Math 0580

June 1999

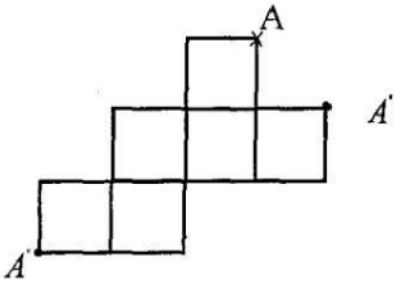
Paper 2

1. $\left(\frac{1}{8}+\frac{1}{2}\right)\div\frac{5}{6}=\frac{3}{4}$

2. (a) $300\,000 = 3\times10^5$

   (b) $4.2\times3\times10^5\times365\times24\times60\times60 = 3.97\times10^{13}\,Km$

3.



4. (a) 3 min 58.2 sec - 0.9 sec = 3 min 57.3 sec

   (Using calculator: 0 ''' 3 ''' 58.2 ''' - 0 ''' 0 ''' 0.9 = shift ''' 0 3 57.3)

   or just 58.2 - 0.9 = 57.3

   (b) 3 min 58.2 sec + 3.1 sec = 4 min 1.3 sec

   (Similar way to (a))

5. (a) 1 mm = 0.1 cm

$$\frac{0.1}{2}=0.05$$

$$5.2 - 0.05 \le AC < 5.2 + 0.05$$

$$5.15 \le AC < 5.25$$

   (b) The least value of AD is $\sqrt{(5.15)^2-(2.35)^2}$ cm.

6. 10 % on administration

   90 % on charitable work

   90 % of income is 234000

$$\text{income} = \frac{234000\times100}{90} = \$260000$$

7. Ratio of volumes is 64 : 1

   Ratio of diameters (or radii) = $\sqrt[3]{64}$: 1 = 4 : 1

   Ratio of surface areas = $(4)^2$: 1 = 16 : 1

8. $x = \sqrt{y^3 + 3}$

$x^2 = y^3 + 3$

$y^3 = x^2 - 3$

$y = \sqrt[3]{x^2 - 3}$

9. Angle ACD = 90° angle of a semicircle

x = 90 - 40 = 50°

y = x alternate angle

y = 50°

$Z = \frac{1}{2}y = \frac{1}{2} \times 50 = 25°$

Angle at centre double angle at circumference

10. Method A: \$ 1 = 4.15 F

? = 1000 F

$\frac{1000 \times 1}{4.15} = \$240.96$

Method B: 1000 - 20 = 980

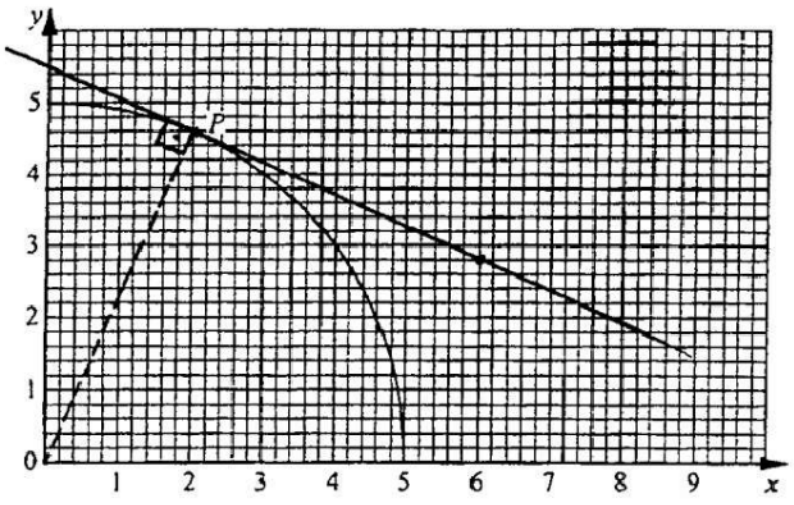
\$ 1 = 4 F

? = 980

$980 \times \frac{1}{4} = \$245$

Method B gives more by 245 - 240.96 = \$ 4.04

11.



