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## IGCSE Mathematics (4400)

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Guidance for teachers for the following topics:

- set language and notation (paragraph number 1.5 of the specification)
- function notation (paragraph number 3.2 of the specification)
- calculus (paragraph number 3.4 of the specification).

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## Support material on sets, functions and calculus for IGCSE Mathematics (4400)

## Introduction

Most of the IGCSE mathematics specification is covered in standard GCSE mathematics textbooks. Examples of such textbooks are given on page 32 of the specification.

The style of examination questions will be similar to that of the UK GCSE. However, there are some differences in content between IGCSE mathematics and GCSE mathematics. Firstly there are some additional topics, about which notes and specimen questions are given below. Secondly, a few topics are omitted from IGCSE. These are listed on page 22.

## Additional Topics

There are three major topics that are not included in the UK GCSE but which do feature in IGCSE. These are

- set language and notation (paragraph number 1.5 of the specification)
- function notation (paragraph number 3.2 of the specification)
- calculus (paragraph number 3.4 of the specification)

A few other smaller topics are also included in IGCSE.

- The intersecting chords theorem
- Finding the gradient of a curve at a point by drawing a tangent
- Quadratic inequalities
- Simple conditional probability
- Modulus of a vector

The following notes and specimen questions on the three major topics provide supplementary information as to how these parts of the specification will be assessed.

Centres should note that these examples are not exhaustive, but are intended to give some indication of the level of difficulty and the types of question which may be expected.

## Notes on Set Language and Notation <br> (paragraph 1.5 of the specification)

## 1. Foundation and Higher tiers

Definition: In words, e.g. $\{$ Cats $\},\{$ Positive integers less than 10$\}$, \{Multiples of 3\}, or as a list of members e.g. $\{2,4,6,8\}$, \{chairs, tables $\}$.

## Typical Questions:

- Given the definition of a set, list all the elements (or members).
- Given a list of all the elements of a set, write the definition.

Symbols: $\mathcal{E}, \varnothing, \epsilon, \cup, \cap$

## Typical Questions:

- Given defined sets $\mathcal{E}, A \& B$,
- describe $A \cap B$,
- list the members of $A \cup B$,
- what is meant by " $6 \in A$ "?
- is it true that $A \cap B=\varnothing$ ? Explain your answer.


## 2. Higher tier only

Definition: Algebraic, e.g. $\{\mathcal{E}=$ Integers $\}, P=\{x: 0 \leq x<10\}$
Venn diagrams: Different cases, e.g.


Symbols: $A^{\prime}$ (the complement of $A$ ), $\subset$ ("is a subset of")

## Typical Questions:

- Given defined sets $\mathcal{E}, A, B$, and $C$,
- draw a Venn diagram
- shade $A \cup B \cap C^{\prime}$,
- list the members of $B^{\prime} \cap C$,
- is it true that $A \subset B$ ?
- Describe a given shaded region in a Venn diagram.
- Draw a Venn diagram in which certain conditions are true.

Symbols: $n(A)$ (the number of members in $A$ ),

## Typical Questions:

- Given a Venn diagram (e.g. Black animals, Cats, Dogs), with numbers inserted, - how many black cats are there?
- Given two or three defined sets, find $n\left(A \cup B^{\prime}\right)$
- Given $n(\mathcal{E})=23, n(A)=16, n(B)=10, n(A \cup B)=20$,
- draw a Venn diagram
- show the number of members in each region.
- Questions involving three sets, where an equation needs to be set up. See Question 16 below.


## Specimen Questions on Set Language and Notation

## Foundation and Higher tiers

1. List the members of the following sets.
(a) \{Days of the week $\}$
(b) $\quad$ Even numbers between 1 and 9$\}$
(c) $\{$ Factors of 18$\}$
(d) $\{$ Colours of the rainbow $\}$
(e) $\{$ Square numbers less than 100$\}$
2. $\mathcal{E}=\{$ Positive integers less than 20$\}$
$P=\{11,13,15,17\}$
$Q=\{12,14,16\}$
$R=\{$ Multiples of 4$\}$
(a) List the members of
(i) $R$
(ii) $P \cup Q$
(iii) $Q \cap R$
(b) What is the set $P \cap R$ ?
3. $\mathcal{E}=\{$ The books in St John's library $\}$
$M=$ \{Mathematics books $\}$
$P=\{$ Paperback books $\}$
$T=\{$ Travel books $\}$
(a) Describe the set $M \cap P$.
(b) What is the set $M \cap T$ ?
(c) One book in St John's library has the title 'Explore'.

