# International General Certificate of Secondary Education UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE MATHEMATICS 0580/4, 0581/4 

PAPER 4
Wednesday 8 NOVEMBER 2000 Morning 2 hours 30 minutes
Additional materials:
Answer paper
Electronic calculator Geometrical instruments
Graph paper (1 sheet)
Mathematical tables (optional)
Tracing paper (optional)
TIME 2 hours 30 minutes

## INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/ answer booklet

Answer all questions.
Write your answers on the separate answer paper provided.
All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.
If you use more than one sheet of paper, fasten the sheets together.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130 .
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.
For $\pi$, use either your calculator value or 3.142.


A company's accounts for 1998 are shown in the pie chart above.
The profit was $\$ 36000$ and the labour costs were $\$ 75000$.
(a) (i) The angle in the profit sector is $x^{\circ}$. Show that $x=72$.
(ii) Calculate the amount paid for materials.
(iii) Find the ratio tax : profit in its lowest terms.
(b) In 1999 the labour costs were $\$ 78000$.
(i) Write 78000 in standard form.
(ii) Calculate the percentage increase in labour costs from 1998 to 1999.
(c) The labour costs of $\$ 78000$ in 1999 were $\mathbf{1 6 0 \%}$ more than the labour costs in 1993. Calculate the labour costs for 1993.
(a) $\left(\begin{array}{rrr}3 & 0 & 0 \\ 9 & 5 & 0 \\ 4 & -3 & 2\end{array}\right)\left(\begin{array}{l}1 \\ q \\ r\end{array}\right)=\left(\begin{array}{c}p \\ -26 \\ 35\end{array}\right)$.

Find the values of $p, q$ and $r$.
(b) $\mathbf{M}=\left(\begin{array}{ll}t & 6 \\ t & 5 t\end{array}\right) \quad$ and $\quad \mathbf{M}^{-1}=\left(\begin{array}{rr}-5 t & 6 \\ t & -t\end{array}\right) \quad$ where $\mathbf{M}^{-1}$ is the inverse of $\mathbf{M}$ and $t \neq 0$.

Write down an equation in $t$ and solve it.
(c) $(x$
2) $\binom{x}{5}=k x$ is a matrix equation.
(i) Find the value of $k$ if $x^{2}+8 x+10=0$.
(ii) Solve the equation $x^{2}+8 x+10=0$.

Show all your working and give your answers correct to 2 decimal places.

## NOT TO



Diagram 1


Diagram 2

The diagrams show a triangular prism $A B C D E F$ and a rough sketch of one of its possible nets. $A B=4 \mathrm{~cm}, B C=5 \mathrm{~cm}, A C=6 \mathrm{~cm}$ and $C F=7 \mathrm{~cm}$.
(a) Calculate, correct to one decimal place, the angle $A B C$ shown in Diagram 1.
(b) Construct accurately the net of the prism shown in Diagram 2.

Label every corner of your net with the appropriate letter $A, B, C, D, E$ or $F$.
(c) Taking any measurements from your net which you may need and showing all your working, calculate
(i) the total surface area of the prism,
(ii) the volume of the prism.

## 4 Answer the whole of this question on a sheet of graph paper.

The equation $h=20 t-5 t^{2}+1$ gives the height $h$ metres above ground level of a stone $t$ seconds after it has been thrown vertically upwards. Some values of $h$ and $t$ are given in the following table.

| $t$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h$ | 1 | 9.75 | 16 | 19.75 | 21 | $a$ | 16 | 9.75 | $b$ | $c$ |

(a) Calculate the values of $a, b$ and $c$.
(b) Using a scale of 2 cm to represent 0.5 seconds on the horizontal $t$-axis and 2 cm to represent 4 m on the vertical $h$-axis, draw the graph of $h=20 t-5 t^{2}+1$ for $0 \leqslant t \leqslant 4.5$.
(c) Use your graph to answer these questions.
(i) What is the value of $t$ when the stone reaches ground level?
(ii) For how long is the stone more than 12 m above the ground? Give your answer in seconds to 1 decimal place.
(iii) How far does the stone travel altogether in the first 3 seconds?
(d) (i) Draw a suitable tangent on your graph and use it to calculate an estimate of the gradient (slope) when $t=1$.
(ii) What quantity does the gradient measure and what are the units for this quantity?