Take two points on the tangent (2, 4.6) and (6, 2.8)

Gradient = $\frac{4.6 - 2.8}{2 - 6} = -0.45$ (any answer from -0.4 to -0.46)

12. (a) 9 litres = 9 x 1000 = 9000 cm^3
0.0009 m^3 = 0.0009 x 100 x 100 x 100 = 900 cm^3
0.0009 m^3 < 7000 cm^3 < 9 litres.

(b) 3 litres = 3000 cm^3
3000 - 900 = 2100
7000 - 3000 = 4000
9000 - 3000 = 6000
closest is 900 cm^3 i.e 0.0009 m^3

13. $\frac{n}{2}\times 150+\frac{n}{2}\times 170=(2n-4)\times 90$

75 n + 85 n = 180 n - 360

360 = 20 n

$n = \frac{360}{20} = 18$

14. (a) (i) f(-5) = 2 (-5) + 1 = -9

(ii) $gf(-5) = g(-9) = (-9)^2 + 3 = 84$

(b) gf(x) = g (2x + 1) = $(2x+1)^2 + 3$

$= 4x^2 + 4x + 4$

15. $2x^2 + 4x - 3 = 0$

a = 2 b = 4 c = -3

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{16 - 4(2)(-3)}}{4}$

$x = \frac{-4 \pm \sqrt{40}}{4}$

x = 0.58 or -2.58

16. (a) Shaded area = large sector - small sector.

$= \frac{60}{360}\pi R^2 - \frac{60}{360}\pi r^2$

$= \frac{\pi}{6}(R^2 - r^2)$

(b) shaded area = $\frac{\pi}{6}(R+r)(R-r)$

17. (a) AM is shorter because the opposite angle is smaller.

(b) $180 - (63 + 65) = 52°$

$$\frac{100}{\sin 52°} = \frac{BM}{\sin 65°}$$

BM = 115 cm.

18. (a) $T = K\,h$

$-5 = K \,.\, 500$ $\qquad K = \frac{-5}{500} = -0.01$

$T = -0.01\ h$

(b) (i) $T = -18$ $\qquad -18 = -0.01\ h$

$$h = \frac{-18}{-0.01} = 1800$$

heght above see level = 1800 + 2500 = 4300 m.

(ii) at sea level h = -2500

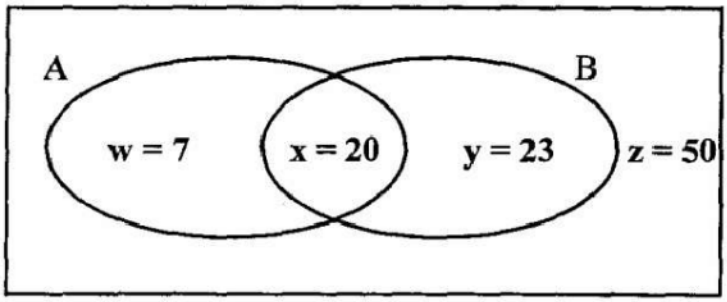
T = -0.01 x -2500 = 25°C

19. (a) 27 + 43 = 70

70 - 50 = 20

people reading both magazines = 20

(b)



(c) $Z = n(A \cup B)'$

20. (a) $(64x^8)^{\frac{1}{2}} = (64)^{\frac{1}{2}}(x^8)^{\frac{1}{2}} = 8x^4$

(b) $$\frac{3x^2}{x^2+3x} = \frac{3x^2}{x(x+3)} = \frac{3x}{x+3}$$

21. (a) AB = C

$$\begin{pmatrix} 4 & x \\ -3 & 6 \end{pmatrix}\begin{pmatrix} 5 & -3 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ y & 21 \end{pmatrix}$$

