

**International General Certificate of Secondary Education
UNIVERSITY OF CAMBRIDGE LOCAL EXAMINATIONS SYNDICATE**

MATHEMATICS

0580/4, 0581/4

PAPER 4

MAY/JUNE SESSION 2000

2 hours 30 minutes

Additional materials:

- Answer paper
- Electronic calculator
- Geometrical instruments
- Graph paper (1 sheet)
- Mathematical tables (optional)
- Tracing paper (optional)

TIME 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

Write your answers and working on the separate answer paper provided.

All working must be clearly shown. It should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 130.

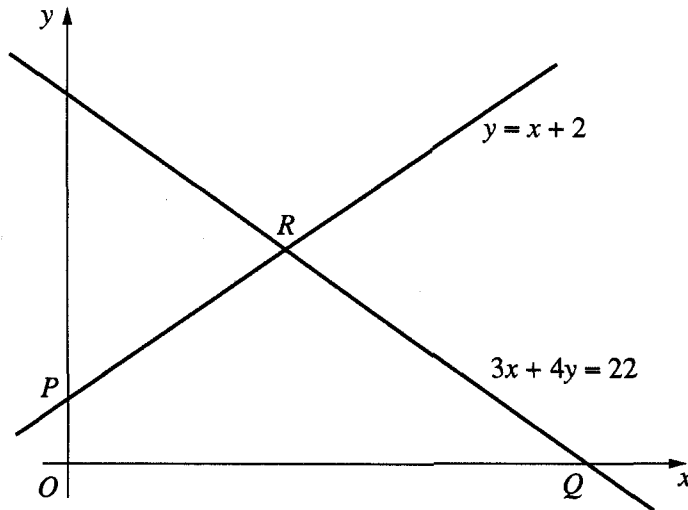
Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

This question paper consists of 8 printed pages.