Given that 'Explore' $\in M \cup T$, what can you say about the book 'Explore'?
4. $\mathcal{E}=$ \{Polygons $\}$
$A=\{$ Three-sided shapes $\}$
$B=\{$ Shapes with two equal sides $\}$
$C=\{$ Shapes with two parallel sides $\}$
(a) What is the mathematical name for the members of $A \cap B$ ?
(b) Which of the following are true?
(i) Kite $\epsilon$ A.
(ii) Trapezium $\in C$.
(iii) $A \cap C=\varnothing$.
5. $R=\{$ Positive odd numbers less than 10$\}$
$S=\{$ Multiples of 3 between 4 and 20 $\}$
$T=\{$ Prime numbers $\}$
(a) List the elements of
(i) $R \cup S$,
(ii) $R \cap S$.
(b) You are told that $x \in R \cap T$. Write down all the possible values of $x$.
(c) Is it true that $S \cap T=\emptyset$ ? Explain your answer.

## Higher tier only

6. $\mathcal{E}=\{$ Positive integers less than 20$\}$
$A=\{x: 0<x \leq 9\}$
$B=\{$ Even numbers $\}$
$C=\{$ Multiples of 5$\}$
(a) List the members of $A \cap B^{\prime}$.
(b) Find the value of $n(A \cup C)$.
(c) Complete the statement $A \cap B \cap C=\ldots$
(d) Is it true that $\left(A \cap C^{\prime}\right) \subset B$ ? Explain your answer.
7. There are 30 people in a group. 17 own a car. 11 own a bicycle. 5 do not own either a car or a bicycle.

Find how many people in this group own a car but not a bicycle.
8. Draw a Venn diagram with circles representing three sets, $A, B$ and $C$.

Shade the region representing $A \cap\left(B \cup C^{\prime}\right)$.
9.


Make two copies of this Venn diagram.
(a) On one diagram draw a circle to represent set $C$, such that

$$
\begin{aligned}
& C \subset A \\
& C \cap B^{\prime}=C .
\end{aligned}
$$

(b) On the other diagram draw a circle to represent set $D$ such that
$D \subset A^{\prime}$,
$D \cap B \neq \emptyset$ and
$D \cup B \neq D$.
10. Draw a Venn diagram with circles representing three sets, $A, B$ and $C$, such that all the following are true:

$$
A \cap C \neq \varnothing, \quad A \cap C^{\prime} \neq \varnothing \quad \text { and } \quad B \subset(A \cup C)^{\prime}
$$

11. $\mathcal{E}=\{x: x$ is an integer and $1 \leq x \leq 30\}$
$A=\{$ Multiples of 3$\}$
$B=\{$ Multiples of 4$\}$
(a) Find the value of $n(A \cap B)$.

Sets $A$ and $B$ are represented by circles in the Venn diagram.

(b) $\quad C=\{$ Odd numbers $\}$
(i) Copy the Venn diagram, and draw on it a circle to represent set $C$.
(ii) Shade the region $A \cap(B \cup C)$.
(ii) Write down all the values of $x$ such that $x \in A \cap(B \cup C)^{\prime}$.
12. In the Venn diagram, the numbers of elements in several regions are shown.


You are also given that $n(\mathcal{E})=25, n(B)=12$ and $n(A)=8$.
(a) Find $\mathrm{n}(B \cap C)$.
(b) Find $\mathrm{n}\left(A \cap C \cap B^{\prime}\right)$.
13. $\mathcal{E}=\{$ Positive integers less than 15$\}$
$E=\{$ Even numbers $\}$
$M=\{$ Multiples of 3$\}$

(a) Copy the Venn diagram and fill in each member of $\mathcal{E}$ in the correct region.
(b) Write down the value of $n\left(E \cap M^{\prime}\right)$.
14. $\mathcal{E}=$ \{Quadrilaterals $\}$
$P=\{$ Parallelograms $\}$
$K=\{$ Kites $\}$
$S=\{$ Squares $\}$
(a) What is the mathematical name for a member of $P \cap K$ ?
(b) Complete the statement $P \cup S=\ldots$.
(c) Draw a Venn diagram showing sets $P, K$ and $S$.
15.