4 x 5 + x (-2) = 6

20 - 2x = 6

$2x = 14$ $\qquad x = 7$

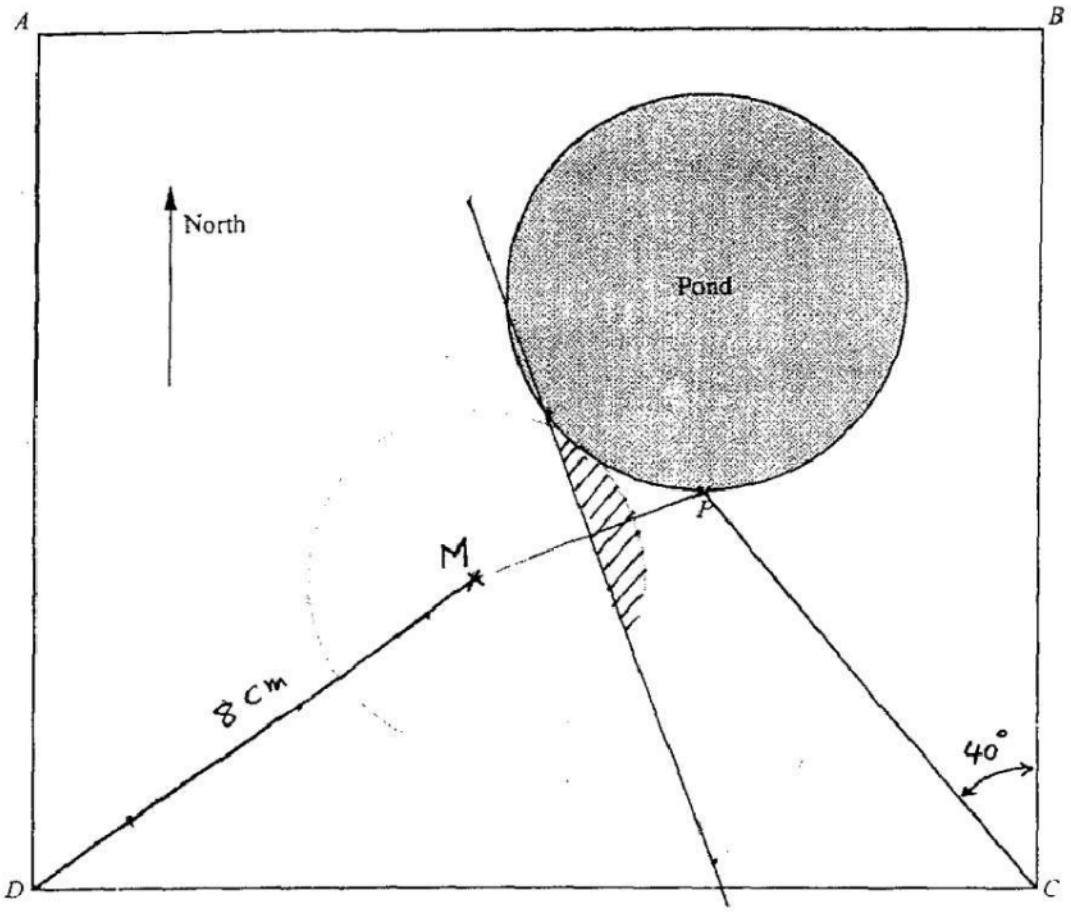
$-3(5) + 6(-2) = y$ $\qquad \therefore\ y = -27$

(b) $B = \begin{pmatrix} 5 & -3 \\ -2 & 2 \end{pmatrix}$

$|B| = 5(2) - (-3)(-2) = 10 - 6 = 4$

$$B^{-1} = \frac{1}{4}\begin{pmatrix} 2 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{5}{4} \end{pmatrix}$$

22.



(a) Bearing of P from C = 360 - 40 = 320°

(b) 80 m = $\frac{80}{10}$ = 8 cm

November 99

Paper 2

1. Sea level $= -2.40 + 1.97$
$= -0.43$

2. $3(x+1) \geq 5 - x$
$3x + 3 \geq 5 - x$
$3x + x \geq 5 - 3$
$4x \geq 2$
$x \geq \frac{1}{2}$

3. $I = \frac{PRT}{100}$
$P = 560 \quad R = 5.5 \quad I = 123.20$
$123.20 = \frac{560 \times 5.5 \times T}{100}$
$T = \frac{123.20 \times 100}{560 \times 5.5} = 4$ years

4. $x = 0.083 \quad y = \frac{84}{991} = 0.08476$
$z = 8.4 \times 10^{-3} = 0.0084$
$z < x < y$

