond length $\propto \dfrac{1}{\text{Bond order}}$

Bond order $CO_3^{2-} < CO_2 < CO$

Bond order in CO = 3 (with the help of molecular orbital theory)

$$\text{Bond order in } CO_2 = \frac{\text{no. of bonds in all possible sides}}{\text{no. of resonating structure}} \quad \text{(By resonance)}$$

$$= \frac{4}{2} = 2$$

$$\text{Bond order in } CO_3^{2-} = \frac{4}{3} = 1.33 \quad \text{(By resonance)}$$

So order of bond length of C—O

$$CO < CO_2 < CO_3^{2-}$$ **Ans. (D)**

4. At higher temperature & low pressure **Ans. (C)**

5. H_3PO_3 is dibasic acid so its mole wt. = 2 × eq. wt.

$\therefore$ For it $1M = 2N$

Thus $0.3M = 0.6N$ **Ans. (D)**

6. Coordination number of a metal crystallizing in a hexagonal close-packing structure is (12). **Ans. (A)**

7. On the basis of reduction potential $(Z > Y > X)$

A spontaneous reaction will have the following characteristics :

Z reduced and X oxidised

Y reduced and X oxidised

Z reduced and Y oxidised

Hence Y will oxidise X and not Z. **Ans. (A)**

8. In these HCl stronger acid.

Aqueous NH_4Cl solution is slightly acidic (pH is lowest)

$$NH_4Cl + H_2O \rightleftharpoons NH_4OH + HCl \qquad (H^+) > (OH^-)$$

Aqueous NaCN solution is basic.

$$NaCN + H_2O \rightleftharpoons NaOH + HCN \qquad (OH^-) > (H^+)$$

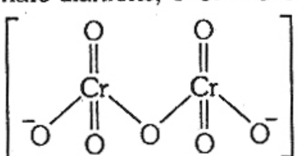
Aqueous NaCl solution is **neutral.**

Hence increasing order of pH.

$$HCl < NH_4Cl < NaCl < NaCN$$ **Ans. (B)**

9. Equilibrium is effected by temperature and pressure due to change in heat as well as change in volume of substances. **Ans. (A)**

10. In the dichromate dianions, 6 Cr – O bonds are equivalent.



(In $Cr_2O_7^{2-}$)

It shows the properties of resonance, so all six Cr-O bonds are equivalent and two bridged Cr-O bond are equivalent. **Ans. (B)**

11. $$Ca_3P_2 + 6H_2O \rightarrow 3Ca(OH)_2 + \underset{\text{two moles}}{2PH_3}$$ **Ans. (C)**

12. ON of S in $S_8 = 0$

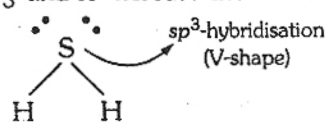
ON of S in $S_2F_2 = +1$

ON of S in $H_2S = -2$ **Ans. (A)**

13. $$(NH_4)_2Cr_2O_7 \rightarrow N_2 + Cr_2O_3 + 4H_2O$$ **Ans. (D)**

14. In it Na_3AlF_3 provides two functions. Hence it is used to decrease the melting point of Al_2O_3 and to increase the conductivity. **Ans. (C)**

15.

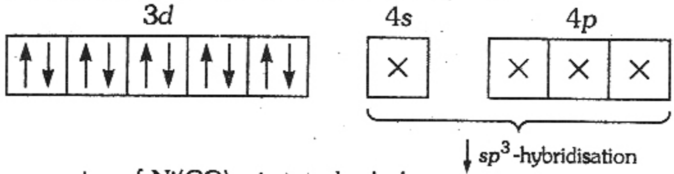


Hence angular geometry of non-zero value of dipole moment. **Ans. (A)**

16. In $Ni(CO)_4$ O.N. of Ni = 0

$$Ni = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^8, 4s^2$$

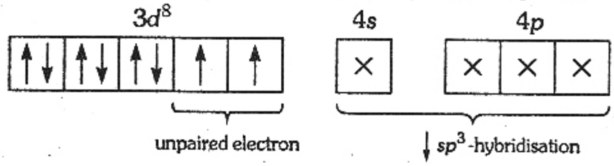
In excitation $Ni = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}$



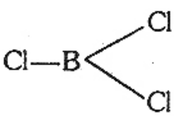
Hence geometry of $Ni(CO)_4$ is tetrahedral.

In $Ni(PPh_3)_2Cl_2$ ON of Ni = + 2 (Its coordination no. = 4)

$$Ni^{2+} = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^8$$

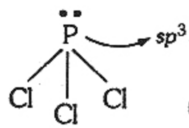


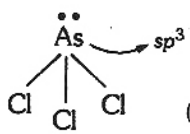
Hence geometry of $Ni(PPh_3)_2Cl_2$ is tetrahedral **Ans. (C)**

17. In BCl_3  (Trigonal geometry)

sp^2-hybridisation

Bond angle = 120°

In PCl_3  (Pyramidal geometry)

In $AsCl_3$  (Pyramidal geometry)

In $BiCl_3$  (Pyramidal geometry)

Bond angle = below 109°28′ and decreases from PCl_3 to $BiCl_3$

In these, order of bond angle $BCl_3 > PCl_3 > AsCl_3 > BiCl_3$ **Ans. (B)**

18. $\overset{1}{C}H_2 = \overset{2}{C}H - \overset{3}{C}H_2 - \overset{4}{C}H_2 - \overset{5}{C} \equiv \overset{6}{C}H$

(C_1: sp^2, C_2: sp^2, C_3: sp^3)

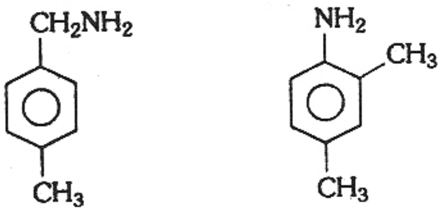
Hence C_2 and C_3 are sp^2 & sp^3-hybrid. **Ans. (D)**

19. $C_2H_5COOH + NaHCO_3 \rightarrow C_2H_5COONa + H_2O + CO_2$ **Ans. (D)**

20. $CH_3-\underset{\underset{O-H}{\|}}{C}=CH_2 \xrightarrow{D_2O} CD_3-\underset{\underset{OD}{|}}{C}=CD_2 \rightleftharpoons CD_3-\underset{\underset{O}{\|}}{C}-CD_3$ **Ans. (B)**

21. Carbylamine test is given by p-amines.

2, 4-Di methylaniline and *p*-methyl benzyl amine

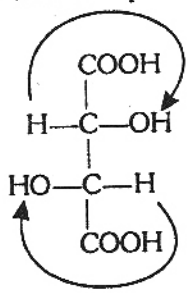


Ans. (B) & (D)

22. D-word is used to represent the arrangement of —OH group in right side as in glyceraldehyde.

$$H-\overset{|}{\underset{|}{C}}-OH$$

and + sign is used to represent the rotation in right side. Hence in D – (+) – tartaric acid



Hence it has a positive optical rotation and it is derived with glyceraldehyde.

Ans. (C)

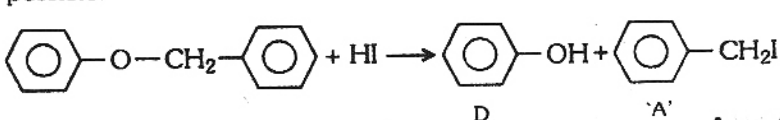
23. $SbCl_5$ is used for the formation of carbocation. **Ans. (D)**

24. $CH_3-CH_2-C\equiv CH + H_2O \xrightarrow[\text{Dil. } H_2SO_4]{HgSO_4} CH_3-CH_2-\underset{\underset{OH}{|}}{C}=CH_2$

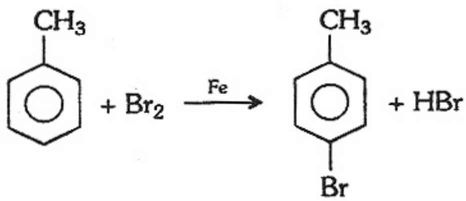
(Because keto form is more stable than enol)

$$CH_3-CH_2-\overset{\overset{O}{\|}}{C}-CH_3$$ **Ans. (A)**

25. In structure 'C' N-atoms five bonds and (+) charged, so the structure is not possible. **Ans. (C)**

26. 

Ans. (A) & (D)

27. 

