June 1997

Paper 4

1. (a) $\frac{80}{100} \times 200 = 160$

$$
\begin{array}{lll}
\text{20 children} & = 20 \times 2.50 = & 50 \\
\text{140 adult} & = 140 \times 5 & = 700 \\
\text{total} & 700 + 50 & = 750
\end{array}
$$

(b) Sale of children tickets $= 2.5\,x$
Sale of adult tickets $= (200 - x)\,5$

$$
\begin{aligned}
2.5x + (200 - x)\,5 &= 905 \\
2.5x + 1000 - 5x &= 905 \\
-2.5x &= -95 \\
x &= 38
\end{aligned}
$$

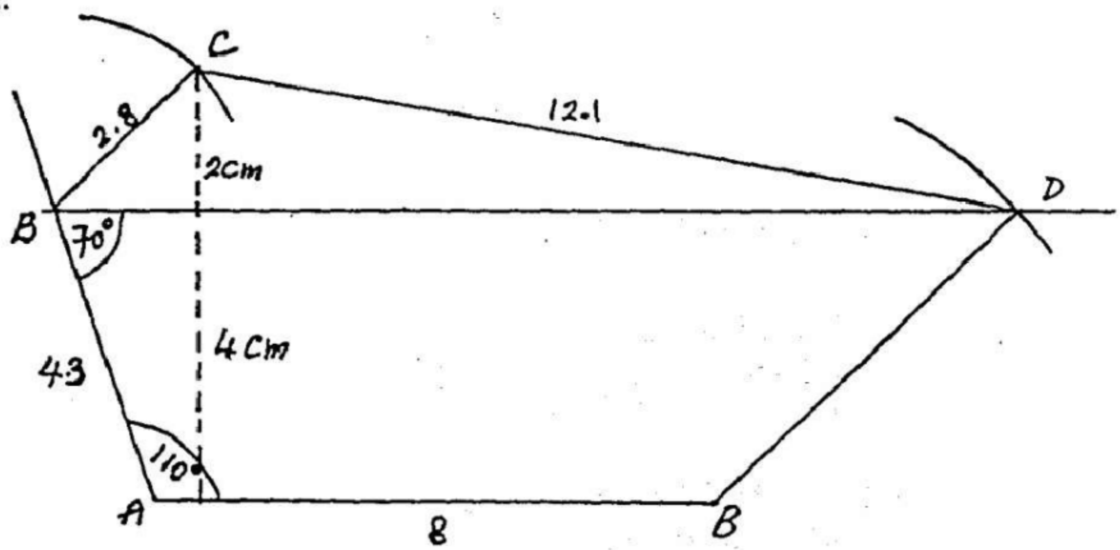
(c) (i)

					total
2	:	3	:	7	12
				?	10800

$$\text{profit} = \frac{7 \times 10800}{12} = 6300$$

(ii) $I = \frac{PRT}{100} = \frac{6300 \times 5 \times \frac{4}{12}}{100} = £\,105$

2.



(a) BD = 13.9

(b) Area of Δ BCD = ½ × 13.9 × 2 = 13.9

Area of trapezium = $\frac{8+13.9}{2} \times 4 = 43.8$

Total area of the pentagon = 13.9 + 43.8 = 57.7 cm^2

3. (a) (i) Square numbers are 1, 4, 9

Prob. $= \frac{3}{12} = \frac{1}{4}$

(ii) Prime numbers or numbers less than 6 are 1, 2, 3, 4, 5, 7, 11

Prob. $= \frac{7}{12}$

(b) (i) 12 + 9, 9 + 12 10 + 11, 11 + 10

(ii) $\frac{1}{12} \times \frac{1}{12} \times 4 = \frac{1}{36}$

(c)

Score	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	1	1	0	1	1	2	3	4	5	6	5
c.f	1	2	3	3	4	5	7	10	14	19	25	30

(i) the mode 11

(ii) the median

median order $\frac{30+1}{2} = 15\frac{1}{2}$

terms numbered 15, 16, 17, 18, 19 are all 10

Median is 10

(iii) the mean $\frac{\sum fx}{30} = 8.9$

(d) (i) area $= \frac{\theta}{360} \pi r^2 = \frac{30}{360} \times 3.142 \times 10^2 = 26.18 \text{ cm}^2$

$= 26.2 \text{ cm}^2$

(ii) Probability $= \frac{\text{Shaded area}}{\text{Area of square}} = \frac{26.18}{30 \times 30} = 0.0291$

4. (a) $\angle ABC = 90 + 25 = 115°$

(b) (i) $AC^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \text{ Cos } 115 = 21.95$

