IGCSE

Mathematics (Specification B)

Teacher's guide

Edexcel IGCSE in Mathematics (Specification B) (4MB0)

First examination 2011

A PEARSON COMPANY

Acknowledgements

This guide has been produced by Edexcel on the basis of consultation with teachers, examiners, consultants and other interested parties. Edexcel would like to thank all those who contributed their time and expertise to its development.

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Introduction

The Edexcel International General Certificate of Secondary Education (IGCSE) in Mathematics (Specification B) is designed for schools and colleges. It is part of a suite of IGCSE qualifications offered by Edexcel.

About this guide

This guide is for teachers who are delivering, or planning to deliver, the Edexcel IGCSE in Mathematics (Specification B) qualification. The guide supports you in delivering the course content and explains how to raise the achievement of your students. The guide provides:

- an outline delivery plan
- outline teaching ideas
- details of Assessment Objectives (AO)
- example questions.

Contents

Why choose this qualification?

The Edexcel IGCSE in Mathematics (Specification B) has been developed to:

- enable students to acquire knowledge and skills with confidence, satisfaction and enjoyment
- provide papers that are balanced in terms of topics and difficulty
- give a basis for students wishing to progress to Edexcel's AS and Advanced GCEs in Mathematics, or equivalent qualifications
- offer grades A* to E.

Go to www.edexcel.com/igcse2009 for more information about this IGCSE and related resources.

Support from Edexcel

We are dedicated to giving you exceptional customer service. Details of our main support services are given below. They will all help you to keep up to date with IGCSE 2009.

Website

Our dedicated microsite www.edexcel.com/igcse2009 is where you will find the resources and information you need to successfully deliver IGCSE qualifications. To stay ahead of all the latest developments visit the microsite and sign up for our email alerts.

Ask Edexcel

Ask Edexcel is our free, comprehensive online enquiry service. Use Ask Edexcel to get the answer to your queries about the administration of all Edexcel qualifications. To ask a question please go to www.edexcel.com/ask and fill out the online form.

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This free service puts teachers in direct contact with over 200 senior examiners, moderators and external verifiers who will respond to subject-specific queries about IGCSE 2009 and other Edexcel qualifications.

You can contact our experts via email or by completing our online form. Visit www.edexcel.com/asktheexpert for contact details.

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If you have any queries about the IGCSE 2009 qualifications, or if you are interested in offering other Edexcel qualifications your Regional Development Manager can help you. Go to www.edexcel.com/international for details of our regional offices.

Head Office — London

If you have a question about IGCSE 2009 and are not sure who you need to ask email us on IGCSE2009@edexcel.com or call our Customer Services Team on +44 (0) 1204770696.

Training

A programme of professional development and training courses, covering various aspects of the specification and examination is available. Go to www.edexcel.com for details.

Section A: Qualification content

Introduction

The Edexcel IGCSE in Mathematics (Specification B) has been designed to replace the legacy Edexcel Ordinary Level GCE in Mathematics B (7361).

There have been no major changes to the specification content and only minimal changes to the style of assessment. This means that you can teach the Edexcel IGCSE in Mathematics (Specification B) without having to spend a lot of time updating schemes of work, and that past papers can still be used as examination practice for your students.

The specification content now includes expanded examples and explanation, so that you can be confident about the skills, knowledge and understanding your students need for this qualification.

The qualification also now offers an A* grade award.

Information for Edexcel centres

Teachers who have taught the O Level GCE in Mathematics B (7361) specification will see that most of the content remains the same. There is a slight re-ordering of the sections; and minor additions and removals have been made and are identified in the following section. Whilst it may seem that there are significant additions in some areas, most of these additions have been examined over the years but were not an explicit part of the specification.

The one significant topic which has been added is the trigonometry of non-right-angled triangles and, as a result, the sine and cosine rule have been introduced. The notes section of the new specification clarifies what students are expected to know and demonstrate. This is particularly evident in the section on *Statistics and Probability* where the key phrases of mutually exclusive events, independent events and conditional probability are introduced.

Changes to content from Edexcel O Level GCE in Mathematics B (7361) to this IGCSE qualification

The table below sets out the relationship between the legacy **O Level GCE in Mathematics B** qualification (7361) and this IGCSE qualification.

Legacy Edexcel qualification content reference	IGCSE content reference	New content/deleted content*
Number Indices	1	**Number** Powers and roots. Simple manipulation of surds. Rationalising the denominator where the denominator is a pure surd.
Mensuration Mensuration of the rectangle, parallelogram, triangle, circle Mensuration of the cylinder, right circular cone and sphere	7	**Mensuration** Trapezium. Cuboid, pyramid and prism.
Sets	2	**Sets** *Binary operations and tables; identity and inverse elements.*
Algebra The manipulation of simple algebraic fractions, the denominators being numerical or linear Variation	3 **4**	**Algebra** Numerical, linear or quadratic. Direct and indirect proportion. $y \propto \frac{1}{x^3}$

Legacy Edexcel qualification content reference	IGCSE content reference	New content/deleted content*
Functions Determination of gradients, rates of change, maxima and minima	4	**Functions** Use functional notations of the form f(x) = …. and f: $x \mapsto$ Stationary points.
Matrices Determinants, singular matrices	5	**Matrices** *Singular matrices.*
Vectors	8	**Vectors** Parallel vectors, unit vectors. Find the resultant of two or more vectors. Apply vector methods to simple geometrical problems.
Geometry Use of Pythagoras's theorem	6	**Geometry** In 2D and 3D. Congruent shapes.
Trigonometry	9	**Trigonometry** Use of the sine and cosine rule. (Non-ambiguous cases)

Legacy Edexcel qualification content reference	IGCSE content reference	New content/deleted content*
Statistics and Probability *Determination of the mean and median of a small number of quantities* Expanded Sum and product rules of probability and their application to simple problems.	10	**Statistics and Probability** Change: Determination of the mean, median and mode for a discrete data set. Determination of a modal class and the median for grouped continuous data. Use of addition rule for two or more mutually exclusive events. Use of the product rule for two or more independent events. Determination of the probability of two or more independent events. Use of simple conditional probability for combined events.

*Any content that has been deleted from the legacy qualification is indicated in italics.

Changes to assessment from Edexcel O Level GCE in Mathematics B (7361) to this IGCSE qualification

There have been no major changes to the scheme of assessment. However, the questions in Paper 1 are now ordered in terms of demand of question, rather than in order of marks.

The A* can now be awarded for students achieving that grade.

Information for centres starting the Edexcel IGCSE in Mathematics (Specification B) for the first time

The table below shows where some mathematical topics appear in the specification. The Edexcel IGCSE in Mathematics (Specification B) is equivalent to an extended curriculum, with grades A*-E.

