PHYSICS - 1998

PART - A

Directions

1. *Section I consists of 40 objective type questions.*
2. *This section should take about one hour to answer.*
3. *Each question in this section carries 2 marks.*

1. A transistor is used in common emitter mode as an amplifier, then :
(A) the base emitter junction is forward biased
(B) the base emitter junction is reverse biased
(C) the input signal is connected in series with the voltage applied to bias the base emitter junction
(D) the input signal is connected in series with the voltage applied to bias the base collector junction.

2. Water from a tap emerges vertically downwards with an initial speed of 1.0 m/s. The cross-sectional area of tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is :
(A) $5.0\times 10^{-4} m^2$ (B) $1.0\times 10^{-4} m^2$
(C) $5.0\times 10^{-5} m^2$ (D) $2.0\times 10^{-5} m^2$

3. A real image of a distant object is formed by a planoconvex lens on its principal axis. Spherical aberration :
(A) is absent
(B) is smaller if the curved surface of the lens faces the object
(C) is smaller if the plane surface of the lens faces the object
(D) is the same whichever side of the lens faces the object.

4. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m. Then :
(A) no molecule can have a energy greater than $\sqrt{2}v_{rms}$
(B) no molecule can have speed less then $v_p/\sqrt{2}$
(C) $v_p < \bar{v} < v_{rms}$
(D) the average kinetic energy of a molecule is $\frac{3}{4}\,m{v_p}^2$

5. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is :
(A) 1 : 1
(B) 1 : 2
(C) 2 : 1
(D) depends on the moment of inertia of the two molecules.

6. A string of length 0.4 m and mass 10^{-2} Kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt, which allows constructive interference between successive pulses, is :

(A) 0.05 s (B) 0.10 s
(C) 0.20 s (D) 0.40 s

7. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length 2R. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is :

(A) $q/2m$ (B) q/m
(C) $2q/m$ (D) $q/\pi m$

8. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value (s) of n from the following :

(A) 1.3 (B) 1.4
(C) 1.5 (D) 1.6

9. Let m_p be the mass of proton, m_n the mass of neutron. M_1 the mass of $^{20}_{10}Ne$ nucleus and M_2 the mass of $^{40}_{20}Ca$ nucleus. Then :

(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$
(C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_p)$

10. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is :

(A) 0 (B) $\pi/2$
(C) π (D) 2π

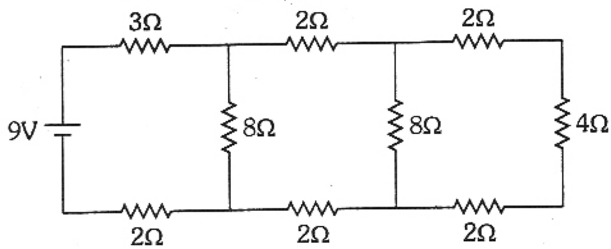
11. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. The time period of the electron in the intial state is eight times that in the final state. The possible values of n_1 and n_2 are :

(A) $n_1 = 4, n_2 = 2$ (B) $n_1 = 8, n_2 = 2$
(C) $n_1 = 8, n_2 = 1$ (D) $n_1 = 6, n_2 = 3$

12. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is :

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$
(C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

13. In the circuit shown in the figure, the current through :



(A) the 3Ω resistor is 0.50 A
(B) the 3Ω resistor is 0.25 A
(C) the 4Ω resistor is 0.50 A
(D) the 4Ω resistor is 0.25 A

14. A dielectric slab of thickness d is inserted in a parellel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to 3d :

(A) the magnitude of the electric field remains the same
(B) the direction of the electric field remains the same
(C) the electric potential increases continuously
(D) the electric potential increases at first, then decreases and again increases.

15. The (x, y) coordinates of the corners of a square plate are (0,0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression (s) for u is (are) (a= positive constant) :

(A) $a \cos(\pi x/2L) \cos(\pi y/2L)$
(B) $a \sin(\pi x/L) \sin(\pi y/L)$
(C) $a \sin(\pi x/L) \sin(2\pi y/L)$
(D) $a \cos(2\pi x/L) \sin(\pi y/L)$

16. A force $\vec{F} = -K(y\,\hat{i} + x\,\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0) and then parallel to the y-axis to the point (a, a). The total work done by the force F on the particle is :

(A) $-2Ka^2$
(B) $2Ka^2$
(C) $-Ka^2$
(D) Ka^2

17. A small square loop of wire of side l is placed inside a large square loop of wire of side L $(L >> l)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to :

(A) l/L
(B) l^2/L
(C) L/l
(D) L^2/l.

18. The half life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that :

(A) no nucleus will decay before $t = 4$ days
(B) no nucleus will decay before $t = 8$ days

(C) all nuclei will decay before $t = 16$ days

(D) a given nucleus may decay at any time after $t = 0$

19. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and $1.5\,\Delta P$ respectively. Then :

(A) $4\,m_A = 9m_B$ (B) $2m_A = 3m_B$

(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

20. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :

(A) $\frac{2}{3}P$ (B) P

(C) $\frac{3}{2}P$ (D) 2P

21. A charge $+q$ is fixed at each of the points $x = x_0, x = 3x_0, x = 5x_0 \ldots \infty$ on the x-axis and a charge $-q$ is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0 \ldots \infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\epsilon_0 r$. Then the potential at the origin due to the above system of charges is :

(A) 0 (B) $\dfrac{q}{8\pi\epsilon_0 x_0 \ln 2}$

(C) ∞ (D) $\dfrac{q \ln(2)}{4\pi\epsilon_0 x_0}$

22. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to :

(A) I (B) $I \sin^2\theta$

(C) $I \cos^2\theta$ (D) $I \cos^2(\theta/2)$

23. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is :

(A) 30 K (B) 18 K

(C) 50 K (D) 42 K

24. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let 0 be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be :

(A) real and will remain at C

(B) real and located at a point between C and ∞

(C) virtual and located at a point between C and 0

(D) real and located at a point between C and 0

25. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following :

(A) the entire rod is at the same electric potential

(B) there is an electric field in the rod

(C) the electric potential is highest at the centre of the rod and decreases towards its ends.

(D) the electric potential is lowest at the centre of the rod and increases towards its ends.

26. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is :

(A) periodic for all values of z_0 satisfying $0 < z_0 < \infty$

(B) simple harmonic for all values of z_0 satisfying $0 < z_0 \leq R$

(C) approximately simple harmonic provided $z_0 << R$

(D) such that P crosses 0 and continues to move along the negative z-axis towards $z = -\infty$

27. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth :

(A) The acceleration of S is always directed towards the centre of the earth.

(B) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remain constant.

(C) the total mechanical energy of S varies periodically with time

(D) The linear momentum of S remains constant in magnitude.

28. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that :

(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.

(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.

(C) the magnitude of $\vec{L}$ does not change with time.

(D) $\vec{L}$ does not change with time.

29. During the melting of a slab of ice at 273K at atmospheric pressure :

(A) positive work is done by the ice-water system on the atmosphere.

(B) positive work is done on the ice-water system by the atmosphere.

(C) the internal energy of the ice-water system increases.

(D) the internal energy of the ice-water system decreases.

30. In a p-n junction diode not connected to any circuit :

(A) the potential is the same everywhere

(B) the p-type side is at a higher potential than the n-type side

(C) there is an electric field at the junction directed from the n-side to the p-type side.

(D) there is an electric field at the junction directed from the p-type side to the n-type side.

31. A spherical surface of radius of curvature R, separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = OQ. The distance PO is equal to :

(A) 5R (B) 3R

(C) 2R (D) 1.5 R

32. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre :

(A) increases as r increases for $r < R$

(B) decreases as r increases for $0 < r < \infty$

(C) decreases as r increases for $R < r < \infty$

(D) is discontinuous at $r = R$

33. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and v = 10 m/s, then λ and f are given by :

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m

(C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

34. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wein constant, $b = 2.88 \times 10^6$ nm-K. Then :

(A) $U_1 = 0$ (B) $U_3 = 0$

(C) $U_1 > U_2$ (D) $U_2 > U_1$

35. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current :

(A) $[\epsilon_0] = [M^{-1}L^{-3}T^2I]$ (B) $[\epsilon_0] = [M^{-1}L^{-3}T^4I^2]$

(C) $[\mu_0] = [MLT^{-2}I^{-2}]$ (D) $[\mu_0] = [ML^2T^{-1}I]$

36. The SI unit of the inductance, the henry can be written as :

(A) Weber/ampere (B) Volt-second/ampere

(C) Joule/(ampere)^2 (D) ohm-second

37. Two very long straight parallel wires carry steady currents I and –I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is :

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) 0

38. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from :

(A) 0 to ∞

(B) λ_{min} to ∞ where $\lambda_{min} > 0$

(C) 0 to λ_{max} where $\lambda_{max} < \propto$

(D) λ_{min} to λ_{max} Where $0 < \lambda_{min} < \lambda_{max} < \propto$

39. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k\,|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is :

(A) proportional to $1/\sqrt{a}$ (B) independent of a

(C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

40. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately :

(A) 540 nm (B) 400 nm

(C) 310 nm (D) 220 nm

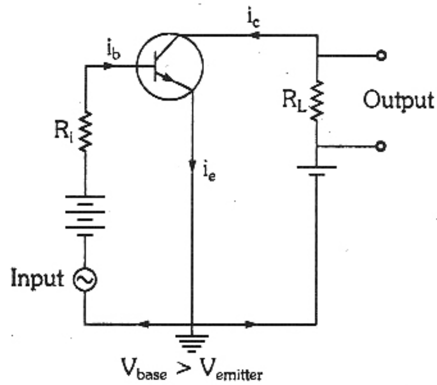
ANSWERS

1. (A), (C)	**2.** (C)	**3.** (B)	**4.** (C), (D)	**5.** (A)	**6.** (B)
7. (A)	**8.** (C), (D)	**9.** (C), (D)	**10.** (D)	**11.** (A), (D)	**12.** (D)
13. (D)	**14.** (B), (C)	**15.** (B), (C)	**16.** (C)	**17.** (B)	**18.** (D)
19. (C)	**20.** (B)	**21.** (D)	**22.** (A)	**23.** (D)	**24.** (D)
25. (B)	**26.** (A), (C)	**27.** (A)	**28.** (A), (B), (C)		**29.** (B), (C)
30. (C)	**31.** (A)	**32.** (A), (C)	**33.** (A), (C)	**34.** (D)	**35.** (B), (C)
36. (A), (B), (C), (D)		**37.** (D)	**38.** (B)	**39.** (A)	**40.** (C)

SOLUTIONS

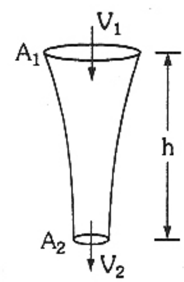
1. **(A, (C)**

The circuit of a common emitter amplifier is as shown below :



This has been shown a n-p-n transistor. Therefore, base-emitter are forward biased and Input signal is connected between base and emitter.

2. **(C)**



From conservation of energy

$$V_2^2 = V_1^2 + 2gh \qquad ...(1)$$

[can also be found by applying Bernouilli's theorem between 1 and 2]

From continuity equation

$$A_1V_1 = A_2V_2; V_2 = \left(\frac{A_1}{A_2}\right) V_1 \qquad ...(2)$$

Substituting value of V_2 from equation (2) in equation (1)

$$\frac{A_1{}^2}{A_2{}^2} . V_1{}^2 = V_1{}^2 + 2gh$$

or $$A_2^2 = \frac{A_1^2V_1^2}{V_1^2 + 2gh}$$

$\therefore$ $$A_2 = \frac{A_1V_1}{\sqrt{V_1{}^2 + 2gh}}$$

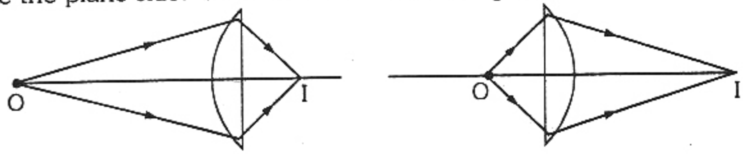
Substituting the given values

$$A_2 = \frac{(10^{-4}\ m^2)(1.0\ m/s)}{\sqrt{(1.0 m/s)^2 + 2(10)(0.15)}}$$

$$A_2 = 5.0 \times 10^{-5}\ m^2$$

3. (B)

In general spherical aberration is minimum when the total deviation produced by the system is equally divided on all refracting surfaces. A planoconvex lens is used for this purpose. In order that the total deviation be equally divided on two surfaces, it is essential that more parallel beam (of the incident and refracted) be incident on the convex side. Thus when the object is far away from the lens, incident rays will be more parallel than the refracted rays, therefore, the object should face the convex side, but if the object is near the lens, the object should face the plane side. This has been shown in figure.



