## *Nov. 1993*

## Paper 4

1- (a) amount received $= \frac{800}{1.68} \times \frac{99}{100} = 471.4$

answer is 471 dollars

(b) (i) amount $= p + \frac{PRT}{100}$

$$= 800 + \frac{800 \times 9 \times \frac{6}{12}}{100} = 836 \text{ DM}$$

(ii) amount in dollars $= \frac{836}{1.87} = 447.1$

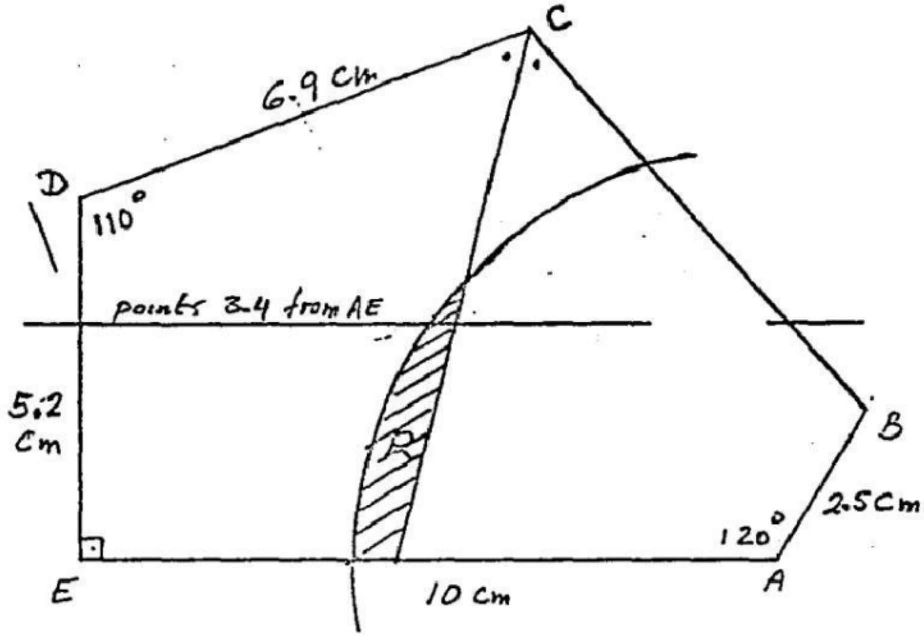
answer is 447 dollars

(c) K Laus , he got the larger amount

(d) amount $= 120 \times 1.72 = 206.4$

$= \text{DM } 206$

(2)




(b) (iii) Yes

3- (a) Four.

(b) A is (4 , 6) and H is (0 , 2)

$$\begin{pmatrix}0\\2\end{pmatrix} - \begin{pmatrix}4\\6\end{pmatrix} = \begin{pmatrix}-4\\-4\end{pmatrix}$$

Vector of translation is $\begin{pmatrix}-4\\-4\end{pmatrix}$

(c) The line through (0 , 6) and (6 , 0) its equation is $x + y = 6$

(d) Reflection on the line $x = 3$

(e) Rotation clockwise by 90° centre point (3 , 3)

(f) $\begin{pmatrix}1 & 3\\-1 & 7\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix}$ i.e. point B