Mathematical content	Specification area where it appears
Number, set notation and language	1: Number and 2: Sets
Squares, square roots and cubes	1: Number
Directed numbers	1: Number
Vulgar and decimal fractions and percentages	1: Number
Standard form	1: Number
The four rules	1: Number
Estimation and limits of accuracy	1: Number
Ratio, proportion, rate	1: Number
Percentages	1: Number
Use of an electronic calculator	1: Number
Measures, time, money, personal and household finance	1: Number
Graphs in practical situations	4: Functions
Graphs of functions and straight line graphs	4: Functions
Algebraic representation and formulae	3: Algebra
Algebraic manipulation	3: Algebra
Functions	4: Functions
Indices	1: Number and 3: Algebra
Inequalities	3: Algebra
Geometrical terms and relationships	6: Geometry
Geometrical constructions, locus	6: Geometry
Symmetry	6: Geometry
Angle properties	6: Geometry
Mensuration	7: Mensuration
Trigonometry	9: Trigonometry
Statistics and probability	10: Statistics and probability
Vectors	8: Vectors
Matrices	5: Matrices
Transformations	6: Geometry and 5: Matrices

Section B: Assessment

Assessment overview

Paper 1	Percentage	Marks	Time	Availability
External examination paper	$33\frac{1}{3}$	100	1 hour and 30 minutes	January and June examination series First assessment June 2011
Paper 2	**Percentage**	**Marks**	**Time**	**Availability**
External examination paper	$66\frac{2}{3}$	100	2 hours and 30 minutes	January and June examination series First assessment June 2011

Assessment Objectives and weightings

	% in IGCSE
AO1: demonstrate knowledge, understanding and skills in number and algebra: • numbers and the numbering system • calculations • solving numerical problems • equations, formulae and expressions • sequences, functions and graphs • matrices	**60%**
AO2: demonstrate knowledge, understanding and skills in shape, space and measures: • geometry • vectors and transformation geometry • trigonometry	**30%**
AO3: demonstrate knowledge, understanding and skills in handling data: • statistics • probability	**10%**
TOTAL	**100%**

Assessment summary

Paper 1	Description	Knowledge and skills
External examination paper	• Calculators are allowed. • 100 marks available. • $33\frac{1}{3}$ % of the final grade. • About 26 to 30 questions. • Shorter questions at the beginning of the paper. • Questions ramped in difficulty through the paper. • Grades A*-E available. • Questions taken from all areas of the specification content.	All areas of content are covered. • Number • Algebra • Shape, space and measure • Data handling All Assessment Objectives are assessed.

Paper 2	Description	Knowledge and skills
External examination paper	• Calculators are allowed. • 100 marks available. • $66\frac{2}{3}$ % of the final grade. • About 12 questions. • Longer questions than in Paper 1. • Questions ramped in difficulty through the paper. • Grades A*-E available. • Questions taken from all areas of the specification content.	All areas of content are covered. • Number • Algebra • Shape, space and measure • Data handling All Assessment Objectives are assessed.

Examination questions

The following four questions illustrate the type and standard of questions that will be set in the examination. Questions 1 and 2 are targeted at grade C students and Questions 3 and 4 are targeted at grade A students. In each question, the required knowledge is identified and a model answer is provided. You should find the extra comments useful, as experience has shown that some students find questions on these topics difficult.

Using the mark scheme

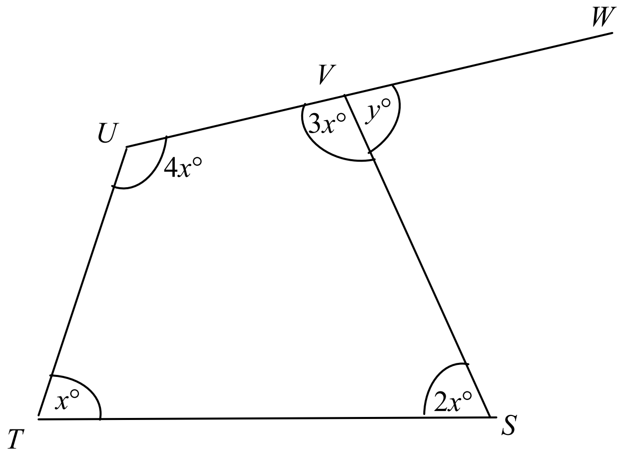
The mark scheme gives the responses we expect from students. Indicative answers are given but during the standardisation of examiners process the mark scheme is updated and expanded to cover unexpected, correct student responses.

General comments on mark schemes

- M marks are ‘method marks’. They are awarded for knowing a method and attempting to apply it. Any formulae used must be correct, either by quoting the formula in its general form before substituting the required numbers, or by having a completely correct substitution.
- A marks are ‘accuracy marks’. They can be awarded only if the relevant method mark(s) have been earned.
- B marks are independent of method marks. They are used, for example, when the question requires students to ‘write something down’ or where a student has made progress in a small part of the question.

Question 1

Targeting grade C: Question 24, January 2004, Paper 1



In quadrilateral *TSVU*, $\angle TUV = 4x°$, $\angle STU = x°$, $\angle TSV = 2x°$ and $\angle SVU = 3x°$.
The point *W* on *UV* produced is such that $\angle SVW = y°$.

Calculate:

(a) x
(b) y.

(c) State, with a reason, what type of quadrilateral is *TSVU*.

(5 marks)

Knowledge needed

1. Angle sum of a quadrilateral.
2. Sum of angles on a straight line.
3. Properties of named quadrilaterals.

Mark scheme and comments

(a) Sum of the internal angles of a quadrilateral = 360°.

So, $x + 2x + 3x + 4x = 360$ which gives $x = 36$.

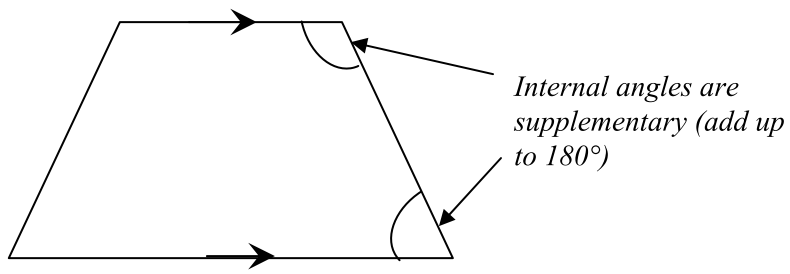
A correct equation earns 1 M mark and the correct answer 1 A mark.

(b) Sum of angles on a straight line = 180°.

So, $3 \times 36 + y = 180$ which gives $y = 72$.

Answer earns 1 B mark.

(c)



As x = 36 so ∠UVS = 3 × 36 = 108° and ∠TSV = 2 × 36 = 72°.
By adding these two angles together we get 180°, so the figure is a trapezium.
Reason: Supplementary angles between parallel lines (or an equivalent correct reason).

Trapezium earns 1 B mark reason earns 1 B mark.

NB: The diagram was deliberately drawn not to look like a trapezium – remember that diagrams in questions are not necessarily drawn to scale. Many students make fundamental errors on geometry questions as they often make incorrect assumptions about the diagram.

As well as angle properties of general polygons, questions are often asked about the angles of regular polygons. The key piece of knowledge for most of these questions is that the sum of all the exterior angles is 360°.