5. $\dfrac{478 \times 49.82}{0.1248}$

Writing each number correct to two significant figures

478 approximated to 480
49.82 approximated to 50
0.1248 approximated to 0.12

$$\frac{480 \times 50}{0.12} = 200\,000$$

6. *Cost in Paris* = 1600 French francs
Cost in London = £ 170 (pounds).
= $170 \times 9.30 = 1581$ French francs
The cycle cost less in London than Paris
OR cost in London = £170 (pounds).

$$\text{cost in Paris} = \frac{1600 \text{ francs}}{9.30}$$

= 172.04 (pounds).
The cycle cost less in London than Paris

7. *Perimeter P is 65cm to the nearest centimeter*
$64.5 \leq P < 65.5$

$P = 3L$ where L is the length of one side $L = \dfrac{P}{3}$

smallest possible length of one side

$$= \frac{64.5}{3} = 21.5 \text{ cm}$$

8. $3x - y = -3$ (1)
$9x + 2y = 1$ (2)
(1)x2 $6x - 2y = -6$
(2) $9x + 2y = 1$
adding $15x = -5$

$$x = \frac{-5}{15} = -\frac{1}{3}$$

substituting in (1)

$$3\left(-\frac{1}{3}\right) - y = -3$$

$-1 - y = -3$
$1 + y = 3$
$y = 2$

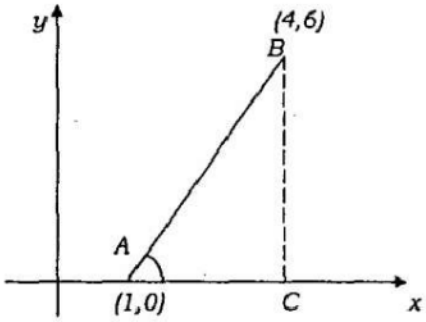
$x = -\frac{1}{3}$ $y = 2$

9.

Distance AC = 4 – 1 = 3
Distance BC = 6 – 0 = 6

$\tan A = \frac{6}{3} = 2$

$A = 63.4°$



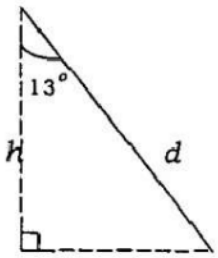
10. (a) angle BCD = 180 – (55 +26) = 180 – 81 = 99 °
(b) angle ACD = angle ABD = 55 ° (same arc)
angle BAC = angle ACD = 55 ° (alternate)
angle BXC = angle BAX + angle ABX
= 55 + 55 = 110 (exterior)
(c) angle ACB = 180 – (26 + 110) = 44 °
angle ADB = angle ACB = 44 °

11.