—CH_3 group is able to activate the benzene ring by hyperconjugation. So —CH_3 group shows *o/p*-directing influence on benzene ring. **Ans. (A) & (D)**

28. The relevant expression is as follows :

(A) $\text{Log } K_P = -\frac{\Delta H}{R} \cdot \frac{1}{T} + I$

(B) $\text{Log}(X) = \text{Log}(X_0)_0 + kt$

(C) $\frac{P}{t}$ = constant (at V-constant)

(d) PV = constant (at T- constant) **Ans. (A), (B) & (D) are correct.**

29. Ans. (B), (C) & (D)

30. Ans. (A) & (D) (By Hund's Rule)

31. Ans. (A) & (D)

32. Longer the (+) charge, lower will be radii. ∴ **Ans. (D)**

33. $2NH_3 + OCl^- \rightarrow NH_2 .NH_2 + H_2O + Cl^-$ **Ans. (C)**

34. (A) & (C) are correct because a buffer solution is prepared by mixing a weak acid/base with salt of its conjugate base/acid. **Ans. (A) & (C)**

35. An aromatic will have :

(B) $(4n + 2)\pi$ electrons (by Huckel's Rule)
(C) planar structure (due to resonance)
(D) cyclic strueture (due to presence of sp^2-hybrid carbon atoms).

Ans. (B), (C) & (D)

MATHEMATICS - 1999

PART - A

Directions : *Select the most appropriate alternative A, B, C or D in questions 1–25.*

1. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to :

(A) $1 - i\sqrt{3}$
(B) $-1 + i\sqrt{3}$
(C) $i\sqrt{3}$
(D) $-i\sqrt{3}$

2. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G. P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) :

(A) lie on a straight line
(B) lie on an ellipse
(C) lie on a circle
(D) are vertices of a triangle

3. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is :

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$
(B) $\frac{1}{2}(1 + \sqrt{1 + 4 \log_2 x})$
(C) $\frac{1}{2}(1 - \sqrt{1 + 4 \log_2 x})$
(D) not defined

4. The harmonic mean of the roots of the equation

$$(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0 \text{ is :}$$

(A) 2
(B) 4
(C) 6
(D) 8

5. The function $f(x) = \sin^4 x + \cos^4 x$ increases if :

(A) $0 < x < \frac{\pi}{8}$
(B) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
(C) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$
(D) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

6. The curve described parametrically by $x = t^2 + t + 1, y = t^2 - t + 1$ represents :

(A) a pair of straight lines
(B) an ellipse
(C) a parabola
(D) a hyperbola

7. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$, then :

(A) $a + b = c$
(B) $b + c = a$
(C) $a + c = b$
(D) $b = c$

8. If for a real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2 \sin x]\, dx$ is :

(A) $-\pi$
(B) 0
(C) $-\frac{\pi}{2}$
(D) $\frac{\pi}{2}$

9. Let $a_1, a_2, \ldots\ldots, a_{10}$ be in A. P. and $h_1, h_2, \ldots\ldots, h_{10}$ be in H. P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is :

(A) 2
(B) 3
(C) 5
(D) 6

10. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30°, then $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| =$

(A) $\frac{2}{3}$
(B) $\frac{3}{2}$
(C) 2
(D) 3

11. The number of real solutions of

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2} \text{ is :}$$

(A) zero
(B) one
(C) two
(D) infinite.

12. Let $P(a \sec\theta, b \tan\theta)$ and $Q(a \sec\phi, b \tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q, then k is equal to :

(A) $\frac{a^2 + b^2}{a}$
(B) $-\left(\frac{a^2 + b^2}{a}\right)$
(C) $\frac{a^2 + b^2}{b}$
(D) $-\left(\frac{a^2 + b^2}{b}\right)$

13. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is :

(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
(B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
(D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

14. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to :

(A) 0
(B) 1
(C) 100
(D) −100

15. The function $f(x) = [x]^2 - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at :

(A) all integers
(B) all integers except 0 and 1
(C) all integers except 0
(D) all integers except 1

16. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then :

(A) $p^2 = q^2$
(B) $p^2 = 8q^2$
(C) $p^2 < 8q^2$
(D) $p^2 > 8q^2$

17. The function $f(x) = (x^2 - 1)\left|x^2 - 3x + 2\right| + \cos(|x|)$ is NOT differentiable at :

(A) −1
(B) 0
(C) 1
(D) 2

18. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then :

(A) $a < 2$
(B) $2 \le a \le 3$
(C) $3 < a \le 4$
(D) $a > 4$

19. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is :

(A) $y = 2$
(B) $y = 2x$
(C) $y = 2x - 4$
(D) $y = 2x^2 - 4$

20. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ is :

(A) 2
(B) −2
(C) $\frac{1}{2}$
(D) $-\frac{1}{2}$

21. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c} =$

(A) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$
(B) $\frac{1}{\sqrt{3}}(-\hat{i} - \hat{j} - \hat{k})$
(C) $\frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$
(D) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

22. If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, then m is :

(A) 6 (B) 9

(C) 12 (D) 24

23. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ is equal to :

(A) 2 (B) -2

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

24. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is :

(A) $9x^2 - 8y^2 + 18x - 9 = 0$

(B) $9x^2 - 8y^2 - 18x + 9 = 0$

(C) $9x^2 - 8y^2 - 18x - 9 = 0$

(D) $9x^2 - 8y^2 + 18x + 9 = 0$

25. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals :

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$

(C) $\frac{1}{8}$ (D) $\frac{1}{49}$

Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answers. All correct answers must be marked to get any credit in these questions :*

26. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ?

(A) $x + y = 0$ (B) $x - y = 0$

(C) $x + 7y = 0$ (D) $x - 7y = 0$

27. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $|\vec{v}|$ is :

(A) $|\vec{u}|$ (B) $|\vec{u}| + |\vec{u} \cdot \vec{a}|$

(C) $|\vec{u}| + |\vec{u} \cdot \vec{b}|$ (D) $|\vec{u}| + \vec{u} \cdot (\vec{a} + \vec{b})$

28. For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots\dots\dots + \frac{1}{(2^n) - 1}$. Then :

(A) $a(100) \leq 100$ (B) $a(100) > (100)$

(C) $a(200) \leq 100$ (D) $a(200) > 100$

29. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5 \, dt$ has a local minimum at $x =$

(A) 0 (B) 1

(C) 2 (D) 3

30. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are :

(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

31. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two, and a 40% chance of passing in exactly two. Which of the following relations are true ?

(A) $p + m + c = \frac{19}{20}$ (B) $p + m + c = \frac{27}{20}$

(C) $pmc = \frac{1}{10}$ (D) $pmc = \frac{1}{4}$

32. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of :

(A) order 1 (B) order 2

(C) degree 3 (D) degree 4

33. Let $S_1, S_2 \dots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1sq. cm?

(A) 7 (B) 8

(C) 9 (D) 10

34. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$?

(A) –4 (B) –2

(C) 2 (D) 4

35. For a positive integer n, let

$$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots(1+\sec 2^n\theta).$$ Then

(A) $f_2\left(\frac{\pi}{16}\right) = 1$

(B) $f_3\left(\frac{\pi}{32}\right) = 1$

(C) $f_4\left(\frac{\pi}{64}\right) = 1$

(D) $f_5\left(\frac{\pi}{128}\right) = 1$

ANSWERS

1. (C)	**2.** (A)	**3.** (B)	**4.** (B)	**5.** (B)	**6.** (C)
7. (A)	**8.** (C)	**9.** (D)	**10.** (B)	**11.** (C)	**12.** (D)
13. (B)	**14.** (A)	**15.** (B)	**16.** (D)	**17.** (D)	**18.** (A)
19. (C)	**20.** (C)	**21.** (A)	**22.** (C)	**23.** (A)	**24.** (B)
25. (A)	**26.** (A), (C)	**27.** (B), (C)	**28.** (A), (D)	**29.** (B), (D)	**30.** (B), (D)
31. (B),	**32.** (A), (C)	**33.** (B), (C), (D)	**34.** (B), (D)	**35.** (A), (B), (C), (D).	

SOLUTIONS

1. **Imp. note :** If in a complex no. $a + ib$, the ratio $a : b$ is $1 : \sqrt{3}$ then always try to convert that complex no. in ω.

Here $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Therefore, $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$

$= 4 + 5\omega^{334} + 3\omega^{365}$

$= 4 + 5\cdot(\omega^3)^{111}\cdot\omega + 3\cdot(\omega^3)^{123}\cdot\omega^2$

$= 4 + 5\omega + 3\omega^2 \qquad \because \omega^3 = 1$

$= 1 + 3 + 2\omega + 3\omega + 3\omega^2$

$= 1 + 2\omega + 3(1 + \omega + \omega^2) = 1 + 2\omega + 3 \times 0 \qquad \because 1 + \omega + \omega^2 = 0$

$= 1 + (-1 + \sqrt{3}i) = \sqrt{3}i$ Therefore, (C) is the answer.