4. (C, D)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\bar{v} = \sqrt{\frac{8}{\pi} \cdot \frac{RT}{M}} \approx \sqrt{\frac{2.5RT}{M}} \text{ and}$$

$$v_p = \sqrt{\frac{2RT}{M}}$$

From these expressions we can see that

$$v_p < \bar{v} < v_{rms}$$

secondly, $v_{rms} = \sqrt{\frac{3}{2}} v_p$

and average kinetic energy of a gas molecule $= \frac{1}{2} m v_{rms}^2$

$$= \frac{1}{2} m \left(\sqrt{\frac{3}{2}} v_p\right)^2 = \frac{3}{4} m v_p^2$$

5. (A)

Average kinetic energy per molecule per degree of freedom $= \frac{1}{2}$ kT. Since both the gases are diatomic and at same temperature (300 K), both will have the same number of rotational degree of freedom i.e. two. Therefore, both the gases will have the same average rotational kinetic energy per molecule $\left(= 2 \times \frac{1}{2} \text{kT or kT}\right)$

Thus ratio will be 1 : 1

6. (B)

Mass per unit length of the string, $m = \frac{10^{-2}}{0.4}$

$$= 2.5 \times 10^{-2} \text{ kg/m}$$

∴ Velocity of wave in the string, $V = \sqrt{T/m} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}}$

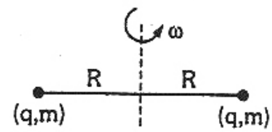
$$V = 8 \text{m/s}$$

For constructive interference between successive pulses :

$$\Delta t_{min} = \frac{2l}{V} = \frac{(2)(0.4)}{8} = 0.10\ s$$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

7. (A)



current, $i =$ (frequency) (charge)

$$= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$$

Magnetic moment, $M = (i)(A)$

$$= \left(\frac{q\omega}{\pi}\right)(\pi R^2)$$

$$= (q\omega R^2)$$

Angular momentum, $L = 2I\omega = 2(mR^2)\omega$

$$\therefore \quad \frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$$

8. (C, D)

For total internal reflection to take place :

Angle of incidence, $i >$ critical angle, θ_c

or $\sin i > \sin\theta_c$

or $\sin 45° > \frac{1}{n}$

or $\frac{1}{\sqrt{2}} > \frac{1}{n}$

or $n > \sqrt{2}$

or $n > 1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

9. (C, D)

Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

$^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of $^{20}_{10}Ne$ nucleus

$$M_1 < 10(m_p + m_n)$$

Also, heavier the nucleus, more is the mass defect.

Thus, $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

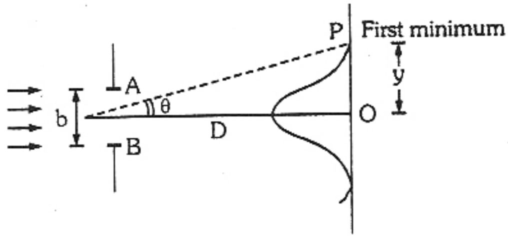
or $10(m_p + m_n) > M_2 - M_1$

or $M_2 < M_1 + 10(m_p + m_n)$

Now since $M_1 < 10(m_p + m_n)$

$\therefore \quad M_2 < 2M_1$

10. (D)



At first minima, $b \sin\theta = \lambda$

or $b\theta = \lambda$

or $b\left(\frac{y}{D}\right) = \lambda$

or $y = \frac{\lambda D}{b}$

or $\frac{yb}{D} = \lambda$ ... (1) $\sin\theta \approx \theta$

Now at P (First minima) path difference between the rays reaching from two edges (A and B) will be

$\Delta x = \frac{yb}{D}$ (Compare with $\Delta x = \frac{yd}{D}$ in YDSE)

or $\Delta x = \lambda$ (From 1)

Corresponding phase difference (ϕ) will be

$$\phi = \left(\frac{2\pi}{\lambda}\right) . \Delta x$$

$$\phi = \frac{2\pi}{\lambda} . \lambda$$

$$\phi = 2\pi$$

11. (A, D)

Time period, $T_n = \frac{2\pi r_n}{V_n}$ (in n^{th} state)

i.e. $T_n \propto \frac{r_n}{V_n}$

But $r_n \propto n^2$

and $V_n \propto \frac{1}{n}$

Therefore $T_n \propto n^3$

Given $T_{n_1} = 8 T_{n_2}$

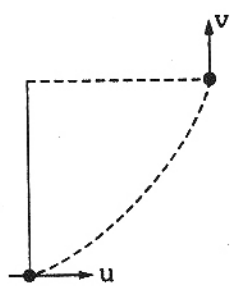
Hence $n_1 = 2n_2$

Therefore, options (A) and (D) are correct.

12. (D)

From energy conservation

$$v^2 = u^2 - 2gL \quad ...(1)$$



Now since the two velocity vectors shown in figure are mutually perpendicular, hence the magnitude of change of velocity will be given by

$$|\Delta \vec{v}| = \sqrt{u^2 + v^2}$$

Substituting value of v^2 from equation (1)

$$|\Delta \vec{v}| = \sqrt{u^2 + u^2 - 2gL}$$

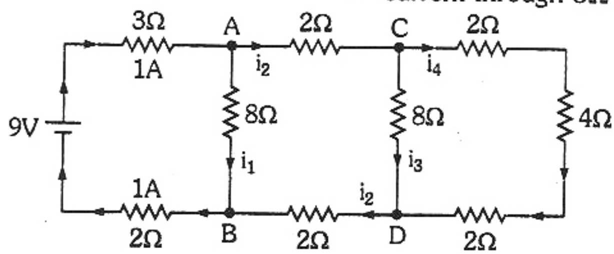
$$|\Delta \vec{v}| = \sqrt{2(u^2 - gL)}$$

13. (D)

Net resistance of the circuit is 9 Ω

$\therefore$ Current drawn from the battery, $i = \frac{9}{9} = 1$ A

= current through 3Ω resistor



Potential difference between A and B is

$$V_A - V_B = 9 - 1(3+2) = 4V = 8i_1$$

$\therefore$ $i_1 = 0.5A$

$\therefore$ $i_2 = 1 - i_1 = 0.5$ A

Similarly, potential difference between C and D

$$V_C - V_D = (V_A - V_B) - i_2(2+2)$$
$$= (4) - 4i_2$$
$$= 4 - 4(0.5)$$
$$= 2V = 8i_3$$

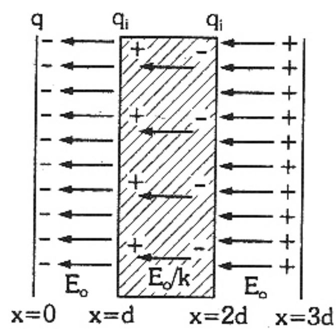
$\therefore$ $i_3 = 0.25A$

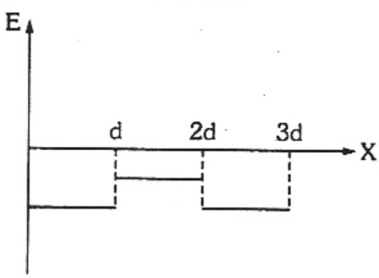
Therefore $i_4 = i_2 - i_3 = 0.5 - 0.25$

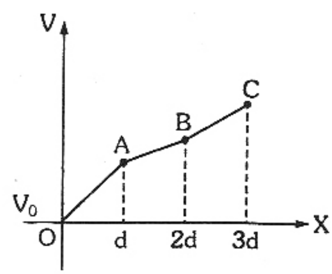
$i_4 = 0.25$ A

14. (B,C)

The magnitude and direction of electric field at different points are shown in figure. The direction of the electric field remains the same. Hence option (B) is correct. Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from x = 0 to x = 3d. Therefore, option (C) is also correct. The variation of electric field (E) and potential (V) with x will be as follows :

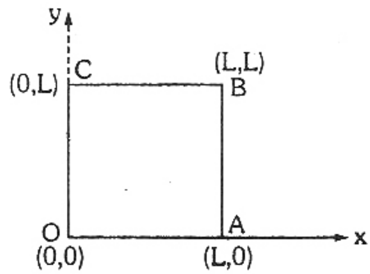






OA || BC and $(\text{Slope})_{OA} > (\text{Slope})_{AB}$
Becouse $E_{o-d} = E_{2d-3d}$ and $E_{o-d} > E_{d-2d}$

15. (B), (C)



Since the edges are clamped, displacement of the edges u (x, y) = 0 for

Line OA i.e. $y = 0$, $0 \le x \le L$
AB i.e. $x = L$, $0 \le y \le L$
BC i.e. $y = L$, $0 \le x \le L$
OC i.e. $x = 0$, $0 \le y \le L$

The above conditions are satisfied only in alternatives (B) and (C).

Note that u (x, y) = 0, for all four values *e.g.* in alternative (D), u (x, y) = 0 for y = 0, y = L but it is not zero for x = 0 or x = L. Similarly in option (A), u (x, y) = 0 at x = L, y = L but it is not zero for x = 0 or y = 0, while in options (B) and (C), u (x, y) = 0 for x = 0, y = 0, x = L and y = L

16. (C)

$$dW = \vec{F} \cdot \vec{ds} \text{ where } \vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

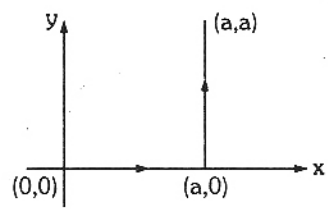
and $$\vec{F} = -K(y\hat{i} + x\hat{j})$$

$\therefore$ $$dW = -K(ydx + xdy) = -k\, d(xy)$$

$\therefore$ $$W = \int_{(0,0)}^{(a,a)} dW = -K\int_{(0,0)}^{(a,a)} d(xy) = -K[xy]_{(0,0)}^{(a,a)}$$

$$W = -Ka^2$$

Alternate Solution



While moving from (0, 0) to (a, 0) along positive x-axis, $y = 0 \therefore \vec{F} = -Kx\hat{j}$ *i.e.* force is in negative y-direction while the displacement is in positive x-direction. Therefore, $W_1 = 0$
(Force ⊥ displacement)

Then it moves from (a, 0) to (a, a) along a line parallel to y-axis (x = + a). During this

$$\vec{F} = -K(y\hat{i} + a\hat{j})$$

The first component of force, $-ky\hat{i}$ will not contribute any work because this component is along negative x-direction $(-\hat{i})$ while displacement is in positive y-direction [(a, 0) to (a, a)].

The second component of force *i.e.* $-Ka\hat{j}$ will perform negative work

$$W_2 = (-ka)(a) = -Ka^2 \quad \begin{bmatrix} \vec{F}_y = -Ka\hat{j} \\ \vec{s} = a\hat{j} \end{bmatrix}$$

$\therefore$ $$W = w_1 + w_2 = -Ka^2$$

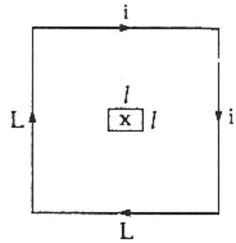
→ *The given force is conservative in nature. Therefore, work done is path independent. It depends only on initial and final positions. Therefore, first method is brief and correct.*

17. (B)

Magnetic field produced by a current i in a large square loop at its centre,

$$B \propto \frac{i}{L}$$

say $\quad B = K\frac{i}{L}$



∴ Magnetic flux linked with smaller loop,

$$\phi = B.S$$

$$\phi = \left(K\frac{i}{L}\right)(l^2)$$

Therefore, the mutual inductance

$$M = \frac{\phi}{i} = K\frac{l^2}{L} \qquad \text{or} \qquad M \propto \frac{l^2}{L}$$

→ *Dimensions of self inductance (L) or mutual inductionce (M) are :*

[Mutual inductance] = [Self inductance]

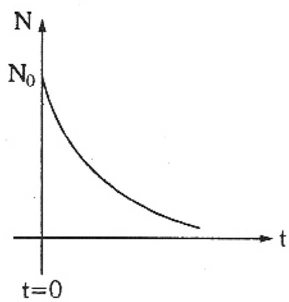
$= [\mu_0]$ [length]

Similarly, Dimensions of capacitance are :

[capacitance] $= [\epsilon_0]$ *[Length]*

From this point of view options B and D may be correct.

18. (D)



Number of nuclei decreases exponentially

$$N = N_0 e^{-\lambda t} \text{ and}$$

Rate of decay $\left(-\frac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto $t = \propto$. Therefore, a given nucleus may decay at any time after $t = 0$

19. (C)

Process is isothermal. Therefore, T = constant. $\left(P \propto \frac{1}{V}\right)$ volume is increasing, therefore, pressure will decrease.

