CANDIDATE
NAME

CENTRE NUMBER

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CANDIDATE NUMBER


Paper 6 (Extended)
May/June 2012
1 hour 30 minutes
Candidates answer on the Question Paper
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
You may use a pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.
Answer both parts $\mathbf{A}$ and $\mathbf{B}$.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40 .

## Answer both parts A and B.

## A INVESTIGATION

## ADDITIONAL TRIPLES (20 marks)

You are advised to spend 45 minutes on part $\mathbf{A}$.
An addition triple has three different numbers.
The numbers $(8,10,18)$ form an addition triple because $8+10=18$.
Some other addition triples are $(10,11,21)$ and $(21,24,45)$.
This investigation explores patterns with addition triples.
1 Nine addition triples can be found from the list of integers 1, 2, 3, 4, 5, 6, 7 . One of these triples is $(3,4,7)$.

Write down the other eight addition triples in the spaces provided.
[Note that $(3,4,7)$ and $(4,3,7)$ are the same addition triple.]
( .......... , .......... , .......... )
( .......... , .......... , .......... )
( ......... , .......... , .......... )
( .......... , .......... , .......... )
( .......... , .......... , ........... )
( .......... , .......... , ........... )
( .......... , .......... , .......... )
( .......... , .......... , .......... )
$(3,4,7)$

2 Complete the table, showing the addition triples for each list of integers.
In the last column write the total number of triples.

| Number <br> of <br> integers | List of integers | Addition triples | Total number <br> of addition <br> triples |
| :---: | :--- | :--- | :---: |
| 3 | $1,2,3$ | $(1,2,3)$ | 1 |
| 4 | $1,2,3,4$ |  | 2 |
| 5 | $1,2,3,4,5$ |  |  |
| 6 | $1,2,3,4,5,6$ |  |  |
| 7 |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| 8 | $1,2,3,4,5,6,7$ |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

3 Look at the pattern in the last column in the table on page 3.
Use it to complete the following table.

| Number of <br> integers | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of <br> addition triples | 1 | 2 |  |  | 9 | 12 | 16 | 20 |  | 30 |  |  |  |

4 Using Question 3, complete the following table when there is an odd number of integers in the list.

| Number of <br> integers | 3 | 5 | 7 | 9 | 11 | 13 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> addition triples | 1 |  | 9 | 16 |  |  |  |

5 For the table in Question 4, the same three arithmetic operations always take you from the number of integers in the list to the corresponding number of addition triples.

The first operation is subtract 1 .
Find the other two operations.
Show that these three operations take you
from 7 integers in the list to 9 addition triples,
and
from 9 integers in the list to 16 addition triples.

6 Using Question 5, find
(a) the number of addition triples when there are 101 integers in the list,
(b) the number of integers in the list when there are 11449 addition triples,
(c) an expression for the number of addition triples when the list has $n$ integers and $n$ is odd.

7 Using patterns in the table in Question 3, find
(a) the number of addition triples when there are 100 integers in the list,
(b) the number of integers in the list when there are 1332 addition triples,
(c) an expression for the number of addition triples when the list has $n$ integers and $n$ is even.

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## B MODELLING <br> REGIOMONTANUS' STATUE (20 marks)

You are advised to spend 45 minutes on part $\mathbf{B}$.
In the $15^{\text {th }}$ century the German mathematician Regiomontanus worked out the best place to stand to view a statue that was on top of a column.

The picture shows a statue of height one metre.
The base $C$ of the statue is one metre above the line of sight $A D$.
Angle $B A C$ is called the angle of view.
The largest angle of view gives the best view of the statue.


1 The diagram models the picture.


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Regiomontanus stands 3 metres from the base of the column so $A D=3 \mathrm{~m}$.
(a) (i) Use the right-angled triangle $\boldsymbol{A} \boldsymbol{D} \boldsymbol{B}$ to show that the length $A B=\sqrt{13}$.
(ii) Use this answer to write down $\sin A B D$ as a fraction.
(b) Show that the length $A C=\sqrt{10}$.
(c) Regiomontanus wrote that, in triangle $A B C$,

$$
\frac{\sin A}{a}=\frac{\sin B}{b}
$$

Show that $\sin B A C=\frac{3}{\sqrt{130}}$.

2 Using the method in Question 1, find $\sin B A C$ when $A D=1 \mathrm{~m}$.

3 Model $\sin B A C$ by letting $\mathrm{AD}=x$ metres.

Show that $\sin B A C=\frac{x}{\sqrt{\left(x^{2}+1\right)\left(x^{2}+4\right)}}$.


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4 (a) Using the model in Question 3, sketch the graph of $\sin B A C$ against $x$.

(b) Find the value of $x$ which makes $\sin B A C$ a maximum.
(c) Find the largest angle of view.

5 (a) Instead of one metre high, the statue is $h$ metres high. The base of the statue is still one metre above the line of sight.

Modify the model in Question 3.
(b) The one metre high statue is replaced by a statue that is 2 metres high. Use your model from part (a) to find the change (if any) in
(i) the largest angle of view,
(ii) the corresponding distance from the column.