Question 2

Targeting grade C: Question 23, May 2001, Paper 1

Given that $\mathbf{a} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \quad \mathbf{c} = 3\mathbf{a} + 4\mathbf{b}.$

Find the modulus of **c**. Give your answer correct to one decimal place.

(4 marks)

Knowledge/methods needed

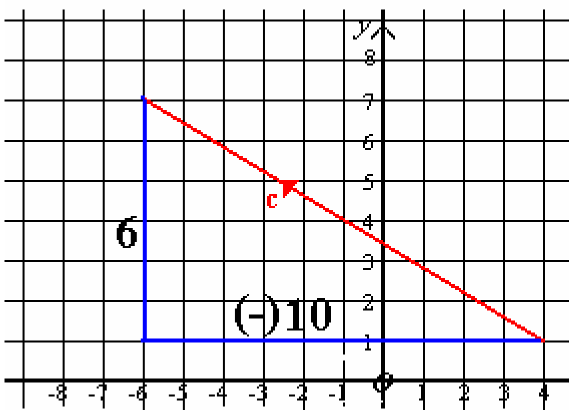
1. Basic technique of multiplying a vector by a scalar.
2. The meaning of the word *modulus* in the context of a vector.
3. Pythagoras's theorem.

Mark scheme and comments

This is a typical 'opening' style question for this topic and the student needs, firstly, to determine the vector **c**.

Method for writing down 3 × a + 4 × b gains 1 M mark. Answer 1 A mark

Now the *modulus* of the vector is the length of the vector.



The figure shows a representation of this vector. Forming the right-angled triangle we have the length of the two shorter sides. Using Pythagoras we can determine the length of **c** as follows:

the modulus (or length) of **c** is

$\sqrt{\{(-10)^2 + (6)^2\}}$

$= \sqrt{136}$

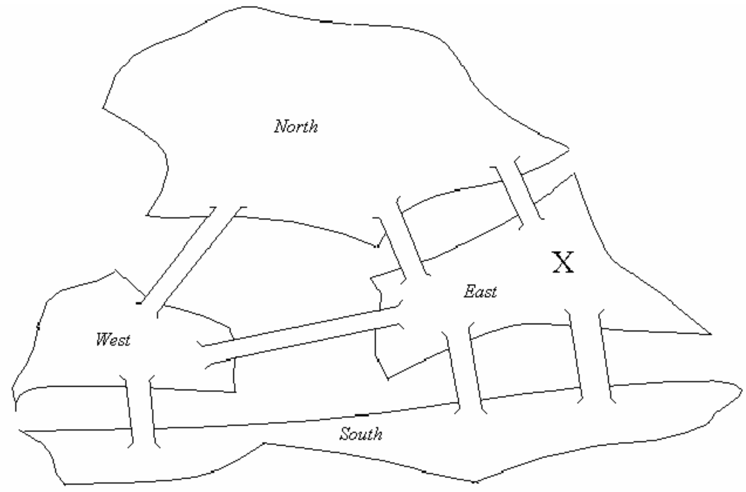
$= 11.7$

Using Pythagoras correctly earns 1 M mark. Answer 1 A mark.

NB: The question asks for an answer to one decimal place. If students ignore these, or similar, instructions on the paper, they could lose marks – remind students to underline requirements for the format or accuracy of the answer, or for units.

Question 3

Targeting grade A: Question 6, May 2002, Paper 2



The republic of Quadrisle is comprised of four islands; North, South, East and West.
The figure shows that these islands are joined by seven bridges.

Fatima lives on East island at location X. On one particular day she decides to go for a walk.
Her walk starts from her home at X, and crosses bridges to adjacent islands.
From any island she chooses a bridge to walk across at random.

Calculate the probability that:

(a) after the first bridge is crossed she will be on North island

(b) after the first bridge is crossed she will not be on West island

(c) after the second bridge is crossed she will be on South island

(d) after the second bridge is crossed she will be back on East island.

(7 marks)

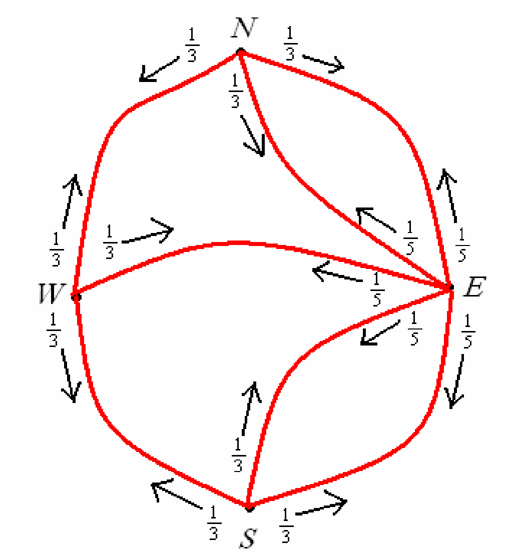
Knowledge/methods needed

1. The numerical value assigned to the likelihood of an event happening is called the *probability* of the event and is usually expressed as a fraction in the range 0 (impossible) to 1 (certainty).
2. When events are equally likely to happen a *sample space* can be drawn to identify all possible events. The probability of an event happening is determined by the number of *favourable* outcomes ÷ number of *possible* outcomes.
3. If the probability of an event happening is p, the probability of the event not happening is $1 - p$.

From each island, a bridge is chosen at random so each bridge is equally likely to be chosen **from those available**.

Because there are seven bridges in total many students use fractions involving $\frac{1}{7}$ ths, rather than looking at each island in turn and applying equal probabilities to the bridges leading from each island.

This type of question (which is proving to be quite popular on examination papers) can be approached by using a network diagram and applying probabilities to the diagram in a similar way to a tree diagram.



Although initially this diagram might look complicated, it indicates, by the lines, the routes from each island to each of the other islands. The black arrows indicate the movement and the fractions indicate the probability that that particular bridge will be chosen for that move.

So how does this help us?

(a) Fatima starts from E and therefore can choose two of five bridges.

The probability that she will be at N after one bridge is $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$. *1 B mark*

Note the addition rule for two mutually exclusive events.

(b) After crossing one bridge, the probability that Fatima will be at W is $\frac{1}{5}$.

So, the probability that Fatima will **not** be at W is $1 - \frac{1}{5} = \frac{4}{5}$. *1 B mark*

Note the use of the complement.

(c) To find the probability that Fatima will be at S after crossing two bridges, we need to identify the only possible route ….. $E \to W \to S$.

The probability that Fatima follows this route is $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$.

Method for combining two correct probabilities together earns 1 M mark. Answer 1 A mark.

Note the use of two independent events.

(d) Like part (c), we need to identify all the possible routes by which Fatima can end up back at E having crossed two bridges. Each route will require multiplying two probabilities together and then we will need to add these compound probabilities together.

The routes are: $E \to W \to E, \quad = \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$.

$+$

$E \to N \to E, \quad = \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$.

$+$

$E \to S \to E, \quad = \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$.

two routes from E to S   two routes from S to E

The required probability is $\frac{1}{15} + \frac{4}{15} + \frac{4}{15} = \frac{9}{15} = \frac{3}{5}$.