$= 22$ km

(ii) $\frac{AC}{\text{Sin B}} = \frac{BC}{\text{Sin A}}$

$\frac{21.95}{\text{Sin A}} = \frac{14}{\text{Sin A}}$

Sin A $= 0.5779$

A $= 35.3°$

(iii) Bearing of C from A $= 25 + 35.3 = 60.3°$

Bearing of A from C $= 180 + 60.3 = 240.3°$

5. (a) ABC and ADE are similar

(i) $\therefore \frac{AC}{AE} = \frac{BC}{DE}$ $\quad \frac{5}{5+2x} = \frac{x+3}{4x+1}$

$(x + 3)(5 + 2x) = 5(4x + 1)$

$2x^2 + 11x + 15 = 20x + 5$

$2x^2 - 9x + 10 = 0$

(ii) $2x^2 - 9x + 10 = (2x - 5)(x - 2)$

(iii) $X = \frac{5}{2}, X = 2$

(iv) Ratio of sides $\frac{x+3}{4x+1} = \frac{2\frac{1}{2}+3}{4\times 2\frac{1}{2}+1} = \frac{5\frac{1}{2}}{11} = \frac{1}{2}$

ratio of areas = $\left(\frac{1}{2}\right)^2$

(b) (i) determinant of M $= (2y+1)(2y+3) - y(3y-4)$
$= 4y^2 + 8y + 3 - 3y^2 + 4y$
$= y^2 + 12y + 3 = 10$
$y^2 + 12y - 7 = 0$

(ii) $y^2 + 12y - 7 = 0$
$y = \frac{-12 \pm \sqrt{144 - 4\times 1\times(-7)}}{2} = \frac{-12 \pm \sqrt{172}}{2}$
$y = 0.557 \;,\; -12.557$
$y = 0.6$ or -12.6

6. (a) $OC = \sqrt{6^2 - (3.6)^2} = 4.8$
$VC = 6 + 4.8 = 10.8$

(b) (i) the volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times 3.142 \times 6^3 = 904.896$
$= 905$

(ii) the volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times 3.142\times 3.6^2 \times 10.8$
$= 146.59 = 147$

(iii) percentage of sphere occupied $= \frac{147}{905} = 16.2\%$
not occupied $= 100 - 16.20 = 83.8\ \%$

(c) (i) $2\pi r = 37.704$

(ii) $\frac{300}{37.628} = 7.957 = 7$

(iii) remaining part of revolution $= 1 - 0.957 = 0.043$
Angle $= 0.043 \times 36 = 15.5°$

