بسم الله الرحمن الرحيم

مقابل هذا المجهود ارجو منكم الدعاء لي بالمغفرة ولابنائي الهداية والنجاح والتوفيق

أرجو ان يساعد هذا المجهود على مساعدة ابنائنا طلبة ال IGCSE للثانوية البريطانية وتحصيلهم على افضل واحسن واعلى الدرجات انشاء الله.
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ابو احمد

للاستفسار والمساعدة اكتب لي على العنوان البريدي التالي :-
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In the name of god

Pry for me and my sons to success, mitigating and proselyting

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IGCSE

Mathematics O.L

Answers to Examination Papers

June1993 - June 2003

Math O.L
Answers
contant

Answers to Examination Paper

2

June 1993

Paper 2

1- (a) $3.5 - (-1.5) = 5$

(b) $3.5 - 4.75 = -1.25$

2- $PQ = \sqrt{(2-0)^2 + (-1-4)^2}$

$= \sqrt{2^2 + 5^2}$

$= \sqrt{29} = 5.385$

$= 5.39$

3- $8x = 12$ adding given equations

$x = \frac{12}{8} = 1.5$ or $1\frac{1}{2}$

4- $2.70 \times 10^8 + 1.02 \times 10^9 = 1.29 \times 10^9$

using calculator 2.7 Exp 8 + 1.02 Exp 9 = 1.29 Exp 9 .

$= 1.29 \times 10^9$

5-

	1990	1991
percent	100	97
actual	?	6.305

$\frac{6.305 \times 100}{97} = 6.5$

6- $010 + 180 = 190°$

7- 4 Swiss Francs $= 4 \times 1.23$ D.M

$= 4.92$ D.M

no. of bottles $= \frac{4.92}{0.55} = 8.945$

$= 8$

8- (a) $\frac{33x^2}{11x^{-4}} = \frac{33x^2.x^4}{11} = 3x^6$

(b) $\left(\frac{27}{64}\right)^{2/3} = \left[\sqrt[3]{\frac{27}{64}}\right]^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

9-

Mark x	5	6	7	8	9	10	
Frequency f	2	0	10	9	5	4	30
fx	10	0	70	72	45	40	237

(a) 7

(b) $\frac{237}{30} = 7.9$

10- (a) $= \frac{\frac{1}{8}+1}{3} = \frac{\frac{9}{8}}{3} = \frac{3}{8}$

(b) x → [+1] → [÷3] → f (x) → OR $y = \frac{x+1}{3}$

$f^{-1} = 3x - 1$ ← [−1] ← [x3] ← x

$3y = x + 1$

$3y - 1 = x$

$x = 3y - 1$

$f^{-1} = 3x - 1$

11- (a) $\frac{10000}{225000} = \frac{10}{225} = \frac{2}{45}$

(b) $\frac{2}{45} \times 100$

$= 4\frac{4}{9}\,\%$ or $4.44\,\%$

12- (a) $2M = 2\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -2 & 6 \end{pmatrix}$

(b) $M^{-1} = \frac{1}{3 - (-2)} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

$= \frac{1}{5} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$ or $\frac{1}{5} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

13- (a) 25.5 cm $\leq d <$ 26.5 cm

(b) $C = 2\pi r = \pi d$

$3 \times 25.5 = 76.5$

$3.2 \times 26.5 = 84.8$

76.5 cm $< C <$ 84.8 cm

14- $P = \dfrac{k}{v}$

$70 = \dfrac{k}{0.5}$

$k = 35$

$P = \dfrac{35}{v}$

$28 = \dfrac{35}{v}$ $\therefore\ v = 1.25$

15- Time $= \dfrac{465}{30} = 15.5$ hours $=$ 15 h 30 min

18 h 40 min + 15 h 30 min = 34 h 10 min

34 h 10 min - 24 h = 10 h 10 min

= 10 : 10

16- (a) \$ 70

(b) p is the intersection of the line with y axis

p = 35

additional cost per hour = 55 - 35 = 20

p = 35

q = 20

17- $\tan\theta = \dfrac{4}{7}$ $\theta = 29.74°$

$< APB = 2\theta = 59.48$

$= 59.5°$

18- $\frac{1}{4}\pi r^2 = 16.5$

$r^2 = \frac{16.5 \times 4}{\pi} = \frac{16.5 \times 4}{3.142} = 21.006$

$r = 4.58$

19- (a) $-5 \le 2x + 1$

$-6 \le 2x$

$-3 \le x$

$2x + 1 < 5$

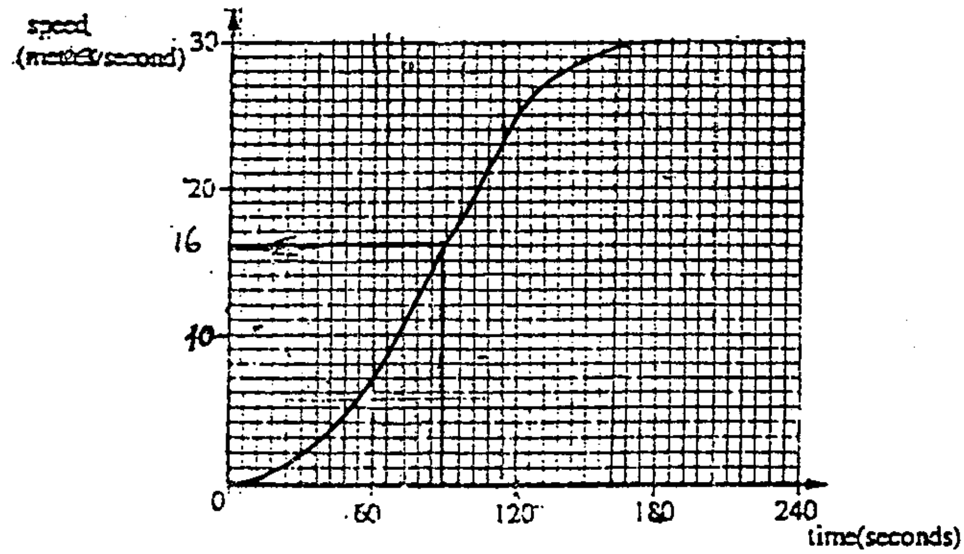
$2x < 4$

$x < 2$

$\{ x : -3 \le x < 2 \}$

(b) $\{ -3, -2, -1, 0, 1 \}$

20-

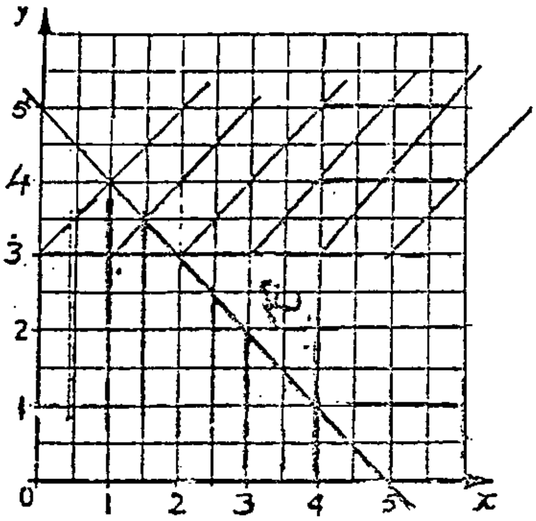


(a) from graph = 16 m/s

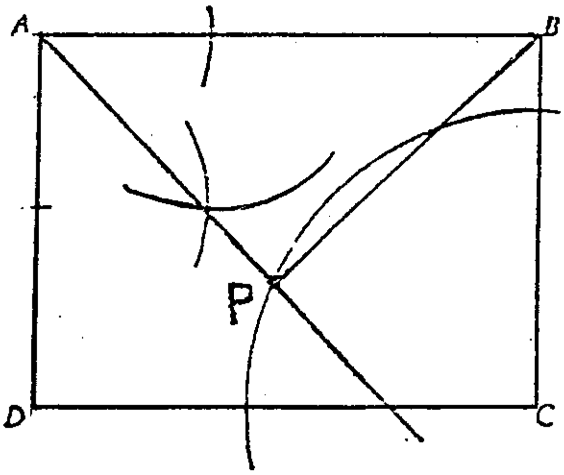
(b) from 60 s to 120 s the graph is a straight line

$\text{acceleration} = \frac{\text{change in speed}}{\text{time}} = \frac{25-7}{60} = \frac{18}{60} = 0.3 \text{ m/s}^2$

21-



22-



(b) BP = 4.9 cm

23-(a) $4x^2(x-2y^2)$

(b) (i) $(2x+3)(x-2)$

(ii) $2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$

$2x + 3 = 0$ or $x - 2 = 0$

$x = -\frac{3}{2}$ or $x = 2$

24- (a) PQ is parallel to OR and equal ½ of it

$\overrightarrow{PQ} = \frac{1}{2} r$

(b) $\overrightarrow{QR} = \overrightarrow{QP} + \overrightarrow{PO} + \overrightarrow{OR}$

$= -\frac{1}{2} r - P - r$

$= \frac{1}{2} r - P$

(c) $\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$

$= r - P$

Nov. 1993

Paper 2

1- (a) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

(b) $3^2 \div 2^{-3}$

$= 9 \div \frac{1}{8} = 72$

or using calculator

2- $\frac{3x-4}{2} = 7.$

$3x - 4 = 14$

$3x = 18$

$x = 6$

3- $1.42 \times 10^9 - 1.5 \times 10^8 = 1.27 \times 10^9$

4- $3\frac{2}{9}$ m $= 3.22222$ m

32.4 cm $= 0.324$ m

32.4 cm < 3.22 m $< 3\frac{2}{9}$ m

5- $\text{Time} = \frac{\text{Distance}}{\text{spead}}$

$\text{maximum time} = \frac{\text{Distance}}{\text{Least speed}}$

$= \frac{575}{11.5} = 50$

6- no. of sides are 7

Sum of all interior angles = (2 x 7 - 4) x 90 = 900

Sum of the five equal angles = 900 - (100 + 100) = 700

each angle = $\frac{700}{5}$ = 140

Angle BCD = 140°

7- (a) $= \frac{3(-4)+2}{-4-1}$

$= \frac{-10}{-5} = 2$

(b) $\frac{3x+2}{x-1} = 4$

3x + 2 = 4x − 4

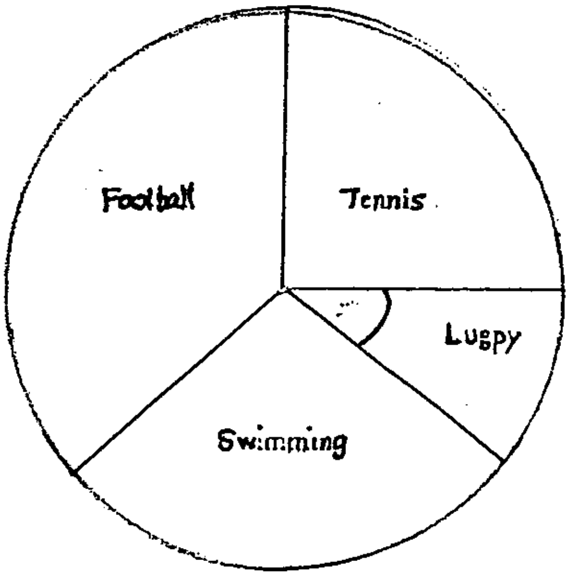
2 + 4 = 4x − 3x = x

x = 6

8- (a) $\frac{1}{4}+\frac{1}{3}+\frac{1}{8}=\frac{6+8+3}{24}=\frac{17}{24}$

$1 - \frac{17}{24} = \frac{7}{24}$

(b) $\frac{1}{8}$ x 360 = 45°



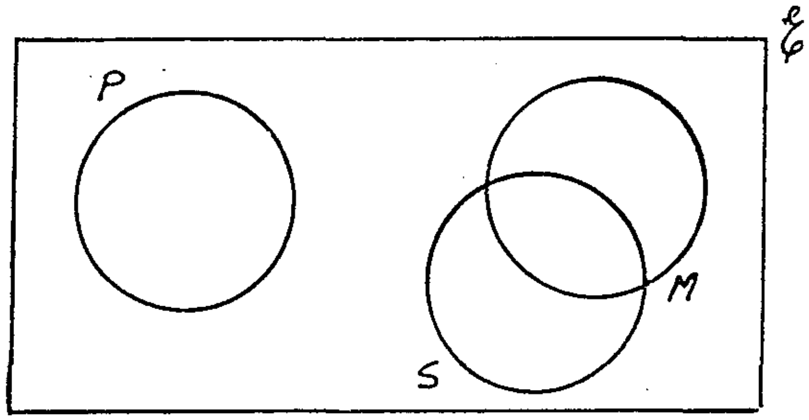
(c) $\frac{1}{3}$ 32

$\frac{1}{4}$?