For any one correct pair of probabilities	*1 M mark*
All three pairs of probabilities	*1 A mark*
Answer	*1 A mark*

Question 4

Targeting grade A: Question 7, January 2004, Paper 2

A coach was hired for a student trip to Paris from London.

The hire cost was £1008 and this was to be divided equally between all students travelling on the trip.

Let x be the number of students who originally planned to go on the trip.

(a) Write down, in terms of x, an expression for the original cost, in £, per student.

Six students were unable to go on the trip.

The total hire cost had then to be divided equally between the remaining students.

(b) Write down, in terms of x, an expression for the new cost, in £, per student.

As a consequence of the six students being unable to go on the trip, the cost per student increased by £ 4.00

(c) Write down an equation in terms of x, and show that it simplifies to the quadratic equation

$$x^2 - 6x - 1512 = 0.$$

(d) Solve the quadratic equation in part (c) to find the number of students who originally planned to go on the trip.

(8 marks)

Mark scheme and comments

Whilst this question is fundamentally solving a quadratic equation, students need to interpret the problem in a literal context.

Knowledge/skills needed

1. Ability to interpret given information into appropriate algebraic expressions.
2. Interpreting the given data to produce an equation.
3. Removing algebraic fractions in an equation.
4. Solving a quadratic equation by factorisation (or by formula).

(a) Cost per student = total cost/number of students

$$= \frac{1008}{x}.$$ *1 B mark*

(b) As there are now six fewer students, the cost per student is

$$= \frac{1008}{x-6}.$$ *1 B mark*

(c) This is the difficult part of the question. We need to form an equation using the two expressions from parts (a) and (b) and the extra piece of information given: *the cost per student increased by £ 4.00.*

This last statement means that the expression 1008/(x – 6) is £ 4.00 more than 1008/x.

So, $$\frac{1008}{x-6} = \frac{1008}{x} + 4$$ *1 M mark*

To remove the algebraic denominators we need to multiply all three terms by $x(x - 6)$.

So, $$x\cancel{(x-6)}.\frac{1008}{\cancel{x-6}} = \cancel{x}(x-6).\frac{1008}{\cancel{x}} + x(x-6).4$$ *(M1 dep)*

$$1008x = 1008(x - 6) + 4x(x - 6)$$

Which gives $1008x = 1008x - 6048 + 4x^2 - 24x$ which simplifies to:

$4x^2 - 24x + 6048 = 0$ *1 A mark*

Now 4 is a common factor of all three terms so this equation reduces to:

$$x^2 - 6x + 1512 = 0.$$ *1 A mark*

This final A mark is for arriving at the required answer correctly – no errors on the way.

(d) Whilst students may feel more comfortable in solving this quadratic by using the formula, it will factorise. So how do we find factors of such a large number, 1512?

Useful hint

Because it is so large, we can find an estimate simply by finding the square root of the number and adding on half the x value. Using this method we arrive at 38.9 + 3 which is approximately 42. Now dividing 1512 by 42 we can find the second factor which is 36.

Note that this method to determine an estimate only works effectively for large constant terms.

Using these two numbers we can factorise the quadratic into the following:

$(x - 42)(x + 36) = 0$ *1 M mark*

So, $x = 42$ or $x = -36$. Because only one of these answers is positive, this leads us to the required answer of 42 students who originally planned to go on the trip. *1 A mark*

Section C: Planning and teaching

Course planner

Each centre where this course is delivered will allocate appropriate time to deliver the qualification content. This will vary from centre to centre and, whilst the specification content is expected to take two years to deliver, you could use a delivery model requiring only one year of tuition, but please note the first assessment opportunity is in the June 2011 series.

The planner given, therefore, is not intended to be prescriptive in terms of time. However, you can use the order of topics as an outline to plan schemes of work. The course planner identifies four strands, Number, Algebra, Geometry and Statistics.

Each of these strands is repeated. For instance, on a two-year course each strand, numbered (1), could be covered in the first year with the strands numbered (2) being covered in the second year.

Number and Algebra provide the understanding that underpins the rest of the course so these must be covered first. Geometry and Statistics can be covered in either order. As the two Algebra strands are content heavy, they can be sub-divided, with the * topics being left to later in the course. However, you should bear in mind that some of the techniques identified by an * in Algebra (1) may be needed in Geometry (1). Similarly, matrices in Algebra (2) may be linked with the work on vectors in Geometry (2).

Year 1

Content area	Topic
Number (1)	• Processes of number manipulation – integers, fractions and decimals • Prime numbers and factors – HCF and LCM • Indices using positive integers only • Percentages • Weights, measures and money
Algebra (1)	• Basic algebraic processes • Manipulation of algebraic fractions, the denominators being numerical or linear • Construction, interpretation and use of formulae • Factorisation of simple algebraic expressions • Solution of linear equations • Solution of simultaneous equations* • Solution of quadratic equations by factorisation* • Variation* • Sequences • Drawing and interpreting graphs from given equations* (see specification for types of equations) • Gradients*
Geometry (1)	• Length, area and volume • The circle • Mensuration of standard two- and three-dimensional shapes • Angle and geometrical properties of lines and polygons • Symmetry • Pythagoras • Simple uses of basic trigonometry • Simple properties of chords and tangents of circles • Concept of a vector and simple manipulation of vectors
Statistics (1)	• Graphical representation of numerical data • Measures of central tendency – mean, median and mode • Use of simple probability

Year 2

Content area	Topic
Number (2)	• Ratio and proportion • Significant figures and standard form • Indices using fractional and negative powers • Surds – to include concept of rationals and irrationals • Sets
Algebra (2)	• Factor theorem • The manipulation of algebraic fractions, the denominators being quadratic • Solution of quadratic equations by methods other than factorisation • Linear inequalities • Applying algebraic techniques to solve problems* • Functions* • Calculus and simple applications* • Matrices and transformations*
Geometry (2)	• Similarity and congruency • Loci and constructions • Angle and intersecting chord properties of a circle • Applying vector methods • Bearings, angles of elevation and depression • Sine and cosine rule • Applying trigonometry to two- and three-dimensional problems
Statistics (2)	• Histograms • Addition and product rules of probability • Conditional probability

Teaching ideas

Notes on set language and notation

Typical questions

Given defined sets ℰ, A & B

a) describe $A \cap B$

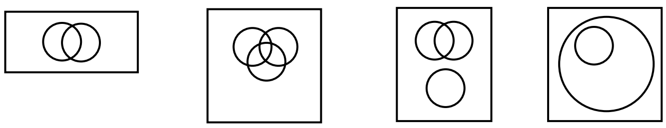
b) list the members of $A \cup B$

c) what is meant by ‘$6 \in A$’?

d) is it true that $A \cap B = \emptyset$? Explain your answer.