Use set notation to describe the shaded region.
16. There are 40 members in a sports club. 2 play all three sports. 23 play squash. 24 play tennis. 18 play golf. 14 play squash and tennis. 8 play tennis and golf.
1 member makes the refreshments and does not play any sport. How many play squash and golf?

## Answers:

1.(a) Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday (b) 2, 4, 6, 8
(c) 1, 2, 3, 6, 9, 18 (d) Red, orange, yellow, green, blue, indigo, violet
(e) $1,4,9,16,25,36,49,64,81$
2.(a)(i) $4,8,12,16$ (ii) $11,12,13,14,15,16,17$ (iii) 12,16 (b) $\varnothing$
3.(a) Paperback maths books in St John's library. (b) Ø (c) It is either a maths or a travel book.
4. (a) Isosceles triangles (b) ii \& iii
5.(a)(i) $1,3,5,6,7,9,12,15,18$ (ii) 9 (b) 3, 5, 7 (c) Yes. No members of $S$ are prime.
6.(a) 1, 3, 5, 7, 9 (b) 11 (c) Ø (d) No. E.g. 3, 7 or 9
7. 14

9.(a)

(b)

10.

11.(a) 2 (b)(i)(ii)

(iii) 6,18
12.(a) 4 (b) 1
13.(a)

(b) 5
14.(a) Rhombus
(b) $P$
(c)

15. $(A \cup B) \cap C^{\prime}$ or $\left(A \cap C^{\prime}\right) \cup\left(B \cap C^{\prime}\right)$
16. 6

## Notes on Function Notation (paragraph 3.2 of the specification)

Notation and definitions: $\quad \mathrm{f}(x)=x^{2} \quad \mathrm{f}: x \mapsto x^{2}$


Notation for particular sets (e.g. I is the set of integers, R is the set of real numbers) is not required.

Domain is all values of $x$ to which the function is applied.
Range is all values of $\mathrm{f}(x)$.
Domain and/or range may be given in words, or as a list, or algebraically e.g. $0 \leq x<10$
If the domain is not given, it is assumed to be $\{x: x$ is any number $\}$.

Vocabulary such as "One to one" and "Many to one" is not required

Codomain is not required

## Which functions?

Usually e.g. linear, quadratic, cubic, $\sqrt{x}, 1 /$ linear.
Sometimes harder functions e.g. $\sqrt{ }$ linear, $1 / \sqrt{ }$ linear, linear/linear, $/$ quadratic, $1 /$ quadratic, $a+b / x, a x+b / x$, trig

Note: " $\sqrt{ }$ " indicates the positive value of the square root.

## Typical Questions:

- Given a function and its domain, find the range.
- Given a function applied to all numbers, find the range.
- Given a function, which values cannot be included in the domain?
- Given $\mathrm{f}(x)$, find $\mathrm{f}(-2)$.
- Given $\mathrm{f}(x)=3$, find the value(s) of $x$ (not necessarily involving the notation $\mathrm{f}^{-1}$ ).


## Composite functions:

$\operatorname{fg}(x)$ means $f(g(x))$, i.e. do $g$ first followed by $f$.

## Typical Questions:

- Given functions $f$ and $g$, find $f g(-3), g f(2)$
- Given functions f and g , find fg in the form $\mathrm{fg}: x \mapsto \ldots$ or $\mathrm{fg}(x)=\ldots$
- Given functions f and g , and the domain of f , find the range of gf .
- Given functions f and g , which values need to be excluded from the domain of gf ?