2. Let $\frac{x_2}{x_1} = \frac{x_3}{x_2} = r$ and $\frac{y_2}{y_1} = \frac{y_3}{y_2} = r$

$\Rightarrow x_2 = x_1 r,\ x_3 = x_1 r^2$ and $y_2 = y_1 r$ and $y_3 = yr^2$.

again, $\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 r & y_1 r & 1 \\ x_1 r^2 & y_1 r^2 & 1 \end{vmatrix}$

Using $R_3 \to R_3 - rR_2$ and $R_2 \to R_2 - rR_1$ $\because$ 3, one's $\Rightarrow$ 2, zero's $\Rightarrow 20^s$

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ 0 & 0 & 1-r \\ 0 & 0 & 1-r \end{vmatrix} = 0 \quad \because \ R_2 \text{ and } R_3 \text{ are identicale}$$

Thus $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ lie on a straight line.
Therefore, (A) is answer.

3. Let $y = 2^{x(x-1)}$ where $y \geq 1$ as $x \geq 1$.

taking $\log_2$ of both side

$$\log_2 y = \log_2 2^{x(x-1)}$$

$$\Rightarrow \quad \log_2 y = x(x-1) \qquad \because \quad \log_a a^x = x$$

$$\Rightarrow \quad x^2 - x - \log_2 y = 0$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1 + 4\log_2 y}}{2}$$

For $y \geq 1, \log_2 y \geq 0 \Rightarrow 4\log_2 y \geq 0 \Rightarrow 1 + 4\log_2 y \geq 1$

$$\Rightarrow \quad \sqrt{1 + 4\log_2 y} \geq 1$$

$$\Rightarrow \quad -\sqrt{1 + 4\log_2 y} \leq -1$$

$$\Rightarrow \quad 1 - \sqrt{1 + 4\log_2 y} \leq 0$$

but $\quad x \geq 1$

so $\quad x = 1 - \sqrt{1 + 4\log_2 y}$ is not possible.

therefore $x = \frac{1}{2}(1 + \sqrt{1 + 4\log_2 y})$

$$\Rightarrow \quad f^{-1}(y) = \frac{1}{2}(1 + \sqrt{1 + 4\log_2 y}) \qquad \because x = f^{-1}(y)$$

$$\Rightarrow \quad f^{-1}(x) = \frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$$

Therefore, (B) is the Answer.

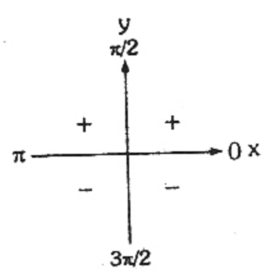
4. Let α, β be the root of given quardratic equation. Then

$$S = \alpha + \beta = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \qquad \text{and} \qquad \alpha\beta = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}$$

Again, H be the Harmonic mean between α and β, then

$$H = \frac{2\alpha\beta}{\alpha + \beta} = \frac{16 + 4\sqrt{5}}{4 + \sqrt{5}} = 4$$ Therefore, (B) is the answer.

5. $f(x) = \sin^4 x + \cos^4 x$



Differentiating w.r.t. x, we get

$$f'(x) = 4\sin^3 x \cdot \cos x - 4\cos^3 x \cdot \sin x$$
$$= 4\sin x \cos x(\sin^2 x - \cos^2 x)$$
$$= 2 \cdot \sin 2x(-\cos 2x)$$
$$= -\sin 4x$$

Now, $f'(x) > 0$ if $\sin 4x < 0$

$\Rightarrow \quad \pi < 4x < 2\pi$

$\Rightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$ ...(1) and (A) is wrong. $\therefore \quad 0 < x < 3\pi/8$

$\Rightarrow$ (A) is not proper subset of (1)

Again (B) is the answer since (B) is proper subset of (1)

Again (C), $\frac{3\pi}{8} < x < \frac{5\pi}{8}$, is not the answer because C is not proper subset of (1)

Again (D) is not answer.

6. $x = t^2 + t + 1$ ...(1)

$y = t^2 - t + 1$ ...(2)

> **Imp. note :** In this, direct substitution in terms of y or x of t is a typical method. So we will use here slight different way.

subtract (2) from (1)

$$x - y = 2t$$

Thus, $x = t^2 + t + 1$

$$\Rightarrow \quad x = \left(\frac{x-y}{2}\right)^2 + \left(\frac{x-y}{2}\right) + 1$$

$$\Rightarrow \quad 4x = (x-y)^2 + 2x - 2y + 4$$

$$\Rightarrow \quad (x-y)^2 = 2(x + y - 2)$$

$$\Rightarrow \quad x^2 + y^2 - 2xy = 2x + 2y - 4$$

$$\Rightarrow \quad x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$$

Now $\Delta = 1.1.4 + 2.(-1)(-1)(-1) - 1 \times (-1)^2 - 1 \times (-1)^2 - 4(-1)^2$

$= 4 - 2 - 1 - 1 - 4$

$= -4$ therefore $\Delta \neq 0$

and $ab - h^2 = 1.1 - (1)^2 = 1 - 1$

$= 0$ so it is equation of a parabola. therefore (C) is the answer.

7. It is given that tan $P/2$ and tan $Q/2$ are the roots of the quadratic equation $ax^2 + bx + c = 0$ and $\angle R = \pi/2$

so $\quad \tan P/2 + \tan Q/2 = -b/a$

$\quad \tan P/2 \tan Q/2 = c/a$

Now $\quad P + Q + R = 180°$

$\Rightarrow \quad P + Q = 90°$

$$\Rightarrow \quad \frac{P+Q}{2} = 45°$$

taking tan of both sides

$$\tan\left(\frac{P+Q}{2}\right) = \tan 45°$$

$$\Rightarrow \quad \frac{\tan P/2 + \tan Q/2}{1 - \tan P/2 \cdot \tan Q/2} = 1$$

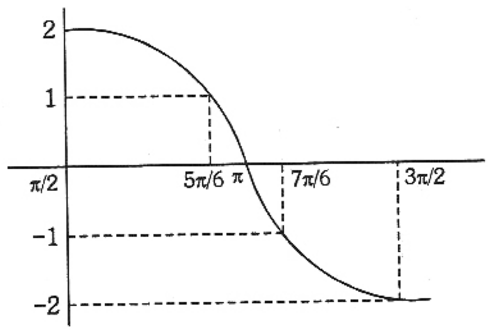
$\Rightarrow \qquad \dfrac{-b/a}{1-c/a} = 1$

$\Rightarrow \qquad \dfrac{-b/a}{\dfrac{a-c}{a}} = 1 \Rightarrow \qquad \dfrac{-b}{a-c} = 1$

$\Rightarrow \qquad -b = a - c$

$\Rightarrow \qquad a + b = c$. Therefore, (A) is the answer.

8. The graph of $y = 2 \sin x$ for $\pi/2 \le x \le 3\pi/2$ is given in Fig. From the graph it is clear that



$$[2 \sin x] = \begin{cases} 2 & \text{if} \quad x = \pi/2 \\ 1 & \text{if} \quad \pi/2 < x \le 5\pi/6 \\ 0 & \text{if} \quad 5\pi/6 < x \le \pi \\ -1 & \text{if} \quad \pi < x \le 7\pi/6 \\ -2 & \text{if} \quad 7\pi/6 < x \le 3\pi/2 \end{cases}$$

Therefore,

$\int_{\pi/2}^{3\pi/2} [2 \sin x]\, dx$

$$= \int_{\pi/2}^{5\pi/6} dx + \int_{5\pi/6}^{\pi} 0\, dx + \int_{\pi}^{7\pi/6} (-1)\, dx + \int_{7\pi/6}^{3\pi/2} (-2)\, dx$$

$$= [x]_{\pi/2}^{5\pi/6} + [-x]_{\pi}^{7\pi/6} + [-2x]_{7\pi/6}^{3\pi/2}$$

$$= \left(\frac{5\pi}{6} - \frac{\pi}{2}\right) + \left(\frac{-7\pi}{6} + \pi\right) + \left(\frac{-2\cdot 3\pi}{2} + \frac{2\cdot 7\pi}{6}\right)$$

$$= \pi\left(\frac{5}{6} - \frac{1}{2}\right) + \pi\left(1 - \frac{7}{6}\right) + \pi\left(\frac{7}{3} - 3\right)$$

$$= \pi\left(\frac{5-3}{6}\right) + \pi\left(-\frac{1}{6}\right) + \pi\left(\frac{7-9}{3}\right) = \frac{-\pi}{2}$$

Therefore, (C) is the answer.