Definition: Algebraic, for example {ℰ = Integers}, $P = \{x: 0 < x < 10\}$

Venn diagrams: Different cases, for example



Symbols: A‘ (the complement of A), $\subset$ (‘is a subset of’)

Typical questions

Given defined sets ℰ, A, B, and C

a) draw a Venn diagram.

b) shade $A \cup B \cap C'$

c) list the members of $B' \cap C$

d) is it true that $A \subset B$?

e) describe a given shaded region in a Venn diagram

f) draw a Venn diagram in which certain conditions are true.

Symbols: n(A) (the number of members in A)

Typical questions

Given n(ℰ) = 23, n(A) = 16, n(B) = 10, $n(A \cup B) = 20$

a) draw a Venn diagram

b) show the number of members in each region.

Specimen questions on set language and notation

1. $\mathcal{E}$ = {Positive integers less than 20}

$A = \{x: 0 < x \leq 9\}$

B = {Even numbers}

C = {Multiples of 5}

(a) List the members of $A \cap B'$.

(b) Find the value of $n(A \cup C)$.

(c) Complete the statement $A \cap B \cap C = \ldots$

(d) Is it true that $(A \cap C') \subset B$? Explain your answer.

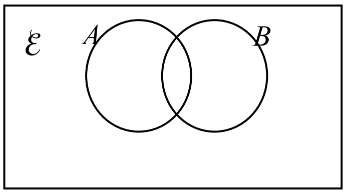
2. There are 30 people in a group. 17 own a car. 11 own a bicycle. Five do not own a car or a bicycle.

Find how many people in this group own a car but not a bicycle.

3. Draw a Venn diagram with circles representing three sets, A, B and C.

Shade the region representing $A \cap (B \cup C')$.

4.



Make two copies of this Venn diagram.

(a) On one diagram draw a circle to represent set C, such that

$C \subset A$ and

$C \cap B' = C$.

(b) On the other diagram draw a circle to represent set D, such that

$D \subset A'$,

$D \cap B \neq \emptyset$ and

$D \cup B \neq D$.

5. Draw a Venn diagram with circles representing three sets, A, B and C, such that all the following are true:

$A \cap C \neq \varnothing$, $A \cap C' \neq \varnothing$ and $B \subset (A \cup C)'$

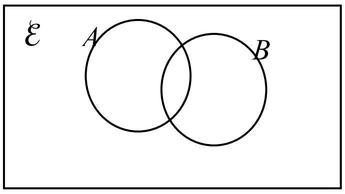
6. $\mathcal{E} = \{ x : x \text{ is an integer and } 1 \leq x \leq 30 \}$

A = {Multiples of 3}

B = {Multiples of 4}

(a) Find the value of $n(A \cap B)$.

Sets A and B are represented by circles in the Venn diagram.



(b) C = {Odd numbers}

(i) Copy the Venn diagram, and draw a circle to represent set C.

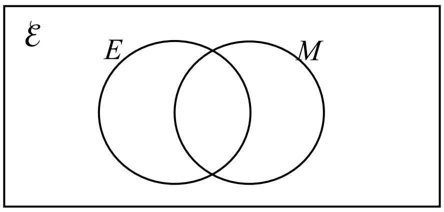
(ii) Shade the region $A \cap (B \cup C)'$.

(ii) Write down all the values of x such that $x \in A \cap (B \cup C)'$.

7. $\mathcal{E}$ = {Positive integers less than 15}

E = {Even numbers}

M = {Multiples of 3}



(a) Copy the Venn diagram and fill in each member of $\mathcal{E}$ in the correct region.

(b) Write down the value of $n(E \cap M')$.

8. $\mathcal{E}$ = {Quadrilaterals}

P = {Parallelograms}

K = {Kites}

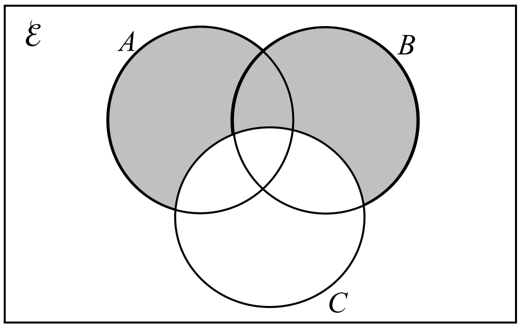
S = {Squares}

(a) What is the mathematical name for a member of $P \cap K$?

(b) Complete the statement $P \cup S = \ldots$

(c) Draw a Venn diagram showing sets P, K and S.

9.



Use set notation to describe the shaded region.

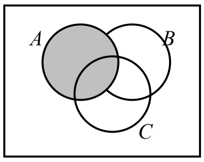
10. There are 40 members in a sports club. Two play all three sports. 23 play squash. 24 play tennis. 18 play golf. 14 play squash and tennis. Eight play tennis and golf.

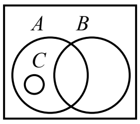
One member makes the refreshments and does not play any sport. How many members play squash and golf?

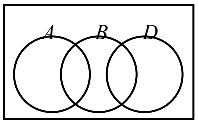
Answers

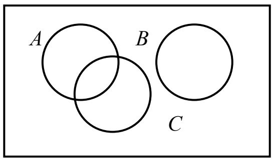
1. (a) 1, 3, 5, 7, 9 (b) 11 (c) Ø (d) No. Eg 3, 7 or 9

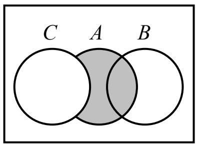
2. 14

3. 

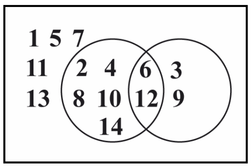
4. (a) 

(b) 

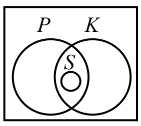
5. 

6. (a) 2 (b)(i)(ii) 

(iii) 6, 18

7. (a) 

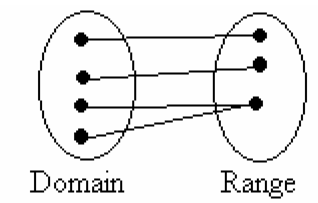
(b) 5

8. (a) Rhombus (b) *P* (c) 

9. $(A \cup B) \cap C'$ or $(A \cap C') \cup (B \cap C')$ **10**. 6

Notes on function notation

Notation and definitions: f(x) = x^2 f: $x \to x^2$ |



Domain is all values of x to which the function is applied.

Range is all values of f(x).

Domain and/or range may be given in words, as a list or algebraically, for example $0 \le x < 10$.
If the domain is not given, it is assumed to be {x: x is any number}.

Which functions?

Usually, for example linear, quadratic, cubic, $\sqrt{x}$, 1/linear.

Sometimes harder functions, for example √linear, 1/√linear, linear/linear, √quadratic, 1/quadratic, $a + b/x$, $ax + b/x$

Note: '$\sqrt{}$' indicates the positive value of the square root.

Typical questions

- Given a function and its domain, find the range.
- Given a function applied to all numbers, find the range.
- Given a function, which values cannot be included in the domain?
- Given f(x), find f(–2).
- Given f(x) = 3, find the value(s) of x (not necessarily involving the notation f^{-1}).

Composite functions

fg(x) means f(g(x)), ie do g first followed by f.