## Inverse functions:

Functions required:
Usually e.g. linear, $1 /$ linear, $\sqrt{x}$ or $x^{2}$ (with domain restricted to positive numbers).
Sometimes harder functions, e.g. /linear, $1 / \sqrt{ }$ linear, linear/linear, $a+b / x, 1 / \sqrt{x}$.
Any method for finding $\mathrm{f}^{-1}$ is acceptable, e.g.
Algebraic: write as $y=\ldots$; rearrange to make $x$ the subject; interchange $x$ and $y$. Flowchart: reverse each operation, in reverse order.

## Typical Questions:

- Given the function $f$, find $\mathrm{f}^{-1}(3)$.
- Given the function f , find $\mathrm{f}^{-1}$ in the form $\mathrm{f}^{-1}: x \mapsto \ldots$ or $\mathrm{f}^{-1}(x)=\ldots$
- Without working, write down the value of $\mathrm{ff}^{-1}(5)$.
- Given functions $f$ and $g$, find the function $f^{-1} g$.
- Given functions f and g , solve the equation $\mathrm{f}(x)=\mathrm{g}^{-1}(x)$.


## Specimen Questions on Function Notation

1. Here are three functions.

$$
\mathrm{f}(x)=3-2 x \quad \mathrm{~g}(x)=\frac{1}{x-2} \quad \mathrm{~h}(x)=\sqrt{3 x+1}
$$

(a) Find (i) $f(-1)$
(ii) $f\left(\frac{3}{4}\right)$
(iii) $g(4.5)$ (iv) $g(-2)$
(v) $h(5) \quad$ (vi) $h\left(2 \frac{2}{3}\right)$
(b) (i) Given that $\mathrm{f}(x)=-7$, find $x$.
(ii) Given that $\mathrm{g}(x)=2$, find $x$.
(iii) Given that $\mathrm{h}(x)=5$, find $x$.
2. Three functions, $\mathrm{p}, \mathrm{q}$ and r , are defined as follows.

$$
\mathrm{p}(x)=x^{2}-3 x+4 \quad \mathrm{q}(x)=\frac{2 x-3}{x+1} \quad \mathrm{r}(x)=\sin x^{0}
$$

(a) Find (i) p(-4)
(ii) $\mathrm{p}\left(\frac{3}{4}\right)$
(iii) $q(4) \quad$ (iv) $q(-2)$
(v) $r(45)$
(vi) $r(180)$
(b) (i) Find the values of $x$ for which $\mathrm{p}(x)=2$.
(ii) Find the value of $x$ for which $\mathrm{q}(x)=\frac{3}{4}$.
(iii) Find the values of $x$, in the domain $0 \leq x \leq 180$, for which $\mathrm{r}(x)=0.5$
3. State which values of $x$ cannot be included in the domain of these functions.
(i) $\mathrm{f}: x \mapsto \sqrt{5-x}$
(ii) $\mathrm{g}: \mathrm{x} \mapsto \frac{5}{2 x-7}$
(iii) $\mathrm{h}: \mapsto \frac{1}{\sqrt{x+3}}$ (iv) $\mathrm{j}: \mathrm{x} \mapsto \sqrt{\left(x^{2}-4\right)}$
(v) $1: x \mapsto 2 x+\frac{1}{x}$
(vi) $\mathrm{k}: x \mapsto \frac{1}{(3 x+2)^{2}}$
(vii) $1: x \rightarrow \sqrt{\frac{x-3}{6-x}}$
4. $\mathrm{f}: \mathrm{x} \mapsto \mathrm{x}^{3} \quad \mathrm{~g}: x \mapsto \frac{1}{x+8}$
(a) Find (i) fg(-4), (ii) $\operatorname{gf}(5)$.
(b) Find (i) $\operatorname{gf}(x)$, (ii) $\operatorname{fg}(x)$.
(c) What value(s) must be excluded from the domain of (i) $\operatorname{gf}(x)$, (ii) $\operatorname{fg}(x)$ ?
(d) Find and simplify $\operatorname{gg}(x)$.
5. Three functions are defined as follows.

$$
\begin{aligned}
& \mathrm{p}(x)=(x+4)^{2} \text { with domain }\{x: x \text { is any number }\} \\
& \mathrm{q}(x)=8-x \quad \text { with domain }\{x: x>0\} \\
& \mathrm{r}(x)=\cos x^{\mathrm{o}} \quad \text { with domain }\{x: 0 \leq x \leq 180\}
\end{aligned}
$$