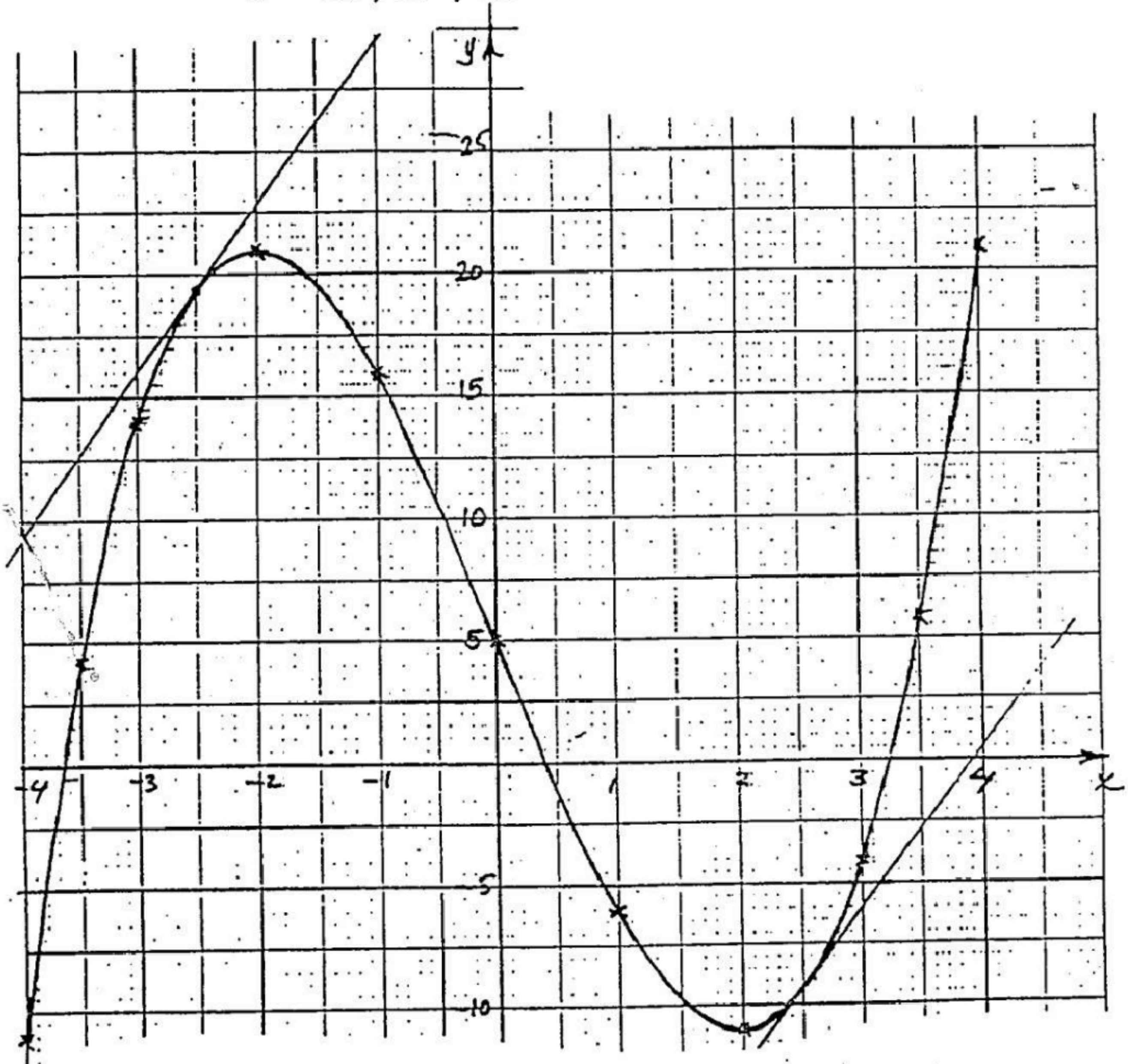
7. $f(x) = x^3 - 12x + 5$

(a) $a = (-1)^3 - 12(-1) + 5 = 16$ b = $(4)^3 - 12(4) + 5 = 21$

(c) (i) $f(x) = 0$ $y = 0$ $x = -3.65, 0.4, 3.25$

(ii) $x^3 - 12x + 10 = 0 \Rightarrow x^3 - 12x + 5 = -5$

$x = -3.8\ ,\ 0.9\ ,\ 2.9$



(d) Tangent passes through $(-3, 16)$ and $(-1, 29.5)$

gradient $= \frac{29.5-16}{(-1)-(-3)} = \frac{13.5}{2}$

$= 6.75$

(iii) To find another point the tangent at which is parallel to the tangent at $x = -2.5$

point is $x = 2.5$

(tangent are parallel)

8. (a) (i) B, translation $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

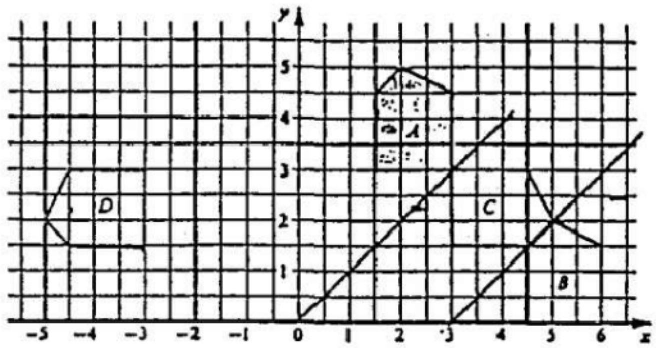
(ii) C, reflection on the line $y = x$

(iii) D, rotation 90° anticlockwise centre origin

(b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(c) reflection on the y axis.

(d) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$



(e) $y = x - 3$

9.

Birthday / Scheme	1st	2nd	3rd	4th	5th	6th	7th
A	\$10	\$20	\$30	\$40	\$50	\$60	\$70
B	\$1	\$2	\$3	\$8	\$16	\$32	\$64
C	\$1	\$4	\$9	\$16	\$25	\$36	\$49

(a) (i) his 7th birthday, 70, 64, 49.

(ii) his *n*th birthday, $10n$, 2^{n-1} , n^2

(b) (i) 550 , 1023 , 385.

(ii) 1710 , 262143 , 2109 A the smallest.

(iii) A and C an equal when

$$5n\ (n + 1) = \frac{n(n+1)(2n+1)}{6}$$

$30 = 2n + 1$ $2n = 29$

$n = 14\frac{1}{2}$

i.e. up to 14th birthday C is smaller

& starting from 15th birthday A is smaller.