Typical questions

- Given functions f and g, find fg(–3), gf(2).
- Given functions f and g, find fg in the form fg: $x \mapsto \ldots$ or fg$(x) = \ldots$
- Given functions f and g, and the domain of f, find the range of gf.
- Given functions f and g, which values need to be excluded from the domain of gf?

Inverse functions

Functions required

Usually, for example linear, 1/linear, or x^2 (with domain restricted to positive numbers).

Sometimes harder functions, for example √linear, linear/linear, $a + b/x$.

Any method for finding f^{-1} is acceptable, for example:

Algebraic: write as $y = \ldots$; rearrange to make x the subject; interchange x and y

Flow chart: reverse each operation, in reverse order.

Typical questions

- Given the function f, find $f^{-1}(3)$.
- Given the function f, find f^{-1} in the form f^{-1}: $x \mapsto \ldots$ or $f^{-1}(x) = \ldots$
- Without working, write down the value of $ff^{-1}(5)$.
- Given functions f and g, find the function $f^{-1}g$.
- Given functions f and g, solve the equation $f(x) = g^{-1}(x)$.

Specimen questions on function notation

1. Here are three functions.

$f(x) = 3 - 2x$ $\qquad g(x) = \dfrac{1}{x-2}$ $\qquad h(x) = \sqrt{3x+1}$

(a) Find (i) f(-1) (ii) f($\frac{3}{4}$) (iii) g(4.5) (iv) g(-2) (v) h(5) (vi) h$\left(2\frac{2}{3}\right)$

(b) (i) Given that f(x) = -7, find x.

(ii) Given that g(x) = 2, find x.

(iii) Given that h(x) = 5, find x.

2. Three functions, p, q and r, are defined as follows.

$p(x) = x^2 - 3x + 4$ $\qquad q(x) = \dfrac{2x-3}{x+1}$ $\qquad r(x) = \sin x^{\circ}$

(a) Find (i) p(–4) (ii) p($\frac{3}{4}$) (iii) q(4) (iv) q(–2) (v) r(45) (vi) r(180)

(b) (i) Find the values of x for which p(x) = 2.

(ii) Find the value of x for which q(x) = $\frac{3}{4}$.

(iii) Find the values of x, in the domain $0 \le x \le 180$, for which r(x) = 0.5

3. State which values of x cannot be included in the domain of these functions.

(i) f: $x \mapsto \sqrt{5-x}$ (ii) g:$x \mapsto \dfrac{5}{2x-7}$ (iii) h: $\mapsto \dfrac{1}{\sqrt{x+3}}$ (iv) j: $x \mapsto \sqrt{(x^2-4)}$

(v) l: $x \mapsto 2x + \dfrac{1}{x}$ (vi) k: $x \mapsto \dfrac{1}{(3x+2)}$ (vii) m: $x \to \sqrt{\dfrac{x-3}{6-x}}$

4. f: $x \mapsto x^3$ $\qquad$ g: $x \mapsto \dfrac{1}{x+8}$

(a) Find (i) fg(–4), (ii) gf(5).

(b) Find (i) gf(x), (ii) fg(x).

(c) What value(s) must be excluded from the domain of (i) gf(x), (ii) fg(x)?

(d) Find and simplify gg(x).

5. Three functions are defined as follows.

$p(x) = (x + 4)^2$ with domain $\{x: x \text{ is any number}\}$

$q(x) = 8 - x$ with domain $\{x: x > 0\}$

$r(x) = \cos x^{\circ}$ with domain $\{x: 0 \leq x \leq 180\}$

(a) Find the range of each of these functions.

(b) Find the values of x such that $p(x) = q(x)$.

6. Find the inverse function of each of the following functions.

(a) $f(x) = 2x - 3$ (b) $g(x) = 5 - x$ (c) $h(x) = \dfrac{1}{3x+4}$ (d) $j(x) = 3 - \dfrac{2}{x}$

(e) $k(x) = \dfrac{2x+1}{5-x}$

7. Find the inverse function of each of the following functions.

(a) $p: x \mapsto \sqrt{3x-2}$ (for $x \geq \frac{2}{3}$) (b) $q: x \mapsto \dfrac{1}{\sqrt{x+2}}$ (for $x > -2$)

(c) $r: x \mapsto x^2 + 5$ (for $x \geq 0$) (d) $s: x \mapsto (x-3)^2$ (for $x \geq 3$)

8. The function $f(x)$ is defined as $f(x) = \dfrac{2}{x+1}$.

Solve the equation $f(x) = f^{-1}(x)$.

9. Here are two functions.

$f(x) = \dfrac{2}{5+x}$ $g(x) = x^2 + 3$

(a) Calculate $g(-2)$.

(b) Given that $f(z) = \frac{1}{8}$, calculate the value of z.

(c) Which value of x must be excluded from the domain of $f(x)$?

(d) Find the inverse function, f^{-1}, in the form $f^{-1} : x \mapsto \ldots$

(e) Calculate $f^{-1}g(1)$.

10. Functions f and g are defined as follows.

f: $x \mapsto 4 + \sqrt{x}$ g: $x \mapsto \dfrac{1}{(x+2)^2}$

(a) Calculate (i) f(25) (ii) g(0.5) (iii) fg(–1).

(b) Given that fg(x) = 4.04, find the value of x.

(c) Find the function $f^{-1}(x)$.

(d) Calculate $gf^{-1}(4)$.

11. $p(x) = \dfrac{2-x}{3+x}$ $q(x) = \dfrac{2-3x}{1+x}$

(a) Find the function pq(x).

(b) Hence describe the relationship between the functions p and q.

(c) Write down the exact value of $pq(\sqrt{2})$.

Answers

NB: In the examination, equivalent answers are acceptable, for example decimal instead of fraction.

1. (a) (i) 5 (ii) $1\frac{1}{2}$ (iii) $\frac{2}{5}$ (iv) –0.25 (v) 4 (vi) 3 (b) (i) 5 (ii) 2.5 (iii) 8

2. (a) (i) 32 (ii) $2\frac{5}{16}$ (iii) 1 (iv) 7 (v) 0.707 (vi) 0 (b) (i) 1 or 2 (ii) 3 (iii) 30 or 150

3. (i) $x > 5$ (ii) $x = 3.5$ (iii) $x \leq -3$ (iv) $-2 < x < 2$ (v) $x = 0$ (vi) $x = -\frac{2}{3}$ (vii) $x < 3$ or $x \geq 6$

4. (a) (i) $\frac{1}{64}$ (ii) $\frac{1}{133}$ (b) (i) $\dfrac{1}{x^3+8}$ (ii) $\dfrac{1}{(x+8)^3}$ (c) (i) $x = -2$ (ii) $x = -8$ (d) $\dfrac{x+8}{8x+65}$

5. (a) p: ≥ 0; q: < 8; *r*: –1 to 1 (b) –8 or –1

6. (a) $\dfrac{x+3}{2}$ (b) $5 - x$ (c) $\dfrac{1-4x}{3x}$ (d) $\dfrac{2}{3-x}$ (e) $\dfrac{5x-1}{2+x}$

7. (a) $\dfrac{x^2+2}{3}$ (b) $\dfrac{1-2}{x^2}$ (c) $\sqrt{x-5}$ (d) $\sqrt{x}+3$

8. 1 or –2

9. (a) 7 (b) 11 (c) –5 (d) $\dfrac{2}{x} - 5$ (e) $-4\frac{1}{2}$

10. (a) (i) 9 (ii) 0.16 (iii) 5 (b) 23 (c) $(x-4)^2$ (d) $\frac{1}{4}$

11. (a) $\text{pq}(x) = x$ (b) Inverses of each other (c) $\sqrt{2}$

Notes on calculus

Basic concepts and notation

Ideas of gradient of tangent and gradient of curve.