(a) Find the range of each of these functions.
(b) Find the values of $x$ such that $\mathrm{p}(x)=\mathrm{q}(x)$.
6. Find the inverse function of each of the following functions.
(a) $\mathrm{f}(\mathrm{x})=2 \mathrm{x}-3$
(b) $g(x)=5-x$
(c) $h(x)=\frac{1}{3 x+4}$
(d) $\mathrm{j}(x)=3-\frac{2}{x}$
(e) $\mathrm{k}(x)=\frac{2 x+1}{5-x}$
7. Find the inverse function of each of the following functions.
(a) p: $x \mapsto \sqrt{3 x-2}\left(\right.$ for $\left.x \geq \frac{2}{3}\right)$
(b) q: $x \mapsto \frac{1}{\sqrt{x+2}}($ for $x>-2)$
(c) r: $x \mapsto x^{2}+5($ for $x \geq 0)$
(d) $\mathrm{s}: x \mapsto(x-3)^{2} \quad($ for $x \geq 3)$
8. The function $\mathrm{f}(x)$ is defined as $\mathrm{f}(x)=\frac{2}{x+1}$.

Solve the equation $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$.
9. Here are two functions.

$$
\mathrm{f}(x)=\frac{2}{5+x} \quad \mathrm{~g}(x)=x^{2}+3
$$

(a) Calculate $\mathrm{g}(-2)$.
(b) Given that $\mathrm{f}(z)=\frac{1}{8}$, calculate the value of $z$.
(c) Which value of $x$ must be excluded from the domain of $\mathrm{f}(x)$ ?
(d) Find the inverse function, $\mathrm{f}^{-1}$, in the form $\mathrm{f}^{-1}: x \mapsto \ldots$
(e) Calculate $\mathrm{f}^{-1} \mathrm{~g}(1)$.
10. Functions $f$ and $g$ are defined as follows.

$$
\mathrm{f}: x \mapsto 4+\sqrt{x} \quad \mathrm{~g}: x \mapsto \frac{1}{(x+2)^{2}}
$$

(a) Calculate (i) $f(25)$ (ii) $g(0.5)$ (iii) $f g(-1)$.
(b) Given that $\operatorname{fg}(x)=4.04$, find the value of $x$.
(c) Find the function $\mathrm{f}^{-1}(x)$.
(d) Calculate $\mathrm{gf}^{-1}(4)$.
11. $\mathrm{p}(x)=\frac{2-x}{3+x} \quad \mathrm{q}(x)=\frac{2-3 x}{1+x}$
(a) Find the function $\mathrm{pq}(x)$.
(b) Hence describe the relationship between the functions p and q .
(c) Write down the exact value of $\mathrm{pq}(\sqrt{2})$

## Answers

NB. In the examination, equivalent answers are acceptable, e.g. decimal instead of fraction.
1.(a)(i) 5 (ii) $1 \frac{1}{2}$ (iii) $\frac{2}{5}$ (iv) -0.25 (v) 4 (vi) 3 (b)(i) 5 (ii) 2.5 (iii) 8
2.(a)(i) 32 (ii) $2 \frac{5}{16}$ (iii) 1 (iv) 7 (v) 0.707 (vi) 0 (b)(i) 1 or 2 (ii) 3 (iii) 30 or 150
3.(i) $x>5$
(ii) $x=3.5$
(iii) $x \leq-3$ (iv) $-2<x<2$
(v) $x=0$
(vi) $x=-\frac{2}{3}$
(vii) $x<3$ or $x \geq 6$
4.(a)(i)
(ii) $\frac{1}{133}$ (b)(i) $\frac{1}{x^{3}+8}$
(ii) $\frac{1}{(x+8)^{3}}$
(c)(i) $x=-2$
(ii) $x=-8$
(d) $\frac{x+8}{8 x+65}$
5.(a) $\mathrm{p}: \geq 0 ; \mathrm{q}:<8 ; r:-1$ to 1 (b) -8 or -1
6.(a) $\frac{x+3}{2}$
(b) $5-x$
(c) $\frac{1-4 x}{3 x}$
(d) $\frac{2}{3-x}$
(e) $\frac{5 x-1}{2+x}$
7.(a) $\frac{x^{2}+2}{3}$
(b) $\frac{1}{x^{2}}-2$
(c) $\sqrt{x-5}$
(d) $\sqrt{x}+3$
8. 1 or -2
9. (a) 7 (b) 11 (c) -5 (d) $\frac{2}{x}-5 \quad$ (e) $-4 \frac{1}{2}$
10.(a)(i) 9 (ii) 0.16 (iii) 5 (b) 23 (c) $(x-4)^{2}$ (d) $\frac{1}{4}$
11.(a) $\operatorname{pq}(x)=x \quad$ (b) Inverses of each other (c) $\sqrt{2}$