A belt PQRSTU passes round two wheels, which turn when the belt moves.
The larger wheel has radius 30 cm and centre $A$. The smaller wheel has radius 10 cm and centre $B$. Angle TAP $=130^{\circ}$.
(a) Explain why the angles at $P, Q, S$ and $T$ are all $90^{\circ}$.
(b) Find the size of angles $P U T, Q B S$ and $Q R S$.
(c) The smaller wheel makes 12 revolutions in a clockwise direction. How many revolutions does the larger wheel make and in which direction?
(d)


This diagram represents part of the diagram at the top of the page.
The perpendicular from $B$ meets $P A$ at $X$.
(i) Work out the length of $A X$ and the size of angle $P A B$.
(ii) Find the distance between the centres $A$ and $B$.

6 One teacher from Argentina, one from Brazil and three from Namibia attend an international conference. One of these five teachers is chosen at random to make a speech, and one of the remaining four is chosen at random to write a report.
(a) Copy and complete the probability tree diagram below, showing the countries from which the teachers were chosen.

## Makes a speech

## Writes a report


(b) Calculate the probability that
(i) both the chosen teachers were from Namibia,
(ii) neither of the chosen teachers was from Namibia,
(iii) the teacher from Brazil was not chosen.
(c) One of the remaining three teachers is chosen at random to chair the conference.

Calculate the probability that this is the teacher from Brazil.

7

(a) Triangle $T$ is mapped onto triangle $U$ by a single transformation. Describe this transformation fully.
(b) Triangle $V$ is a rotation of triangle $U$ about the origin by $\theta^{\circ}$ anticlockwise. Calculate $\theta$ correct to 1 decimal place.
(c) Triangle $W$ is an enlargement of triangle $V$, centre $(a, 0)$, with scale factor $k$.
(i) Calculate the value of $k$.
(ii) Find the value of $a$.
(d) (i) Find the area of triangle $U$.
(ii) Find the equation of the hypotenuse of triangle $U$.
(e) A triangle similar to $T$ has an area 64 times larger than that of $T$. Calculate the length of the hypotenuse of this triangle.

8 On television a weather forecaster uses a cloud symbol shown in the diagram.
Its perimeter consists of a straight line $A E$, two semicircular arcs $A P B$ and $D Q E$ and the major arc $B R D$ of a circle, centre $C$.
$A E=7.5 \mathrm{~cm}, A B=D E=3 \mathrm{~cm}$ and $B C=C D=2.8 \mathrm{~cm}$.
Angle $B A E=$ angle $D E A=70^{\circ}$ and $X$ is the midpoint of $B D$.
[For $\pi$, use either your calculator value or 3.142.]

(a) (i) Use the trapezium $A B D E$ to show that $B X=2.724 \mathrm{~cm}$.
(ii) Calculate angle $B C X$.
(b) Calculate
(i) the area of triangle $B C D$,
(ii) the area of the trapezium $A B D E$,
(iii) the area of the major sector $B C D$,
(iv) the total area of the cloud symbol.

9 A teacher asks four students to write down an expression using each of the integers $1,2,3$ and $n$ exactly once.

Ahmed's expression was $(3 n+1)^{2}$.
Bumni's expression was $(2 n+1)^{3}$.
Cesar's expression was $(2 n)^{3+1}$.
Dan's expression was $(3+1)^{2 n}$.
The value of each expression has been worked out for $n=1$ and put in the table below.
(a) Copy and complete this table, giving the values for each student's expression for $n=2,0,-1$ and -2 .

|  | $n=2$ | $n=1$ | $n=0$ | $n=-1$ | $n=-2$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Ahmed |  | 16 |  |  |  |
| Bumni |  | 27 |  |  |  |
| Cesar |  | 16 |  |  |  |
| Dan |  | 16 |  |  |  |

(b) Whose expression will always give the greatest value
(i) if $n<-2$,
(ii) if $n>2$ ?
(c) Cesar's expression $(2 n)^{3+1}$ can be written as $a n^{b}$ and Dan's expression $(3+1)^{2 n}$ can be written as $c^{n}$. Find the values of $a, b$ and $c$.
(d) Find any expression, using 1, 2, 3 and $n$ exactly once, which will always be greater than 1 for any value of $n$.