$= \frac{\frac{1}{4} \times 32}{1/3} = \frac{8 \times 3}{1}$

= 24

9-



(a) $\mathcal{E} = \{x : 20 < x < 40\}$

$p = \{x : x \text{ is a prime number}\} = \{23, 29, 31, 37\}$

$M = \{x : x \text{ is a multiple of } 3\} = \{21, 24, 27, 30, 36, 39\}$

$S = \{x : x \text{ is a square number}\} = \{25, 36\}$

(b) $P \cap S = \Phi$

(c) $M \cup S = \{21, 24, 25, 27, 30, 33, 36, 39\}$

$n(M \cup S) = 8$

10- $2v = hk(a + B)$

$$\frac{2v}{hk} = a + b$$

$$\frac{2v}{hk} - b = a$$

11- (a) Δ_s DCB and DEA are similar

(b) $7x = 1.7x + 8.5$

$5 + 3x = 8.5 \qquad x = \frac{8.5}{5.3} = 1.604 \doteq 1.6$

12- (a) 4 .

(b) (i) Area = $\pi R^2 - 4\pi r^2$

(ii) $\pi (R^2 - 4 r^{2)}$

$= \pi (R + 2 r)(R - 2 r)$

13- (a) $3\begin{pmatrix}3\\5\end{pmatrix} - 4\begin{pmatrix}-2\\3\end{pmatrix}$

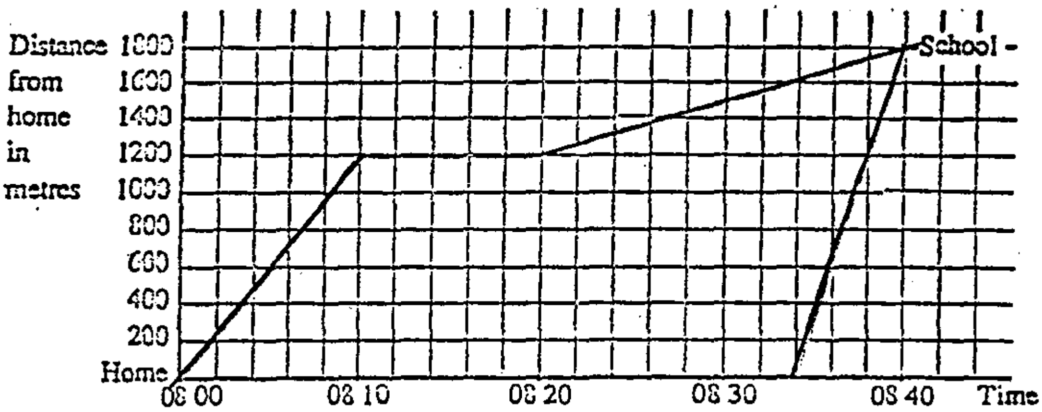
$= \begin{pmatrix}9\\15\end{pmatrix} - \begin{pmatrix}8\\12\end{pmatrix} = \begin{pmatrix}9+8\\15-12\end{pmatrix} = \begin{pmatrix}17\\3\end{pmatrix}$

(b) $= \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{13}$

$\sqrt{13} = 3.61$

14-



(a) Speed = $\dfrac{\text{distance}}{\text{time}} = \dfrac{1200}{10 \times 60} = 2$ m/s

(b) (ii) Time = $\dfrac{1800}{5}$ = 360 sec

360 sec = 6 min

time of departure = 8 : 40 − 6 min = 8 : 34

15- (a) distance = $\sqrt{(7-11)^2+(4-1)^2}$

$= \sqrt{16+9}$

$= 5$

(b) CS = 5

greatest distance = 5 + 3 = 8

16- (a) (i) < CAB = < CDB

= x°

(ii) < AED = x + y

exterior angle of a Δ

< AED = (x + y)°

(b) $\frac{\text{area } \Delta \text{ ABE}}{\text{area } \Delta \text{ DCE}} = \left(\frac{BE}{CE}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

Δ ABE : Δ DCE = 16 : 25

17- $\overrightarrow{OC} = \overrightarrow{3OP} = 3P$

$\overrightarrow{OD} = \overrightarrow{4OQ} = 4q$

(a) $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$

$= 4q - 3p$

(b) $\overrightarrow{OM} = ½(\overrightarrow{OC} + \overrightarrow{OD})$

$= ½(3p + 4q)$

$= 1½p + 2q$

18- (a) $MN = (1 \quad 2)\begin{pmatrix} -1 & 2 & 3 \\ 2 & -1 & 2 \end{pmatrix} = (3 \quad 0 \quad 7)$

(b) $P = \begin{pmatrix} 3 & 4 \\ 8 & 12 \end{pmatrix}$

$$P^{-1} = \frac{1}{3x12 - 4x8}\begin{pmatrix} 12 & -4 \\ -8 & 3 \end{pmatrix}$$

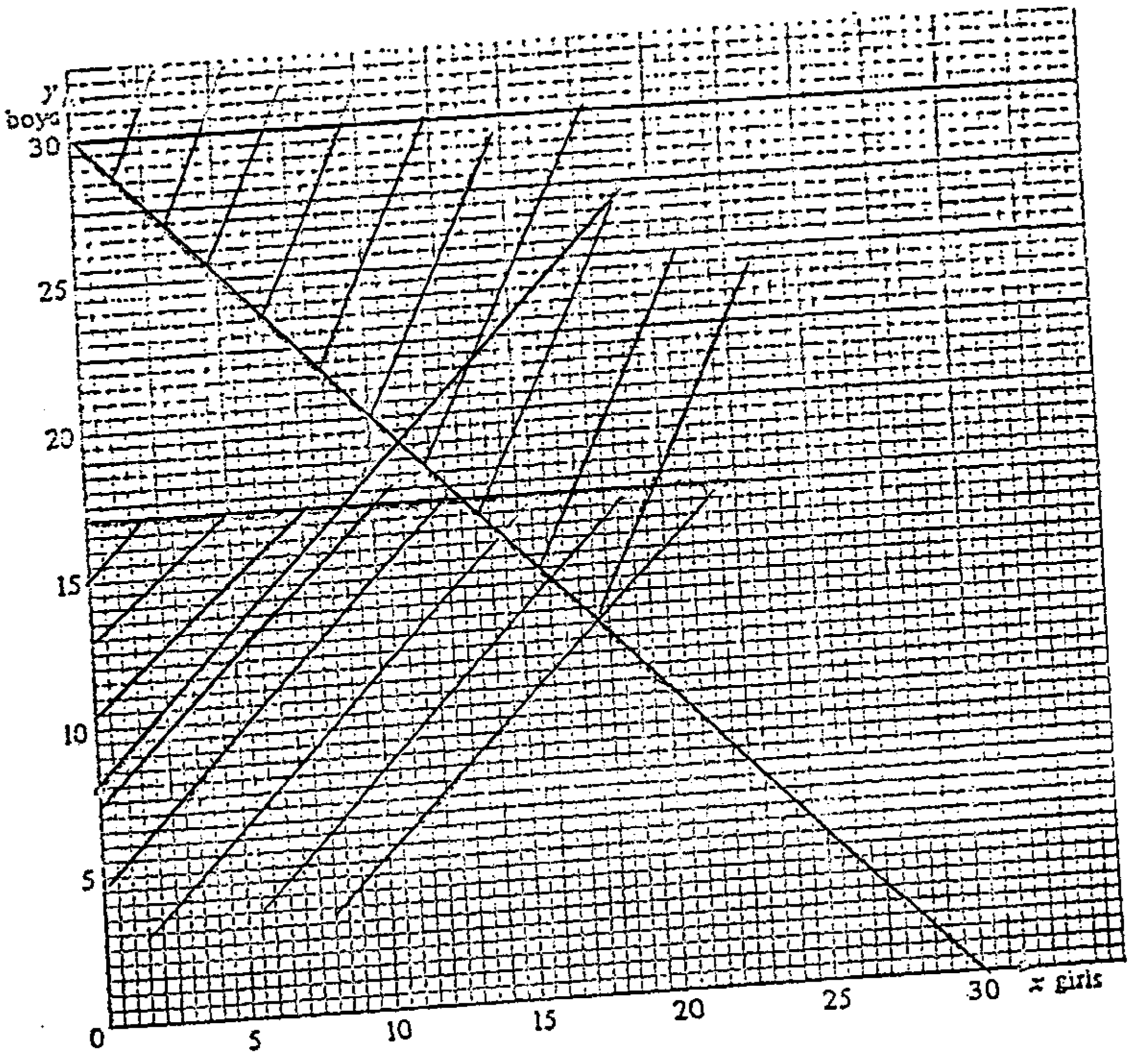
$$P^{-1} = \frac{1}{4}\begin{pmatrix} 12 & -4 \\ -8 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -2 & \frac{3}{4} \end{pmatrix}$$

(c) When det $P = 0$

$3 \times 12 - 4k = 0$

$K = \frac{36}{4} = 9$

19-



(a) $x + y < 30$

$y > 17$

(c) (i) $y - x = 8$

(d) Solution is the point marked above

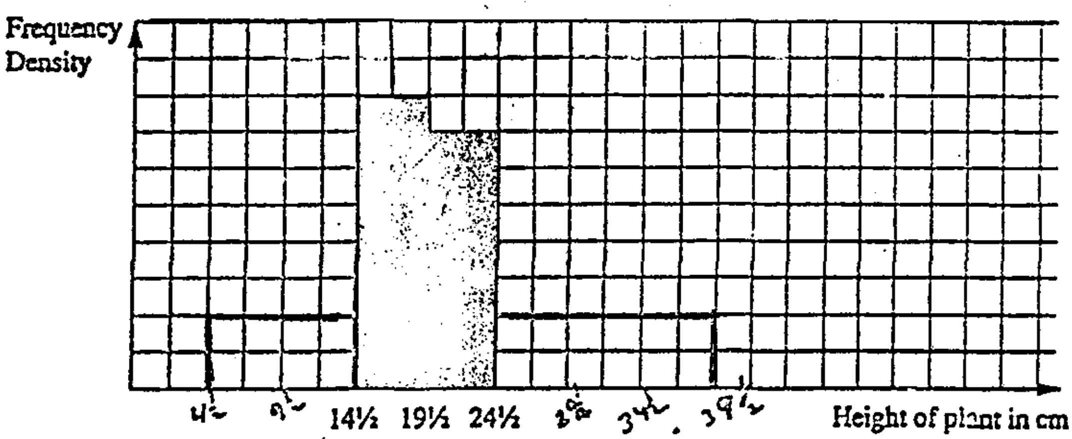
$x = 10$ $y = 18$

$10 + 18 = 28$

20-

Height in cm	Number of plants	Frequency density	Comulative frequency
5 - 14 4½ - 14½	4	$\frac{4}{10} = 0.4$	4
15 - 19 14½ - 19½	8	$\frac{8}{5} = 1.6$	12
20 - 24 19½ - 24½	7	$\frac{7}{5} = 1.4$	19
25 - 39 24½ - 39½	6	$\frac{6}{15} = 0.4$	25