## Notes on Calculus (paragraph 3.4 of the specification)

## Basic concepts and notation

Ideas of gradient of tangent and gradient of curve.
$y=x^{\mathrm{n}} \Rightarrow \operatorname{grad}=\frac{\mathrm{d} y}{\mathrm{~d} x}=n x^{\mathrm{n}-1}$,
firstly for + ve integer $n$; then also $n=0,-1,-2$.
Differentiation of polynomials
Usually no rearrangement will be required.
If rearrangement is required, this will usually be asked for explicitly.

Differentiation from first principles is not required.

If teachers wish to give an introduction to the concept of a limiting gradient, the following is adequate, though it will NOT be tested:

On the curve $\mathrm{y}=x^{2}$,
$P\left(3,3^{2}\right) ; Q_{1}\left(3.1,(3.1)^{2}\right) ; Q_{2}\left(3.01,(3.01)^{2}\right)$; etc Find gradients of $P Q_{1}, P Q_{2}, P Q_{3} \ldots$

## Typical Questions:

- Differentiate $x^{5}-3 x^{2}+5$ or $x^{2}+3 x-4$.
- Given $y=\frac{5 x+3}{2}$, find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.

The notation $f^{\prime}(x)$ and the terms "derivative" and "derived function" are not required.

- Given $y=\ldots$, find the gradient for a given $x$,
find $x$ for a given gradient.
- $y=(x+3)^{2}$. Expand and find $\frac{d y}{d x}$.


## Turning Points

Turning Points
At turning points, $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$.

The language used will be "turning points", "maximum", "minimum"; not "stationary points". Points of inflexion are not required.

Find TPs for quadratic, cubic, $a x \pm b / x$.
Distinguish max/min by rough shape,
e.g. shape of $y=a x^{2}+b x+c$ is $\cap$ when $a<0$.

For $a x \pm b / x$, if distinguishing $\mathrm{max} / \mathrm{min}$ is required, the question will ask for the curve to be drawn first.

Consideration of the gradient on either side is not required. $\mathrm{d}^{2} y$ is not required. $\mathrm{d} x^{2}$
But candidates may use these methods if they wish.

## Typical Questions:

- $y=$ quadratic or cubic. Find the TP(s). State, with a reason, whether each is a max or min.
- $y=a x+b / x$. See Question 13 below.


## Rate of change

Know that $\underline{\mathrm{d} y}$ is the rate of change of $y$ with respect to $x$. $\mathrm{d} x$

## Typical Question:

- See Question 14 below.


## Kinematics

Quadratic, cubic, at $\pm b / t$ only. Notation $\frac{\mathrm{ds}}{\mathrm{d} t}$ and $\frac{\mathrm{d} v}{\mathrm{~d} t}$.

```
not}\frac{\mp@subsup{d}{}{2}s}{d\mp@subsup{t}{}{2}
```


## Typical Questions:

- Given $s$ in terms of $t$, find $v$ and/or $a$ at time $t$ or at given time.
- find max distance from starting point.
- find $t$ for given $s, v$, or $a$ (requiring solutions of equations only within specification).


## Practical problems

## Typical Questions:

- Easier type: See Question 12 below
- Hardest type: See Question 16 below


## Applications to coordinate geometry

Only very simple applications will be tested, possibly requiring understanding of $y=m x+c$.
Usually candidates will be led through step by step. See Questions 7, 15 below.