4- (a) $\overline{OS}^2 = 400^2 + 850^2 - 2 \times 400 \times 850 \cos 110$

$= 1115073.7$

OS = 1056 km

(b) $\dfrac{850}{\sin < SOT} = \dfrac{1056}{\sin 110}$

$\sin < SOT = 0.7564$

$< SOT = 49°$

(c) The bearing of Tokyo from Osaka

= 30 + 49 = 79

∴ The bearing of Osaka from Tokyo

= 180 + 79 = 259°

(d) Time of journey $= \dfrac{850}{500} = 1.7$ h = 1 h 42 min

time of arrival = 9 h 30 min + 1 h 42 mmin

= 11 h 12 min i.e 11 12

5- (a) amount $= 10000 \times 15 + 20000 \times 8 = \$\ 310000$

(b) number of standing places replaced

$= 20000 - 4000 = 16000$

number of the extra seats $= \dfrac{16000}{2} = 8000$

(i) number of seats now $= 10000 + 8000 = 18000$

(ii) amount $= 18000 \times 15 + 4000 \times 8 = \$\ 302000$

(iii) number of seats $= \dfrac{200000-(4000 \times 8)}{15} = 11200$

(c) (i) x standing places remain

$(20000 - x)$ standing places replaced to $\dfrac{20000-x}{2}$ seats

number of seats now $= 10000 + \dfrac{20000-x}{2}$

$= \dfrac{20000+20000-x}{2} = 20000 - \dfrac{x}{2}$

(ii) $20000 - \dfrac{x}{2} = 2x$

$40000 = 5x \Rightarrow x = 8000$

Total number of places $= 8000 + \left(20000 - \dfrac{8000}{2}\right)$

$= 24000$

maximum number of spectators $= 24000$

6- (a) Area of the circle radius 10 cm $= \pi \times 10^2 = 100\pi$

Area of the circle radius 20 cm $= \pi \times (20)^2 = 400\pi$

Area of the circle radius 30 cm $= \pi \times (30)^2 = 900\pi$

Area of bull $= 100\pi$

Area of inner $= 400\pi - 100\pi = 300\pi$

Area of outer $= 900\pi - 400\pi = 500\pi$

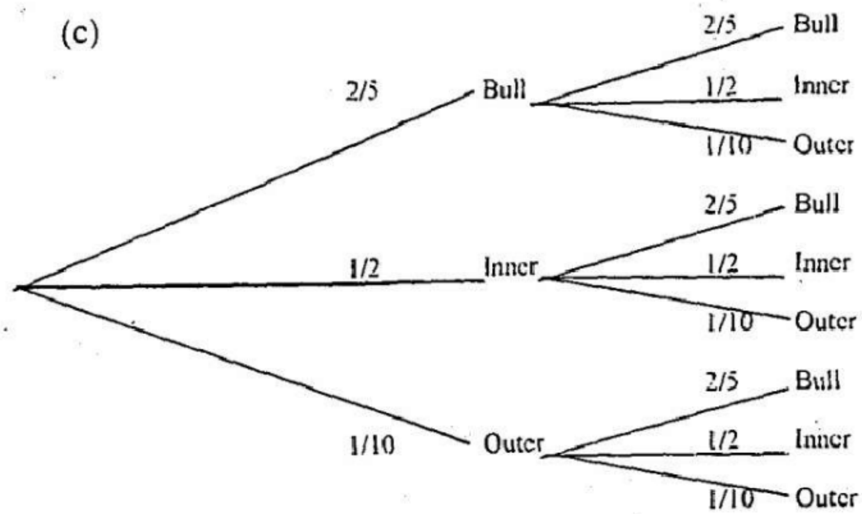
$\therefore$ ratio of the areas of bull : inner : outer

is $100\pi : 300\pi : 500\pi$

i.e. $1 : 3 : 5$

(b) Probability $= \dfrac{\text{area of bull}}{\text{total area}} = \dfrac{1}{1+3+5} = \dfrac{1}{9}$

(c)




(i) Probability $= \dfrac{1}{10} \times \dfrac{1}{10} = \dfrac{1}{100}$

(ii) to win \$ 12 means to get one bull and one inner.

Probability $= \dfrac{2}{5} \times \dfrac{1}{2} + \dfrac{1}{2} \times \dfrac{2}{5} = \dfrac{2}{5}$

(iii) To win \$ 30 means hitting the bull all three times, therefore,

Probability $= \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} = \dfrac{8}{125}$

7- (a) (i) $< VTO = 90$, since VB is targent to the circle and OT is a radius.

(ii) $< TOV = 90 - 20 = 70$

$< TOS = 2 \times 70 = 140°$

(iii) $< TPS = \frac{1}{2} < TOS = 70°$ (theorem)

(b) (i) $\sin 20 = \frac{OT}{VO} = \frac{10}{VO}$

$VO = \frac{10}{\sin 20^\circ} = 29.2$ cm

(ii) $VP = VO + OP = 29.2 + 10 = 39.2$ cm

$\therefore$ height of the cone = 39.2 cm

(iii) $\tan 20^\circ = \frac{AP}{VP} = \frac{AP}{39.2}$

$AP = 39.2 \times \tan 20^\circ = 14.3$

$\therefore \quad R = 14.3$ cm

(c) (i) Volume of the cone $= \frac{1}{3} \pi R^2 h$

$= \frac{1}{3} \times 3.142 \times (14.3)^2 \times 39.2$

$= 8395.4 = 8400 \text{ cm}^3$

(ii) Volume of the sphere $= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.142 \times (10)^3 = 4189.3$

$= 4190 \text{ cm}^3$

(iii) Volume of empty space

$= 8395.4 - 4189.3 = 4206.1$

percentage of the volume of the cone

$= \frac{4206.1}{8395.4} \times 100 = 50.1$ %

8- $y = 4 + 2x - x^2$

(a) At A $y = 1$

$\therefore \quad 4 + 2x - x^2 = 1$

$x^2 - 2x - 3 = 0$

(b) $x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$\therefore x = 3$ or $x = -1$ (rejected)