In chamber A →

$$\Delta P = (P_A)_i - (P_A)_f = \frac{n_A RT}{V} - \frac{n_A RT}{2V} = \frac{n_A RT}{2V} \qquad \ldots(1)$$

In chamber B →

$$1.5\,\Delta P = (P_B)_i - (P_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V} = \frac{n_B RT}{2V} \qquad \ldots(2)$$

From (1) and (2)

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3}$$

or $$\frac{m_A/M}{m_B/M} = \frac{2}{3}$$

or $$\frac{m_A}{m_B} = \frac{2}{3}$$

or $$3m_A = 2m_B$$

20. (B)

In isothermal process

$$PV = \text{ constant}$$

$\therefore$ $$PdV + VdP = 0$$

or $$\left(\frac{dP}{dV}\right) = -\left(\frac{P}{V}\right)$$

$\therefore$ $$\text{Bulk modulus, } B = -\left(\frac{dP}{dV/V}\right) = -\left(\frac{dP}{dV}\right) V$$

$\therefore$ $$B = -\left[\left(-\frac{P}{V}\right) V\right] = P$$

$\therefore$ $$B = P$$

→ *Adiabatic bulk modulus is given by $B = \gamma P$*

21. (D)

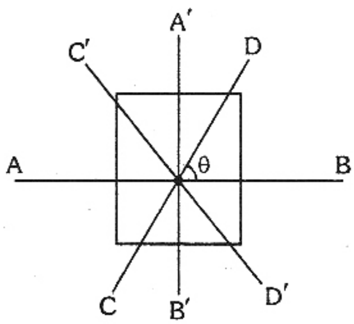
Potential at origin will be given by

$$V = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4}x_0 + ...\right]$$

$$= \frac{q}{4\pi \epsilon_0} \cdot \frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right]$$

$$= \frac{q}{4\pi \epsilon_0 x_0} \ln(2)$$

22. (A)



$A'B' \perp AB$ and $C'D' \perp CD$

From symmetry $I_{AB} = I_{A'B'}$

and $I_{CD} = I_{C'D'}$

From theorem of perpendicular axes,

$$I_{ZZ} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$$

$$= 2I_{AB} = 2I_{CD}$$

$\therefore$ $$I_{AB} = I_{CD}$$

Alternate Solution

The relation between I_{AB} and I_{CD} should be true for all values of θ.

and at $\theta = 0, I_{CD} = I_{AB}$

Similarly at $\theta = \pi/2, I_{CD} = I_{AB}$ (by symmetry)

Keeping these things in mind, only option (A) is correct.

23. (D)

A is free to move, therefore, heat will be supplied at constant pressure

$\therefore \quad dQ_A = nC_P dT_A \quad \ldots(1)$

B is held fixed, therefore heat will be supplied at constant volume.

$\therefore \quad dQ_B = nC_V dT_B \quad \ldots(2)$

But $\quad dQ_A = dQ_B \quad$ (Given)

$\therefore \quad nC_P dT_A = nC_V dT_B$

$\therefore \quad dT_B = \left(\frac{C_P}{C_V}\right) dT_A$

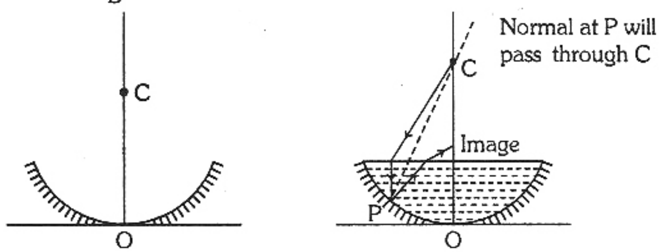
$= \gamma\,(dT_A) \quad \gamma = 1.4$ (diatomic)

$dT_A = 30K$

$= (1.4)\,(30K)$

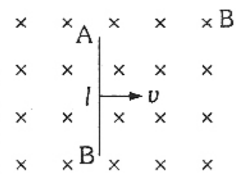
$\therefore \quad dT_B = 42K$

24. (D)



The ray diagram is shown in figure. Therefore, the image will be real and between C and D.

25. (B)



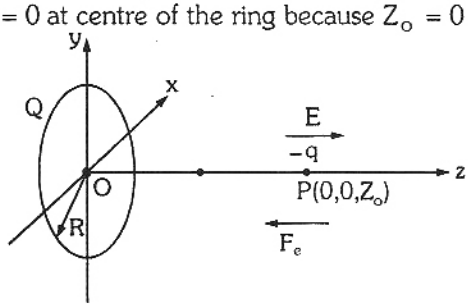
A motional emf, $e = Blv$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.

26. (A, C)

Let θ be the charge on the ring, the negative charge –q is released from point P (0, 0, Z_0). The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{QZ_0}{(R^2 + Z_0^2)^{3/2}}$$

E = 0 at centre of the ring because $Z_o = 0$



Therefore, force on charge P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi \epsilon_0} \cdot \frac{Qq}{(R^2 + Z_0{}^2)^{3/2}} \cdot Z_0 \qquad \ldots(1)$$

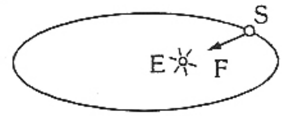
Similarly, when it crosses the origin, the force is again towards centre O. Thus the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.

Secondly if $Z_0 << R$, $(R_2 + Z_o^2)^{3/2} \approx R^3$

$$F_e \approx \frac{1}{4\pi \epsilon_0} \cdot \frac{Qq}{R^3} \cdot Z_0 \qquad \text{(From equation 1)}$$

i.e. the restoring force $F_e \propto - Z_0$. Hence the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position.)

27. (A)



Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

28. (A, B, C)

(A) $\vec{\tau} = \vec{A} \times \vec{L}$

i.e. $\dfrac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$

This relation implies that $\dfrac{d\vec{L}}{dt}$ is perpendicular to both $\vec{A}$ and $\vec{L}$. Therefore, option (A) is correct.

(C) $\vec{L} \cdot \vec{L} = L^2$ Here

Differentiating with repect to time, we get

$$\vec{L} \cdot \frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt} \cdot \vec{L} = 2L\frac{dL}{dt}$$

$$\Rightarrow \quad 2\vec{L} \cdot \frac{d\vec{L}}{dt} = 2L\frac{dL}{dt} \qquad \ldots(1)$$

But since $\vec{L} \perp \dfrac{d\vec{L}}{dt}$

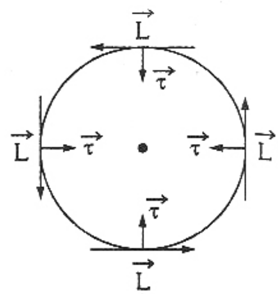
$$\therefore \quad \vec{L} \cdot \frac{d\vec{L}}{dt} = 0$$

Therefore, from equation (1) $\frac{dL}{dt} = 0$

or magnitude of $\vec{L}$ *i.e.* L does not change with time.

(B) So far we are confirm about two points :

(1) $\vec{\tau}$ or $\frac{d\vec{L}}{dt} \perp \vec{L}$ and



(2) $|\vec{L}|$ or L is not changing with time, therefore it is a case when direction of $\vec{L}$ is changing but its magnitude is constant and $\vec{\tau}$ is perpendicular to $\vec{L}$ at all points.

This can be written as :

If $\vec{L} = (a \cos \theta)\ \vec{i} + (a \sin \theta)\ \vec{j}$ Here a = positive constant

then $\vec{\tau} = (a \sin \theta)\ \hat{i} - (a \cos \theta)\ \hat{j}$

so that $\vec{L} \cdot \vec{\tau} = 0$ and $\vec{L} \perp \vec{\tau}$

Now $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$. Thus $\vec{A}$ can be written as

$$\vec{A} = A\hat{k}$$

We can see that $\vec{L} \cdot \vec{A} = 0$

i.e. $\vec{L} \perp \vec{A}$ also.

Thus we can say that component of $\vec{L}$ along $\vec{A}$ is zero or component of $\vec{L}$ along $\vec{A}$ is always constant.

Finally we conclude $\vec{\tau}$, $\vec{A}$ and $\vec{L}$ are always mutually perpendicular.

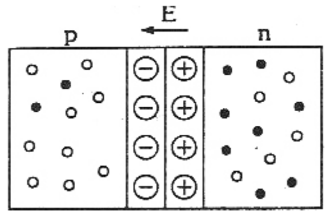
29. (B, C)

There is a decrease is volume during melting on an ice slab at 273K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence option (B) is correct. Secondly heat is absorbed during melting (*i.e.* dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative). Therefore, from first law of thermodynamics

$$dU = dQ - dW$$

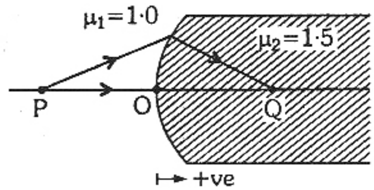
change in internal energy of ice-water system, dU will be positive or internal energy will increase.

30. (C)



At junction a potential barrier/depletion layer is formed as shown, with n-side at higher potential and p-side at lower potential. Therefore, there is an electric field at the junction directed from the n-side to p-side.

31. (A)



Let us say $\quad PO = OQ = X$

Applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values with sign

$$\frac{1.5}{+X} - \frac{1.0}{-X} = \frac{1.5 - 1.0}{+R}$$

(Distances are measured from O and are taken as positive in the direction of ray of light)

$$\therefore \quad \frac{2.5}{X} = \frac{0.5}{R}$$

$$\therefore \quad X = 5R$$

32. (A, C)

Inside the sphere

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^3} r$$

$$\Rightarrow \quad E \propto r \quad \text{for} \quad r \le R$$

i.e. E at centre = 0 $\quad (r = 0)$

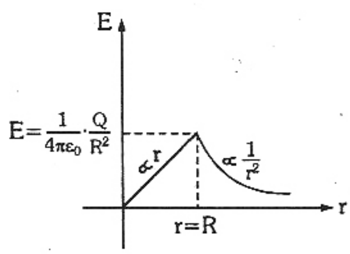
and $\quad$ E at surface $= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^2} \quad (r = R)$

Outside the sphere

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} \quad r \ge R$$

or $\quad E \propto \frac{1}{r^2}$

Thus variation of electric field (E) with distance (r) from the centre will be as follows :



33. (A, C)

Maximum speed of any point on the string = $a\omega$

$$= a\,(2\pi f)$$

$$= v/10 \qquad \text{(Given)}$$

$$= 10/10 \qquad (v = 10\ m/s)$$

$$= 1$$

$\therefore$ $$2\pi af = 1$$

$\therefore$ $$f = \frac{1}{2\pi a}$$

$$a = 10^{-3}\ m \qquad \text{(Given)}$$

$\therefore$ $$f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}\ Hz$$

Speed of wave $v = f\,\lambda$

$\therefore$ $$(10\ m/s) = \left(\frac{10^3}{2\pi}\ S^{-1}\right)\lambda$$

$\therefore$ $$\lambda = 2\pi \times 10^{-2}\ m$$

34. (D)

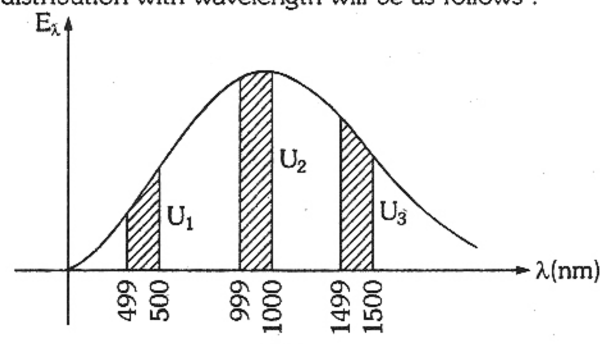
Wein's displacement law is :

$$\lambda_m T = b \qquad (b = \text{wein's constant})$$

$\therefore$ $$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6\ nm\text{-}k}{2880\ nm}$$

$\therefore$ $$\lambda_m = 1000\ nm$$

Energy distribution with wavelength will be as follows :



$\lambda = 1000nm$

From the graph it is clear that

$$U_2 > U_1$$

(In fact U_2 is maximum)

35. (B,C)

$$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$\therefore$ $$[\epsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[IT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4I^2]$$

Speed of light, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

$$\therefore \quad [\mu_0] = \frac{1}{[\epsilon_0][c]^2}$$

$$= \frac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-1}]^2}$$

$$= [MLT^{-2}I^{-2}]$$

36. (A, B, C, D)

(A) $L = \frac{\phi}{i}$ or Henry = $\frac{\text{Weber}}{\text{Ampere}}$

(B) $e = -L\left(\frac{di}{dt}\right)$

$\therefore$ $L = -\frac{e}{(di/dt)}$ or Henry = $\frac{\text{Volt-second}}{\text{Ampere}}$

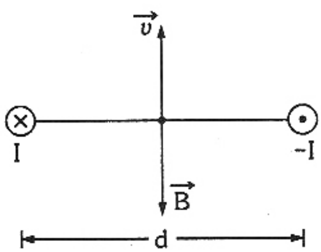
(C) $U = \frac{1}{2}Li^2$

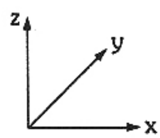
$\therefore$ $L = \frac{2U}{i^2}$ or Henry = $\frac{\text{Joule}}{(\text{Ampere})^2}$

(D) $U = \frac{1}{2}Li^2 = i^2Rt$

$\therefore$ $L \equiv R \,.\, t$ or Henry = ohm-sceond

37. (D)





Wires are in x-y plane
and velocity in z-direction

Net magnetic field due to both the wires will be downward as shown above.