(a)



(b) median is No. $\frac{25+1}{2} = 13$

median $= 19\frac{1}{2} + \frac{13-12}{19-12} \times (24\frac{1}{2} - 19\frac{1}{2}) = 20.2$

(c)

Height in cm	Mid-interval value (x)	Frequency (f)	fx
5 - 14	9½	4	38
15 - 19	17	8	136
20 - 24	22	7	154
25 - 39	32	6	192
		25	520

Mean $= \frac{520}{25} = 20.8$ cm

June 1994

Paper 2

1- (a) $280 + 50 = 330$

(b) $330 - 220 = 110$

2- $\frac{500}{530} \times 100 = 94.3\ \%$

3- $\frac{5}{9} \times 225 = \$\ 125$

$\frac{4}{9} \times 225 = \$\ 100$

4- $\frac{1}{f} = \frac{1}{\left(\frac{1}{4}\right)} + \frac{1}{\left(\frac{2}{3}\right)}$

$\frac{1}{f} = 4 + \frac{3}{2} = \frac{11}{2}$

$f = \frac{2}{11}$

5-

	1980	increase	1990
percent	100	140	240
actual	?		180

$\frac{180 \times 100}{240} = 75$

answer = \$ 75

6- 57 hours = 2 x 24 + 9

i.e. two days and 9 hours

<u>Fri. 16.30</u>	<u>Sat. 17.30</u>	<u>Sun. 17.30</u>
7/2	8/2	9/2 + 9 hours
		26.30
		<u>− 24</u>
		2.30 next day

Answer : Time : 2.30 Day : Monday Date : 10th Feb.

7- $\dfrac{\text{Larger capacity}}{\text{Smaller capacity}} = \left(\dfrac{12}{7}\right)^3$

$\dfrac{x}{300} = \left(\dfrac{12}{7}\right)^3$

$x = 300 \times \left(\dfrac{12}{7}\right)^3 = 1511$ Answer : 1510 ml

8- (a) $149.5 < 150 < 150.5$ and $39.5 < 40 < 40.5$

Least possible distance = 149.5 + 39.5 = 189 km

(b) $145 < 150 < 155$ $35 < 40 < 45$

Least possible distance = 145 + 35 = 180 km

9- (a) $(8 - 5) \times 3 = 9$,

(b) $8 - (5 \times 3) = {}^-7$.

10- $\mathscr{E}$ = {x 2 ≤ x ≤ 12 and x is an integer}. A = {prime numbers}. and B = {factors of 12}

$\mathscr{E}$ = {2 3 4 5 6 7 8 9 10 11 12}. A = {2 3 5 7 11} B = {2. 3. 4. 6. 12}

A' = {4. 6. 8. 10. 12}

(a) (i) $A \cap B = \{2, 3\}$ (ii) $\{2, 3, 4, 5, 6, 7, 11, 12\}$

(b) $n(A') = 6$

11- $p + 2q = 1$ x - 3

$3p + 4q = 0$

$\underline{-3p - 6q = -3}$

$-2q = -3$

$q = \frac{-3}{-2} = \frac{3}{2} = 1\frac{1}{2}$

$p + 2 \times \frac{3}{2} = 1$

$p + 3 = 1$

$p = 1 - 3$

$p = -2$

12- (a) $= \frac{1}{3} \times 12 \times 12 \times 9.5$

$= 456 \text{ cm}^3$

(b) $= 4$

13- $2x(x^2 - 4y^2)$

$2x(x + 2y)(x - 2y)$

14- (a) 1

(b) $4x^6$

15- $I = \frac{K}{R}$

$6 = \frac{K}{2}$

$\therefore K = 12$ $I = \frac{12}{R}$

$= \frac{12}{\frac{1}{2}} = 24$

16- $\frac{620}{3.14} = 197.452$

$197.452 - 192$

$= £\ 5.45$

17- $-5 \le x \le -3$ and $-1 \le y \le 2.$

$x = \{-5, -4, -3\}, \quad y = \{-1, 0, 1, 2\}$

(a) $x + y = -3 + 2 = -1$

(b) $xy = -5 \times -1 = 5$

(c) $x^2 y = (-5)^2 \times 2 = 50$

18- (a) $\frac{2}{x} - \frac{1}{x+1} = \frac{2(x+1) - x}{x(x+1)}$

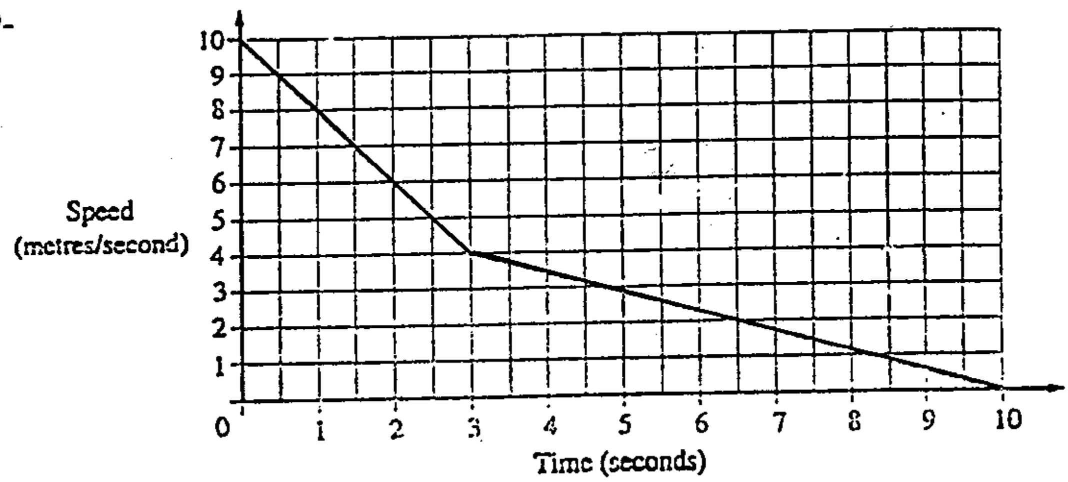
$= \frac{2x + 2 - x}{x(x+1)} = \frac{x+2}{x(x+1)}$

(b) $\frac{x+2}{x(x+1)} = 0$

$x + 2 = 0$

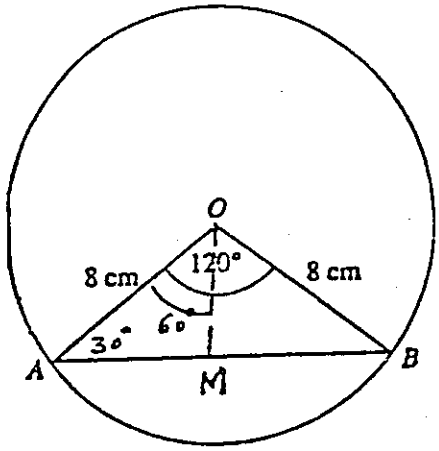
$x = -2$

19-



(a) $\frac{10-4}{3} = 2 \text{ m/s}^2$

(b) $\frac{6x3}{2} + 4x3 + \frac{7x4}{2} = 35 \text{ m}$



NOT TO SCALE

20- (a) AM = 8 sin 60

or 8 cos 30

AB = 2 x 8 cos 30

= 16 cos 30

= 13.9 cm

(b) $\frac{120}{360} \times 2 \times 3.142 \times 8 = 16.8 \text{ cm}$

21- (a) $BA = (-3 \ \ 2)\begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix} = (-9 \ \ -7)$

(b) $A^{-1} = \frac{1}{-6}\begin{pmatrix} -2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ 0 & -\frac{1}{2} \end{pmatrix}$

22- (a) $\overrightarrow{OM} = \overline{OQ} + \overline{QM}$

$= q + ½(\overline{QP})$

$= q + ½(-q + P)$

$= q - ½q + ½P$

$= ½P + ½q$

(b) $\overline{NQ} + \overline{QM}$

$= ¼q + ½(-q + P)$

$= ¼q - ½q + ½P = ½p - ¼q$

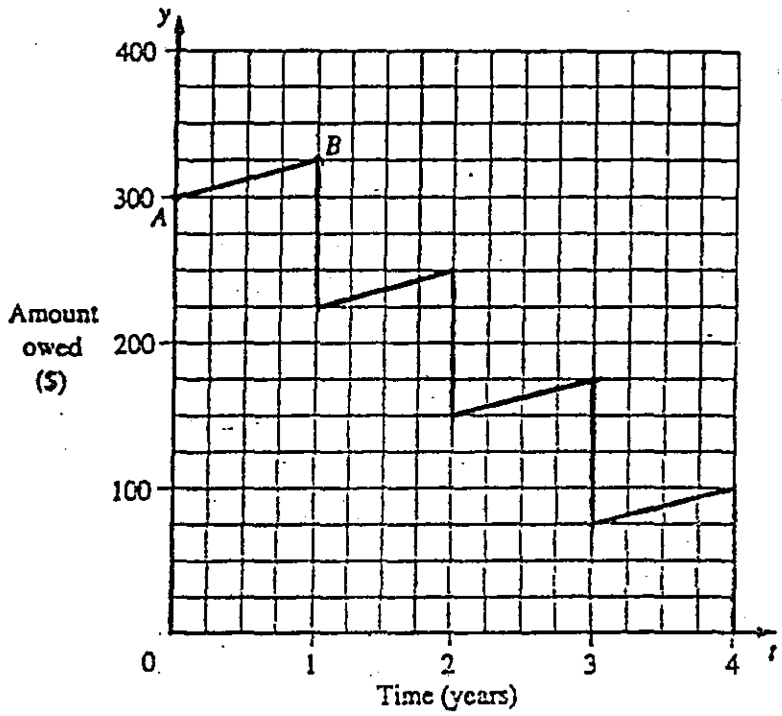
$\overline{NM} = ½P - ¼q$

23- (a) Shear in the x – direction, scalefactor 2

x – axis invariant

(b) Stretch [redacted] to the y – axis scale factor 2

24-



(a) \$ 162 ½

(b) \$ 100

(c) 100 x 4 – 300 = \$ 100

(d) C = 300

intersection with y axis

when t = 1 y = 325

∴ 325 = m + 300

m = 25

Nov. 1994

Paper 2

1. Time of arrival = 23 40 + 7 30 − 5
   = 26 10 − 24
   = 02 10 (next day)

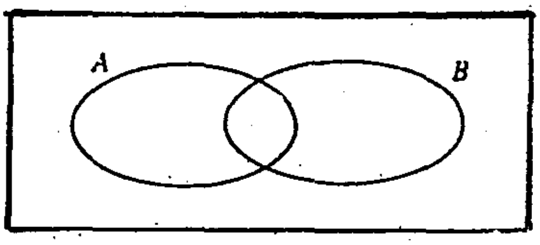
2. Original reduction sale price
   100 15 85
   ? 27.20

   Cost before the sale $= \dfrac{100 \times 27.20}{85} = \$\ 32$

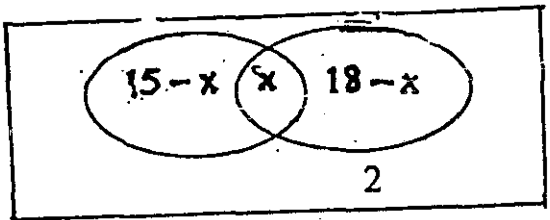
3. $\sin 18° = \dfrac{BC}{280}$

   $BC = 280 \times \sin 18 = 86.5$ m

4. (a)