## Specimen Questions on Calculus (paragraph 3.4 of the specification)

1. Differentiate
(a) $x^{3}+x^{2}-5 x-4$
(b) $2 x^{4}-5 x^{2}+2 x-3$
(c) $3 x^{5}+7 x^{3}-x+2.5$
(d) $5-2 x+4 x^{2}-2 x^{3}$
(e) $\frac{x^{3}}{6}+\frac{3 x^{2}}{4}-\frac{2 x}{3}$
(f) $\frac{7-x^{2}}{2}$
2. Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ for the following.
(a) $y=2 x^{3}+4 x^{2}+x^{-1}$
(b) $y=6 x+3-4 x^{-1}+3 x^{-2}$
(c) $y=\frac{2}{x}-\frac{6}{x^{2}}$
3. Find an expression for the gradient of each of these curves.
(a) $y=x^{5}-3 x^{3}+2 x-4$
(b) $y=3 x+\frac{4}{x^{2}}$
(c) $y=\frac{3 x^{2}+2 x-4}{3}$
4. Find the gradient of the tangent at the given point on each of the following curves.
(a) $y=x^{2}-5 x-6$, at the point where $x=2$
(b) $y=x^{3}-2 x^{2}-3 x$, at the point $(-4,-52)$
(c) $y=3 x-\frac{4}{x^{2}}$, at the point where $x=1 / 2$
(d) $y=\frac{x^{2}+3 x}{12}$ at the point $(3,1.5)$
5. Expand and differentiate
(a) $(x+3)^{2}$
(b) $(2 x-3)(x+5)$
(c) $(4-x)(2+3 x)$
(d) $x^{2}(4-2 x)$
6. A curve has equation $y=x^{2}-3 x+5$.
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(b) Find the gradient of the curve at the point with coordinates $(2,3)$.
(c) Find the coordinates of the point on the curve where the gradient $=-5$.
7. A curve has equation $y=x^{3}-6 x^{2}+9 x-2$.
(a) Find the coordinates of the point on this curve at which the tangent is parallel to the line $y=-3 x+5$
(b) Find the coordinates of the two turning points on this curve.
8. For the curve with equation $y=x^{2}-4 x+5$
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$,
(b) Find the turning point,
(c) State, with a reason, whether this turning point is a maximum or a minimum.
9. Find the maximum value of $y$ where $y=3+6 x-2 x^{2}$. Explain how you know that it is a maximum.
10. A publisher has to choose a price, $£ x$, for a new book.

The total amount of money she will receive from sales is $£ y$, where

$$
y=20000 x-5000 x^{2} .
$$

(a) Find the price which gives the maximum amount of money from sales.
(b) Find the maximum amount of money from sales.
11. The temperature, $T^{\mathrm{o}}$, of a liquid at time $t$ seconds is $t^{2}-6 t+9$.
(a) Find the rate of change of the temperature after 2 seconds.
(b) Find the time when the rate of change of temperature is $-3 \%$ second.
12. A car is moving along a straight road. It passes a point $O$. After $t$ seconds its distance, $s \mathrm{~m}$, from $O$ is given by

$$
s=10 t-t^{2} \quad \text { for } 0 \leq t \leq 10
$$

(a) Find the time when the car passes through $O$ again.
(b) Find $\frac{\mathrm{d} s}{\mathrm{~d} t}$.
(c) Find the maximum distance of the car from $O$.
(d) Find the speed of the car 3 seconds after passing $O$.
(e) Find the acceleration of the car.
13. A curve has equation $y=2 x+\frac{8}{x}$.
(a) Find the turning points.
(b) Copy and complete the table of values for $y=2 x+\frac{8}{x}$.

| $x$ | -4 | -3 | -2 | -1 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ |  | -8.7 | -8 |  | 10 |  |  |  |

(c) Copy the grid and draw the curve for $-4 \leq x \leq 4$.