Since angle between $\vec{v}$ and $\vec{B}$ is 180°

Therefore, magnetic force

$$\vec{F}_m = q(\vec{v} \times \vec{B}) = 0$$

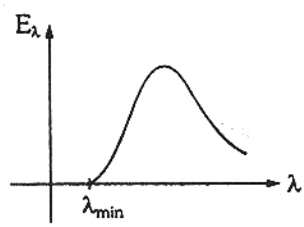
38. (B)

The continuous x-ray spectrum is shown in figure.

All wavelengths > λ_{min} are found, where

$$\lambda_{min} = \frac{12375}{V \text{ (in volts)}} \text{ Å}$$

Here V is the applied voltage.



39. (A)

$$U(x) = k\,|\,x\,|^3$$

$\therefore$
$$[k] = \frac{[U]}{[x^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

Now time period may depend on

$T \propto (\text{mass})^x\ (\text{amplitude})^y\ (k)^z$

$\therefore$
$$[M^0L^0T] = [M]^x\ [L]^y\ [ML^{-1}T^{-2}]^z$$
$$= [M^{x+z}\ L^{y-z}\ T^{-2z}]$$

Equating the powers, we get

$$-2z = 1 \quad \text{or} \quad z = -1/2$$
$$y - z = 0 \quad \text{or} \quad y = z = -1/2$$

Hence $\quad T \propto (\text{amplitude})^{-1/2}$

$$\propto (a)^{-1/2}$$

or
$$T \propto \frac{1}{\sqrt{a}}$$

40. (C)

$$\lambda\ (\text{in A}^\circ) = \frac{12375}{W\,(\text{eV})} = \frac{12375}{4.0}\,\text{Å} \approx 3093\ \text{Å}$$

or $\quad \lambda \approx 309.3$ nm $\quad$ or $\quad$ 310 nm.

⟶ $\lambda\ (in\ A^\circ) = \dfrac{12375}{W\,(eV)}$ *comes from* $W = \dfrac{hc}{\lambda}$

CHEMISTRY - 1998

PART - A

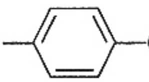
1. *Read questions 1 to 28 carefully and choose from amongst the alternatives given below each question the correct lettered choice (s). A question may have one or more correct alternatives. In order to secure any marks for a given question, all correct lettered alternative(s) must be chosen.*

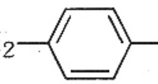
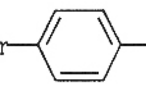
1. Which of the following statement(s) is(are) correct when a mixture of NaCl and $K_2Cr_2O_7$ is gently warmed with conc. H_2SO_4 :

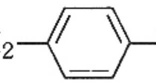
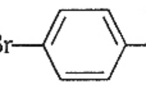
(A) A deep red vapour is evolved.
(B) The vapour when passed into NaOH solution gives a yellow solution of Na_2CrO_4
(C) Chlorine gas is evolved
(D) Chromyl chloride is formed.

2. Highly pure dilute solution of sodium in liquid ammonia :

(A) shows blue colour
(B) exhibits electrical conductivity
(C) produces sodium amide
(D) produces hydrogen gas

3. The reaction $CH_3CH = CH$—  —OH with HBr gives :

(A) $CH_3CHBrCH_2$—  —OH
(B) CH_3CH_2CHBr—  —OH

(C) $CH_3CHBrCH_2$—  —Br
(D) CH_3CH_2CHBr—  —Br

4. p-Chloroaniline and anilinium hydrochloride can be distinguished by :

(A) Sandmeyer reaction
(B) $NaHCO_3$
(C) $AgNO_3$
(D) Carbylamine test

5. The energy of an electron in the first Bohr orbit of H atom is -13.6 eV. The possible energy value(s) of the excited state(s) for electrons in Bohr orbits of hydrogen is(are) :

(A) -3.4 eV
(B) -4.2 eV
(C) -6.8 eV
(D) $+6.8$ eV

6. In nitroprusside ion the iron and NO exist as Fe^{II} and NO^+ rather than Fe^{III} and NO. These forms can be differentiated by :

(A) estimating the concentration of iron.
(B) measuring the concentration of CN.
(C) measuring the solid state magnetic moment.
(D) thermally decomposing the compound.

7. Which of the following statement(s) is(are) correct :

(A) The coordination number of each type of ion in CsCl crystal is 8.

(B) A metal that crystallizes in bcc structure has a coordination number of 12.

(C) A unit cell of an ionic crystal shares some of its ions with other unit cells.

(D) The length of the unit cell in NaCl is 552 pm. (r_{Na^+} = 95 pm; r_{Cl^-} = 181 pm)

8. Sodium nitrate decomposes above –800°C to give :

(A) N_2 (B) O_2

(C) NO_2 (D) Na_2O

9. Which of the following statement(s) is (are) correct with reference to the ferrous and ferric ions :

(A) Fe^{3+} gives brown colour with potassium ferricyanide.

(B) Fe^{2+} gives blue precipitate with potassium ferricyanide.

(C) Fe^{3+} gives red colour with potassium thiocyanate.

(D) Fe^{2+} gives brown colour with ammonium thiocyanate.

10. Which of the following statement(s) is(are) correct :

(A) The electronic configuration of Cr is [Ar] $3d^5 4s^1$. (Atomic Number of Cr = 24).

(B) The magnetic quantum number may have a negative value.

(C) In silver atom, 23 electrons have a spin of one type and 24 of the opposite type. (Atomic Number of Ag = 47)

(D) The oxidation state of nitrogen in HN_3 is –3.

11. A new carbon-carbon bond formation is possible in :

(A) Cannizzaro reaction (B) Friedel-Crafts alkylation

(C) Clemmensen reduction (D) Reimer-Tiemann reaction

12. White phosphorus (P_4) has :

(A) six P–P single bonds (B) four P–P single bonds

(C) four lone pairs of electrons (D) PPP angle of 60°.

13. Which of the following will react with water :

(A) $CHCl_3$ (B) Cl_3CCHO

(C) CCl_4 (D) $ClCH_2CH_2Cl$

14. The standard reduction potential values of three metallic cations, X, Y, Z are 0.52, –3.03 and –1.18V respectively. The order of reducing power of the corresponding metals is :

(A) $Y > Z > X$ (B) $X > Y > Z$

(C) $Z > Y > X$ (D) $Z > X > Y$

15. Among the following compounds, which will react with acetone to give a product containing $> C = N -$:

(A) $C_6H_5NH_2$ (B) $(CH_3)_3N$

(C) $C_6H_5NHC_6H_5$ (D) $C_6H_5NHNH_2$

16. Which of the following compounds will show geometrical isomerism :

(A) 2-butene (B) propene

(C) 1-phenylpropene (D) 2-methyl-2-butene

17. The geometry and the type of hybrid orbital present about the central atom in BF_3 is :

(A) linear, sp (B) trigonal planar, sp^2

(C) tetrahedral, sp^3 (D) pyramidal, sp^3

18. Benzyl chloride ($C_6H_5CH_2Cl$) can be prepared from toluene by chlorination with :

(A) SO_2Cl_2 (B) $SOCl_2$

(C) Cl_2 (D) NaOCl

19. Which of the following will undergo aldol condensation :

(A) acetaldehyde (B) propanaldehyde

(C) benzaldehyde (D) trideuteroacetaldehyde

20. Addition of high proportions of manganese makes steel useful in making rails of railroads, because manganese :

(A) gives hardness to steel.

(B) helps the formation of oxides of iron.

(C) can remove oxygen and sulphur.

(D) can show highest oxidation state of +7.

21. Decrease in atomic number is observed during :

(A) alpha emission (B) beta emission

(C) positron emission (D) electron capture.

22. Benzenediazonium chloride on reaction with phenol in weakly basic medium gives :

(A) diphenyl ether (B) p-hydroxyazobenzene

(C) chlorobenzene (D) benzene

23. Among the following compounds, the strongest acid is :

(A) $HC \equiv CH$ (B) C_6H_6

(C) C_2H_6 (D) CH_3OH

24. For a first order reaction :

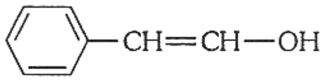
(A) the degree of dissociation is equal to $(1 - e^{-kt})$.

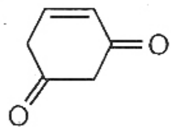
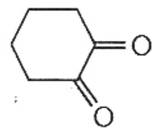
(B) a plot of reciprocal concentration of the reactant vs. time gives a straight line.

(C) the time taken for the completion of 75% reaction is thrice the $t_{1/2}$ of the reaction.

(D) the pre-exponential factor in the Arrhenius equation has the dimension of time, T^{-1}.

25. Tautomerism is exhibited by :

(A)    (B) 

(C)    (D) 

26. According to Graham's law, at a given temperature the ratio of the rates of diffusion r_A/r_B of gases A and B is given by :

(A) $(P_A/P_B)\,(M_A/M_B)^{1/2}$

(B) $(M_A/M_B)\,(P_A/P_B)^{1/2}$

(C) $(P_A/P_B)\,(M_B/M_A)^{1/2}$

(D) $(M_A/M_B)\,(P_B/P_A)^{1/2}$

(Where P and M are pressures and molecular weights of gases A and B respectively.)

27. For the reaction $CO(g) + H_2O(g)$ $\quad$ $CO_2(g) + H_2(g)$ at a given temperature the equilibrium amount of $CO_2(g)$ can be increased by :

(A) adding a suitable catalyst.
(B) adding an inert gas.
(C) decreasing the volume of the container.
(D) increasing the amount of CO (g).

28. Which of the following statement(s) is(are) correct :

(A) The pH of 1.0×10^{-8} M solution of HCl is 8.

(B) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}.

(C) Autoprotolysis constant of water increases with temperature.

(D) When a solution of a weak monoprotic acid is titrated against a strong base, at half-neutralisation point $pH = (1/2)\, pK_a$.

ASSERTION-REASON TYPE QUESTIONS

Directions : *The questions below (29 to 40) consist of an assertion in column 1 and the reason in column 2. Against the specific question number, write in the appropriate space.*

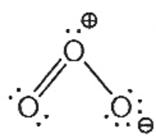
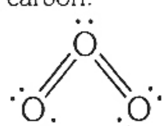
(A) If both *assertion* and *reason* are correct, and *reason* is the correct explanation of the *assertion.*

(B) If both *assertion* and *reason* are correct, but *reason* is not the correct explanation of the *assertion.*

(C) If *assertion* is correct but *reason* is incorrect.

(D) If *assertion* is incorrect but *reason* is correct.

Example :

	Assertion	Reason
	F–F bond in F_2 molecule is strong. Answer : (D)	F atom is small in size.
29.	Benzonitrile is prepared by the reaction of chlorobenzene with potassium cyanide.	Cyanide (CN^-) is a strong nucleophile.
30.	F atom has a less negative electron affinity than Cl atom.	Additional electrons are repelled more effectively by 3p electrons in Cl atom than by 2p electrons in F atom.
31.	Nuclide $^{30}_{13}Al$ is less stable than $^{40}_{20}Ca$.	Nuclides having odd number of protons and neutrons are generally unstable.
32.	Al $(OH)_3$ is amphoteric in nature.	Al—O and O—H bonds can be broken with equal ease in $Al(OH)_3$.
33.	The value of Van der Waals' constant '*a*' is larger for ammonia than for nitrogen.	Hydrogen bonding is present in ammonia.
34.	Zn^{2+} is diamagnetic.	The electrons are lost from 4s orbital to form Zn^{2+}.
35.	Addition of Br_2 to 1 –butene gives two optical isomers.	The product contains one asymmetric carbon.
36.	The electronic structure of O_3 is 	 structure is not allowed because octet around O cannot be expanded.
37.	LiCl is predominantly a covalent compound.	Electronegativity difference between Li and Cl is too small.
38.	HNO_3 is a stronger acid than HNO_2.	In HNO_3 there are two nitrogen-to-oxygen bonds whereas in HNO_2 there is only one.
39.	Sulphate is estimated as $BaSO_4$ and not as $MgSO_4$.	Ionic radius of Mg^{2+} is smaller than that of Ba^{2+}.
40.	Acetic acid does not undergo haloform reaction.	Acetic acid has no alpha hydrogens.