1

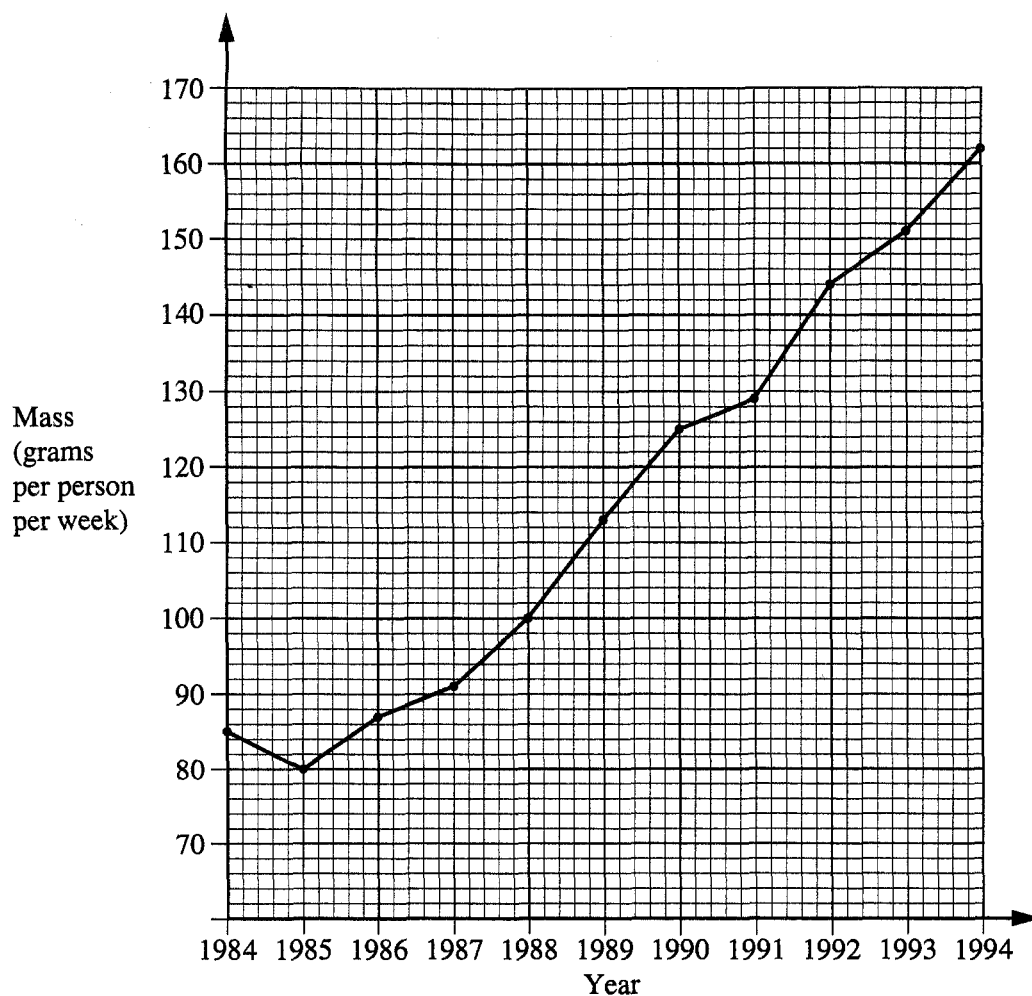
NOT TO
SCALE

The line $y = x + 2$ cuts the y -axis at P . The line $3x + 4y = 22$ cuts the x -axis at Q .
The two lines intersect at R .

- (a) Find the coordinates of (i) P , [1]
(ii) Q , [2]
(iii) R . [3]
- (b) $x \geq 0$ is one of the four inequalities which define the region $OPRQ$.
Write down the other three inequalities. [3]

2 Winston and Anthony take a driving test.
The probability that Winston will pass is $\frac{3}{4}$ and the probability that Anthony will pass is $\frac{2}{3}$.

- (a) Which of them is more likely to pass? [1]
- (b) Calculate the probability that they will both fail. [2]
- (c) Calculate the probability that only one of them will pass. [3]
- (d) If Winston fails he will take the test again.
The probability that he will pass at **any** future attempt is $\frac{4}{5}$.
- (i) Draw a tree diagram to show the probabilities of Winston passing or failing on each of his first three attempts. [3]
- (ii) Calculate the probability that he will pass at his third attempt. [2]
- (iii) Calculate the probability that he will **not** need more than three attempts to pass. [3]



The line graph shows how many grams of bananas were eaten per person per week in Europe between 1984 and 1994.

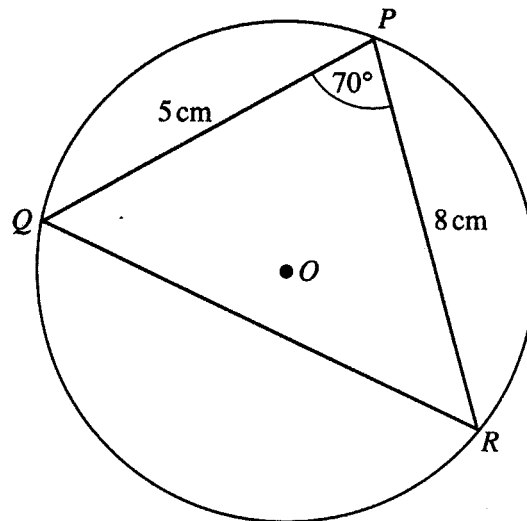
(a) Between which two years did the largest yearly increase take place? [1]

(b) What was the percentage increase between 1984 and 1994? [2]

(c) Work out the fraction $\frac{\text{mass of bananas eaten per person per week in 1992}}{\text{mass of bananas eaten per person per week in 1994}}$

Give your answer as a fraction in its lowest terms. [2]

(d) There were 497 million people in Europe in 1990.
Calculate the total mass of bananas eaten in Europe in 1990.
Give your answer in tonnes, correct to two significant figures. [3]



NOT TO
SCALE

The points P , Q and R lie on the circumference of a circle, centre O .
 $PQ = 5$ cm, $PR = 8$ cm and angle $QPR = 70^\circ$.