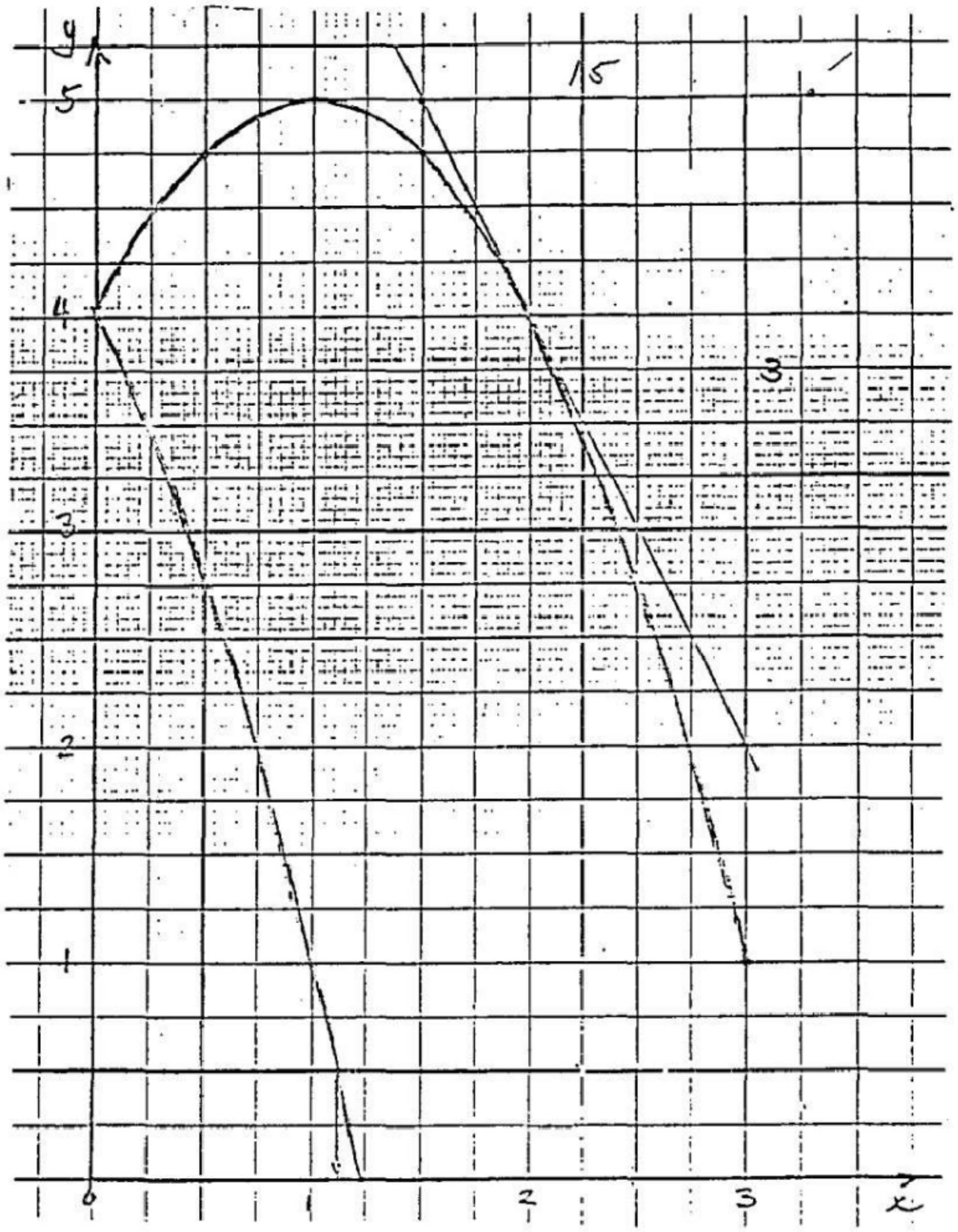
x coordinate of A is 3

(c) $x^2 + 2x - 4 = 0$

$$x = \frac{-2 \pm \sqrt{4+16}}{2} = \frac{-2 \pm \sqrt{20}}{2}$$

$= 1.24$ (only positive root)

8-




(e) 1.12 m

(f) gradient = − 2 or gradient using point (3 , 7) , (1½ , 5)

gradient = $\frac{5-2}{1^1-3}$ = − 2

9- (a) $\frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$

(b)

| n | $\frac{1}{n} - \frac{1}{n+1}$ | $\frac{1}{n(n+1)}$ |
|---|---|---|
| 1 | $\frac{1}{1} - \frac{1}{2}$ | $\frac{1}{1 \times 2}$ |
| 2 | $\frac{1}{2} - \frac{1}{3}$ | $\frac{1}{2 \times 3}$ |
| 3 | $\frac{1}{3} - \frac{1}{4}$ | $\frac{1}{3 \times 4}$ |
| 4 | $\frac{1}{4} - \frac{1}{5}$ | $\frac{1}{4 \times 5}$ |
| ↓ | ↓ | ↓ |
| 99 | $\frac{1}{99} - \frac{1}{100}$ | $\frac{1}{99 \times 100}$ |
| 100 | $\frac{1}{100} - \frac{1}{101}$ | $\frac{1}{100 \times 101}$ |

(c) adding all terms of column 2 and column 3

$$\therefore \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} \ldots\ldots\ldots - \frac{1}{100} + \frac{1}{100} - \frac{1}{101}$$

$$= \frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} \ldots\ldots\ldots + \frac{1}{100x101}$$

$$\therefore \frac{1}{1} - \frac{1}{101} = \frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \ldots\ldots\ldots + \frac{1}{100x101}$$

$$\therefore \frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \ldots\ldots + \frac{1}{100x101} = 1 - \frac{1}{101} = \frac{100}{101}$$