   (b)



   $15 - x + x + 18 - x + 2 = 30$
   $x = 5$
   number of students studying both Art and Biology = 5

5. AMBULANCE | E C N A L U B M A (mirror image)

6. (a) 13.50 x 3.90 = 52.92 Ryals
   (b) 400 ÷ 3.90 = \$ 102.04

7.
$$\frac{x+9}{3} - \frac{x-11}{4} = \frac{4(x+9) - 3(x-11)}{12}$$
$$= \frac{4x+36-3x+33}{12} = \frac{x+69}{12}$$

8. (a) Angle DEF = 130°

(b) Sum of interior angles = (2x6 − 4) x90 = 720

$$\text{Angle BCD} = \frac{720-(2\text{x}130+2\text{x}120)}{2} = 110$$

<u>OR</u> Angle BCD = (180 − 130) + (180 − 120) = 50 + 60 = 110

9. (a) $\sqrt{250}$ (b) 25 (c) 29

10. (a) $\frac{3}{5} = 0.6$, $\frac{7}{12} = 0.583$, $\frac{17}{30} = 0.567$

$$\frac{17}{30} < \frac{7}{12} < \frac{3}{5}$$

(b) $\frac{215}{360} = 0.597$

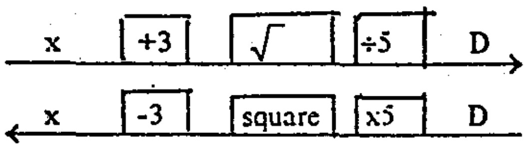
closest estimate is 0.6 i.e. $\frac{3}{5}$

11. (a) $40\,000 \times 10^6 \times \frac{2}{1000} = 8.00 \times 10^7$ kg

(b) $\frac{8 \times 10^7}{1000} = 8 \times 10^4 = 80\,000$ hectares

12.
$$D = \frac{\sqrt{x+3}}{5}$$
$$D^2 = \frac{x+3}{25}$$
$$x + 3 = 25\,D^2$$
$$x = 25\,D^2 - 3$$

<u>OR</u>



$x = (5D)^2 - 3 = 25D^2 - 3$

13. Least speed $= \frac{860-50}{3\text{x}60} = \frac{810}{180} = 4.5$ m/s

14. Capacity of the model $= \dfrac{10000}{(50)^3}$ Litres

$= 0.08 \times 1000 = 80$ mL

15. (a) (i) angle OBD $= 180 - 130 = 50°$

(ii) angle OAC $= \dfrac{130}{2} = 65°$

(iii) angle BDC $= \dfrac{360-130}{2} = 115°$

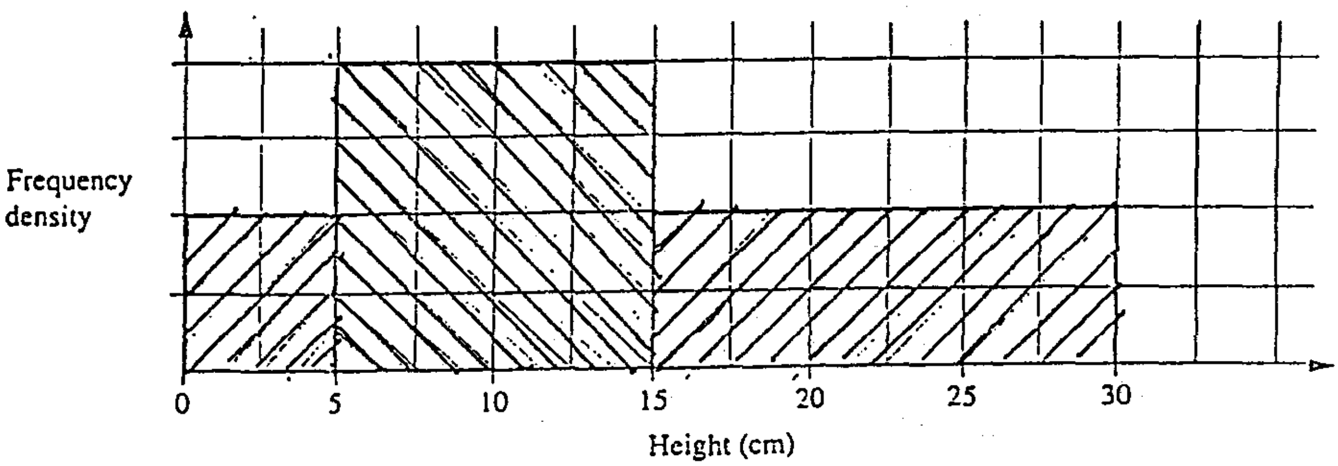
(b) $< OBD + < BDC = 50 + 115 = 165$

As the sum is not equal 180°, therefore AB is not parallel to CD .

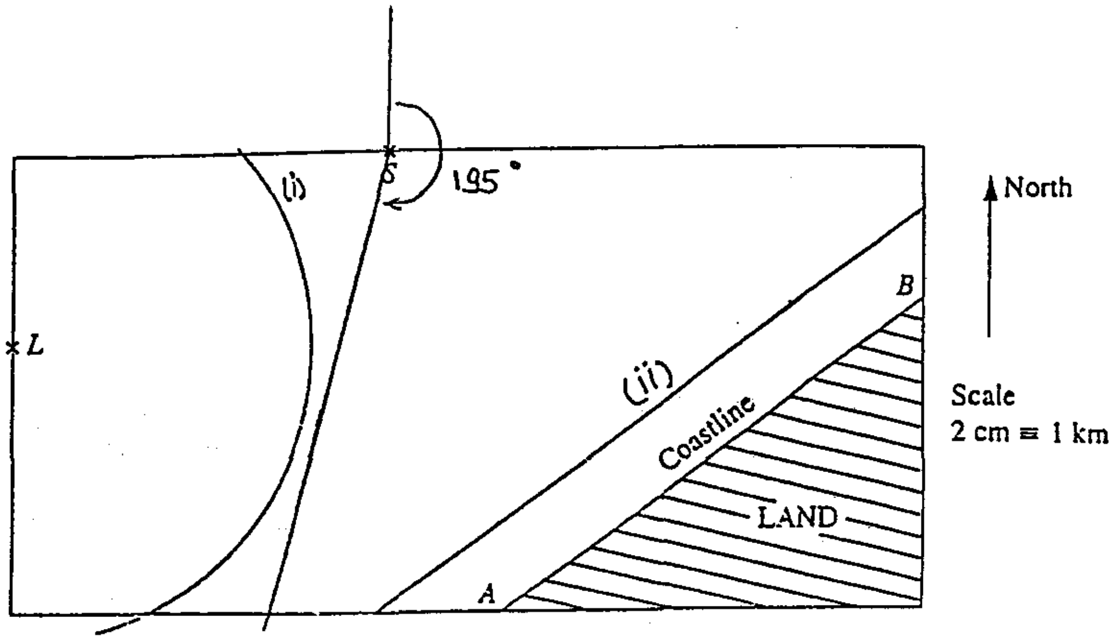
16. (a) $\dfrac{15}{5+20+15} \times 360° = \dfrac{15}{40} \times 360 = 135°$

(b)

Height & width	$0 < h \leq 5$ 5	$5 < h \leq 15$ $15 - 5 = 10$	$15 < h \leq 30$ $30 - 15 = 15$
Frequency	5	20	15
Frequency density	$\frac{5}{5} = 1$	$\frac{20}{10} = 1$	$\frac{15}{15} = 1$



17. (a)



(b) (ii) Yes.

18. (a) $f(x) = 7$

$$\frac{2(x-5)}{3} = 7$$

$$2x - 10 = 3 \times 7 = 21$$

$$2x = 21 + 10 = 31$$

$$x = \frac{31}{2} = 15.5$$

(b) $f^{-1}(x)$

x → $\boxed{-5}$ → $\boxed{\times 2}$ → $\boxed{\div 3}$ → $f(x)$

$f^{-1}(x)$ ← $\boxed{+5}$ ← $\boxed{\div 2}$ ← $\boxed{\times 3}$ ← x

$$f^{-1}(x) = \frac{3x}{2} + 5 = \frac{3x + 10}{2}$$

OR $y = \frac{2(x-5)}{3}$

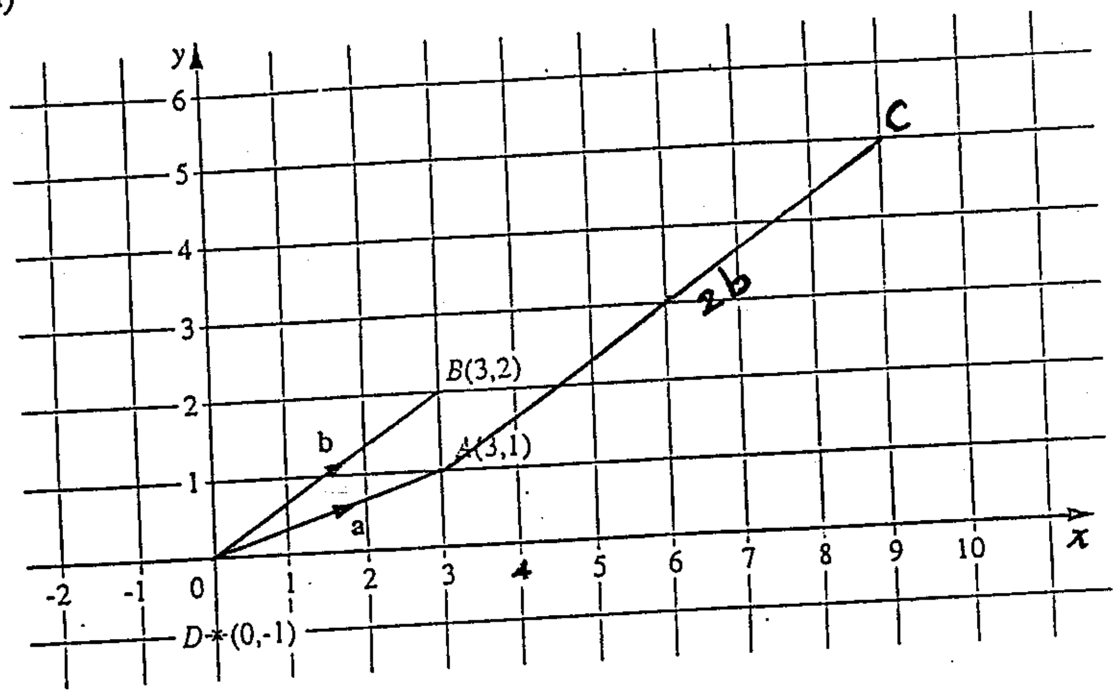
$3y = 2x - 10$

$2x = 3y + 10$

$x = \frac{3y+10}{2}$

$f^{-1}(x) = \frac{3x+10}{2}$

19. (a)