November 1997
Paper 4

1-(a) Volume = $\pi r^2 h = 3.142 \times \left(\frac{8}{2}\right)^2 \times 11 = 552.992 = 553\ cm^3$

(b) (i) Length = $4 \times 8 = 32\ cm$
Width = $3 \times 8 = 24\ cm$

(ii) Volume of the box = $32 \times 24 \times 11 = 8448\ cm^3$
Volume occupied by the tins = $12 \times 552.992 = 6635.904$
Volume not occupied = 8448 - 6635.904 = 1812.096
Percentage not occupied = $\frac{1812.096}{8448} \times 100 = 21.45\% = 21.5\%$

(c)

Cost Price	Profit	Selling Price
100	25	125
?		0.60

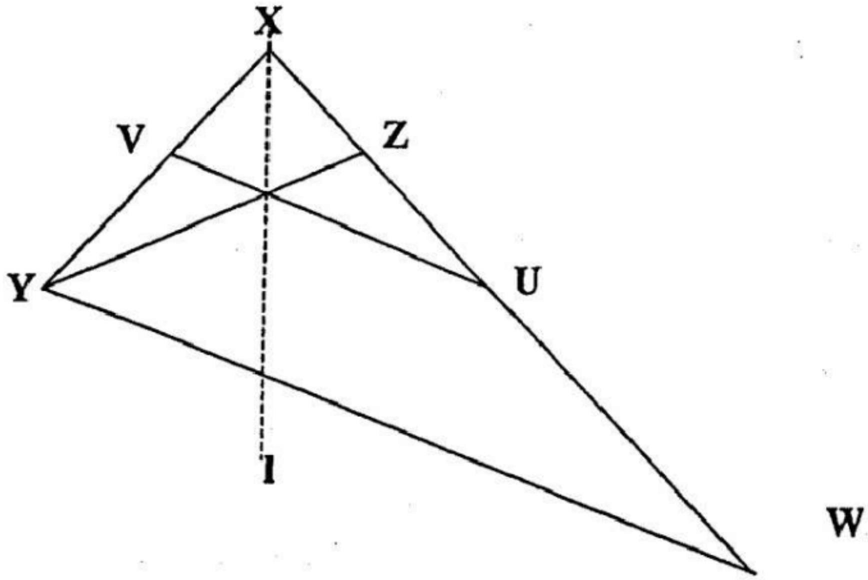
Cost price of one tin = $\frac{100 \times 0.60}{125} = 0.48$

Cost price of a box 12 tins = $12 \times 0.48 = \$5.76$

(d) (i) Selling price for a box = $12 \times 0.60 = \$7.20$
Saving per box = 7.20 - 6.49 = \$0.71

(ii) Cost price of a box = \$5.76
new selling price of a box = \$6.49
profit per box = 6.49 - 5.76 = 0.73
percentage profit = $\frac{0.73}{5.76} \times 100 = 12.7\%$

2-



(a) (i) M(Z) = V

(ii) XU = XY = 5 cm

XV = XZ = 2 cm

(iii) Scale factor of enlargement = $\frac{XY}{XV} = \frac{5}{2} = 2.5$

$XW = XU \times 2.5 = 5 \times 2.5 = 12.5cm$

VU = YZ = 6 cm

$YW = 6 \times 2.5 = 15cm$

(v) $\angle XYZ = \angle XUV = \angle XWY$

(b) $\cos \angle YXZ = \frac{5^2 + 2^2 - 6^2}{2 \times 5 \times 2} = \frac{-7}{20} = -0.35$

$\angle YXZ = 110.5°$

3-(a) (i) 11,12,13,14,21,22,23,24,31,32,33,34

(ii)(a) outcomes multiples of 4 are 12,24,32

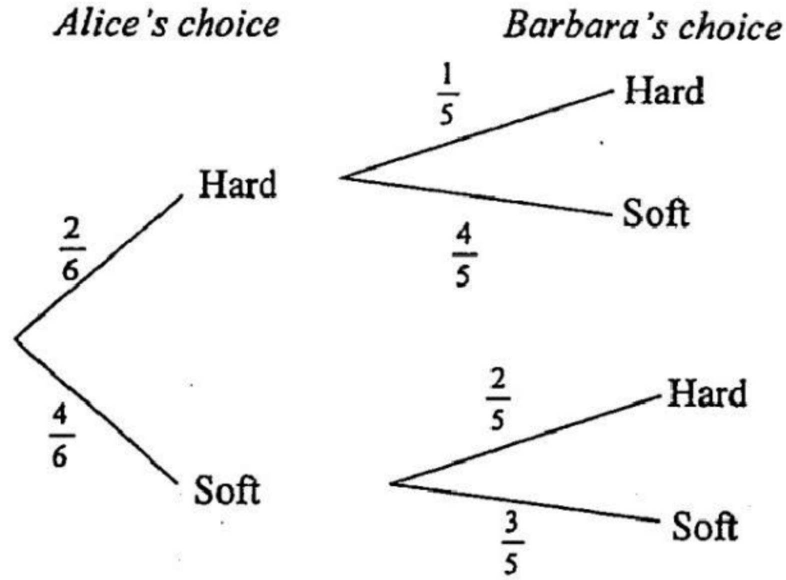
probability = $\frac{3}{12} = \frac{1}{4}$