9. $a_1, a_2, a_3 \ldots. a_{10}$ be in A.P.

so, $\qquad a_{10} = a_1 + 9d$

$\Rightarrow \qquad 3 = a_1 + 9d$

$\Rightarrow \qquad 3 = 2 + 9d$

$\Rightarrow \quad d = 1/9$

Now, $\quad a_4 = a_1 + 3d$

$\Rightarrow \quad a_4 = 2 + 3(1/9) = 2 + 1/3 = 7/3$

Again $h_1, h_2, h_3 \ldots h_{10}$ be in H.P.

$\Rightarrow \quad \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}, \ldots \frac{1}{h_{10}}$ be in A.P.

$h_1 = 2, \quad h_{10} = 3$ (given).

so, $$\frac{1}{h_{10}} = \frac{1}{h_1} + 9d_1$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1}{2} + 9d_1$$

$$\Rightarrow \quad \frac{1}{3} - \frac{1}{2} = 9d_1$$

$$\Rightarrow \quad -\frac{1}{6} = 9d_1$$

$$\Rightarrow \quad d_1 = -\frac{1}{54}$$

Now, $\frac{1}{h_7} = \frac{1}{h_1} + 6d_1$

$$\frac{1}{h_7} = \frac{1}{2} + \frac{6 \times 1}{-54}$$

$$\frac{1}{h_7} = \frac{1}{2} - \frac{1}{9}$$

$$\frac{1}{h_7} = \frac{9-2}{18}$$

$$h_7 = \frac{18}{7}$$

So $a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$. Therefore, (D) is the answer.

10. **Imp. note :** In this Question vector $\vec{c}$ is not given, therefore, we cannot apply the formulae of $\vec{a} \times \vec{b} \times \vec{c}$ (vector triple product).

Now $$\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = \left|\vec{a} \times \vec{b}\right| \left|\vec{c}\right| \sin 30°$$

So we need now $\left|\vec{a} \times \vec{b}\right|$ and $\left|\vec{c}\right|$

again $$\left|\vec{a} \times \vec{b}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow \quad \left|\vec{a} \times \vec{b}\right| = \sqrt{2^2 + (-2)^2 + 1} = \sqrt{4+4+1} = \sqrt{9} = 3$$

Next, $\left|\vec{c} - \vec{a}\right| = 2\sqrt{2}$

$\Rightarrow \quad \left|\vec{c} - \vec{a}\right|^2 = 8$

$\Rightarrow \quad (\vec{c} - \vec{a}) \cdot (\vec{c} - \vec{a}) = 8$

$\Rightarrow \quad \vec{c} \cdot \vec{c} - \vec{c} \cdot \vec{a} - \vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{a} = 8$

$\Rightarrow \quad \left|\vec{c}\right|^2 + \left|\vec{a}\right|^2 - 2\,\vec{a} \cdot \vec{c} = 8$

$\Rightarrow \quad \left|\vec{c}\right|^2 + 9 - 2\left|\vec{c}\right| = 8 \quad \therefore \vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ (given), $\vec{a} \cdot \vec{c} = \left|\vec{c}\right|$ (given)

$\Rightarrow \quad \left|\vec{c}\right|^2 - 2\left|\vec{c}\right| + 1 = 0$

$\Rightarrow \quad \left(\left|\vec{c}\right| - 1\right)^2 = 0$

$\Rightarrow \quad \left|\vec{c}\right| = 1$

Now putting in

$$\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = \left|\vec{a} \times \vec{b}\right|\left|\vec{c}\right| \sin 30°$$

$$= (3)\,(1) \cdot \left(\frac{1}{2}\right) = \frac{3}{2}$$. Therefore, (B) is the Ans.

11. From function it is clear that

(1) $x\,(x + 1) \geq 0 \quad \because$ Domain of square root function.

(2) $x^2 + x + 1 \geq 0 \quad \because$ Domain of square root function.

(3) $x^2 + x + 1 \leq 1 \quad \because \quad \sqrt{x^2 + x + 1} \leq 1 \quad \therefore$ Domain of $\sin^{-1}$ function.

From (2) and (3)

$$0 \leq x^2 + x + 1 \leq 1 \cap x^2 + x \geq 0$$

$\Rightarrow \quad 0 \leq x^2 + x + 1 \leq 1 \cap x^2 + x + 1 \geq 1$

$\Rightarrow \quad x^2 + x + 1 = 1$

$\Rightarrow \quad x^2 + x = 0$

$\Rightarrow \quad x\,(x + 1) = 0$

$\Rightarrow \quad x = 0,\ x = -1$, Therefore, (C) is the answer.

12. Firstly we obtain the slope of normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a \sec\theta, b \tan\theta)$.

Differentiating w.r.t. x, we get

$$\frac{2x}{a^2} - \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{b^2}{a^2} \cdot \frac{x}{y}$$

Slope of the normal at the point $(a \sec \theta, b \tan \theta)$ will be equal to

$$-\left(\frac{dx}{dy}\right)_{(a \sec \theta, b \tan \theta)} = -\frac{a^2}{b^2} \frac{b \tan \theta}{a \sec \theta} = -\frac{a}{b} \sin \theta$$

$\therefore$ equation of normal at $(a \sec \theta, b \tan \theta)$ is

$$y - b \tan \theta = -\frac{a}{b} \sin \theta (x - a \sec \theta)$$

$$\Rightarrow \qquad (a \sin \theta)\, x + by = (a^2 + b^2) \tan \theta$$

$$\Rightarrow \qquad ax + b \operatorname{cosec} y = (a^2 + b^2) \sec \theta \qquad \ldots(1)$$

Similarly equation of normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a \sec \phi, b \tan \phi)$ is

$$ax + b \operatorname{cosec} \phi\, y = (a^2 + b^2) \sec \phi \qquad \ldots(2)$$

Subtracting (2) from (i) we get

$b (\operatorname{cosec} \theta - \operatorname{cosec} \phi)\, y = (a^2 + b^2)(\sec \theta - \sec \phi)$

$$\Rightarrow \qquad y = \frac{a^2 + b^2}{b} \cdot \frac{\sec \theta - \sec \phi}{\operatorname{cosec} \theta - \operatorname{cosec} \phi}$$

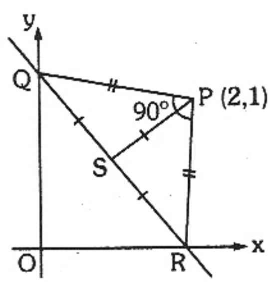
But $\frac{\sec \theta - \sec \phi}{\operatorname{cosec} \theta - \operatorname{cosec} \phi} = \frac{\sec \theta - \sec (\pi/2 - \theta)}{\operatorname{cosec} \theta - \operatorname{cosec} (\pi/2 - \theta)}$ $\qquad [\phi + \theta = \pi/2]$

$$= \frac{\sec \theta - \operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sec \theta} = 1$$

Thus, $y = -\frac{a^2 + b^2}{b}$ i.e. $k = -\frac{a^2 + b^2}{b}$ therefore (D) is the ans.

13. Let S be the mid-point of QR and $\Delta\, PQR$ is isoseles (given).

Therefore $PS \perp QR$ and S is mid-point of hypotenuse, therefore, S is equidistant from P, Q, R $\therefore PS = QS = RS$



Now $\angle P = 90°$ and $\angle Q = \angle R$

But $\angle P + \angle Q + \angle R = 180°$

So, $90° + \angle Q + \angle R = 180°$

$\therefore$ $\angle Q = 45°$ and $\angle R = 45°$

Now slope of QR is -2 (given).

$QR \perp PS$ $\therefore$ slope of PS is $+1/2$.

Now Let m be the slope of PQ.

Therefore, $\tan (\pm 45°) = \frac{m - 1/2}{1 - m(-1/2)}$

$$\Rightarrow \qquad \pm 1 = \frac{2m - 1}{2 + m}$$

$$\Rightarrow \qquad m = 3, -1/3$$

$\therefore$ Equation of PQ and PR are

$$y - 1 = 3(x - 2) \text{ and } y - 1 = -\frac{1}{3}(x - 2) \text{ or } 3(y - 1) + (x - 2) = 0.$$

Therefore, joint equations of PQ and PR are

$$[3(x - 2) - (y - 1)][(x - 2) + 3(y - 1)] = 0$$

$$\Rightarrow \quad 3(x - 2)^2 - 3(y - 1)^2 + 8(x - 2)(y - 1) = 0$$

$$\Rightarrow \quad 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$$

Therefore, Ans. is (B).