(b) $\overrightarrow{OD} = \overrightarrow{BA} = a - b$

(c) $|a| = \sqrt{3^2+1^2} = \sqrt{10} = 3.16$

20. (a) total surface area to be painted

$= (30 + 20) \times 2 \times 4 + 30 \times 20 - 200$

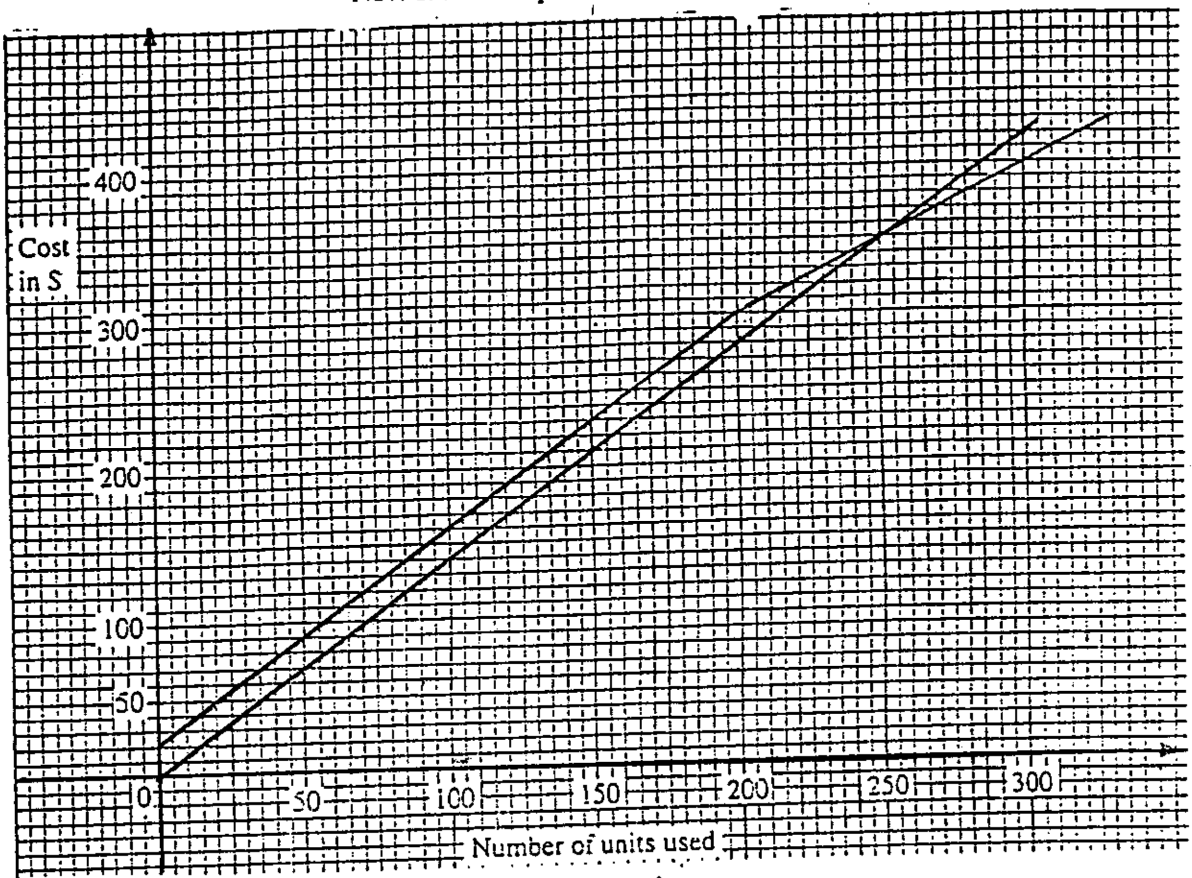
$= 400 + 600 - 200 = 800 \text{ m}^2$

(b) Number of Litres $= \frac{800}{18} = 44.44$

Nuber of tins $= \frac{44.44}{5} = 8.89$

Number of tins required = 9

21.



(a) C = 20 intersection with Cost axis

$$m = \frac{300-20}{200} = \frac{280}{200} = 1.4 \quad \text{(gradient)}$$

(b) for 100 units cost is $ 140
for 200 units cost is $ 280
then join the points with the origin as shown above.

(c) point of intersection of the two lines at 250 units.

22. (a) acceleration $= \frac{\text{change in velocity}}{\text{time}} = \frac{20}{20} = 1\text{m}/\text{s}^2$

(b) distance = area under the line (from $t = 50$ to $t = 60$)
$= \frac{1}{2} \times 10 \times 20 = 100$ m

(c) total distance = total area of trapezium $= \frac{30+60}{2} \times 20 = 900$ m

(d) Average speed $= \frac{900}{60} = 15$ m/s

June 1995

Paper 2

1. $\dfrac{3+\sqrt{3}}{2.9} = 1.63174 = 1.632$ (3 d.p)

2. (a) 12 000 000 miles per minute $= \dfrac{12\,000\,000}{60}$ miles per sec.

   $= 200\,000$ miles per sec.

   error $= 200\,000 - 186\,000 = 14\,000$ miles per sec.

   (b) percentage error $= \dfrac{14\,000}{186\,000} \times 100 = 7.53\ \%$

3. $300 \times 4\frac{1}{2} \times 60 = 81\,000$

   $= 8.1 \times 10^4$

4. interior angle $= 156°$

   exterior angle $= 180° - 156° = 24°$

   number of sides $= \dfrac{360}{24} = 15$ sides

5.

1994	increase	1995
100	5	105
?		840

   Answer $= \dfrac{100 \times 840}{105}$ 800

6. (a) 3 divisions out of 20 $= \dfrac{3}{20}$

   (b) $\dfrac{3}{4} \times 40 = 30$ Litres

   $\dfrac{3}{20} \times 40 = 6$ Litres

   Litres to be added $= 30 - 6 = 24$ Litres

7. (a) $\angle ADB = \frac{114}{2} = 57°$

(b) $\angle OAC = \angle OBC = 90°$

$\therefore \angle ACB = 360 - (114 + 90 + 90) = 66°$

(c) $\angle BAC = \angle ADB = 57°$

or $\angle BAC = \frac{180 - 66}{2} = 57°$

8. (a) $\frac{1}{2}(p + q) = \frac{1}{2}\overline{OR} = \overline{OM}$

(b) $q - p = \overline{PQ}$

(c) $\frac{1}{2}(p - q) = \frac{1}{2}(\overline{QP}) = \overline{QM}$

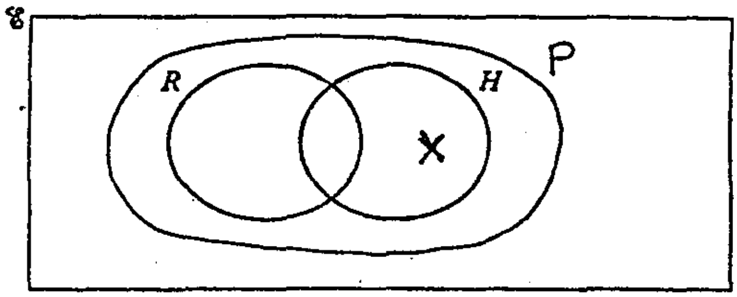
9. (a) $\left(\frac{8}{3}\right)^{-2} = \left(\frac{3}{8}\right)^{2} = \frac{9}{64}$

(b) $\left(27x^{27}\right)^{\frac{1}{3}} = \sqrt[3]{27}\; x^{\frac{27}{3}} = 3x^9$

10. (a) Squares.

(b)

(c)



11. (a) $2.5 \le c < 3.5$

(b) Maximum number $= \frac{100}{2.5} = 40$

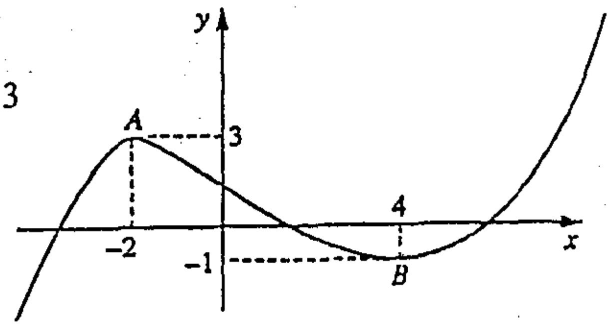
Minimum number $= \frac{100}{3.5} = 28.57$

Minimum number of complete revolutions = 28

12. (a) $-2 < x < 4$

(b) K less than -1 or more than 3

i.e. $K < -1$ or $K > 3$



13. (a) $\frac{1000}{1.48} = £\,675.68$ (2 d.p.)

(b) $675.68 - 400 = £\,275.68$

$275.68 \times 1.56 = \$\,430.06$

Percentage Left $= \frac{430.06}{1000} = 43\ \%$

14. (a) Prism

(b) Volume = Cross sectional area x Length

$= (\frac{1}{2} \times 3 \times 4) \times 10$

$= 60\ \text{cm}^3$

15. (a) $a = \frac{k}{r}$

(b) $a = \frac{k}{r}$ $\quad 2 = \frac{k}{24} \Rightarrow k = 48$

$a = \frac{48}{r}$

$10 = \frac{48}{r} \Rightarrow r = 4.8$

or $a_1 r_1 = a_2 r_2$ inversely proportional

$2 \times 24 = 10 \times r$

$r = \frac{48}{10} = 4.8$

16. $V = 2K + \frac{h^2}{5}$

$V - 2K = \frac{h^2}{5}$

$h^2 = 5(V - 2K)$ or $5V - 10K$

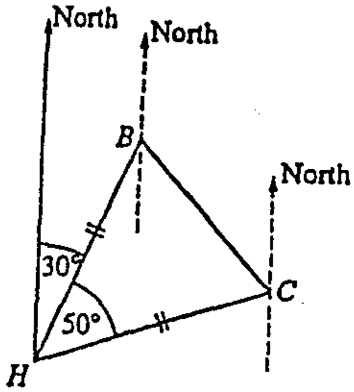
$h = \sqrt{5(V-2K)}$ or $\sqrt{5V - 10K}$

OR

h → [square] → [÷ 5] → [add 2k] →

← h ← [sq. root] ← [x5] ← [Subt. 2k]

$h = \sqrt{5(V-2K)}$



17. (a) Bearing of C from H
$= 30 + 50 = 80$

Bearing of H from C
$= 180 + 80 = 260°$

(b) $< HBC = \frac{180 - 50}{2} = 65°$
Bearing of C from B $= 360 - (180 - 30) - 65 = 145°$

18. (a) AC

(b) $(1 \quad 2)\begin{pmatrix} -3 \\ 4 \end{pmatrix} = (5)$

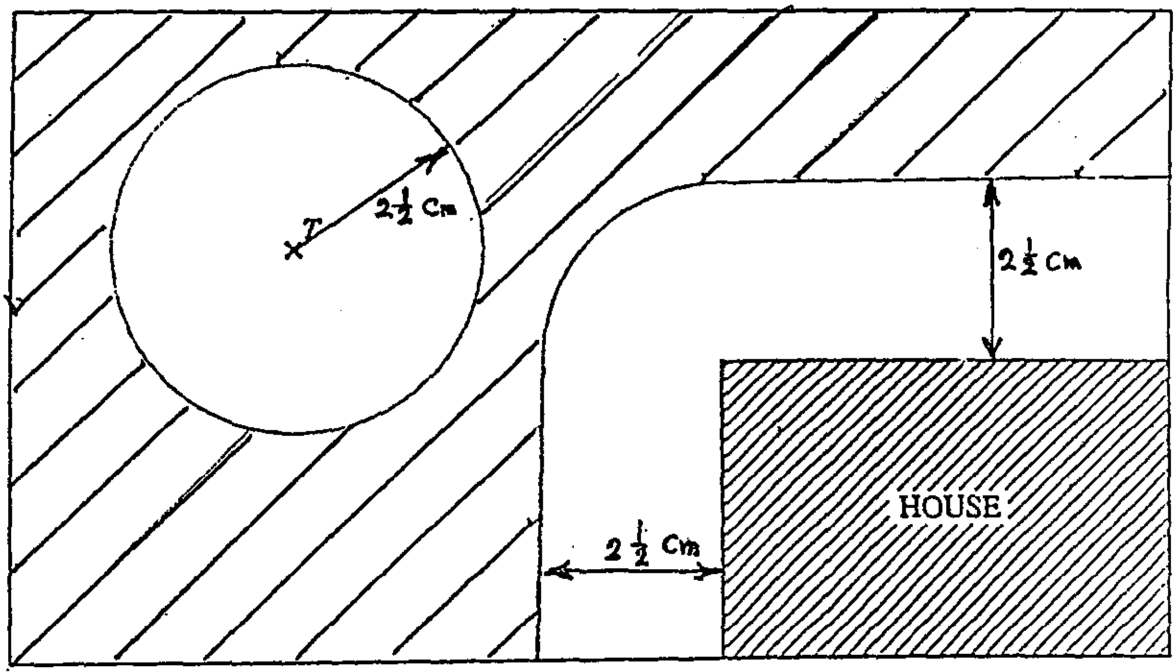
(c) $C^{-1} = \frac{1}{-2x6 + 3x5}\begin{pmatrix} 6 & -5 \\ 3 & -2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 6 & -5 \\ 3 & -2 \end{pmatrix}$

19. $< ABC = 80 - 59 = 21°$
Using sine rule
$$\frac{24}{\text{Sin } 21°} = \frac{AB}{\text{Sin } 100°}$$

$AB = 65.95 = 66.0$ m

20. (a) $< ACB = 90°$
(b) (i) $< BCF = 180 - x$
(ii) $< ACF = 180 - y$
(c) $180 - x + 180 - y + 90 = 360$
$180 + 180 + 90 - 360 = x + y$
$x + y = 90$

21.