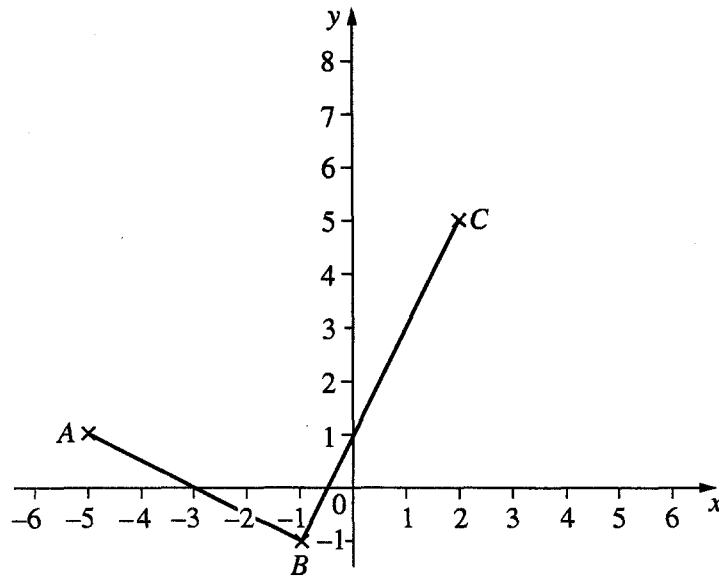
- (a) Calculate the area of triangle PQR . [2]
- (b) Calculate the length of the chord QR . [4]
- (c) Find the size of the obtuse angle QOR . [1]
- (d) Show that the radius of the circle is 4.18 cm, correct to three significant figures. [4]
- (e) Taking the radius of the circle as 4.18 cm, calculate the length of the minor arc QR . [3]
- (f) Find the size of the reflex angle QOR . [1]

- 5 (a) (i) Write down the value of $\cos 295^\circ$. [1]
- (ii) If $\sin x$ and $\cos x$ are both negative, between what values must x lie? [2]

- (b) The depth of water (d metres) in a harbour is given by the formula

$$d = 5 + 4 \sin 30t^\circ, \quad \text{where } t \text{ is the time in hours after midnight.}$$

- (i) Find the depth of water at midnight. [1]
- (ii) Find the depth of water at 10 a.m. [2]
- (iii) What is the greatest depth of water in the harbour? [2]
- (iv) At what times of day is the depth of water greatest? [2]
- (v) What is the least depth of water in the harbour? [1]

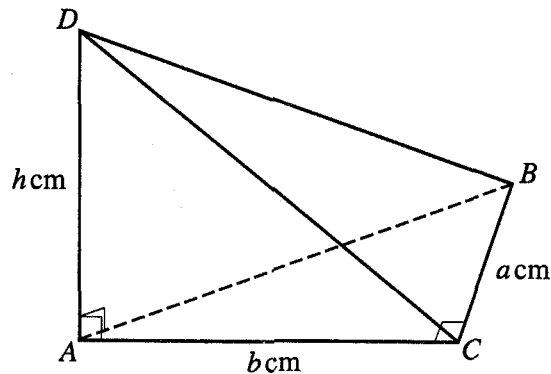


The points $A(-5, 1)$, $B(-1, -1)$ and $C(2, 5)$ are three vertices of a rectangle $ABCD$.

- (a) (i) Write down the coordinates of D . [1]
 (ii) Calculate the length of the line segment AB . [2]
 (iii) Calculate the area of the rectangle $ABCD$. [2]
- (b) (i) Find the equation of the line in which AD is the reflection of BC . [3]
 (ii) Describe fully a single transformation (**not** a reflection) which maps A onto D and B onto C . [2]
- (c) The matrix $\mathbf{M} = \begin{pmatrix} x^2 & 2x + 5 \\ 1 & 10 \end{pmatrix}$.
- (i) The transformation represented by the matrix \mathbf{M} maps A onto C .
 Show that $5x^2 - 2x - 3 = 0$. [2]
 (ii) Solve the equation $5x^2 - 2x - 3 = 0$. [2]
 (iii) Calculate the inverse matrix \mathbf{M}^{-1} when $x = 1$. [2]

7 The formula for the volume of a pyramid is $\frac{1}{3}$ base area \times perpendicular height.

(a)