14. $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Imp. Note : Observe in R_1 that $a_{11} + a_{12} = a_{13}$ Check this trend in R_2 and R_3

apply $R_3 \to R_3 - (R_1 + R_2)$

$$= \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix} = 0$$

$\therefore f(x) = 0 \Rightarrow f(100) = 0$. Therefore, (A) is the answer.

15. **Imp. note :** All Integers are critical point for greatest Integer function.

So, Case 1 : $x \in I$,

$$f(x) = [x]^2 - [x^2] = x^2 - x^2 = 0$$

Case 2 : $x \notin I$.

if $0 < x < 1$, then $[x] = 0$

and $0 < x^2 < 1$, then $[x^2] = 0$

Next, if $1 < x^2 < 2 \Rightarrow 1 < x < \sqrt{2} \Rightarrow [x] = 1$ and $[x^2] = 1$

Therefore, $f(x) = [x]^2 - [x^2] = 0$ if $1 < x < \sqrt{2}$

Therefore, $f(x) = 0$, if $0 \le x < \sqrt{2}$

This shows that $f(x)$ is continuous at $x = 1$

Therefore, $f(x)$ is discontinuous in $(-\infty, 0) \cup [\sqrt{2}, \infty)$ on many other points.

Therefore, (B) is the answer.

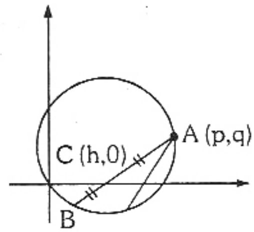
16. **Imp. note :** In solving a line and a circle there often generate a quadratic equation and further we have to apply condition of Discriminant so question convert from coordinate to quadratic equation.

Ans. From equation of circle it is clear that circle passes through origin.

Let AB is chord of the circle

$A \equiv (p, q)$. C is mid-point (h, o)

Then coordinates of B are $(-p + 2h, -q)$.
And B lies on the circle



$$x^2 + y^2 = px + qy, \text{ we have}$$

$$(-p + 2h)^2 + (-q)^2 = p(-p + 2h) + q(-q)$$

$$\Rightarrow \quad p^2 + 4h^2 - 4ph + q^2 = -p^2 + 2ph - q^2$$

$$\Rightarrow \quad 2p^2 + 2q^2 - 6ph + 4h^2 = 0$$

$$\Rightarrow \quad 2h^2 - 3ph + p^2 + q^2 = 0 \qquad ...(1)$$

There are given two distinct chords which are bisected at x axis then there will be two distinct values of h satisfying (1).
So discriminant of this quadratic equation must be > 0.

$$\Rightarrow \quad D > 0$$

$$\Rightarrow \quad (-3p)^2 - 4 \cdot 2.(p^2 + q^2) > 0$$

$$\Rightarrow \quad 9p^2 - 8p^2 - 8q^2 > 0$$

$$\Rightarrow \quad p^2 - 8q^2 > 0$$

$$\Rightarrow \quad p^2 > 8q^2$$ Therefore, (D) is the Answer.

17. Function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ ...(1)

> **Imp. note :** In differentiability of $|f(x)|$ we have to consider critical points for which $f(x) = 0$.

$|x|$ is not differentiable at $x = 0$

but
$$\cos|x| = \begin{cases} \cos(-x) & \text{if } x < 0 \\ \cos x & \text{if } x \geq 0 \end{cases}$$

$$\Rightarrow \quad \cos|x| = \cos x \quad \text{if } x < 0$$
$$= \cos x \quad \text{if } x \geq 0.$$ Therefore it is differentiable at $x = 0$.

Next, $|x^2 - 3x + 2| = |(x-1)(x-2)|$

$$= \begin{cases} (x-1)(x-2) & \text{if } x < 1 \\ (x-1)(2-x) & \text{if } 1 \leq x < 2 \\ (x-1)(x-2) & \text{if } 2 \leq x \end{cases}$$

Therefore,

$$f(x) = \begin{cases} (x^2-1)(x-1)(x-2) + \cos x & \text{if } -\infty < x < 1 \\ -(x^2-1)(x-1)(x-2) + \cos x & \text{if } 1 \leq x < 2 \\ (x^2-1)(x-1)(x-2) + \cos x & \text{if } 2 \leq x < \infty \end{cases}$$

Now $x = 1, 2$ are critical point for differentiability.
Because $f(x)$ is differentiable on other points in its domain.

Differentiability at x = 1

$$L\ f'(1) = \lim_{x \to 1-0} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1-0} \left[(x^2 - 1)(x-2) + \frac{\cos x - \cos 1}{x-1}\right]$$

$$= 0 - \sin 1 = -\sin 1$$

$$\because \lim_{x \to 1-0} \frac{\cos x - \cos 1}{x-1} = \frac{d}{dx}(\cos x)\ at\ x = 1-0$$

$$= -\sin x \ \text{ at } x = 1-0$$

$$= -\sin x \ \text{ at } x = 1$$

$$= -\sin 1$$

and $$Rf'(1) = \lim_{x \to 1+0} \frac{f(x) - f(1)}{x-1}$$

$$= \lim_{x \to 1+} \left[-(x^2 - 1)(x-2) + \frac{\cos x - \cos 1}{x-1}\right]$$

$$= 0 - \sin 1 = -\sin 1 \qquad \text{(same approach)}$$

$\because$ $L\ f'(1) = R\ f'(1)$. Therefore, function is differentiable at $x = 1$.

Again $$Lf'(2) = \lim_{x \to 2-0} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2-0} \left[-(x^2 - 1)(x-1) + \frac{\cos x - \cos 2}{x-2}\right]$$

$$= -(4-1)(2-1) - \sin 2 = -3 - \sin 2$$

and $$R\ f'(2) = \lim_{x \to 2+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2+} \left[(x^2 - 1)(x-1) + \frac{\cos x - \cos 2}{x-2}\right]$$

$$= (2^2 - 1)(2-1) - \sin 2 = 3 - \sin 2$$

So $L\ f'(2) \neq R\ f'(2)$, f is not differentiable at $x = 2$.

Therefore, (D) is the Ans

18. Both root less than 3 (given).

$\Rightarrow$ $\alpha < 3,\ \beta < 3$ ...(A)

$\Rightarrow$ $S = \alpha + \beta < 6$

$\Rightarrow$ $\frac{\alpha + \beta}{2} < 3$

$\Rightarrow$ $+\frac{2a}{2} < 3 \quad \Rightarrow \quad a < 3$ ...(1)

again $P = \alpha\beta$

$\Rightarrow$ $P < 9$

$\Rightarrow$ $\alpha\beta < 9$

$\Rightarrow$ $a^2 + a - 3 < 9$

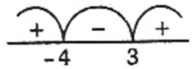
$\Rightarrow \quad a^2 + a - 12 < 0$

$\Rightarrow \quad a^2 + 4a - 3a - 12 < 0$

$\Rightarrow \quad a(a+4) - 3(a+4) < 0$

$\Rightarrow \quad (a-3)(a+4) < 0$

$\Rightarrow$



$\Rightarrow \quad -4 < a < 3$ ...(2)

again $D = B^2 - 4AC \geq 0$

$\Rightarrow \quad (-2a)^2 - 4 \cdot 1 (a^2 + a - 3) \geq 0$

$\Rightarrow \quad 4a^2 - 4a^2 - 4a + 12 \geq 0$

$\Rightarrow \quad -4a + 12 \geq 0$

$\Rightarrow \quad a - 3 \leq 0$

$\Rightarrow \quad a \leq 3$ ...(3)

again, $a\, f(3) > 0$

$\Rightarrow \quad 1 \cdot f(3) > 0$

$\Rightarrow \quad (3)^2 - 2a(3) + a^2 + a - 3 > 0$

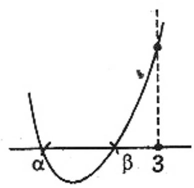
$\Rightarrow \quad 9 - 6a + a^2 + a - 3 > 0$

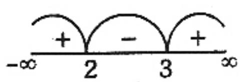
$\Rightarrow \quad a^2 - 5a + 6 > 0$

$\Rightarrow \quad a^2 - 3a - 2a + 6 > 0$

$\Rightarrow \quad a(a-3) - 2(a-3) > 0$

$\Rightarrow \quad (a-2)(a-3) > 0$





$\therefore \quad a \in (-\infty, 2) \cup (3, \infty)$ ...(4)

Collecting (1), (2), (3) and (4)

$\Rightarrow \quad a \in (-4, 2)$. Therefore, (A) is the answer.