(b) no outcome is a multiple of 5

$\therefore$ probability = Zero

(b)

(i)



(ii)(a) $\frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$

(b) Hard and Soft or Soft and Hard

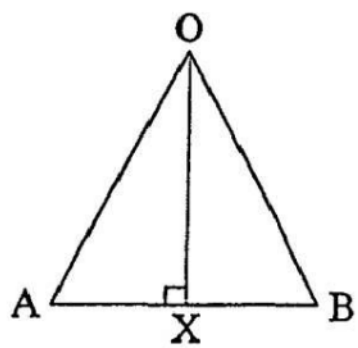
$= \frac{2}{6} \times \frac{4}{5} + \frac{4}{6} \times \frac{2}{5} = \frac{8}{15}$

(c) Hard and Hard or Soft and Hard

$= \frac{2}{6} \times \frac{1}{5} + \frac{4}{6} \times \frac{2}{5} = \frac{2}{30} + \frac{8}{30} = \frac{1}{3}$

4-(a) $\angle AOB = \frac{360}{7} = 51.429$

$\angle OAB = \frac{180 - 51.429}{2} = 64.29°$



(b)(i) $Sin\angle DAB = \frac{OX}{OA}$

$OX = 1.5 \sin 64.29° = 1.35cm$

(ii) AB = 2 AX

$\cos 64.29 = \frac{AX}{1.5}$

$AX = 0.65$

$AB = 2 \times 0.65 = 1.30cm$