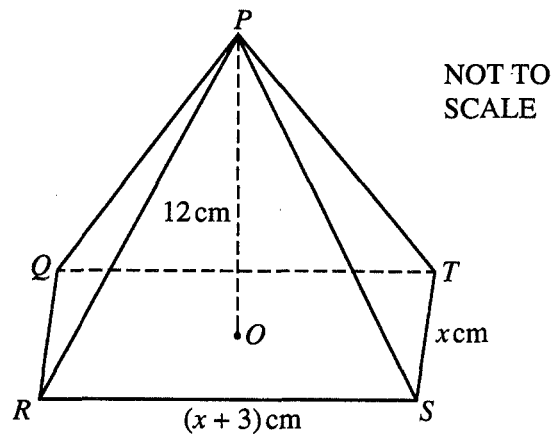


In the pyramid $ABCD$, ABC is the base and D is the vertex.
 Angle $BCA = \text{angle } DAC = \text{angle } DAB = 90^\circ$.
 $AD = h$ cm, $AC = b$ cm and $BC = a$ cm.

(i) Write down a formula for the volume of the pyramid $ABCD$ in terms of a , b and h . [2]

(ii) Calculate the volume of pyramid $ABCD$ when $a = 6$, $b = 5$ and $h = 8$. [2]

(b)



The pyramid $PQRST$ has a rectangular base with $ST = x$ cm and $RS = (x + 3)$ cm.
 The height of the pyramid, OP , is 12 cm, where O is the centre of the rectangle.

(i) Write down a formula for the volume of this pyramid in terms of x . [2]

(ii) When the volume is numerically equal to the perimeter of the rectangular base, show that

$$2x^2 + 4x - 3 = 0. \quad [2]$$

(iii) Solve the equation $2x^2 + 4x - 3 = 0$, giving your answers correct to 2 decimal places.

Show all your working. [5]

(iv) Use your answer to part (iii) to write down the length of RS . [1]

(v) M is the midpoint of ST . Calculate angle PMO . [3]

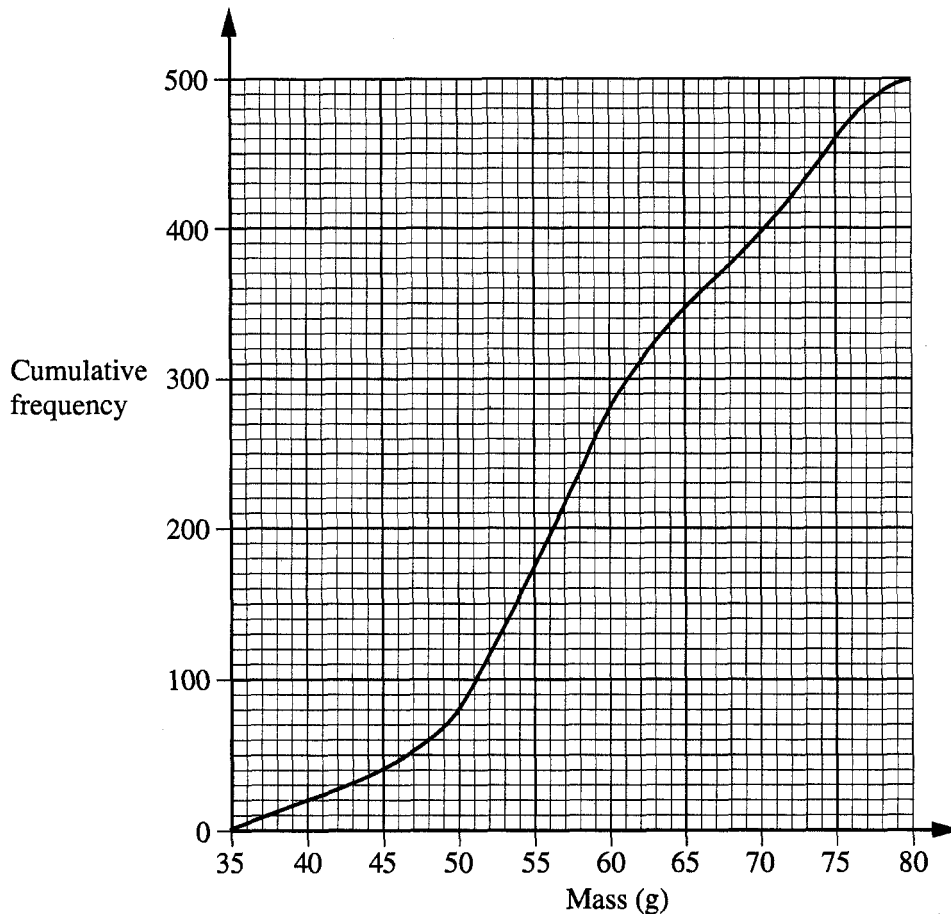
(c) (i) What is the name given to a pyramid with a circular base? [1]

(ii) Write down a formula for the volume of this pyramid if the radius of the circular base is r and the perpendicular height is h . [1]

- 8 Answer the whole of this question on a sheet of graph paper.
500 eggs were sorted by mass into five different sizes.