Imp. Note : There is correction in Ans. $a < 2$ should be $-4 < a < 2$

19. Given differential equation is

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0 \qquad \text{...(1)}$$

(A) $\quad y = 2 \Rightarrow \quad \frac{dy}{dx} = 0$

putting in (1)

$$(0)^2 - x \cdot (0) + y = 0$$

$\Rightarrow \quad y = 0$ which is not satisfied.

(B) $y = 2x \quad \Rightarrow \quad \frac{dy}{dx} = 2$

putting in (1)

$$(2)^2 - x \cdot 2 + y = 0$$

$\Rightarrow \quad 4 - 2x + y = 0$

$\Rightarrow$ $y = 2x - 4$ which is not satisfied but (C) is itself the answer.

(D) $y = 2x^2 - 4$

$$\frac{dy}{dx} = 4x$$

putting in (1)

$$\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$$

$\Rightarrow \quad (4x)^2 - x \cdot 4x + y = 0$

$\Rightarrow \quad y = 0$ which is not satisfied.

Therefore, (C) is the answer.

20. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$

> **Imp. Note :** In trigonometry try to make all trigonometric functions in same angle. It is called **3rd Golden Rule** of trigonometry.

$$= \lim_{x \to 0} \frac{x \frac{2\tan x}{1 - \tan^2 x} - 2x \cdot \tan x}{(2\sin^2 x)^2}$$

$$= \lim_{x \to 0} \frac{2x \tan x \cdot \left[\frac{1}{1 - \tan^2 x} - 1\right]}{4 \cdot \sin^4 x}$$

$$= \lim_{x \to 0} \frac{2x \tan x \cdot \left[\frac{1 - 1 + \tan^2 x}{1 - \tan^2 x}\right]}{4 \sin^4 x}$$

$$= \lim_{x \to 0} \frac{1 \cdot x \cdot \tan^3 x}{2 \cdot \sin^4 x\,(1 - \tan^2 x)} = \lim_{x \to 0} \frac{1}{2} \cdot \frac{x \cdot \left(\frac{\tan x}{x}\right)^3 \cdot x^3}{\sin^4 x\,(1 - \tan^2 x)}$$

$$= \lim_{x \to 0} \frac{1\left(\frac{\tan x}{x}\right)^3}{2\left(\frac{\sin x}{x}\right)^4 \cdot (1 - \tan^2 x)} = \frac{1 \cdot (1)^3}{2 \cdot (1)^4 \cdot (1 - 0)} = \frac{1}{2}$$

Therefore, (C) is the answer.

21. It is given that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, we take

$$\vec{c} = p\,\vec{a} + q\,\vec{b} \qquad \ldots(1)$$

where p, q are scalars.

again $\vec{c} \perp \vec{a}$, (given)

$$\Rightarrow \quad \vec{c} \cdot \vec{a} = 0$$

taking dot product of $\vec{a}$ in (1)

$$\Rightarrow \quad \vec{c} \cdot \vec{a} = p\,\vec{a} \cdot \vec{a} + q\,\vec{b} \cdot \vec{a} \qquad \because \vec{a} = 2\hat{i} + \hat{j} + \hat{k},$$

$$\Rightarrow \quad 0 = p\left|\vec{a}\right|^2 + q\left|\vec{b} \cdot \vec{a}\right| \qquad \Rightarrow \left|\vec{a}\right| = \sqrt{2^2 + 1 + 1} = \sqrt{6}$$

$$\Rightarrow \quad 0 = p \cdot 6 + q\,.3 \qquad \vec{a} \cdot \vec{b} = (2\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} - \hat{k})$$

$$\Rightarrow \quad q = -2p \qquad = 2 \cdot 1 + 1 \cdot 2 + 1 \cdot (-1) = 4 - 3 = 3$$

putting in (1)

$$\Rightarrow \quad \vec{c} = p\,\vec{a} + \vec{b}\,(-2p)$$

$$\Rightarrow \quad \vec{c} = p\,\vec{a} - 2p\,\vec{b}$$

$$\Rightarrow \quad \vec{c} = p\left(\vec{a} - 2\,\vec{b}\right)$$

$$\Rightarrow \quad \vec{c} = p\,\{(2\hat{i} + \hat{j} + \hat{k}) - 2\,(\hat{i} + 2\hat{j} - \hat{k})\}$$

$$\Rightarrow \quad \vec{c} = p\,\{-3\hat{j} + 3\hat{k}\}$$

again $\left|\vec{c}\right| = 1$ (given) $\Rightarrow \left|\vec{c}\right| = p\sqrt{(-3)^2 + 3^2}$

$$\Rightarrow \quad \left|\vec{c}\right|^2 = p^2\,(\sqrt{18})^2$$

$$\Rightarrow \quad \left|\vec{c}\right|^2 = p^2 \cdot 18$$

$$\Rightarrow \quad 1 = p^2 \cdot 18 \Rightarrow p^2 = \frac{1}{18} \Rightarrow p = \pm\frac{1}{3\sqrt{2}}$$

$$\therefore \quad \vec{c} = \pm\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$$

Therefore, (A) is the answer.

22. We have $(1 + x)^m\,(1 - x)^n = \left[1 + mx + \frac{m\,(m-1)}{2} \cdot x^2 + \ldots\right]$

$$\left[1 - nx + \frac{n\,(n-1)}{2}\,x^2 - \ldots\ldots\right]$$

$$= 1 + (m - n)x + \left[\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn\right]x^2 + \ldots.$$

term containing power of $x \geq 3$.

(coefficient of $x = 3$ given) ...(1)

Now, $m - n = 3$

and $\frac{1}{2}m(m-1) + \frac{1}{2}n(n-1) - mn = -6$

or $m(m-1) + n(n-1) - 2mn = -12$

$\Rightarrow m^2 - m + n^2 - n - 2mn = -12$

$\Rightarrow (m-n)^2 - (m+n) = -12$

$\Rightarrow m + n = 9 + 12 = 21$ ...(2)

Solving (1) and (2)

$m = 12$. Therefore, (C) is the answer.

23. $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ ...(1)

$= \int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos(\pi - x)}$ $\because \int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$

$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1 - \cos x}$ ...(2)

adding (1) and (2)

$$2I = \int_{\pi/4}^{3\pi/4} \left(\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}\right) dx$$

$$2I = \int_{\pi/4}^{3\pi/4} \left(\frac{2}{1 - \cos^2 x}\right) dx$$

$$2I = 2\int_{\pi/4}^{3\pi/4} \frac{1}{\sin^2 x}\, dx$$

$$I = \int_{\pi/4}^{3\pi/4} \operatorname{cosec}^2 x\, dx = [-\cot x]_{\pi/4}^{3\pi/4} = \left[-\cot\frac{3\pi}{4} + \cot\frac{\pi}{4}\right]$$

$I = -(-1) + 1 = 2$. Therefore (A) is the answer

24. Let h, k be point whose chord of contact w.r.t. to hyperbola $x^2 - y^2 = 9$ is $x = 9$.

We know that chord of contact of (h, k) w.r.t. hyperbola $x^2 - y^2 = 9$ is

$T = 0 \Rightarrow h \cdot x + k(-y) - 9 = 0$

$\therefore hx - ky - 9 = 0$ but it is the equation of the line $x = 9$.

This is possible when $h = 1, k = 0$ (by comparing both equations). Again equation of pair of tangent is $T^2 = SS_1$

$\Rightarrow (x - 9)^2 = (x^2 - y^2 - 9)(1^2 - 0^2 - 9)$

$\Rightarrow x^2 - 18x - 81 = (x^2 - y^2 - 9)(-8)$

$\Rightarrow x^2 - 18x - 81 = -8x^2 + 8y^2 + 72$

$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0.$

Therefore, (B) is the answer.

25. **Imp. point :** power of prime numbers have cyclic numbers in their unit place.

$7^1 = 7, 7^2 = 49, 7^3 = 343, \ 7^4 = 2401$ ....

Therefore, for $7^r, r \in N$ the no. ends at unit place 7, 9, 3, 1, 7, ...

$\therefore \quad 7^m + 7^n$ will be divisible by 5 if it end at 5 or 0.

but it cannot end at 5

and also cannot end at 0

For this m and n should be as follows :

	m	n
1	$4r$	$4r+2$
2	$4r+1$	$4r+3$
3	$4r+2$	$4r$
4	$4r+3$	$4r+1$

For any given value of m, there will be 25 values of n.

Hence, the probability of the required event is $\dfrac{100 \times 25}{100 \times 100} = \dfrac{1}{4}$

Therefore (A) is the ans.

26. Let equation of line L_1 be $y = mx$. Intercepts made by L_1 and L_2 on the circle will be equal i.e. L_1 and L_2 are at the same distance from the centre of the circle.