OR Use Cosine rule

$AB^2 = 1.5^2 + 1.5^2 - 2(1.5)(1.5)\cos 51.429$

(iii) area of $\Delta AOB = \frac{1}{2} base \times height$

$= \frac{1}{2} \times AB \times AX = \frac{1}{2} \times 1.30 \times 1.35$

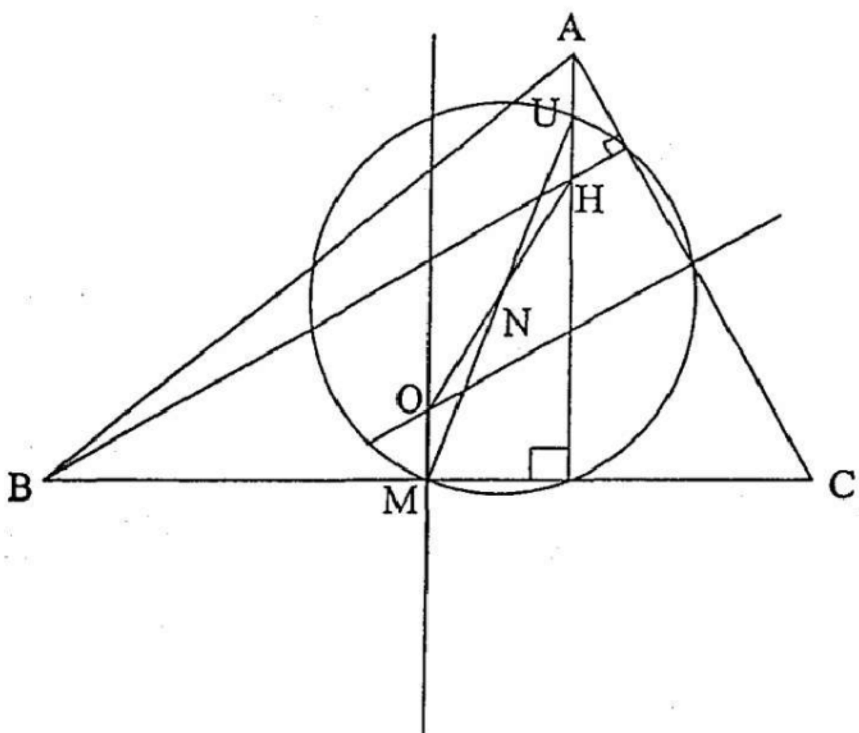
$= 0.8775 \approx 0.878 cm^2$

(iv) area of the whole face $= 7 \times 0.8775 = 6.14 cm^2$

(c) Volume = Area x thickness

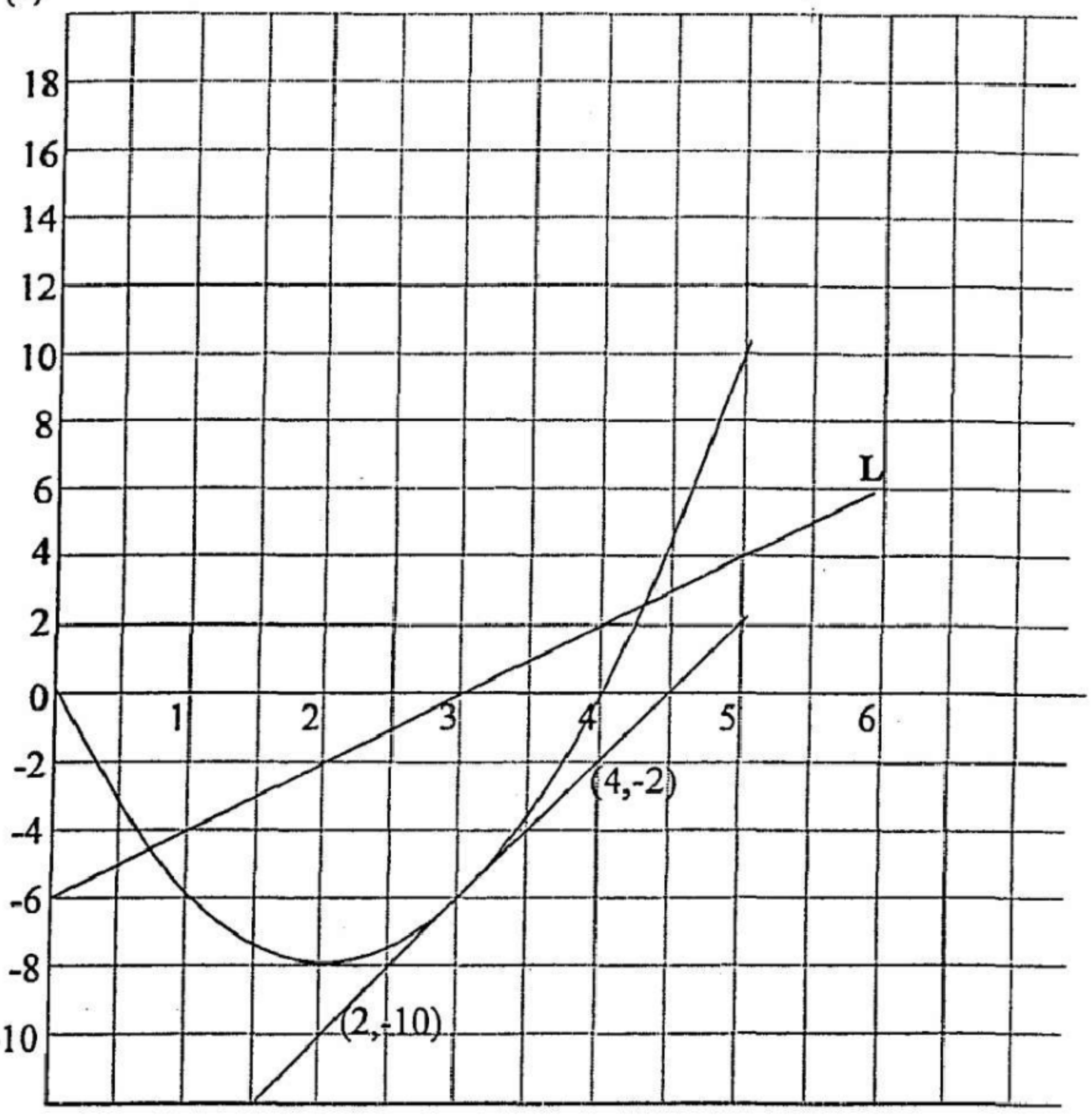
$= 6.14 \times \frac{3}{10} = 1.84 cm^3$

5-



(f) all equal.
(g) congruent
(h) radius = 2.6 cm

6- (a)



(b)(i) Line through (3,0) gradient 2, for each 1 unit along x
y increases by 2 i.e. Line passes through (4,2) , (5,4) ....

(ii) Line intercepts y axis at –6 equation $y = 2x - 6$

(iii) (0.7 , -4.6) , (4.3 , 2.6)