	Mass (m grams)	Frequency
Small	$35 \leq m < 40$	20
Medium	$40 \leq m < 50$	60
Standard	$50 \leq m < 60$	200
Large	$60 \leq m < 75$	180
Extra large	$75 \leq m < 80$	40

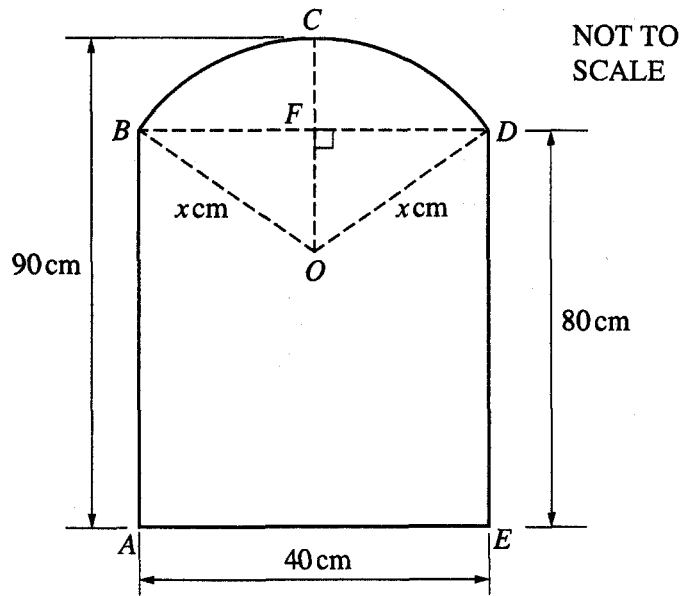
- (a) Draw an accurate histogram to represent this information.
Use a scale of 2 cm to represent 5 grams on the horizontal axis, and an area scale of 1 square centimetre to represent 5 eggs. [6]
- (b) Calculate an estimate of the mean mass of these eggs. [4]
- (c)



This cumulative frequency curve has been drawn using the information in the table above.

- (i) Explain why the point (60, 280) is on the curve. [2]
- (ii) Estimate the median mass of the eggs. [1]
- (iii) Estimate the interquartile range of the masses of the eggs. [2]

9



The diagram shows a window $ABCDE$. $ABDE$ is a rectangle. BCD is an arc of a circle with centre O and radius x cm. The total height of the window is 90 cm. $AB = ED = 80$ cm and $AE = BD = 40$ cm. The line OC is perpendicular to BD , and $BF = FD$.

- (a) (i) Write down, in terms of x , the length of OC and the length of OF . [1]
 (ii) Use Pythagoras' Theorem in triangle OFD to write down an equation in x . [2]
 (iii) By solving the equation, show that $x = 25$. [1]
- (b) Using a scale of 1 cm to represent 10 cm, construct an accurate drawing of the window. [4]
- (c) Find the area of the window. (For π , use either your calculator value or 3.142.) [5]
- (d) The window is made of glass 2 mm thick. The mass of 1 cm^3 of the glass is 6.5 grams. Calculate the mass of glass in the window, giving your answer in kilograms. [3]

10 Throughout this question, remember that 1 is **not** a prime number.

- (a) Find a prime number which can be written as the sum of two prime numbers. [1]
- (b) Consider the statement

"All even numbers greater than 15 can be written as the sum of two different prime numbers in at least two different ways."

For example, $20 = 3 + 17 = 7 + 13$.

- (i) Show that the above statement is true for 16. [1]
 (ii) Find a number between 30 and 50 which shows that the statement is false. [2]
- (c) Show that 16 can be written as the sum of three different prime numbers. [1]
- (d) Consider the statement

"All odd numbers greater than 3 can be written as the sum of two prime numbers".

Is it true or false? Justify your answer. [2]