$\cos 13° = \frac{h}{d}$

$h = d \cos 13°$
$= 1800 \cos 13°$
$= 1753.87$
$= 1754\ m$

vertical distance = 1754 m



12. $\frac{ax - ay}{px - py + qx - qy} = \frac{a(x-y)}{p(x-y)+q(x-y)}$

$= \frac{a(x-y)}{(x-y)(p+q)} = \frac{a}{p+q}$

13. $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

$\frac{24}{3} = \left(\frac{h_1}{15.5}\right)^3$

$8 = \left(\frac{h_1}{15.5}\right)^3$

$\frac{h_1}{15.5} = \sqrt[3]{8} = 2$

$h = 15.5 \times 2 = 31$ cm

14. $F = K V^2$

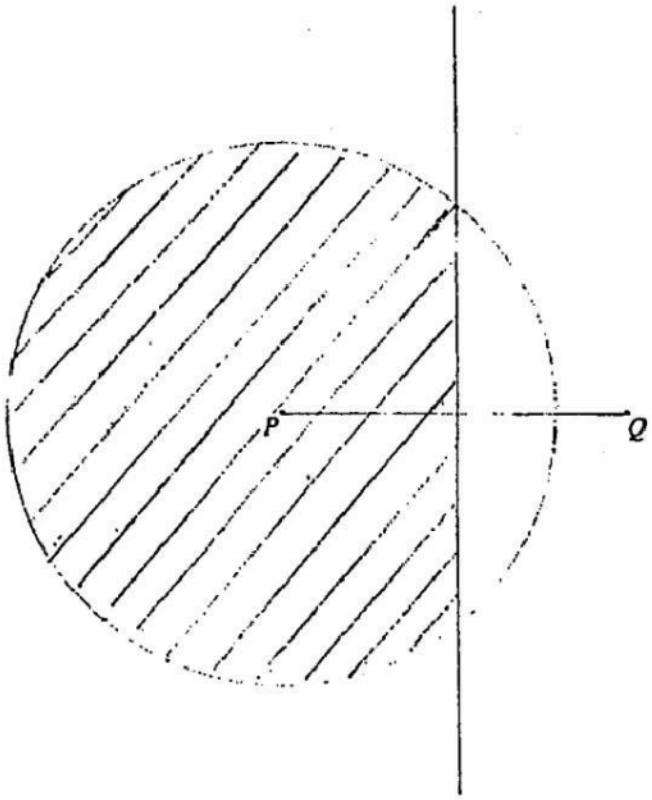
$180 = K(6)^2 = 36K$

$K = \frac{180}{36} = 5$

$F = 5 V^2$

$F = 5 (3)^2 = 5 \times 9 = 45$

15.



16. (a) $2x^{4} \times 5x = 10x^{5}$

(b) $x^{2} \div x^{\frac{1}{2}} = x^{2-\frac{1}{2}} = x^{1\frac{1}{2}}$

(c) $\left(\sqrt{2x}\right)^{6} = \left[(2x)^{\frac{1}{2}}\right]^{6} = (2x)^{3} = 8x^{3}$

17. $x^{2} - 2x - 5 = 0$

$a = 1 \qquad b = -2 \qquad c = -5$

$$x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(-5)}}{2}$$

$$= \frac{2 \pm \sqrt{24}}{2} = \frac{2 \pm 4.899}{2}$$

$= 3.45 \quad or - 1.45$

18. $M = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix} \qquad N = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

(a) $MN = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -11 \\ -17 \end{pmatrix}$

(b) $|M| = 2(-5) - (-3)(4) = 2$

$$M^{-1} = \frac{1}{2}\begin{pmatrix} -5 & 3 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} & \frac{3}{2} \\ -2 & 1 \end{pmatrix}$$

19. $f : x \to 2x - 7 \qquad g : x \to \frac{x+1}{x}$

(a) $fg(2) = f\left(\frac{2+1}{2}\right) = f\left(\frac{3}{2}\right)$

$$= 2\left(\frac{3}{2}\right) - 7 = 3 - 7 = -4$$

(b) $fg(x) = 2\left(\frac{x+1}{x}\right) - 7$

$$= \frac{2x+2}{x} - 7 = \frac{2x+2-7x}{x}$$

$$= \frac{2-5x}{x}$$

20. (a) Area $= \frac{4.6+5}{2} \times \left(\frac{8}{10}\right) = \frac{9.6}{2} \times 0.8$

$= 3.84\ cm^2$

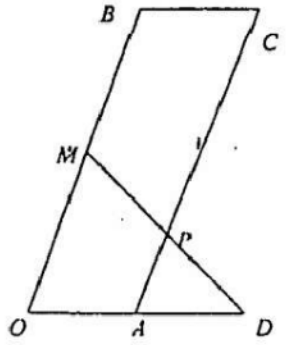
(b) Volume = Area × length

$= 3.84 \times 9.5 \quad = 36.48$

$= 36\ cm^3$

(c) Two planes of symmetry .

21.



(a) $\overrightarrow{DM} = \overrightarrow{DO} + \overrightarrow{OM}$

$= -2a + \frac{1}{2}b$

(b) similar triangles

(c) A is mid OD

AP is parallel to OB

$\overrightarrow{AP} = \frac{1}{2}\overrightarrow{OM} = \frac{1}{2}\left(\frac{1}{2}b\right) = \frac{1}{4}b$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= a + \frac{1}{4}b$

22. (a) deceleration $= \frac{\text{change in velocity}}{\text{time}}$

$= \frac{8-3}{2} = \frac{5}{2} = 2.5m/s^2$

(b) distance = Area under the graph

$= \frac{8+3}{2} \times 2 = 11m$

(c) speed when $t = 0$ is 10 m/s

$10m/s = \frac{10}{1000} \times 60 \times 60$

$= 36\ Km/h$