(c) gradient of tangent = $\frac{-2-(-10)}{4-2} = 4$

(d) $y = ax^2 + bx$

$x = 4$ $\qquad$ $y = 0$

$16a + 4b = 0$

$b = -4a$

$x = 1$ $\qquad$ $y = -6$

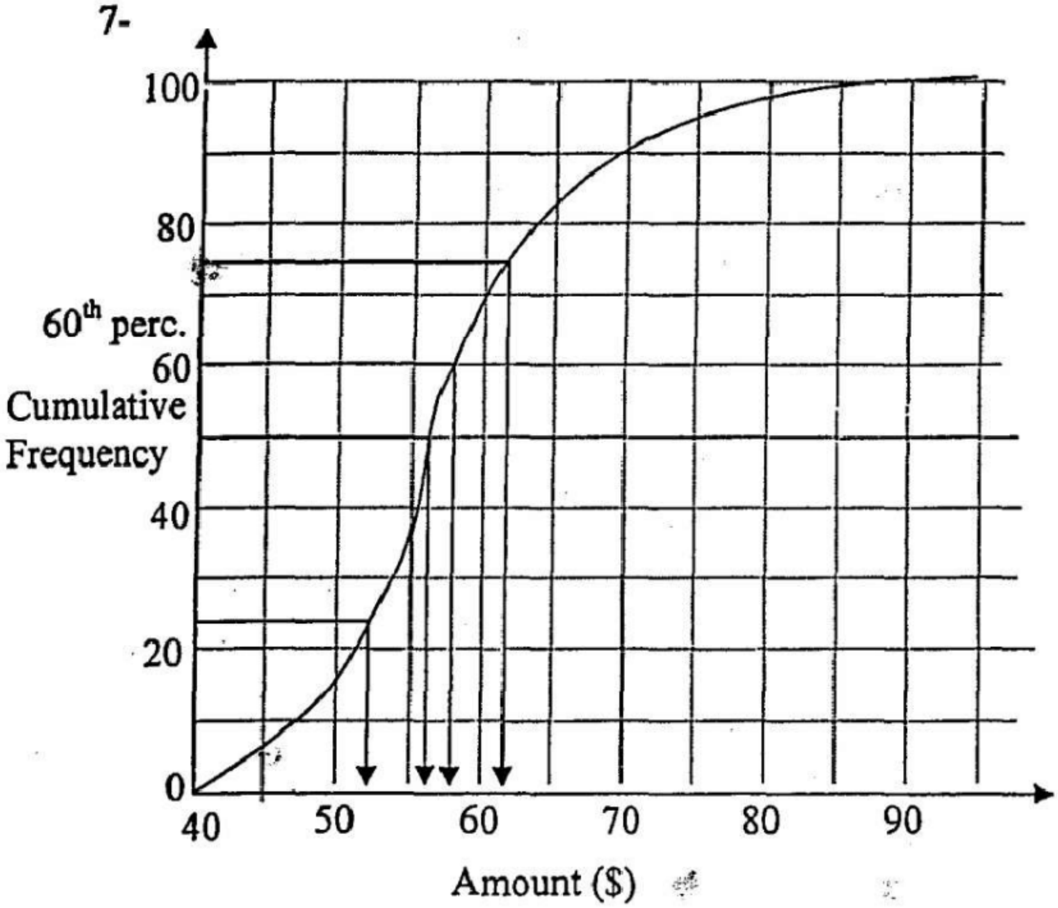
$-6 = a + b$

$-6 = a - 4a$

$-6 = -3a$

$a = 2$ $\qquad$ $b = -8$

7-



(a)(i) Median = 56.5 ≈ \$ 56 or \$ 57 .

(ii) Upper quartile = \$ 61

Lower quartile = \$ 53

(iii) 60^{th} percentile = \$ 58

(b)(i) Interquartile range = Upper quartile – Lower quartile

$= 61 - 53 = \$\,8$

(ii) Percentage = $\frac{8}{50} \times 100 = 16\%$

(c)(i)

Weekly amount \$ x	Frequency	Midclass x	fx
$40 < x \le 50$	14	45	630
$50 < x \le 60$	$72 - 14 = 58$	55	3190
$60 < x \le 70$	$92 - 72 = 20$	65	1300
$70 < x \le 80$	$98 - 92 = 6$	75	450
$80 < x \le 90$	$100 - 98 = 2$	85	170
			5740

(ii) Modal class is $50 < x \le 60$

(iii) Mean = $\frac{\sum fx}{\sum f} = \frac{5740}{100} = 57.4$

(iv) Using smaller class intervals i.e. 40 – 42, 42 – 44, …..
Or 40 – 45, 45 – 50 etc.

8- (b)(i) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 & 6 \\ 2 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 6 \\ 6 & 8 & 16 \end{pmatrix}$