22. (i) $x \geq 2$
(ii) $x + y < 6$
(iii) $y \geq \frac{1}{2} x$

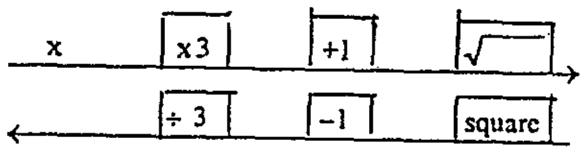
23. $f(x) = \sqrt{3x+1}$

(a) $f\left(3\frac{3}{4}\right) = 3.5$

(b) $f(x) = 5$

$$\sqrt{3x+1} = 5$$
$$3x + 1 = 5^2 = 25$$
$$3x = 24$$
$$x = 8$$

(c) $f^{-1}(x)$



$$f^{-1}(x) = \frac{x^2 - 1}{3}$$

or $y = \sqrt{3x+1}$ $y^2 = 3x + 1$

$3x = y^2 - 1$ $x = \frac{y^2 - 1}{3}$

$$f^{-1}(x) = \frac{x^2 - 1}{3}$$

Nov. 1995

Paper 2

1. $\text{Average} = \dfrac{1-2-4-5+0+2+1}{7} = -1$

   Answer: -1 °C.

2. *Answer (a)* Prism
   Answer (b) 9

3. $3x^2 - 7x + 2 = (3x-1)(x-2)$

   Answer $(3x-1)(x-2)$

4. 6 : 5
   ? : 4.5

   $\dfrac{4.5 \times 6}{5} = 5.4$ *Answer* 5.4 kg

5. 01 10 = 25 10

   25 10 − 7.5 is done by the calculator as follows :

   25 [°′″] 10 [°′″] [-] 7.5 [=] [SHIFT] [°′″] 17 40 0

   . *Answer* is 17 40

6. (a) $(0.2)^2 = 4 \times 10^{-2}$,

   (b) $\frac{37}{73} < 0.507$.

7. $\frac{5}{6}\left(\frac{1}{4}+\frac{1}{8}\right) = \frac{5}{6}\left(\frac{2}{8}+\frac{1}{8}\right) = \frac{5}{6} \times \frac{3}{8} = \frac{5}{16}$

   Answer $\frac{5}{16}$

8. The three sides of the triangle a, b and c

$10.5 \le a < 11.5$ perimeter P

$12.5 \le b < 13.5$ $10.5 + 12.5 + 13.5 \le P < 11.5 + 13.5 + 14.5$

$13.5 \le c < 14.5$ $26.5 \le P < 29.5$

Answer $36.5 \le p < 39.5$

9.
$$3x + 4y = 0 \ (x - 3)$$
$$-9x - 12y = 0$$
$$\underline{9x + 10y = -1}$$
$$-2y = -1$$
$$y = \tfrac{1}{2}$$

$$3x + 4x\,(\tfrac{1}{2}) = 0$$
$$3x = -2$$
$$x = -\tfrac{2}{3}$$

Answer $x = -\frac{2}{3}$

$y = \frac{1}{2}$

10.

Dutch Guilders		Swiss Francs
100		81.20
9.80		?
$\frac{81.20 \times 9.80}{100}$	=	7.9576 = 7.95 to the nearest 0.05

Answer 7.95 Swiss Francs

11.

Cost price	Profit	Selling price
100	20	120
?		684

Cost price = 570

Answer \$ 570

12. (a) $(81)^{\frac{3}{4}} = \left(3^4\right)^{\frac{3}{4}} = 3^3 = 27$

(b) $$\frac{3x^{-\frac{2}{3}}}{6x^{\frac{1}{3}}} = \frac{1}{2}x^{-\frac{2}{3}-\frac{1}{3}} = \frac{1}{2}x^{-1} = \frac{1}{2x}$$

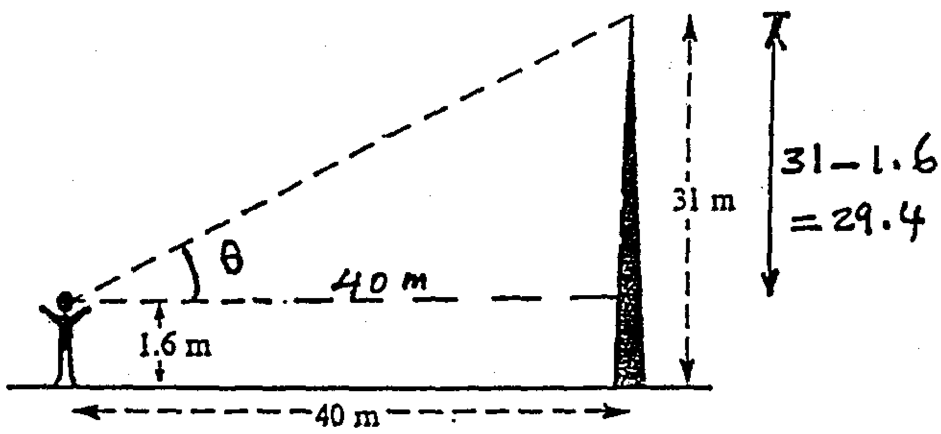
Answer $\frac{1}{2x}$

13. *Answer (a)* Isosceles
Answer (b) Rhombus
Answer (c) Parallelogram

14. $\mathcal{E} = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{3, 6, 9\}$
$B = \{2, 3, 5, 7\}$
$A \cap B = \{3\}$
$A \cup B = \{2, 3, 5, 6, 7, 9\}$
$(A \cup B)' = \{4, 8, 10\}$

Answer (a) $A \cap B = \{3\}$
Answer (b) $A \cup B = \{2, 3, 5, 6, 7, 9\}$
Answer (c) $(A \cup B)' = \{4, 8, 10\}$

15.



$$\tan\theta = \frac{29.4}{40} = 0.735$$
$$\theta = 36.3°$$

Answer 36.3°

16. (a) $f(x) = \dfrac{x-2}{3}$

$f(-4) = \dfrac{-4-2}{3} = \dfrac{-6}{3} = -2$

Answer (a) -2

(b) Two possible methods

(1) $y = \dfrac{x-2}{3}$

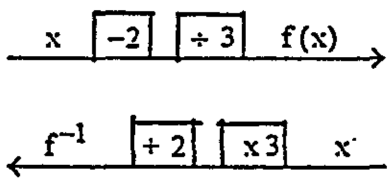
$3y = x - 2$

$3y + 2 = x$

$x = 3y + 2$

$\therefore\ f^{-1}(x) = 3x + 2$

(2) Using flow diagram



$f^{-1}(x) = 3x + 2$

Answer (b) $f^{-1} : x \mapsto 3x + 2$

17. Since the mode is 2 therefore, the largest frequency corresponds to Mark 2.

$\therefore$ x must be less than 10.

Since the median mark is 3 , then $x + 6 + 3$ must be at least one more than $1 + 3 + 10\ (= 14)$.

$9 + x \geq 15$

$x \geq 6$

possible values of x are 6, 7, 8, 9. *Answer* 6, 7, 8, 9.

18. (a) $x^2y = k$

$x = 3\ ,\ y = 10$

$3^2 \times 10 = k$

$k = 90$

When $x = 2$ $\quad 2^2 \times y = 90 \Rightarrow y = \frac{90}{4} = 22.5$ *Answer (a)* $y = 22.5$

(b) $x = 3$ decreased by 50 %.

new value of $x = \frac{50}{100} \times 3 = 1.5$

using $x^2 y = 90$

$(1.5)^2 y = 90 \Rightarrow y = 40$

increase in value of y is $40 - 10 = 30$

percentage increase $= \frac{30}{10} \times 100 = 300\ \%$

<u>OR</u> Let $x = 100$ and $y = 100$

$\therefore \quad k = x^2 y = 1000\,000$

now x is 50 find y

$(50)^2 \times y = 1000\,000 \quad \Rightarrow \quad y = 400$

i.e. y increased by 300 %

Answer (b) increased by 300 %.

19. (a) Scheme A Cost $= 15 + 0.60 \times 80 = \$\,63$

Scheme B Cost $= 2 + 0.80 \times 80 = \$\,66$

Difference $= 66 - 63 = \$\,3$

Answer (a) \$ 3

(b) (i) Scheme A $15 + 0.6\,x$

Scheme B $2 + 0.8\,x$

$15 + 0.6\,x = 2 + 0.8\,x$

Answer (b) (i) $15 + 0.6\,x = 2 + 0.8\,x$

(ii) $15 + 0.6\,x = 2 + 0.8\,x$

$15 - 2 = 0.8\,x - 0.6\,x$

$13 = 0.2\,x$

$x = \frac{13}{0.2} = 65$ units

Answer (b) ii) $x = 65$ units

20. (a) L_1 : $x = 7$

L_2 : through the origin gradient $= \frac{3}{6} = \frac{1}{2}$

equation is $y = \frac{1}{2}x$

L_3 : Through points (0, 5) , (10, 0)

gradient $= \dfrac{5-0}{0-10} = -\dfrac{1}{2}$ and $C = 5$

equation is $y = -\frac{1}{2}x + 5$

Answer (a) $L_1 : x = 7$
$L_2 : y = \frac{1}{2}x$
$L_3 : y = -\frac{1}{2}x + 5$

(b) *Answer (b)* $x \le 7$
$y \le \frac{1}{2}x$ or $2y \le x$
$y + \frac{1}{2}x \ge 5$ or $2y + x \ge 10$