ANSWERS

1. (B), (C), (D)	**2.** (A), (B)	**3.** (B)	**4.** (C)	**5.** (A)	**6.** (C)
7. (A), (C), (D)	**8.** (A), (B)	**9.** (B), (C)	**10.** (A), (B), (C)	**11.** (B), (D)	**12.** (A), (C), (D)
13. (B)	**14.** (A)	**15.** (A), (D)	**16.** (A), (C)	**17.** (B)	**18.** (C)
19. (A), (B), (D)	**20.** (A), (C)	**21.** (A), (C), (D)	**22.** (B)	**23.** (D)	**24.** (A), (D)
25. (A), (C), (D)	**26.** (C)	**27.** (D)	**28.** (B), (C)	**29.** (D)	**30.** (C)
31. (C)	**32.** (A)	**33.** (A)	**34.** (B)	**35.** (A)	**36.** (A)
37. (C)	**38.** (A)	**39.** (B)	**40.** (C)		

SOLUTIONS

Reason of correctness

1. The reactions are

$$4NaCl + K_2Cr_2O_7 + 6H_2SO_4 \rightarrow 2CrO_2Cl_2 + 4NaHSO_4 + 2KHSO_4 + 3H_2O$$

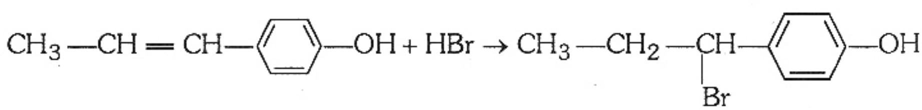
$$\underset{\text{chromyl chloride}}{CrO_2Cl_2} + 4NaOH \rightarrow \underset{\text{yellow solution}}{Na_2CrO_4} + 2NaCl + 2H_2O$$

Ans. (B), (C), (D)

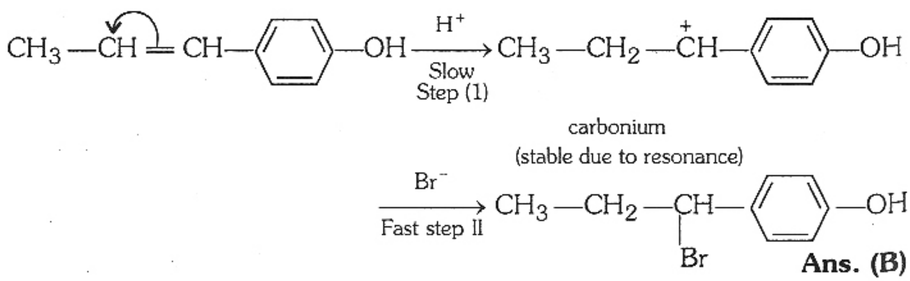
2. $$Na + (x + y)\, NH_{3(l)} \rightarrow \underset{\text{Solvated cation}}{[Na(NH_3)_x]^+} + \underset{\substack{\text{Solvated electrons} \\ \text{(Blue colour)}}}{(l^{\circ} \cdot y(NH_3))^-}$$

Due to formation of solvated electron it shows blue colour and electrical conductance exhibits due to both ions. **Ans. (A) & (B)**

3. The reaction of $CH_3—CH = CH—$$—OH$ with HBr is given as follows.



The mechanism of this reaction is represented as follows.



Ans. (B)

4. Anilium hydrochloride gives the white precipitates of AgCl with $AgNO_3$.

$$C_6H_5NH_3{}^+Cl^- + AgNO_3 \rightarrow C_6H_5NH_3{}^+NO_3^- + AgCl\downarrow$$ **Ans. (C)**

5. The energy of an electron on Bohr orbits of hydrogen atoms is given by the expression.

$$E_n = -\frac{\text{Constant}}{n^2}$$

Where n takes only integral values. For the first Bohr orbit, n = 1 and it is given that $E_1 = -13.6$ eV

$$\text{Hence } E_n = -\frac{13.6 \text{ eV}}{n^2}$$

of the given values of energy, only –3.4 eV can be obtained by substituting n = 2 in the above expression. **Ans. (A)**

6. The magnetic moment (μ) of a species is related to its number of unpaired electrons (n) in form of following expressions.

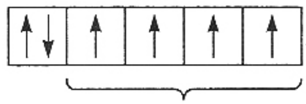
$$\mu = \sqrt{n(n+2)} \text{ B.M.}$$

(B.M. = Bohr Magnetons)

The number of unpaired electrons in the given pairs are as follows :

$Fe^{2+} = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^6\ 3d^6$ or

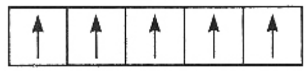
3*d*



n = 4

$Fe^{3+} = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^6\ 3p^5$ or

3*d*



n = 5

$\overset{+}{N}O$ or $\overset{+}{\underset{\times\times}{N}}=\ddot{O}:$ n = 0

NO or $\overset{\times}{\underset{\times}{}}N \overset{\times\bullet\bullet}{=} O:$ n = 1

This given combinations differ in the number of unpaired electrons. Hence these can be differentiated by the measurement on the solid state magnetic moment of nitroprusside ion. **Ans. (C)**

7. The crystals of CsCl has body-centred cubic unit cell. Hence, each ion in this structure has coordination number of eight.

In case of crystals of NaCl two interpenetrating face-centred cubic lattices are present is these one composed entirely of Na^+ ions and the other of Cl^- ions. Each Na^+ ion is located half way between two Cl^- ions and viceversa. A unit cell of NaCl crystal has Cl^- ions at the corners as well as at the face centres and Na^+ ions are located in octahedral voids. On each edge of cubic unit cell, there are two Cl^- ions and one Na^+ ions.

$$\text{Hence } a = 2(r_{Na^+} + r_{Cl^-})$$

$$= 2(95 \text{ pm} + 181 \text{ pm}) = 552 \text{ pm}$$ **Ans. (A), (C), (D)**

8. Sodium nitrate on decomposition upto 500°C to give $NaNO_2$ and oxygen.

$$2NaNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2 \uparrow$$

while at higher temperature (i.e. above to 800°C) this $NaNO_2$ is decomposed into N_2O, N_2 and O_2.

$$2NaNO_2 \xrightarrow{800°C} Na_2O + 3/2\ O_2 \uparrow + N_2 \uparrow$$ **Ans. (A), (B)**

9. The blue precipitate of Fe^{2+} ions with potassium ferricyanide is due to formation of **Turnbull's blue** K Fe^{II} $[Fe^{III}(CN)_6]$

$$Fe^{2+} + K_3[Fe(CN)_6] \rightarrow \underset{\text{Potassium Ferro Ferricyanide}}{K.Fe^{II}[Fe^{III}(CN)_6]} + 2K^+$$

The red colouration of Fe^{3+} ions with potassium thiocyanate is due to the formation of $[Fe(CNS)]^{2+}$.

$$Fe^{3+} + 3KCNS \rightarrow \underset{\substack{\text{Ferric thiocyanate} \\ \text{(Red colour)}}}{[Fe(CNS)_3]} + 3K^+$$

Ans. (B), (C)

10. $_{24}Cr = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^6\ 3d^5, 4s^1$

$= [Ar]\ 3d^5, 4s^1$

For magnetic quantum number (m) negative value be possible.

For s-subhell $l = 0$, hence $m = 0$

for p-subshell $l = 1$ hence $m = -1, 0, +1$

$$_{47}Ag = 1s^2, 2s^2\ 2p^6, 3s^2 3p^6\ 3d^{10}, 4s^2\ 4p^6\ 4d^{10}, 5s^1$$

Hence 23 electrons have a spin of one type and 24 of the opposite type.

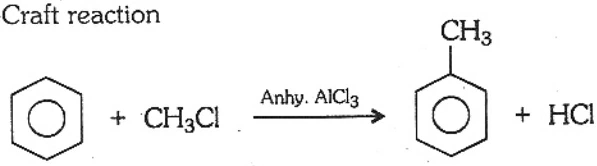
Oxidation state of N in HN_3 is $-1/3$. **Ans. (A), (B), (C)**

11. In Cannizzaro's reactions

$$2HCHO + NaOH_{(conc)} \longrightarrow CH_3OH + HCOONa$$

new C—C bond is not formed.

In Friedal-Craft reaction



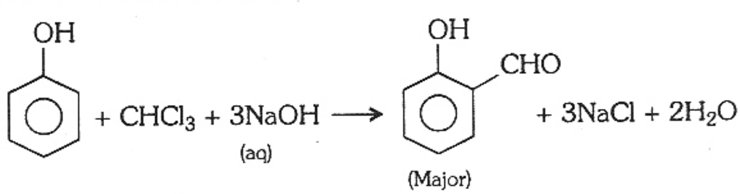
new C—C bond is formed

In Clemmenson reduction

$$\underset{H}{\overset{R}{>}}C=O + 4H \xrightarrow[+conc-HCl]{Zn-Hg} R-CH_3 + H_2O$$

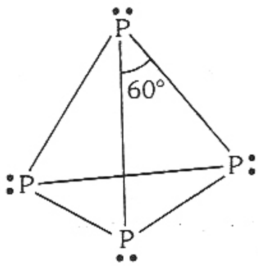
new C—C bond is not formed.

In Reimer Tiemann reactions



New C—C bond is formed. **Ans. (B), (D)**

12. The structure of P_4 is tetrahedral which is given as follows.



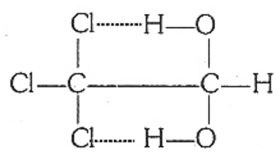
Hence six P — P single bonds, four lone pairs of electrons and P—P—P angle of 60° are present.

Ans. (A), (C), (D)

13. CCl_3CHO reacts with water to give stable chloral monohydrate.

$$CCl_3CHO \xrightarrow{H_2O} CCl_3CH(OH)_2$$

or



chloral monohydrate
(stability is due to intramolecular H-bonding)

Ans. (B)

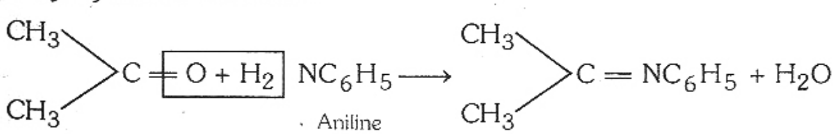
14. On decreasing the negative value of reducing electrode potential of metals, the reducing character is regularly decrease due to decreasing cation formation tendency. (i.e. electron donating tendency)

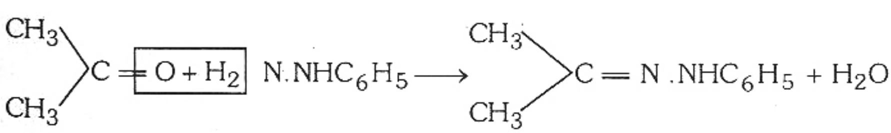
X,	Y,	Z
+0.55	–3.03	–1.18 V

Hence correct order Y > Z > X

Ans. (A)

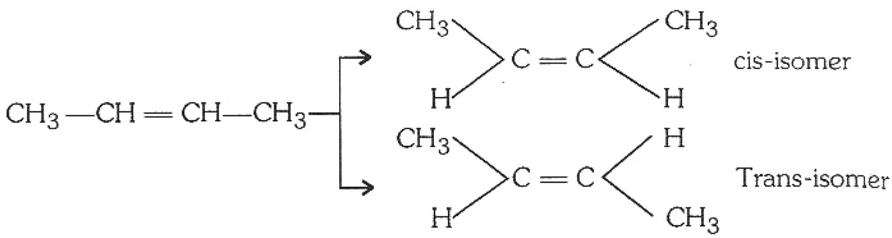
15. Carbonyl compound (acetone) forms condensation product with hydrazine, phenyl hydrazine aniline etc.



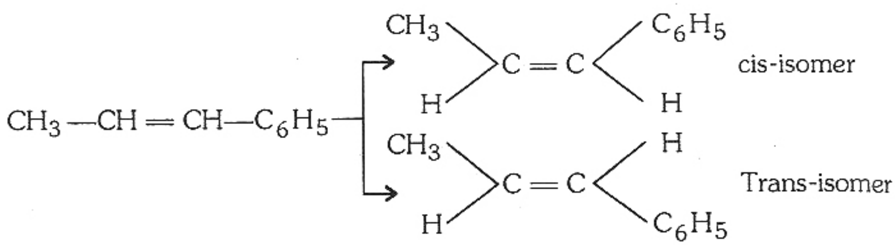


Hence in these reaction > C = N bonds are formed in products.