$y = x^n \Rightarrow \text{grad} = \dfrac{dy}{dx}$.

$= nx^{n-1}$,

firstly for +ve integer n; then also $n = 0, -1, -2$.

Differentiation of polynomials.

Usually no rearrangement will be required.

If rearrangement is required, this will usually be asked for explicitly.

> Differentiation from first principles is not required.
>
> If you wish to give an introduction to the concept of a limiting gradient, the following is adequate, though it will NOT be tested:
>
> On the curve $y = x^2$,
>
> $P(3, 3^2)$; $Q_1(3.1, (3.1)^2)$; $Q_2(3.01, (3.01)^2)$; etc
>
> Find gradients of PQ_1, PQ_2, PQ_3 . . .

Typical questions

- Differentiate $x^5 - 3x^2 + 5$ or $x^2 + 3x - 4$.
- Given $y = \dfrac{5x+3}{2}$, find $\dfrac{dy}{dx}$.
- $y = (x + 3)^2$. Expand and find $\dfrac{dy}{dx}$.

> The notation $f'(x)$ and the terms 'derivative' and 'derived function' are not required.

Turning points

At turning points, $\dfrac{dy}{dx} = 0$.

> The language used will be 'turning points', 'maximum', 'minimum'; not 'stationary points'.
>
> Points of inflexion are not required.

Find TPs for quadratic, cubic, $ax \pm \dfrac{b}{x}$

Distinguish max/min by rough shape,

For example, shape of $y = ax^2 + bx + c$ is ∩ when $a < 0$.

For $ax \pm \dfrac{b}{x}$, if distinguishing max/min is required, the question will ask for the curve to be drawn first.

> Consideration of the gradient on either side is not required.
>
> $\dfrac{d^2y}{dx^2}$ is not required.
>
> But students may use these methods if they wish.

Typical questions

- y = quadratic or cubic. Find the TP(s). State, with a reason, whether each is a max or min.

Rate of change

Know that $\frac{\mathrm{d}y}{\mathrm{d}x}$ **is the rate of change of** ***y*** **with respect to** ***x*****.**

Kinematics

Quadratic, cubic, $at \pm b/t$ **only.** **Notation** $\frac{\mathrm{d}s}{\mathrm{d}t}$ **and** $\frac{\mathrm{dv}}{\mathrm{dt}}$

not $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

Typical questions

Given s in terms of *t*, find *v* and/or *a* at time t or at given time.

Find maximum distance from a starting point.

Find *t* for given *s*, *v*, or *a* (requiring solutions of equations only within specification).

Applications to coordinate geometry

Only very simple applications will be tested, possibly requiring understanding of $y = mx + c$.

Usually students will be led through step-by-step.

Specimen questions on calculus

1. Differentiate

(a) $x^3 + x^2 - 5x - 4$ (b) $2x^4 - 5x^2 + 2x - 3$ (c) $3x^5 + 7x^3 - x + 2.5$

(d) $5 - 2x + 4x^2 - 2x^3$ (e) $\frac{x^3}{6} + \frac{3x^2}{4} - \frac{2x}{3}$ (f) $\frac{7 - x^2}{2}$

2. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the following.

(a) $y = 2x^3 + 4x^2 + x^{-1}$ (b) $y = 6x + 3 - 4x^{-1} + 3x^{-2}$ (c) $y = \frac{2}{x} - \frac{6}{x^2}$

3. Find an expression for the gradient of each of these curves.

(a) $y = x^5 - 3x^3 + 2x - 4$ (b) $y = 3x + \frac{4}{x^2}$ (c) $y = \frac{3x^2 + 2x - 4}{3}$

4. Find the gradient of the tangent at the given point on each of the following curves.

(a) $y = x^2 - 5x - 6$, at the point where $x = 2$ (b) $y = x^3 - 2x^2 - 3x$, at the point $(-4, -52)$

(c) $y = 3x - \frac{4}{x^2}$, at the point where $x = \frac{1}{2}$ (d) $y = \frac{x^2 + 3x}{12}$ at the point $(3, 1.5)$

5. Expand and differentiate

(a) $(x + 3)^2$ (b) $(2x - 3)(x + 5)$ (c) $(4 - x)(2 + 3x)$ (d) $x^2(4 - 2x)$

6. A curve has equation $y = x^2 - 3x + 5$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(b) Find the gradient of the curve at the point with coordinates (2, 3).

(c) Find the coordinates of the point on the curve where the gradient = –5.

7. A curve has equation $y = x^3 - 6x^2 + 9x - 2$.

(a) Find the coordinates of the point on this curve at which the tangent is parallel to the line $y = -3x + 5$.

(b) Find the coordinates of the two turning points on this curve.

8. For the curve with equation $y = x^2 - 4x + 5$

(a) find $\frac{\mathrm{d}y}{\mathrm{d}x}$

(b) find the turning point

(c) state, with a reason, whether this turning point is a maximum or a minimum.

9. Find the maximum value of y where $y = 3 + 6x - 2x^2$. Explain how you know that it is a maximum.

10. A publisher has to choose a price, £x, for a new book.

The total amount of money she will receive from sales is £y, where $y = 20\,000x - 5000x^2$.

(a) Find the price which gives the maximum amount of money from sales.

(b) Find the maximum amount of money from sales.

11. The temperature, T°, of a liquid at time t seconds is $t^2 - 6t + 9$.

(a) Find the rate of change of the temperature after two seconds.

(b) Find the time when the rate of change of temperature is –3 °/second.

12. A car is moving along a straight road. It passes a point O.

After t seconds its distance, s m, from O is given by

$s = 10t - t^2$ for $0 \leq t \leq 10$

(a) Find the time when the car passes through O again.

(b) Find $\frac{\mathrm{d}s}{\mathrm{d}t}$.

(c) Find the maximum distance of the car from O.

(d) Find the speed of the car three seconds after passing O.

(e) Find the acceleration of the car.