(ii) Area of S is the same as T

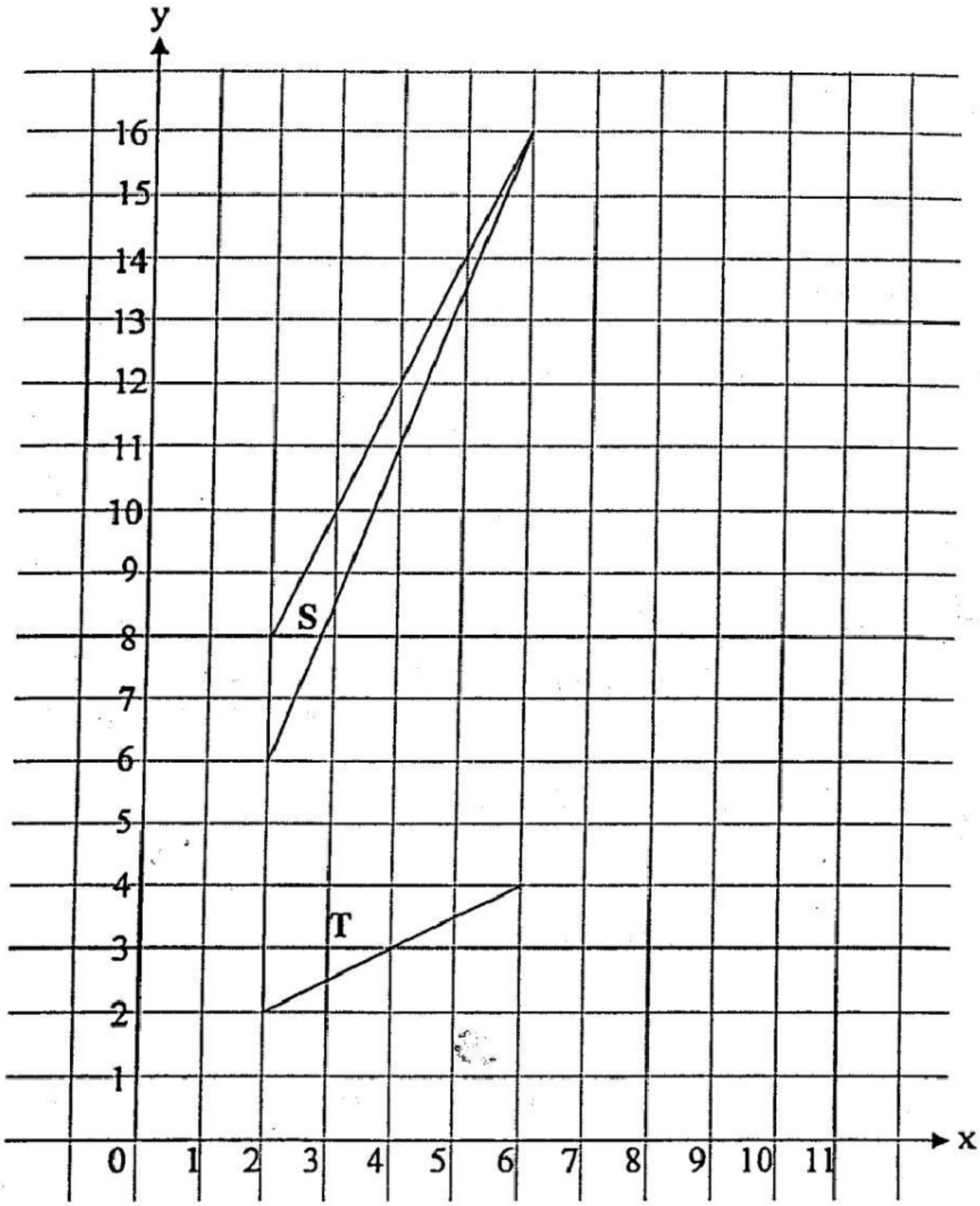
which equal = $\frac{1}{2} \times 2 \times 4 = 4$

(iii) transformation is a shear parallel to the y axis (y axis invariant)

(c)(i) $M = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

$M^{-1} = \frac{1}{1}\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$

(ii) Image of S under the transformation M^{-1} is T



9- (a) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

(b) $24^2 = 576$

$25^2 = 625$

$25^2 - 24^2 = 625 - 576 = 49 = 7^2$

Pythagorean triple is 7 , 24 , 25

(c) (i) $y^2 = x^2 - (x-2)^2$

$= x^2 - (x^2 - 4x + 4)$

$= 4x - 4$

$y = \sqrt{4x-4}$

(ii) $x = 50$ $\quad\quad y = \sqrt{4 \times 50 - 4} = \sqrt{196} = 14$

$x - 2 = 50 - 2 = 48$

other two numbers are 48 , 14

(iii) $x = 101$ $\quad\quad y = \sqrt{4 \times 101 - 4} = 20$

$101 - 2 = 99$

other two numbers are 99 , 20

(iv) since $\quad y = \sqrt{4x-4} = \sqrt{4(x-1)} = 2\sqrt{x-1}$

In order to get y whole number, x should be taken such that (x-1) is a perfect square.

Possible values of x are $9 + 1 = 10$ or $16 + 1 = 17$, $25 + 1 = 26$, $36 + 1 = 37$

for each x , x − 2 and y can be calculated

i.e.	$x = 10$	$x - 2 = 8$	$y = 2\sqrt{10-2} = 6$	{ 6,8,10 }
	$x = 17$	$x - 2 = 15$	$y = 2\sqrt{17-1} = 8$	{ 8,15,17 }
	$x = 26$	$x - 2 = 24$	$y = 2\sqrt{26-1} = 10$	{ 10,24,26 }
	$x = 37$	$x - 2 = 35$	$y = 2\sqrt{37-1} = 12$	{ 12,35,37 }

Any one set is a possible answer.