Centre of the given circle is $(1/2, -3/2)$. Therefore,

$$\frac{|1/2 - 3/2 - 1|}{\sqrt{1+1}} = \left|\frac{3m/2 + 1/2}{\sqrt{m^2+1}}\right| \quad \Rightarrow \quad \frac{2}{\sqrt{2}} = \frac{|3m+1|}{2\sqrt{m^2+1}}$$

$$\Rightarrow \quad 8(m^2+1) = (3m+1)^2 \qquad \Rightarrow \quad m^2 + 6m - 7 = 0$$

$$\Rightarrow \quad (m+7)(m-1) = 0 \qquad \Rightarrow \quad m = -7, \ m = 1$$

Thus two chords are $y + 7x = 0$ and $y - x = 0$

Therefore, (B) and (C) is the Ans.

27. Let θ be the angle between $\vec{a}$ and $\vec{b}$. As $\vec{a}$ and $\vec{b}$ are non-collinear, $\theta \neq 0$ and $\theta \neq \pi$.

We have $\quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$$= \cos\theta \quad |\vec{a}| = 1, \quad |\vec{b}| = 1 \text{ given}$$

Now, $\quad \vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$

Taking modulus

$$|\vec{u}| = \left|\vec{a} - (\vec{a} \cdot \vec{b})\vec{b}\right|$$

$$\Rightarrow \quad \left|\vec{u}\right|^2 = \left|\vec{a} - \left(\vec{a}\cdot\vec{b}\right)\vec{b}\right|^2$$

$$\Rightarrow \quad \left|\vec{u}\right|^2 = \left|\vec{a} - \cos\theta\,\vec{b}\right|^2$$

$$\Rightarrow \quad \left|\vec{u}\right|^2 = \left|\vec{a}\right|^2 + \cos^2\theta\left|\vec{b}\right|^2 - 2\cos\theta\,(\vec{a}\cdot\vec{b})$$

$$\Rightarrow \quad \left|\vec{a}\right|^2 = 1 + \cos^2\theta - 2\cos^2\theta$$

$$\Rightarrow \quad \left|\vec{u}\right|^2 = 1 - \cos^2\theta$$

$$\Rightarrow \quad \left|\vec{u}\right|^2 = \sin^2\theta$$

Also $\vec{v} = \vec{a} \times \vec{b}$ (given)

$$\Rightarrow \quad \left|\vec{v}\right| = \left|\vec{a}\times\vec{b}\right|$$

$$\Rightarrow \quad \left|\vec{v}\right|^2 = \left|\vec{a}\times\vec{b}\right|^2$$

$$\Rightarrow \quad \left|\vec{v}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2 \sin^2\theta$$

$$\Rightarrow \quad \left|\vec{v}\right|^2 = \sin^2\theta$$

$$\therefore \quad \left|\vec{u}\right|^2 = \left|\vec{v}\right|^2$$

Also, $\vec{u}\cdot\vec{a} = \left[\vec{a} - \left(\vec{a}\cdot\vec{b}\right)\vec{b}\right]\cdot\vec{a}$

$$= \vec{a}\cdot\vec{a} - \left(\vec{a}\cdot\vec{b}\right)\left(\vec{b}\cdot\vec{a}\right)$$

$$= \left(\vec{a}\right)^2 - \cos^2\theta$$

$$= 1 - \cos^2\theta = \sin^2\theta$$

$$\therefore \left|\vec{u}\right| + \left|\vec{u}\cdot\vec{a}\right| = \sin\theta + \sin^2\theta \neq \left|\vec{u}\right|$$

Next $\vec{u} \cdot \vec{b} = \left(\vec{a} - \left(\vec{a} \cdot \vec{b}\right) \cdot \vec{b}\right) \cdot \vec{b}$

$$= \vec{a} \cdot \vec{b} - \left(\vec{a} \cdot \vec{b}\right)\left(\vec{b} \cdot \vec{b}\right)$$

$$= \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} \left|\vec{b}\right|^2$$

$$= \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} = 0 \qquad \ldots(1)$$

$\therefore \qquad \left|\vec{u}\right| + \left|\vec{u} \cdot \vec{b}\right| = \left|\vec{u}\right| + 0 = \left|\vec{u}\right| = \left|\vec{v}\right|$

Also $\qquad \vec{u} \cdot \left(\vec{a} + \vec{b}\right) = \vec{u} \cdot \vec{a} + \vec{u} \cdot \vec{b} = \vec{u} \cdot \vec{a}$

$\Rightarrow \quad \left|\vec{u}\right| + \vec{u} \cdot \left(\vec{a} + \vec{b}\right) = \left|\vec{u}\right| + \vec{u} \cdot \vec{a} \neq \left|\vec{v}\right|$

Therefore, (A) and (D) are not the Ans. and (B) and (C) are the Ans.

28. $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots.. + \frac{1}{2^n - 1}$ (given)

$$= 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \ldots \frac{1}{7}\right) + \left(\frac{1}{8} + \ldots.. \frac{1}{15}\right) + \ldots.. \left(\frac{1}{2^{n-1}+1} \ldots.. \frac{1}{2^n - 1}\right)$$

$$< 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \ldots \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \ldots \frac{1}{8}\right)$$

$$+ \ldots. \left(\frac{1}{2^{n-1}+1} + \frac{1}{2^{n-1}+1} + \ldots \frac{1}{2^{n-1}+1}\right)$$

$$= 1 + \frac{2}{2} + \frac{4}{4} + \frac{8}{8} + \ldots. \frac{2^{n-1}}{2^{n-1}} = \frac{1 + 1 + 1 + 1 + \ldots.. + 1}{(n) \text{ times}} = n$$

Thus, $a(100) < 100$, Therefore, (A) is the Ans.

Next,

$$a(n) = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \ldots. \frac{1}{8}\right) + \ldots \frac{1}{2^{n-1}+1} + \ldots \frac{1}{2^n - 1}$$

$$> 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} \ldots. \frac{1}{8}\right) + \ldots \left(\frac{1}{2^n - 1} + \frac{1}{2^n - 1} + \ldots \frac{1}{2^n - 1}\right)$$

$$= 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \ldots. \frac{2^{n-1}}{2^n} - \frac{1}{2^n}$$

$$= 1 + \frac{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots\ldots\ldots \frac{1}{2}}{n \text{ times}} - \frac{1}{2^n}$$

$$= \left(1 - \frac{1}{2^n}\right) + \frac{n}{2}$$

Therefore, $a(200) > \left(1 - \frac{1}{2^{100}}\right) + \frac{200}{2} > 100$

Therefore, (D) is also the Ans.

29. $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3 (t-3)^5 \, dt$

$f'(x) = \frac{d}{dx}\int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3 (t-3)^5 \, dt$

$= x(e^x - 1)(x-1)(x-2)^3 (x-3)^5 \times 1 - x(e^x - 1)(x-1)(x-2)^3 (x-3)^5 \times 0$

$\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)\, dt = f(\psi(x))\, \psi'(x) - f(\phi(x))\, \phi'(x)$ Formula

For local minimum, $f'(x) = 0$

$\Rightarrow \quad x = 0, 1, 2, 3.$ Let $f'(x) = g(x) = x(e^x - 1)(x-1)(x-2)^3 (x-3)^5$

$g(x) =$ $-\infty$ (−) 0 (−) 1 (+) 2 (−) 3 (+) ∞

$\Rightarrow \quad f'(x) < 0$ if $x < 0$

< 0 if $0 < x < 1$

> 0 if $1 < x < 2$

< 0 if $2 < x < 3$

> 0 if $x > 3$

This shows that $f(x)$ has a local minimum at $x = 1$ and $x = 3$

Therefore, (B) and (D) are the Answer.

30. $4x^2 + 9y^2 = 1$ (given) ...(1)

Differentiating w.r.t. x, we get

$$8x + 18y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{8x}{18y} = -\frac{4x}{9y}$$

The tangent at point (h, k) will be parallel to $8x = 9y$, then

$$\frac{-4h}{9k} = \frac{8}{9}$$

$$\Rightarrow \quad h = -2k \quad ...(2)$$

Substituting h, k in (1) since h, k lies in (1)

$$4h^2 + 9k^2 = 1$$

putting value of h in (2)

$$4(-2k)^2 + 9k^2 = 1$$

$$16k^2 + 9k^2 = 1 \Rightarrow 25k^2 = 1 \Rightarrow k^2 = 1/25$$

$$\Rightarrow k = \pm 1/5$$

Thus, the points where the tangents are parallel to $8x = 9y$ are $(-2/5, 1/5)$ and $(2/5, -1/5)$. Therefore (B) (D) are the Ans.