21. *Answer (a)* With constant speed.
Answer (b) (i) A straight line graph.
(b) (ii) Deceleration = gradient

$$= \frac{10-0}{13-5} = \frac{10}{8}$$

$$= 1.25 \text{ m/s}^2$$

Answer (b) (ii) 1.25 m/s^2

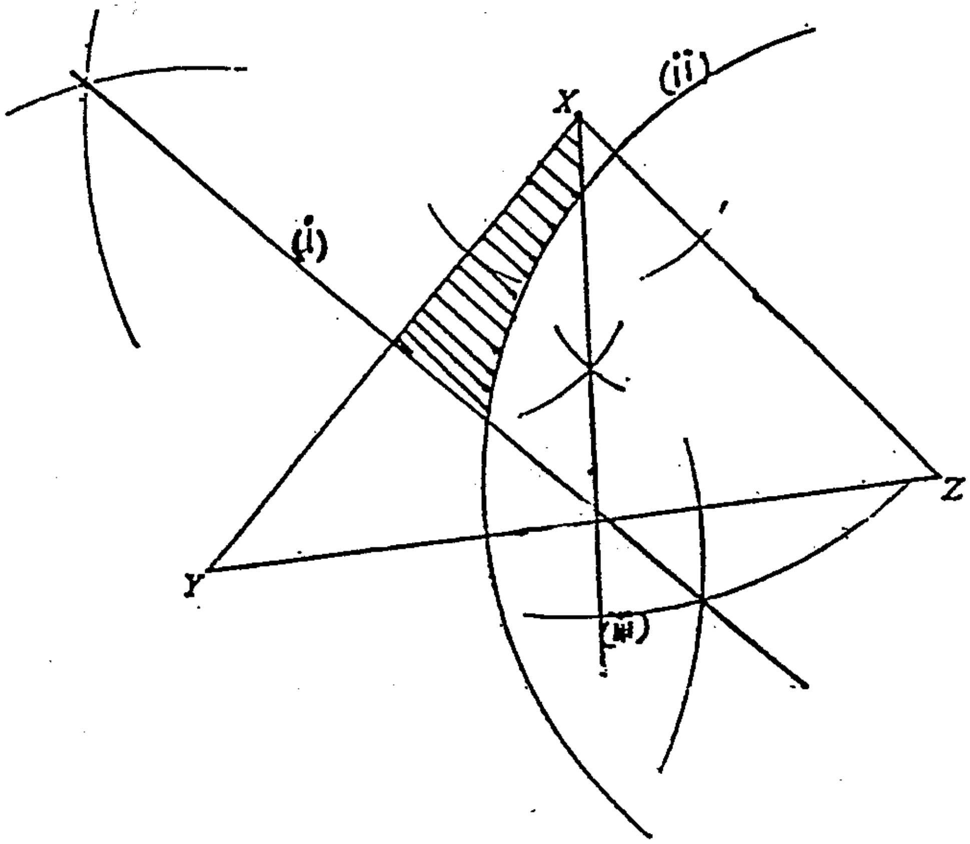
(c) Distance = area

$$= \frac{5+13}{2} \times 10$$

$$= 90 \text{ m}$$

Answer (c) 90 m

22.



June 1996

Paper 2

1. (a) $72 \times 365 \times 24 \times 60 = 37\ 843\ 200$

Answer (a) : 37 843 200

(b)

Answer (b) : 3.8×10^7

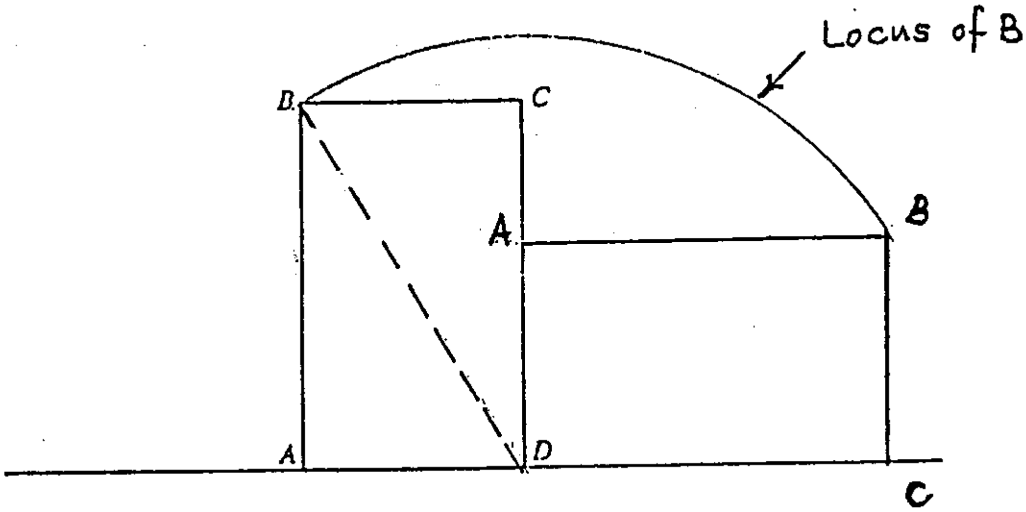
2. $3x + 5y = 21 - 17$

$8x = 4$

$x = \frac{4}{8} = \frac{1}{2}$

Answer $x = \frac{1}{2}$

3.



4. $\frac{3}{17} = 0.17647$

$\frac{39}{233} = 0.16738$

$\frac{1}{6} = 0.16667$

$\frac{85}{512} = 0.16602$

Answer (a) : Smallest is $\frac{85}{512}$

Answer (b) : Largest is $\frac{3}{17}$

5. (a) Using caculator

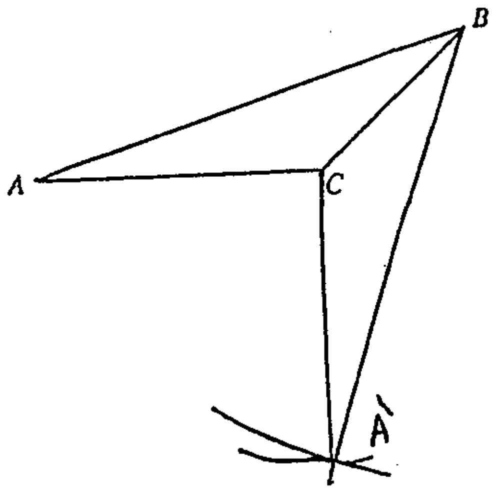
10 [0, , ,] 19 [0, , ,] – ½ = shift + [0, , ,] 0949

(b) Using caculator

11 [0, , ,] 5 [0, , ,] – 10 [0, , ,] 19 [0, , ,] = shift [0, , ,] 46 min

6. *Answer* Angle $ABC = 69°$

7.



8.

1994	increase	1995
100	20	120
?		150

Answer : $\dfrac{150 \times 100}{120} = 125$ kg

9. 1000 Swiss Francs = $1000 \times 1105 = 1105000$ lire

$1105000 - 716000 = 389000$ lire

389000 lire $= \dfrac{389000}{1105} = 352$ Swiss Francs

10. *Answer (a)* : Trapeziem.

Answer (b) : area $= 9 \times 5 = 45$ cm^2

11. (a) $(3x+2)(4x-3) = 12x^2 - 9x + 8x - 6$

Answer (a) $= 12x^2 - x - 6$

(b) $5x^2 - 31x + 6 = 0$

$(5x - 1)(x - 6 = 0$

Answer (b) : $x = \frac{1}{5}$ or $x = 6$

12. (a) A {2, 3, 5, 7}

B {3, 6}

C {2, 4, 8}

Answer (a) (i) : $A \cap C = \{2\}$

$A \cup B = \{2, 3, 5, 6, 7\}$

Answer (a) (ii) : $(A \cup B)' = \{4, 8\}$

(b) *Answer (b) :* $n(A) = 4$

13. $P = \dfrac{Q+3R}{T}$

$PT = Q + 3R$

$PT - Q = 3R$

$R = \dfrac{PT - Q}{3}$

14. (a) real length $= 10 \times 50000$ cm $= 500000$ cm $= \dfrac{500000}{100 \times 1000} = 5$ km

(b) $\dfrac{\text{Actual area}}{\text{Map area}} = (\text{Scale})^2$

$$\frac{\text{Actual area}}{6} = (50000)^2$$

$$\text{Actual area} = 6 \times (50000)^2$$

$$= 15000000000 cm^2$$

$$= \frac{15000000000}{100 \times 100 \times 10000}$$

$$= 150 \text{ hectars}$$

15. Sum of all interior angles = $(2n - 4) \times 90$
$= (2 \times 6 - 4) \times 90$
$= 720$

$< CDE = \dfrac{720 - 160}{5} = 112°$

16. (a) $8x^{\frac{1}{3}} = 8\,(27)^{\frac{1}{3}} = 8 \times 3 = 24$

(b) $\left(\frac{y}{z}\right)^{-2} = \left(\frac{z}{y}\right)^2 = \left(\frac{2}{1/3}\right)^2 = 6^2 = 36$

(c) $(xy)^0 = 1$

17. (a) $x = 360 - (120 + 135) = 105°$

(b) $\dfrac{135}{360} = \dfrac{3}{8}$

(c)

	A	B
	120°	135°
	720	?

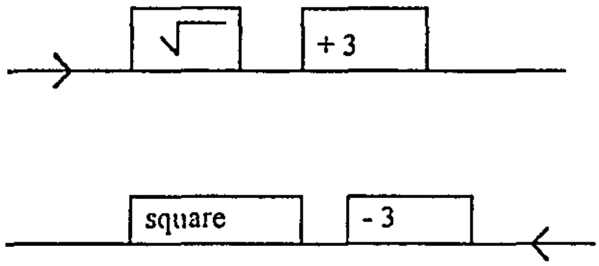
number voted for B $= \dfrac{135 \times 720}{120} = 810$

18. (a) Cos. $< VAO = \dfrac{5}{13}$
$< VAO = 67.4°$

(b) Curved surface Area $= \pi r l = 3.142 \times 5 \times 13$
$= 204.23 = 204 \text{ cm}^2$

19. (a) $3 + \sqrt{x} = 7$ $\qquad \sqrt{x} = 7 - 3 = 4$
$x = 4^2 = 16$

(b)



$f^{-1}(x) = (x-3)^2$

<u>OR</u>

$y = 3 + \sqrt{x}$

$\sqrt{x} = y - 3$

$x = (y-3)^2$

$f^{-1}(x) = (x-3)^2$

20.

(a) $m + 2n = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

(b) $\vec{OQ} = \vec{OP} + \vec{PQ}$

$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

(c) $|m| = \sqrt{2^2 + 3^2} = \sqrt{13} = 3.61$

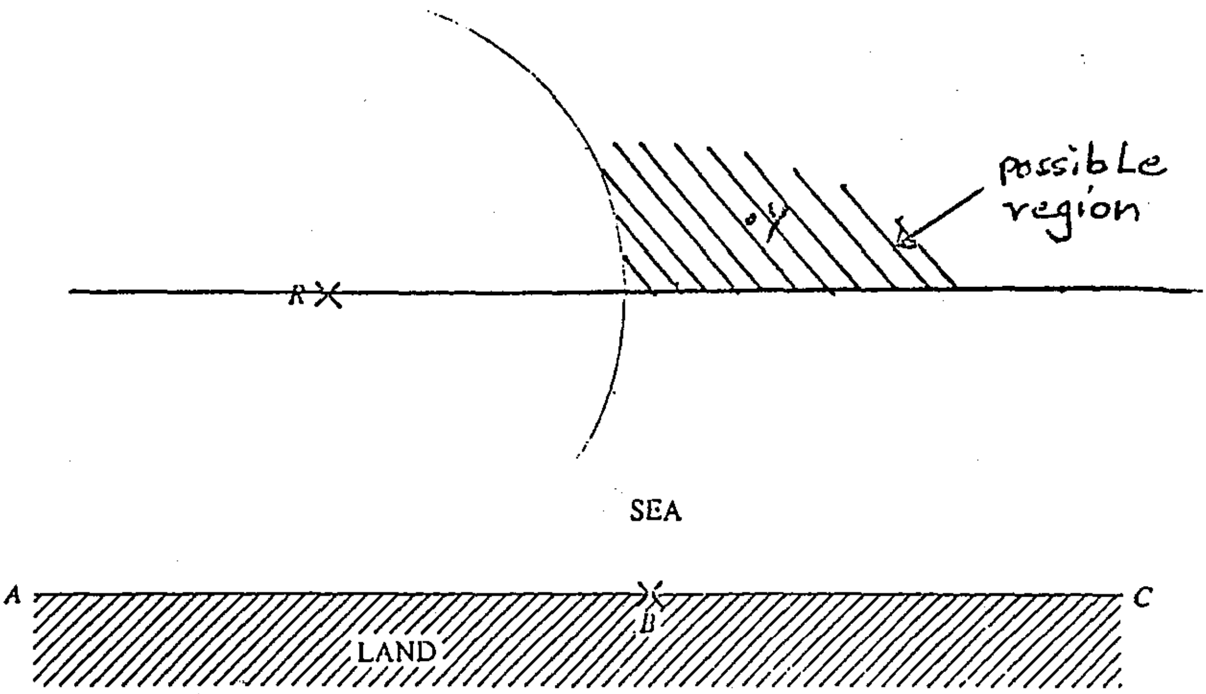
21. (a) *Answer (a) :* $B\hat{D}O = 20$

(b) *Answer (b) :* $B\hat{D}A = 90°$

(c) *Answer (c) :* $O\hat{A}D = 70°$

(d) *Answer (d) :* $B\hat{C}D = 110°$

22.