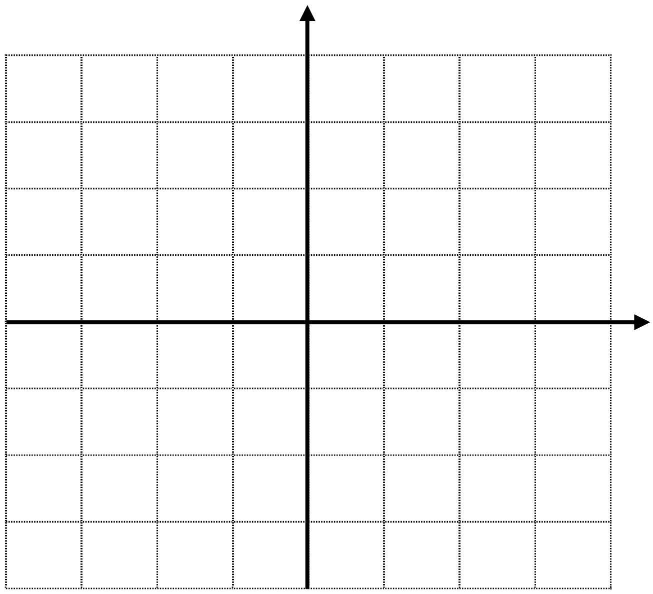
13. A curve has equation $y = 2x + \dfrac{8}{x}$.

(a) Find the turning points.

(b) Copy and complete the table of values for $y = 2x + \dfrac{8}{x}$.

x	–4	–3	–2	–1	1	2	3	4
y		–8.7	–8		10			

(c) Copy the grid and draw the curve for $-4 \le x \le 4$.



(d) State which of the turning points is a maximum.

14. A curve has equation $y = x^3 - 3x^2 + 2x$.

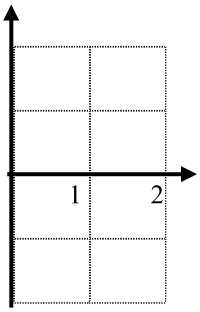
(a) Find $\frac{dy}{dx}$.

(b) Find the x coordinates of the turning points, giving your answers correct to two decimal places.

(c) Copy and complete the table of values for $y = x^3 - 3x^2 + 2x$.

x	0	1	2
y			

(d) Copy the grid and draw the graph of $y = x^3 - 3x^2 + 2x$ for $0 \leq x \leq 2$.



15. A curve has equation $y = x^2 + 3x + 2$

(a) Find $\frac{dy}{dx}$.

The curve cuts the y axis at A.

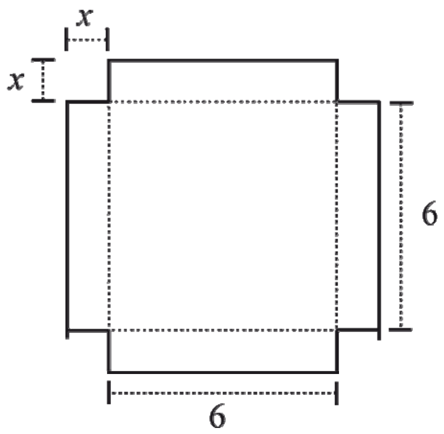
(b) (i) Write down the coordinates of A.

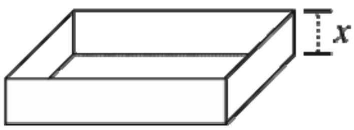
(ii) Find the gradient of the tangent at A.

(iii) Write down the equation of the tangent at A.

16. Square corners, with side x cm, are cut from a square card with side 6 cm.

Then the edges are folded up to make a box.





(a) Show that the volume of the box is V cm^3 where $V = 36x - 24x^2 + 4x^3$.

(b) Find $\frac{dV}{dx}$.

(c) Find the maximum possible volume of the box.

Answers

1. (a) $3x^2 + 2x - 5$ (b) $8x^3 - 10x + 2$ (c) $15x^4 + 21x^2 - 1$ (d) $-2 + 8x - 6x^2$

(e) $\frac{x^2}{2} + \frac{3x}{2} - \frac{2}{3}$ (f) $-x$

2. (a) $6x^2 + 8x - x^{-2}$ (b) $6 + 4x^{-2} - 6x^{-3}$ (c) $-\frac{2}{x^2} + \frac{12}{x^3}$

3. (a) $5x^4 - 9x^2 + 2$ (b) $3 - \frac{8}{x^3}$ (c) $2x + \frac{2}{3}$

4. (a) –1 (b) 61 (c) 67 (d) 0.75

5. (a) $2x + 6$ (b) $4x + 7$ (c) $10 - 6x$ (d) $8x - 6x^2$

6. (a) $2x - 3$ (b) 1 (c) (-1, 9)

7. (a) (2, 0) (b) (1, 2) (3, –2)

8. (a) $2x - 4$ (b) (2, 1) (c) Min. Quadratic with positive coeff of x^2

9. 7.5. Max because quadratic with negative coeff of x^2

10. (a) £2 (b) £20 000

11. (a) – 2 °/sec (b) 1.5 secs

12. (a) 10s (b) $10 - 2t$ (c) 25m (d) 4m/s (e) – 2 m/s^2

13. (a) (–2, –8) (2, 8) (b) –10, –10, 8, 8.7, 10 (d) (–2, –8)

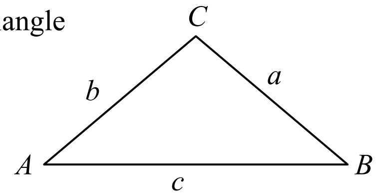
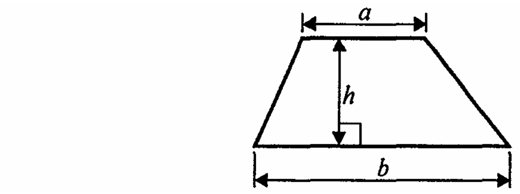
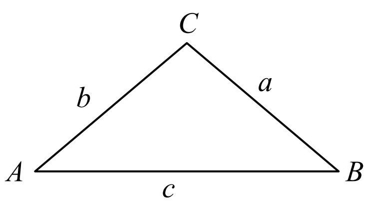
14. (a) $3x^2 - 6x + 2$ (b) 0.42, 1.58 (c) 0, (d) 

15. (a) $2x + 3$ (b) (i) (0, 2) (ii) 3 (iii) $y = 3x + 2$

16. (b) $36 - 48x + 12x^2$ (c) 16 cm^3

Appendices

Appendix 1: Formulae for Paper 2

Circumference of a circle	$2\pi r$
Area of a circle	πr^2
Area of a triangle 	$\frac{1}{2}bc\sin A$
Area of a trapezium 	$\frac{1}{2}(a+b)h$
Curved surface area of right circular cylinder	$2\pi rh$
Curved surface area of right circular cone	πrl
Surface area of sphere	$4\pi r^2$
Volume of pyramid	$\frac{1}{3}$ × base area × height
Volume of right circular cone	$\frac{1}{3}\pi r^2 h$
Volume of sphere	$\frac{4}{3}\pi r^3$
Sum of interior angles of polygon	$(2n-4)$ right angles
Solutions of $ax^2+bx+c=0$	$x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$
Determinant of matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	$ad - bc$
Inverse of matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
	Sine rule $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
	Cosine rule $a^2=b^2+c^2-2bc\cos A$

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