Ans. (A),(D)

16. 

$CH_3-CH=CH_2$ does not show the property of geometrical isomerism.

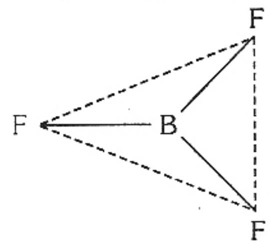


$CH_3-\underset{\substack{|\\CH_3}}{C}=CH-CH_3$ does not show the property of geometrical isomerism. **Ans. (A), (C)**

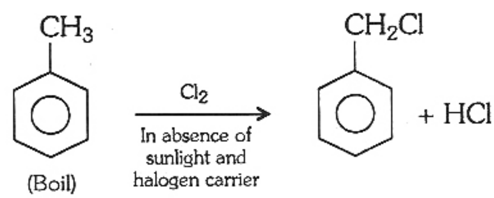
17. In BF_3, Boron is sp^2–hybrid.

$$\underset{5}{B} = 1s^2,\ \underbrace{2s^1\ 2p_x^1\ 2p_y^1}_{sp^2\text{—hybridisation}}$$

These three sp^2–hybrid orbitals are attached to each other trigonally with an angle of $120°$ and they are overlapped with three p-orbitals of three F-atoms on their axes. Hence the geometry of BF_3 molecule is trigonal planar.



Ans. (B)

18. 

Ans. (C)

19. Those aldehydes give aldol condensation which have α-hydrogen atoms.

$$2\overset{\alpha}{C}H_3-CHO \xrightarrow[\text{Dil}]{\text{NaOH}} CH_3-\underset{\displaystyle OH}{\underset{|}{C}H}-CH_2-CHO$$

All — Acetaldol

$$CH_3-CH_2-CH\boxed{O+H}-\underset{\displaystyle CH_3}{\underset{|}{C}H}-CH_3 \xrightarrow[\text{Dil}]{\text{NaOH}} CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{C}H}-\underset{\displaystyle CH_3}{\underset{|}{C}H}-CH_3$$

Propanaldehyde

Benzaldehyde (C_6H_5CHO) does not have α-hydrogen atom.

$$2CD_3CHO \xrightarrow[\text{Dil}]{\text{NaOH}} CD_3-\underset{\displaystyle OD}{\underset{|}{C}H}-CD_2CHO$$

Ans. (A), (B), (D)

20. Manganese (Mn) imparts hardness to steel as well as removes oxygen and sulphur from steel by forming slag as $MnSiO_3$.

$$Fe_2O_3 + 3Mn \rightarrow 3MnO + 2Fe$$

$$MnO + SiO_2 \rightarrow \underset{\text{(slag)}}{MnSiO_3}$$

Ans. (A), (C)

21.

$${}_ZX^A \xrightarrow{-\alpha} {}_{Z-2}Y^{A-4} \quad (\alpha\text{-emission})$$

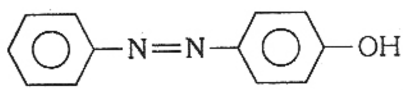
$${}_ZX^A \xrightarrow{-\beta} {}_{Z+1}Z^A \quad (\beta\text{-emission})$$

$${}_ZX^A \xrightarrow{{}_{-1}l^0} {}_{Z-1}D^A \quad (\text{positron-emission})$$

$${}_ZX^m + {}_{-1}e^0 \rightarrow {}_{Z-1}D^A \quad (\text{Electron-emission})$$

Ans. (A), (C), (D)

22. $C_6H_5N_2Cl + C_6H_5OH \xrightarrow[\text{Coupling reaction}]{OH^-}$



p-Hydroxyazobenzene

Ans. (B)

23. Order of acidic strength

$$CH_3OH > CH \equiv CH > C_6H_6 > C_2H_6$$

Although all are neutral towards the litmus paper. **Ans. (D)**

24. In first order reaction, if α is the degree of dissociation therefore

$$k = t \log_e \frac{1}{(1-\alpha)}$$

or $$kt = \log_e \frac{1}{(1-\alpha)} = -\log_e (1-\alpha)$$

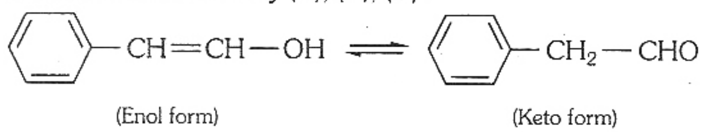
or $$e^{-kt} = 1 - \alpha$$

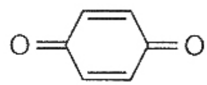
$\therefore$ $$\alpha = \mathbf{1 - e^{-kt}}$$

Plot of reciprocal concentration of the reactant Vs.time is linear for completion of 75%of reaction is first order, $2 \times T_{1/2}$ time is required for first order reaction dimensions of pre-exponential factor 'A' are dimensions of **k i.e. T^{-1}**

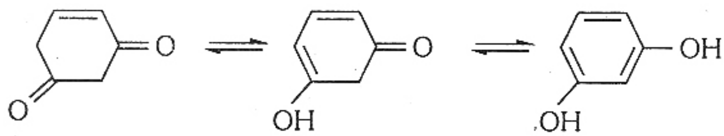
Ans. (A), (D)

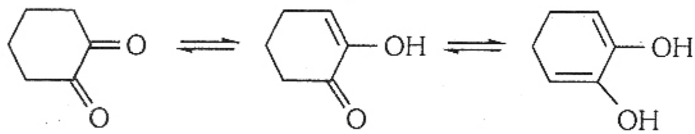
25. Tautomerism is exhibited by (A), (C), (D).





Tautomerism is not possible





Ans. (A), (C), (D)

26. According to Graham's law of diffusion for two gases undergoing diffusion at different pressures through same hole.

$$\frac{r_A}{r_B} = \sqrt{\frac{M_B}{M_A}} \times \frac{P_A}{P_B} \quad \left(r \propto P \times \sqrt{\frac{1}{M}} \text{ At constant temperature.} \right)$$

Ans. (C)

27. $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$

$$K_C = \frac{[CO_2][H_2]}{[CO][H_2O]}$$

A catalyst simply helps in attaining the equilibrium earlier.
Addition of inert gas has no effect on a reaction because in it $\Delta n = 0$.
This equilibrium is not based upon volume because in it $\Delta n = 0$.

On increasing the amount of CO, K_C will be decreased but it is constant at constant temperature, so for maintaining the constant value of K_C, the amount of CO_2 increases. **Ans. (D)**

28. pH of 1×10^{-8} M is below to 7.

$$H_2PO_4^- + H_2O \rightleftharpoons HPO_4^{2-} + H_3O^+$$

conjugate base of $H_2PO_4^-$ acid

$$H_2O + H_2O \rightleftharpoons OH^- + H_3O^+$$

K (Auto protolysis constant of water i.e. with ionic product of water.) increases with temperature.

For half neutralisation of a weak acid by a weak base $pH = pK_a + \log \frac{[salt]}{[acid]}$

$$[Salt] = [Acid] \quad \therefore pH = pK_a$$

MATHEMATICS - 1998

PART - A

Directions : *Read questions 1 to 40 carefully and choose from amongst the alternatives given below each question the correct lettered choice(s). A question may have ONE OR MORE correct alternatives. In order to secure any marks for a given question, ALL correct lettered alternative(s) must be chosen.*

1. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals :

(A) $128\,\omega$ (B) $-128\,\omega$

(C) $128\,\omega^2$ (D) $-128\,\omega^2$.

2. Let T_r be the r^{th} term of an A.P., for $r = 1, 2, 3, \ldots\ldots$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals :

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$

(C) 1 (D) 0

3. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is :

(A) at least 30 (B) at most 20

(C) exactly 25 (D) none of the above

4. The diagonals of a parallelogram *PQRS* are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then *PQRS* must be a :

(A) rectangle (B) square

(C) cyclic quadrilateral (D) rhombus.

5. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is :

(A) 0 (B) 1

(C) 3 (D) 4

6. Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is :

(A) 1 (B) 2

(C) 0 (D) $\frac{1}{2}$

7. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals :

(A) 8 (B) 6

(C) 10 (D) 12

8. If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then :

(A) $a = 2, b = 4$
(B) $a = 3, b = 4$
(C) $a = 2, b = 3$
(D) $a = 3, b = 5$

9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then :

(A) $\alpha = 1, \beta = -1$
(B) $\alpha = 1, \beta = \pm 1$
(C) $\alpha = -1, \beta = \pm 1$
(D) $\alpha = \pm 1, \beta = 1$

10. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is :

(A) $\frac{13}{32}$
(B) $\frac{1}{4}$
(C) $\frac{1}{32}$
(D) $\frac{3}{16}$

11. The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals :

(A) i
(B) $i - 1$
(C) $-i$
(D) 0

12. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2}x)$ attains its maximum is :

(A) 0
(B) 1
(C) 2
(D) infinite

13. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f :

(A) does not exist because f is unbounded.
(B) is not attained even though f is bounded
(C) is equal to 1
(D) is equal to -1

14. Number of divisors of the form $4n + 2$ $(n \geq 0)$ of the integer 240 is :

(A) 4
(B) 8
(C) 10
(D) 3

15. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$:

(A) exists and it equals $\sqrt{2}$
(B) exists and it equals $-\sqrt{2}$
(C) does not exist because $x - 1 \to 0$
(D) does not exist because left hand limit is not equal to right hand limit

16. If in a triangle PQR, $\sin P$, $\sin Q$, $\sin R$ are in A. P., then :

(A) the altitudes are in A. P.
(B) the altitudes are in H. P.
(C) the medians are in G. P.
(D) the medians are in A. P.

17. If $a_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{^nC_r}$ equals :

(A) $(n-1)a_n$
(B) na_n
(C) $\frac{1}{2}na_n$
(D) None of the above

18. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/(are) always rational point(s).

(A) centroid
(B) incentre
(C) circumcentre
(D) orthocentre

(A rational point is a point both of whose co-ordinates are rational numbers)

19. The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is :

(A) 0
(B) 1
(C) 2
(D) infinite.

20. If $x > 1, y > 1, z > 1$ are in G. P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in :

(A) A.P.
(B) H.P.
(C) G.P.
(D) None of the above

21. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is :

(A) 0
(B) 5
(C) 6
(D) 10

22. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is :

(A) 5
(B) 4
(C) 3
(D) 2

23. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then :

(A) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
(B) $f(x) = \sin x, g(x) = |x|$
(C) $f(x) = x^2, g(x) = \sin\sqrt{x}$
(D) f and g cannot be determined

24. Let $A_0 A_1 A_2 A_3 A_4 A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is :

(A) $\frac{3}{4}$
(B) $3\sqrt{3}$
(C) 3
(D) $\frac{3\sqrt{3}}{2}$

25. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$

(B) $(\vec{v} \times \vec{w}) \cdot \vec{u}$

(C) $\vec{v} \cdot (\vec{u} \times \vec{w})$

I(D) $(\vec{u} \times \vec{v}) \cdot \vec{w}$

26. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is :

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

27. Let $h(x) = \min \{x, x^2\}$, for every real number of x. Then :

(A) h is continuous for all x

(B) h is differentiable for all x

(C) $h'(x) = 1$, for all $x > 1$

(D) h is not differentiable at two values of x

28. If $f(x) = 3x - 5$, then $f^{-1}(x)$:

(A) is given by $\frac{1}{3x - 5}$

(B) is given by $\frac{x + 5}{3}$

(C) does not exist because f is not one-one

(D) does not exist because f is not onto.

29. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then.