(d) State which of the turning points is a maximum.
14. A curve has equation $y=x^{3}-3 x^{2}+2 x$.
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(b) Find the $x$ coordinates of the turning points, giving your answers correct to 2 decimal places.
(c) Copy and complete the table of values for $y=x^{3}-3 x^{2}+2 x$.

| $x$ | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: |
| $y$ |  |  |  |

(d) Copy the grid and draw the graph of $y=x^{3}-3 x^{2}+2 x$ for $0 \leq x \leq 2$.

15. A curve has equation $y=x^{2}+3 x+2$
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.

The curve cuts the $y$ axis at $A$.
(b) (i) Write down the coordinates of $A$.
(ii) Find the gradient of the tangent at $A$.
(iii) Write down the equation of the tangent at $A$.
16. Square corners, with side $x \mathrm{~cm}$, are cut from a square card with side 6 cm . Then the edges are folded up to make a box.

(a) Show that the volume of the box is $V \mathrm{~cm}^{3}$ where $V=36 x-24 x^{2}+4 x^{3}$.
(b) Find $\frac{\mathrm{d} V}{\mathrm{~d} x}$.
(c) Find the maximum possible volume of the box.

## Answers

1.(a) $3 x^{2}+2 x-5$
(b) $8 x^{3}-10 x+2$
(c) $15 x^{4}+21 x^{2}-1$
(d) $-2+8 x-6 x^{2}$
(e) $\frac{x^{2}}{2}+\frac{3 x}{2}-\frac{2}{3}$
(f) $-x$
2.(a) $6 x^{2}+8 x-x^{-2}$
(b) $6+4 x^{-2}-6 x^{-3}$
(c) $-\frac{2}{x^{2}}+\frac{12}{x^{3}}$
3. (a) $5 x^{4}-9 x^{2}+2$
(b) $3-\frac{8}{x^{3}}$
(c) $2 x+\frac{2}{3}$
4.(a) -1 (b) 61
(c) 67
(d) 0.75
5. (a) $2 x+6$
(b) $4 x+7$
(c) $10-6 x$
(d) $8 x-6 x^{2}$
6.(a) $2 x-3$
(b) 1 (c) $(-1,9)$
7. (a) $(2,0)$
(b) $(1,2)(3,-2)$
8.(a) $2 x-4$ (b) $(2,1)$ (c) Min. Quadratic with positive coeff of $x^{2}$
9. 7.5. Max because quadratic with negative coeff of $x^{2}$
10.(a) £2 (b) £20 000
11.(a) $-2^{\circ} / \mathrm{sec}$ (b) 1.5 secs
12.(a) 10 s (b) $10-2 t$ (c) 25 m (d) $4 \mathrm{~m} / \mathrm{s}$ (e) $-2 \mathrm{~m} / \mathrm{s}^{2}$
13.(a) $(-2,-8)(2,8)(b)-10,-10,8,8.7,10$ (d) $(-2,-8)$
14.(a) $3 x^{2}-6 x+2$ (b) $0.42,1.58$ (c) $0,0,0$ (d)
(d) $\bigcup$
15. (a) $2 x+3$ (b)(i) $(0,2)$ (ii) 3 (iii) $y=3 x+2$
16.(b) $36-48 x+12 x^{2}$ (c) $16 \mathrm{~cm}^{3}$

## GCSE topics which are omitted from IGCSE

A few topics that are included in GCSE are not included in IGCSE. If a standard GCSE textbook is used, it is important to refer to the IGCSE syllabus and note which topics are not required. Similarly, if past GCSE papers are used, care should be taken to exclude questions on topics that are not required. The relevant topics are largely covered by the following list.

- Exponential growth
- Repeated proportional changes
- Checking by estimation
- Completing the square
- Trial and improvement
- Gradients of perpendicular lines
- Exponential functions
- Transformations of graphs
- Equation of a circle
- SAS, AAS etc
- Proofs of circle theorems
- Trigonometry graphs
- Angles greater than $180^{\circ}$
- Frustum of a cone
- Construct a perpendicular from a point to a line
- Loci
- Negative scale factor
- Plans \& Elevations
- Dimensions
- Metric/Imperial conversion
- Collecting data
- Two-way tables
- Time series. Moving average
- Seasonality and trends
- Scatter graph. Line of best fit
- Correlation
- Box plot. Stem \& leaf