31. Let A, B and C respectively denote the events that the student passes in Maths, Physics and Chemistry.

It is given :

$$P(A) = m, \quad P(B) = p \text{ and } P(C) = c$$

and P (passing in at least one's) = $P(A \cup B \cup C) = 0\cdot75$

$\Rightarrow 1 - P(A' \cap B' \cap C') = 0\cdot75$ $\quad\because P(A) = 1 - P(\overline{A})$ and $P(\overline{A \cup B \cup C}) = P(\overline{A} \cap \overline{B} \cap \overline{C})$

$\Rightarrow 1 - P(A')\cdot P(B')\cdot P(C') = 0\cdot75$ $\quad\because A, B, C$ are independent events therefore, A', B' and C' are independent events.

$\Rightarrow \quad 0\cdot75 = 1 - (1-m)(1-p)(1-c)$

$\Rightarrow \quad 0\cdot25 = (1-m)(1-p)(1-c)$ ...(1)

Also P (passing exactly in two subjects) = $0\cdot4$

$\Rightarrow \quad P(A \cap B \cap \overline{C} \;\cup\; A \cap \overline{B} \cap C \;\cup\; \overline{A} \cap B \cap C) = 0\cdot4$

$\Rightarrow \quad P(A \cap B \cap \overline{C}) \cup P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) = 0\cdot4$

$\Rightarrow \quad P(A)\cdot P(B)\cdot P(\overline{C}) + P(A)\,P(\overline{B})\,P(C) + P(\overline{A})\,P(B)\,P(C) = 0\cdot4$

$\Rightarrow \quad p\,m(1-c) + p(1-m)c + (1-p)\,mc = 0\cdot4$

$\Rightarrow \quad pm - pmc + pc - pmc + mc - pmc = 0\cdot4$ ...(2)

again P (passing at least in two subjects) = $0\cdot5$

$\Rightarrow \quad P(A \cap B \cap \overline{C}) + P(A \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap C) + P(A \cap B \cap C) = 0\cdot5$

$\Rightarrow \quad pm(1-c) + pc(1-m) + cm(1-p) + pcm = 0\cdot5$

$\Rightarrow \quad pm - pcm + pc - pcm + cm - pcm + pcm = 0\cdot5$

$\Rightarrow \quad (pm + pc + mc) - pcm = 0\cdot5$ ...(3)

from (2), we get

$pm + pc + mc - 3pcm = 0\cdot4$ ...(4)

from (1) we get

$0\cdot25 = 1 - (m + p + c) + (pm + pc + cm) - pmc$ ...(5)

solving (3), (4), (5) we get

$p + m + c = 1\cdot35 = 27/20$. Therefore, (B) is the Answer.

32. $y^2 = 2c(x + \sqrt{c})$ ...(1)

Differentiating w.r.t. x we get

$$2y\frac{dy}{dx} = 2c \qquad \Rightarrow \qquad c = y\frac{dy}{dx}$$

putting this value of c in (1) we get

$$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$$

$$\Rightarrow \qquad y = 2\frac{dy}{dx}\cdot x + 2y^{1/2}\left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \qquad y - 2x\frac{dy}{dx} = 2\sqrt{y}\left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \qquad \left(y - 2x\frac{dy}{dx}\right)^2 = 8y^3\left(\frac{dy}{dx}\right)^3$$

Therefore, order of this differential equation is 1 and degree is 3.

Therefore (A), (C) is the Ans.

33. **Imp. note :** sequence is imp. for future considration in IIT exam.

Let a_r denote the length of side of the square S_r.

We are given a_n = length of diagonal of S_{n+1}.

$\Rightarrow \quad a_n = \sqrt{2}\, a_{n+1}$

$\Rightarrow \quad a_{n+1} = \dfrac{a_n}{\sqrt{2}}$

This shows that a_1, a_2, a_3 ....form a G.P. with common ratio $1/\sqrt{2}$.

Therefore, $a_n = a_1 \left(\dfrac{1}{\sqrt{2}}\right)^{n-1}$

$\Rightarrow \quad a_n = 10\left(\dfrac{1}{\sqrt{2}}\right)^{n-1} \qquad \because a_1 = 10$ given,

$\Rightarrow \quad a_n^2 = 100\left(\dfrac{1}{\sqrt{2}}\right)^{2(n-1)}$

$\Rightarrow \quad \dfrac{100}{2^{n-1}} \le 1 \qquad \because a_n^2 \le 1$ given

$\Rightarrow \quad 100 \le 2^{n-1}$

This is possible for $n \ge 8$.

so (B), (C), (D) are the Answer.

34. **Case 1.** $m = 0$

In this case $y = x - x^2$ ...(1)

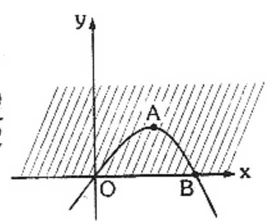
$y = 0$ ...(2)

are two given curves. $y > 0$ is total region above x-axis.

Therefore, area between

$y = x - x^2$ and $y = 0$

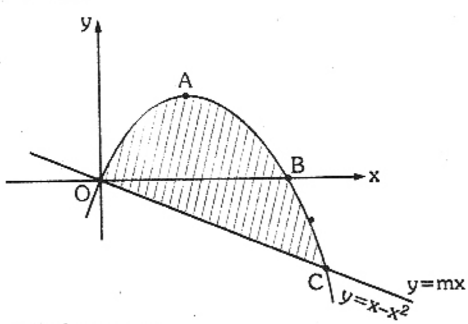
is area between $y = x - x^2$ and above the x-axis.



$$A = \int_0^1 (x - x^2)\, dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \ne \frac{9}{2}$$

Hence no solution.

Case 2. $m < 0$



In this case area between

$y = x - x^2$ and $y = mx$ is

OABCO and points of intersection are (0, 0) and $\{1 - m,\ m(1 - m)\}$

Area $OABCO = \int_0^{1-m} [x - x^2 - mx]\, dx.$

Imp. note : Area *OBCO* considered automatically because *m* is a parameter

$$= \left[(1 - m)\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1-m}$$

$$= \frac{1}{2}(1 - m)^3 - \frac{1}{3}(1 - m)^3$$

$$= \frac{1}{6}(1 - m)^3$$

Put Area $OABCO = 9/2$ (given)

$\therefore \quad \frac{1}{6}(1 - m)^3 = \frac{9}{2}$

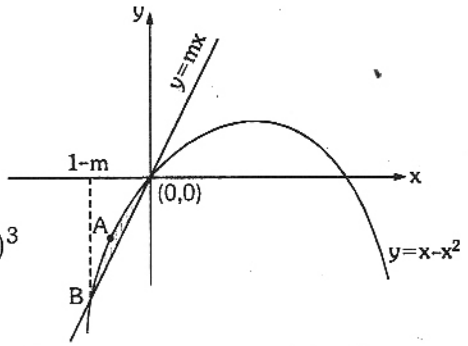
$\Rightarrow \quad (1 - m)^3 = 27$

$\Rightarrow \quad 1 - m = 3$

$\Rightarrow \quad m = -2$

Case 3. $m > 0$

In this case $y = mx$ and $y = x - x^2$ intersect in (0, 0) and $\{(1 - m), m(1 - m)\}$ as shown in Fig.



Area of shaded region

$$= \int_{1-m}^{0} (x - x^2 - mx)\, dx$$

$$= \left[(1 - m)\frac{x^2}{2} - \frac{x^3}{3}\right]_{1-m}^{0}$$

$$= -\frac{1}{2}(1 - m)(1 - m)^2 + \frac{1}{3}(1 - m)^3$$

$$= -\frac{1}{6}(1 - m)^3$$

Area of shaded region = 9/2 square unit (given)

$\Rightarrow \quad \frac{9}{2} = -\frac{1}{6}(1 - m)^3$

$\Rightarrow \quad (1 - m)^3 = -27$

$\Rightarrow \quad (1 - m) = -3$

$\Rightarrow \quad m = 3 + 1 = 4$. Therefore, (B) and (D) are the Answers.

35. **Imp. note : multiplicative loop** is very imp. approch in IIT mathematics.

$$\left(\tan\frac{\theta}{2}\right)(1 + \sec\theta) = \frac{\sin\theta/2}{\cos\theta/2}\cdot\left[1 + \frac{1}{\cos\theta}\right]$$

$$= \frac{\sin\theta/2}{\cos\theta/2} \times \frac{(1 + \cos\theta)}{\cos\theta} = \frac{\sin\theta/2 \cdot 2\cos^2\theta/2}{\cos\theta/2 \cdot \cos\theta}$$

$$= \frac{2\sin\theta/2 \cdot \cos\theta/2}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$