(A) $P(E/F) + P(\overline{E}/F) = 1$

(B) $P(E/F) + P(E/\overline{F}) = 1$

(C) $P(\overline{E}/F) + P(E/\overline{F}) = 1$

(D) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

30. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then :

(A) $x = 3, y = 1$

(B) $x = 1, y = 3$

(C) $x = 0, y = 3$

(D) $x = 0, y = 0$

31. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :

(A) $\frac{1}{2}$

(B) $\frac{1}{32}$

(C) $\frac{31}{32}$

(D) $\frac{1}{5}$

32. An $n-$ digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is :

(A) 6 (B) 7

(C) 8 (D) 9

33. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals :

(A) $\frac{1}{2}$ (B) $\frac{7}{15}$

(C) $\frac{2}{15}$ (D) $\frac{1}{3}$

34. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then :

(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$

(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n^2 - 3n + 3$

35. Which of the following number(s) is/are rational ?

(A) $\sin 15°$ (B) $\cos 15°$

(C) $\sin 15° \cos 15°$ (D) $\sin 15° \cos 75°$

36. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then :

(A) $x_1 + x_2 + x_3 + x_4 = 0$ (B) $y_1 + y_2 + y_3 + y_4 = 0$

(C) $x_1 x_2 x_3 x_4 = c^4$ (D) $y_1 y_2 y_3 y_4 = c^4$

37. If E and F are events with $P(E) \leq P(F)$ and $P(E \cap F) > 0$, then :

(A) occurrence of $E \Rightarrow$ occurrence of F

(B) occurrence of $F \Rightarrow$ occurrence of E

(C) non-occurrence of $E \Rightarrow$ non-occurrence of F

(D) none of the above implications holds

38. Which of the following expressions are meaningful question

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

(C) $(\vec{u} \cdot \vec{v})\, \vec{w}$ (D) $\vec{u} \times (\vec{v} \cdot \vec{w})$

39. If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$, then the value of $f(1)$ is :

(A) $\frac{1}{2}$ (B) 0

(C) 1 (D) $-\frac{1}{2}$

40. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then :

(A) h is increasing whenever f is increasing

(B) h is increasing whenever f is decreasing

(C) h is decreasing whenever f is decreasing

(D) nothing can be said in general.

ANSWERS

1. (D)	**2.** (C)	**3.** (C)	**4.** (D)	**5.** (B)	**6.** (A)
7. (C)	**8.** (C)	**9.** (D)	**10.** (A)	**11.** (B)	**12.** (A)
13. (D)	**14.** (A)	**15.** (D)	**16** (B)	**17.** (C)	**18.** (A)
19. (C)	**20.** (B)	**21.** (C)	**22.** (C)	**23.** (A)	**24.** (C)
25. (C)	**26.** (B)	**27.** (A), (C), (D)	**28.** (B)	**29.** (A), (D)	**30.** (D)
31. (A)	**32.** (B)	**33.** (B)	**34.** (B)	**35.** (C)	
36. (A), (B), (C), (D)		**37.** (D)	**38.** (A), (C)	**39.** (A)	**40.** (A), (C)

SOLUTIONS

1. $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

$= (-2\omega^2)^7 = (-2)^7 (\omega^2)^7 = -128 \cdot \omega^{14} = -128\,\omega^2$

Therefore, (D) is the Ans.

2. Let $T_m = a + (m-1)\,d = \dfrac{1}{n}$

and $T_n = a + (n-1)\,d = \dfrac{1}{m}$

$\Rightarrow \quad (m-n)\,d = \dfrac{1}{n} - \dfrac{1}{m} = \dfrac{m-n}{mn} \quad \Rightarrow \quad d = \dfrac{1}{mn}$

Again $T_{mn} = a + (mn-1)\,d$

$= a + (mn - n + n - 1)\,d$

$= a + (n-1)\,d + (mn-n)\,d$

$= T_n + n(m-1) \cdot \dfrac{1}{mn}$

$= \dfrac{1}{m} + \dfrac{(m-1)}{m} = \dfrac{1}{m} + 1 - \dfrac{1}{m} = 1$

Therefore, (C) is the Ans.

3. Let number of newspaper which are read be n. Then

$60n = 300 \times 5$

$\Rightarrow \quad n = 25$

Therefore, (C) is the Ans.

4. Slope of $x + 3y = 4$ is $-1/3$

and slope of $6x - 2y = 7$ is 3.

therefore, these two lines are perpendicular which show that both diagonals are perpendicular. Hence $PQRS$ must be a rhombus.

5. $x^2 + y^2 = 4$ (given)

centre $\equiv c_1 \equiv (0, 0)$ and $R_1 = 2$.

Again $x^2 + y^2 - 6x - 8y - 24 = 0$ then $c_2 \equiv 3, 4$

and $R_2 = 7$.

Again $c_1 c_2 = 5 = R_2 - R_1$

Therefore, the given circles touch in ternally such that they can have just one common tangents at the point of contact.

Therefore, (B) is the Ans.

6. $\int_{-1}^{1} f(x)\, dx = \int_{-1}^{1} (x - [x])\, dx = \int_{-1}^{1} x\, dx - \int_{-1}^{1} [x]\, dx$

$$= 0 - \int_{-1}^{1} [x]\, dx \qquad [\because x \text{ is an odd function}]$$

But $$[x] = \begin{cases} -1 & \text{if} \quad -1 \le x < 0 \\ 0 & \text{if} \quad 0 \le x < 1 \\ 1 & \text{if} \quad x = 1 \end{cases}$$

$$\therefore \int_{-1}^{1} [x]\, dx = \int_{-1}^{0} [x]\, dx + \int_{0}^{1} [x]\, dx$$

$$= \int_{-1}^{0} (-1)\, dx + \int_{0}^{1} 0\, dx$$

$$= -[x]_{-1}^{0} + 0 = 1$$

Thus $\int_{-1}^{1} f(x)\, dx = 1$.

Therefore, (A) is the Ans.

7. $16x^2 + 25y^2 = 400$ (given)

$\Rightarrow \quad \dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

Here $a^2 = 25, \quad b^2 = 16$

But $b^2 = a^2 (1 - e^2)$

$\Rightarrow \quad 16 = 25 (1 - e^2) \quad \Rightarrow \quad \dfrac{16}{25} = 1 - e^2$

$\Rightarrow \quad e^2 = 1 - \dfrac{16}{25} = \dfrac{9}{25}$

$\Rightarrow \quad e = 3/5$

Now foci of the ellipse are $\pm ae, 0 \equiv \pm 3, 0$

we have $3 = a \cdot \dfrac{3}{5} \quad \Rightarrow \quad a = 5$

Now $PF_1 + PF_2 =$ focal distance $= 2a = 2 \times 5 = 10$

Therefore, (C) is the Ans.

8. $PQRS$ is a parallelogram if and only if the mid-point of PR is same as that if the mid-point of QS. That is, if and only if

$$\frac{1+5}{2} = \frac{4+a}{2} \qquad \text{and} \qquad \frac{2+7}{2} = \frac{6+b}{2}$$

$\Rightarrow$ $a = 2$ and $b = 3$. Therefore, (C) is the Ans.

9. It is given that $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent $\Rightarrow [\vec{a} \,.\, \vec{b} \,.\, \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{vmatrix} = 0 \text{ Apply } C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$$

Now expanding along R_1,

$\Rightarrow$ $-(\beta - 1) = 0 \Rightarrow \beta = 1$

Also $|\vec{c}| = \sqrt{3}$ (given) where $c = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ (given)

$\Rightarrow$ $1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow$ $1 + \alpha^2 + 1 = 3 \Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

Therefore, (D) is the correct Ans.

10. P (2 white and 1 black) $= P(W_1 W_2 B_3$ or $W_1 B_2 W_3$ or $B_1 W_2 W_3)$

$= P(W_1 W_2 B_3) + P(W_1 B_2 W_3) + P(B_1 W_2 W_3)$

$= P(W_1) P(W_2) P(B_3) + P(W_1) P(B_2) P(W_3) + P(B_1)(W_2)(W_3)$

$= \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4}$

$= \frac{1}{32}(9 + 3 + 1) = \frac{13}{32}$

Therefore, (A) is the Ans.

11. $\sum_{i=1}^{13} (i^n + i^{n+1}) = \sum_{i=1}^{13} i^n (1 + i) = (1 + i) \sum_{i=1}^{13} i^n$

$= (1 + i)(i + i^2 + i^3 + \ldots. i^{13}) = (1 + i) \left\{ \frac{i(1 - i)}{1 - i} \right\}$

$= (1 + i)\, i = -1 + i$, Therefore, (B) is the Ans.0

12. The maximum value of $f(x) = \cos x + \cos(\sqrt{2}x)$ is 2 which occurs at $x = 0$. Also, there is no value of x for which this value will be attained again.

Imp. note : This question can be solved by calculus also.

13. $f(x) = \frac{x^2 - 1}{x^2 + 1} = 1 - \frac{2}{x^2 + 1}$

$f(x)$ will be minimum when $\frac{2}{x^2 + 1}$ is maximum, i.e., when $x^2 + 1$ is minimum

i.e. at $x = 0$.

$\therefore$ Minimum value of $f(x)$ is $f(0) = -1$.

Therefore, (D) is the Ans.

$$\frac{1+5}{2} = \frac{4+a}{2} \quad \text{and} \quad \frac{2+7}{2} = \frac{6+b}{2}$$

$\Rightarrow$ $a = 2$ and $b = 3$. Therefore, (C) is the Ans.

9. It is given that $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent $\Rightarrow [\vec{a} \,.\, \vec{b} \,.\, \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{vmatrix} = 0 \text{ Apply } C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$$

Now expanding along R_1,

$\Rightarrow$ $-(\beta - 1) = 0 \Rightarrow \beta = 1$

Also $|\vec{c}| = \sqrt{3}$ (given) where $c = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ (given)

$\Rightarrow$ $1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow$ $1 + \alpha^2 + 1 = 3 \Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

Therefore, (D) is the correct Ans.

10. P (2 white and 1 black) $= P(W_1 W_2 B_3 \text{ or } W_1 B_2 W_3 \text{ or } B_1 W_2 W_3)$

$= P(W_1 W_2 B_3) + P(W_1 B_2 W_3) + P(B_1 W_2 W_3)$

$= P(W_1) P(W_2) P(B_3) + P(W_1) P(B_2) P(W_3) + P(B_1)(W_2)(W_3)$

$= \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{2}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{2}{4} \cdot \frac{1}{4}$

$= \frac{1}{32}(9 + 3 + 1) = \frac{13}{32}$

Therefore, (A) is the Ans.

11. $\sum_{i=1}^{13} (i^n + i^{n+1}) = \sum_{i=1}^{13} i^n (1+i) = (1+i) \sum_{i=1}^{13} i^n$

$= (1+i)(i + i^2 + i^3 + \ldots . i^{13}) = (1+i)\left\{\frac{i(1-i)}{1-i}\right\}$

$= (1+i)\, i = -1 + i$, Therefore, (B) is the Ans.0

12. The maximum value of $f(x) = \cos x + \cos(\sqrt{2}x)$ is 2 which occurs at $x = 0$. Also, there is no value of x for which this value will be attained again.

Imp. note : This question can be solved by calculus also.

13. $f(x) = \frac{x^2 - 1}{x^2 + 1} = 1 - \frac{2}{x^2 + 1}$

$f(x)$ will be minimum when $\frac{2}{x^2 + 1}$ is maximum, i.e., when $x^2 + 1$ is minimum

i.e. at $x = 0$.

$\therefore$ Minimum value of $f(x)$ is $f(0) = -1$.

Therefore, (D) is the Ans.

19. For ellipse condition of tangency is $c^2 = a^2m^2 + b^2$

$\Rightarrow \quad c^2 = 4 \times 4 + 1 = 17$

$\Rightarrow \quad c = \pm\sqrt{17}$ therefore, (C) is the Ans.

20. Let the common ratio of the G.P. be r. Then

$$y = xr \quad \text{and} \quad z = xr^2$$

$\Rightarrow$ $\ln y = \ln x + \ln r$ and $\ln z = \ln x + 2\ln r$

Putting $A = 1 + \ln x, D = \ln r$

Then $\dfrac{1}{1+\ln x} = \dfrac{1}{A}, \dfrac{1}{1+\ln y} = \dfrac{1}{1+\ln xr} = \dfrac{1}{1+\ln x+\ln r} = \dfrac{1}{A+D}$

and $\dfrac{1}{1+\ln z} = \dfrac{1}{1+\ln x+2\ln r} = \dfrac{1}{A+2D}$

Therefore, $\dfrac{1}{1+\ln x}, \dfrac{1}{1+\ln y}, \dfrac{1}{1+\ln z}$ are in H.P.

So (B) is the Ans.

21. $3\sin^2 x - 7\sin x + 2 = 0$ (given)

$\Rightarrow \quad 3\sin^2 x - 6\sin x - \sin x + 2 = 0$

$\Rightarrow \quad 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$

$\Rightarrow \quad (3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \quad \sin x = 1/3$ or $\sin x = 2$ (sin x = 2 is rejected).

$\Rightarrow \quad x = n\pi + (-1)^n \sin^{-1}\dfrac{1}{3}, \ n \in I$

For $0 \le n \le 5, \quad x \in [0, 5\pi]$

$\therefore$ There are six values of $x \in [0, 5\pi]$ which satisfy the equation

$$3\sin^2 x - 7\sin x + 2 = 0$$

Therefore, (C) is the Ans.

22. $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$ (given) ...(1)

$\Rightarrow \quad y = (c_1 + c_2)\cos(x + c_3) - c_4 e^x \cdot e^{c_5}$

Now let $c_1 + c_2 = A, \quad c_3 = B, \quad c_4 e^{c_5} = C$

$\Rightarrow \quad y = A\cos(x + B) - Ce^x$ ...(2)

differentiate w.r.t. x

$\Rightarrow \quad \dfrac{dy}{dx} = -A\sin(x + B) - Ce^x$ ...(3)

differentiate again w.r.t. x

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -A\cos(x + B) - Ce^x$ ...(4)

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = -y - 2Ce^x$ from (2) ...(5)

$\Rightarrow \quad \dfrac{d^2y}{dx^2} + y = -2Ce^x$

differentiate again w.r.t. x

$\Rightarrow \quad \dfrac{d^3y}{dx^3} + \dfrac{dy}{dx} = -2Ce^x$ ...(6)

$$\Rightarrow \qquad \frac{d^3y}{dx^3} + \frac{dy}{dx} = \frac{d^2y}{dx^2} \qquad \text{from (5)}$$

which is a differential equation of order 3. Therefore, (C) is the Ans.

23. Let $f(x) = \sin^2 x$ and $g(x) = \sqrt{x}$

Now $fog(x) = f(g(x)) = f(\sqrt{x}) = \sin^2 \sqrt{x}$

and $gof(x) = g(f(x)) = g(\sin^2 x) = \sqrt{\sin^2 x} = |\sin x|$

again, $f(x) = \sin x,\ g(x) = |x|$

$fog(x) = f[g(x)] = f(|x|) = \sin|x| \neq (\sin\sqrt{x})^2$

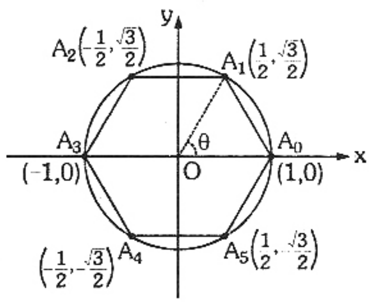
When $f(x) = x^2,\ g(x) = \sin\sqrt{x}$

$fog(x) = f[g(x)] = f(\sin\sqrt{x}) = (\sin\sqrt{x})^2$

and $(gof)(x) = g[f(x)] = g(x^2) = \sin\sqrt{x^2} = \sin|x| \neq |\sin x|$

Therefore, (A) is the Ans.

24.



$$A_0A_1^2 = \left(1 - \frac{1}{2}\right)^2 + \left(0 - \frac{\sqrt{3}}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$\Rightarrow \quad A_0A_1 = 1$

Next, $A_0A_2^2 = \left(1 + \frac{1}{2}\right)^2 + \left(0 - \frac{\sqrt{3}}{2}\right)^2 = \left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4} + \frac{3}{4}$

$= \frac{12}{4} = 3$

$\Rightarrow \quad A_0A_2 = \sqrt{3}$

Next $A_0A_4^2 = \left(1 + \frac{1}{2}\right)^2 + \left(0 + \frac{\sqrt{3}}{2}\right)^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{3}{4}\right) = \frac{9}{4} + \frac{3}{4} = \frac{12}{4} = 3$

$\Rightarrow \quad A_0A_4 = \sqrt{3}$

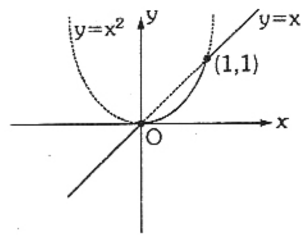
Thus, $(A_0A_1)(A_0A_2)(A_0A_4) = 3$.

25. $[\vec{m}\ \vec{n}\ \vec{a}] - = [\vec{a}\ \vec{n}\ \vec{m}] = [\vec{n}\ \vec{m}\ \vec{a}] = [\vec{m}\ \vec{a}\ \vec{n}]$

Therefore, (C) is the Ans.

26. The probability that only two tests are needed = probability that the first machine tested is faulty × (probability that the second machine tested is faulty given that the first machine tested is faulty $= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$. Therefore, (B) is the Ans.

27. $h(x) = \min\{x, x^2\}$ (given).



We will trace $h(x) = x$ and $h(x) = x^2$ separately.

From fig it is clear that

$$h(x) = \begin{cases} x, & \text{if} \quad x \le 0 \\ x^2, & \text{if} \quad 0 \le x \le 1 \\ x, & \text{if} \quad x \ge 1 \end{cases}$$

From the graph it is clear that h is continuous for all $x \in R$ and $h'(x) = 1$ for all $x > 1$ and h is not differentiable at $x = 0$ and $x = 1$. Therefore, (A), (C) and (D) are the answers.

28. $f(x) = 3x - 5$ (given)

Let $y = f(x) = 3x - 5 \Rightarrow y + 5 = 3x \Rightarrow x = \dfrac{y+5}{3}$ ...(1)

and $y = f(x) \Rightarrow x = f^{-1}(y)$ ...(2)

From (1) and (2)

$$f^{-1}(y) = \frac{y+5}{3} \Rightarrow f^{-1}(x) = \frac{x+5}{3}$$

Therefore, (B) is the Ans.

29. **A.** $P(E/F) + P(\bar{E}/F) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(\bar{E} \cap F)}{P(F)}$

$$= \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)}$$

$= \dfrac{P(F)}{P(F)} = 1$ therefore, A is the Ans.

B. $P(E/F) + P(E/\bar{F}) = \dfrac{P(E \cap F)}{P(F)} + \dfrac{P(E \cap \bar{F})}{P(\bar{F})}$

$$= \frac{P(E \cap F)}{P(F)} + \frac{P(E \cap \bar{F})}{1 - P(F)} \neq 1$$

Therefore, (B) is not the Ans.

C. $P(\bar{E}/F) + P(E/\bar{F}) = \dfrac{P(\bar{E} \cap F)}{P(\bar{F})} + \dfrac{P(E \cap \bar{F})}{P(\bar{F})}$

$$= \frac{P(\bar{E} \cap F)}{P(F)} + \frac{P(E \cap \bar{F})}{1 - P(F)}$$

Therefore, (C) is not the Ans.

D. $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = \dfrac{P(E \cap \bar{F})}{P(\bar{F})} + \dfrac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}$

$$= \frac{P(E \cap \bar{F}) + P(\bar{E} \cap \bar{F})}{P(\bar{F})}$$

$= \dfrac{P(\bar{F})}{P(\bar{F})} = 1$. Therefore, (D) is true.

So (A), (D) are the Ans.

30. $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ (given)

$\Rightarrow \quad -3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = 0$ [$\Rightarrow C_2$ and C_3 are proportional].

$\Rightarrow \quad x + iy = 0 \quad \Rightarrow \quad x = 0, y = 0$. Therefore, (D) is the Ans.

31. The event that the fifth toss results in a head is independent of the event that the first four tosses result in tails.

$\therefore$ Probability of the required event = 1/2

Therefore, (A) is the Ans.

32. Distinct n digit numbers which can be formed using digits 2, 5 and 7 are 3^n. We have to find n so that $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$

$\Rightarrow \quad n - 2 \geq 5 \quad \Rightarrow \quad n \geq 7$ so the least value of n is 7.

Therefore, (B) is the Ans.

33. The no. of ways of placing 3 black balls without any restriction is $^{10}C_3$. Since we have total 10 places of putting 10 balls in a row and firstly we will put 3 black balls. Now the no. of ways in which no two black balls put together is equal to the no. of ways of choosing 3 palaces marked—out of eight places.

— W — W — W — W — W — W — W —

This can be done in $^8c_3$ ways. Thus, probability of the required event

$= \frac{^8c_3}{^{10}c_3} = \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$. Therefore, (B) is the Ans.

34. $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ (given)

Now put $\theta = 0$, we get $\quad 0 = b_0$

$\therefore \quad \sin n\theta = \sum_{r=1}^{n} b_r \sin^r \theta$ is true

$\Rightarrow \quad \frac{\sin n\theta}{\sin \theta} = \sum_{r=1}^{n} b_r (\sin \theta)^{r-1}$

taking limit as $\theta \to 0$

$\Rightarrow \quad \lim_{\theta \to 0} \frac{\sin n\theta}{\sin \theta} = \lim_{\theta \to 0} \sum_{r=1}^{n} b_r (\sin \theta)^{r-1}$

$\Rightarrow \quad \lim_{\theta \to 0} \frac{n\theta \cdot \frac{\sin n\theta}{n\theta}}{\theta \cdot \frac{\sin\theta}{\theta}} = b_1 + 0 + 0 + 0 \ldots\ldots$

Other values becomes zero for higher powers of $\sin \theta$.

$\Rightarrow \quad \frac{n \cdot 1}{1} = b_1$

$\Rightarrow \quad b_1 = n$. Therefore, (B) is the Ans.

35. $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$ (formula)

and $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ (formula)

and $\sin 15° \cos 75° = \sin 15° . \sin 15° = \frac{1}{4}(2-\sqrt{3})$. Therefore, all these values are irrational and

$\sin 15° \cos 15° = \frac{1}{2} \cdot 2 \sin 15° \cos 15° = \frac{1}{2} \cdot \sin 30° = \frac{1}{4}$ which is rational.

therefore, (C) is the Ans.

36. It is given that $x^2 + y^2 = a^2$ ...(1)

and $xy = c^2$ ...(2)

We obtain $x^2 + c^4/x^2 = a^2$

$\Rightarrow \quad x^4 - a^2x^2 + c^4 = 0$ ...(3)

Now x_1, x_2, x_3, x_4 will be roots of (3).

Therefore $\Sigma x_1 = x_1 + x_2 + x_3 + x_4 = 0$

and product of the roots $x_1 x_2 x_3 x_4 = c^4$

Similarly, $y_1 + y_2 + y_3 + y_4 = 0$ and $y_1 y_2 y_3 y_4 = c^4$

Therefore, (a), (b), (c) and (d) are the answers.

37. It is given that $P(E) \leq P(F) \Rightarrow E \subseteq F$ ...(1)

and $P(E \cap F) > 0 \Rightarrow E \neq F$ ...(2)

(A) occurrence of $E \Rightarrow$ occurrence of F from (2)

(B) occurrence of $F \Rightarrow$ occurrence of E from (2)

(C) non-occurrence of $E \Rightarrow$ non-occurrence of F from (1)

Therefore, (D) is the Ans.

38. **A.** $\vec{u} \cdot (\vec{v} \times \vec{w})$ is a meaningful operation, therefore, (A) is the Ans.

B. $\vec{u} \cdot (\vec{v} \cdot \vec{w})$ is not meaningful since $\vec{v} . \vec{w}$ is a scalar quantity and for dot product both quantities should be vector. Therefore, (B) is not the Ans.

C. $(\vec{u} \cdot \vec{v})\, \vec{w}$ is meaningful since it is a simple multiplication of vector and scalar quantity. Therefore, (C) is not the Ans.

D. $\vec{u} \times (\vec{v} \cdot \vec{w})$ is not meaningful since $\vec{v} . \vec{w}$ is a scalar quantity and for cross product, both quantity should be vector. Therefore, (D) is the Ans Hence (A) and (C) are the Ans.

39. $\int_0^x f(t)\,dt = x + \int_x^1 t\, f(t)\,dt,$ (given)

Differentiating both sides w.r.t. x, we get

$$f(x) \cdot 1 = 1 - x\, f(x) \cdot 1$$

$$\Rightarrow \quad (1 + x)\, f(x) = 1$$

$$\Rightarrow \quad f(x) = 1/(1 + x)$$

$$\Rightarrow \quad f(1) = \frac{1}{1+1} = \frac{1}{2}$$

Therefore, (A) is the Ans.

40. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$

Differentiate the whole equation w.r.t. x

$$h'(x) = f'(x) - 2f(x) \cdot f'(x) + 3\, f^2(x) \cdot f'(x)$$

$$= f'(x)\,[1 - 2\, f(x) + 3f^2(x)]$$

$$= 3f'(x)\left[(f(x))^2 - \frac{2}{3} f(x) + \frac{1}{3}\right]$$

$$= 3f'(x)\left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{1}{3} - \frac{1}{9}\right]$$

$$= 3f'(x)\left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{3-1}{9}\right]$$

$$= 3f'(x)\left[\left(f(x) - \frac{1}{3}\right)^2 + \frac{2}{9}\right]$$

Note that $h'(x) < 0$ if $f'(x) < 0$ and $h'(x) > 0$ if $f'(x) > 0$

Therefore, $h(x)$ is increasing function if $f(x)$ is increasing function, and $h(x)$ is decreasing function if $f(x)$ is decreasing function.

Therefore, (A) and (C) are the Ans.