23. (a) (i) acceleration $= \dfrac{7}{4} = 1.75$ m/s

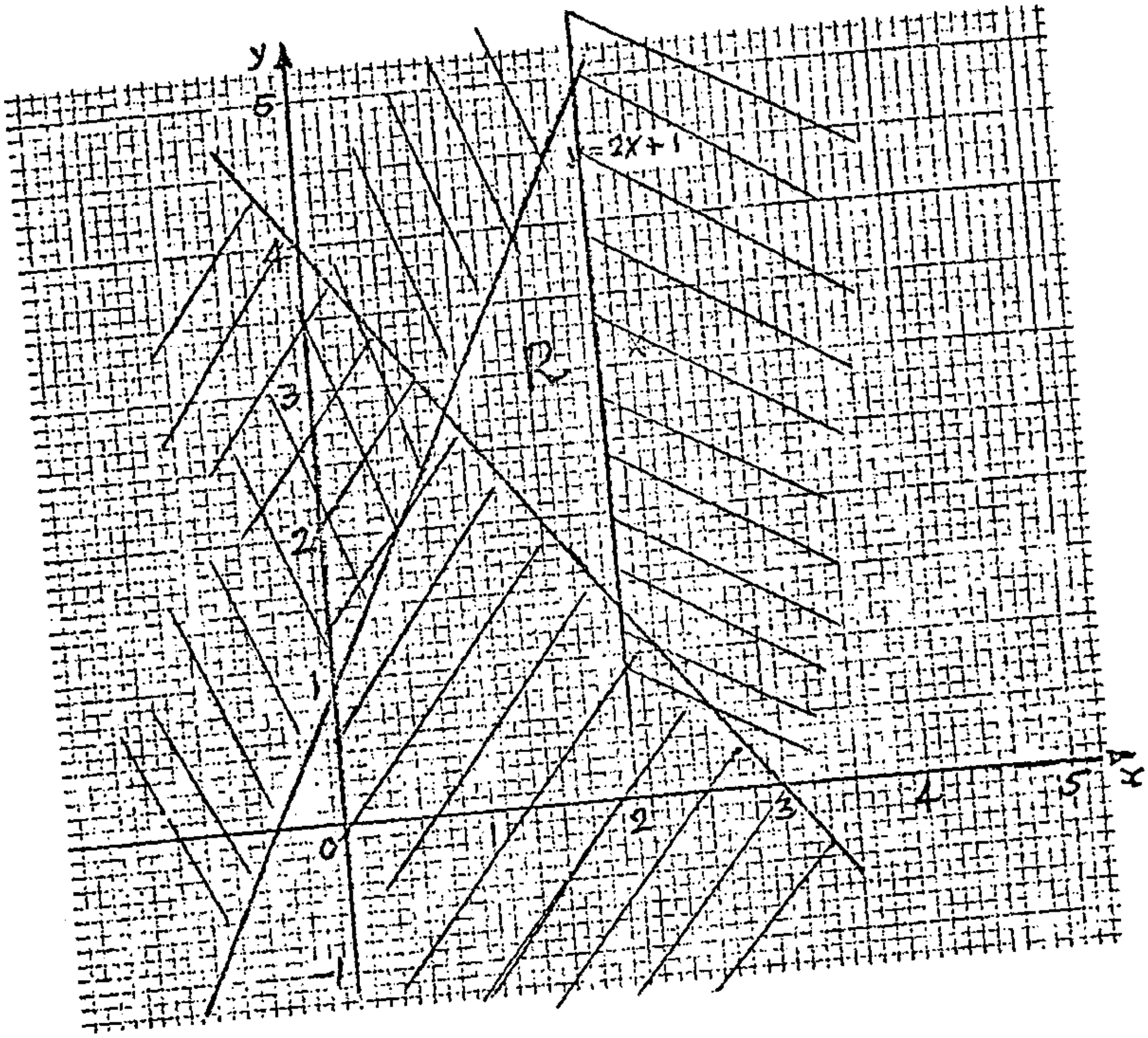
(ii) acceleration = zero

(b) Distance = area $= \dfrac{12+16}{2} \times 7 = 98$ m

(c) Distance is rough by equal the area of a triangle base (24 − 16) and height 7

distance $= \frac{1}{2} \times 8 \times 7 = 28$ m.

24.



(a) 3y + 4x = 12
x = 0
y = 0

y = 4
x = 3

(014)
(310)

(b) Solution is the point intersection i.e. x = 0.9 , y = 2.8

Nov. 96

Paper 2

1.

grams	calories
84	60
98	?

$$\frac{98 \times 60}{84} = 70 \text{ calories}$$

2. $3 - 4x < 11$
   $-4x < 11 - 3$
   $-4x < 8$
   $x > \frac{8}{-4}$
   $x > -2$

3. (a) $12 \times 1\frac{3}{4} = 21$ pints

   (b) $8 \div 1\frac{3}{4} = 4\frac{4}{7} = 4.57$ Litres

4. Translation to the right (parallel to x axis) of magnitude 4.

5. $y = k x^n$
   $x = 1 \quad y = 0.5$
   $0.5 = k(1)^n = k$
   $k = 0.5$
   $y = 0.5\, x^n$
   $n = 3$

   $x = 2 \quad y = 4$
   $4 = 0.5(2)^n$.
   $\frac{4}{0.5} = 2^n$
   $8 = 2^n \Rightarrow n = 3$

6. $x(x+1) = 756$

$x^2 + x - 756 = 0$

$(x+28)(x-27) = 0$

$x = 27$

numbers are 27 & 28

<u>OR</u> Using calculator find $\sqrt{756} = 27.5$

multiply $27 \times 28 = 756$

7. (a)

cost	tax	bill
100	15	115
?		48.30

$$\text{Cost} = \frac{48.30 \times 100}{115} = \$\,42$$

(b) $\text{tip} = \frac{8}{100} \times 48.30 = 3.864 \cong \$\,4$

8. (a) $\text{speed} = \frac{207}{3 \times 60} = 1.15 \text{ m/min}$

(b) $\text{speed} = \frac{207 \times 100}{3 \times 60 \times 60} = 1.92 \text{ cm/s}$

9. (a) $27^{2/3} = 9$

(b) $x^{-3} = 8$

$\frac{1}{x^3} = 8$

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$

10. (a) $x^2 - (x-4)^2 = 112$

(b) $x^2 - (x^2 - 8x + 16) = 112$

$8x - 16 = 112$

$8x = 128$

$x = 16$

11. (a) 65 grams

(b) Least $M = 50 + 4 \times 65 = 310$
Greatest $M = 50 + 6 \times 75 = 500$
$310 \le M < 500$

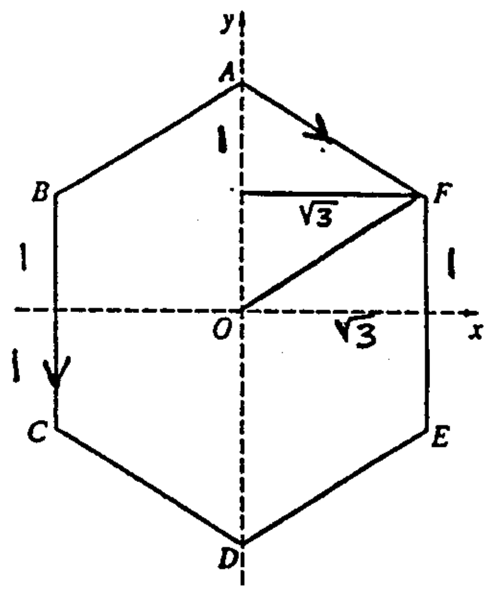
12. (a) has a rotational symmetry of order 4.

(b) $\frac{3}{4}$

13. (a) $|\bar{OF}| = \sqrt{(\sqrt{3})^2 + (1)^2}$
$= \sqrt{4} = 2$

(b) (i) $\bar{AF} = \begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}$

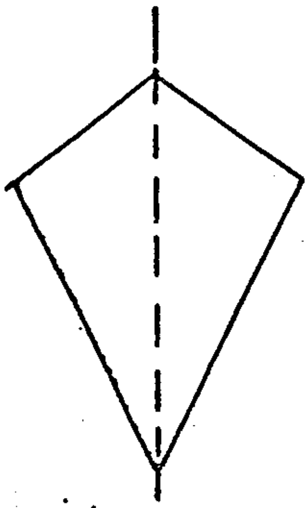
(ii) $\bar{BC} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$



14. (a) $E = mc^2 = 20 \times (3 \times 10^8)^2$
$E = 1.8 \times 10^{18}$

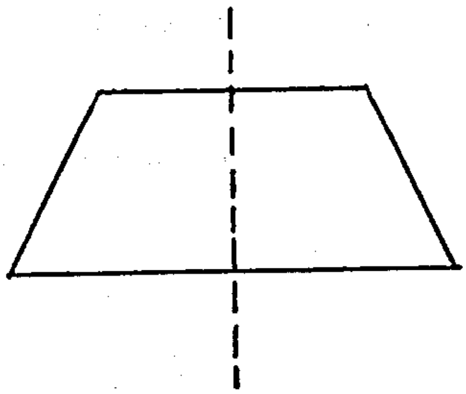
(b) $E = mc^2$
$C^2 = \frac{E}{m}$
$C = \sqrt{\frac{E}{m}}$

15. QUADRILATERAL *A*



Name: Kite

QUADRILATERAL *B*



Name: Trapezium

16. (a) $f\left(\frac{1}{6}\right) = 3\left(\frac{1}{6}\right)^2 - 3\left(\frac{1}{6}\right) + 1 = \frac{7}{12}$

(b) $f(1-x) = 3(1-x)^2 - 3(1-x) + 1$

$= 3(1-2x+x^2) - 3 + 3x + 1$

$= 3 - 6x + 3x^2 - 3 + 3x + 1$

$= 3x^2 - 3x + 1 = f(x)$

(c) $f\left(\frac{5}{6}\right) = f\left(\frac{1}{6}\right)$ (from (b)) $= \frac{7}{12}$

Or $f\left(\frac{5}{6}\right) = 3\left(\frac{5}{6}\right)^2 - 3\left(\frac{5}{6}\right) + 1 = \frac{7}{12}$

17. (a) (i) $\dfrac{ST}{\sin 33°} = \dfrac{5}{\sin 12°}$

$ST = \dfrac{5 \times \sin 33°}{\sin 12°} = 13.1$ km

(ii) Speed $= \dfrac{13.1}{(\frac{1}{2})} = 26.2$ km/h

(b) Bearing = 270 − 12 = 258°

18. Regular 24 sided polyzon

$$\text{exterior angle} = \frac{360}{24} = 15°$$

$$\text{interior angle} = 180 - 15 = 165°$$

Regular octagon

$$\text{exterior angle} = \frac{360}{8} = 45°$$

$$\text{interior angle} = 180 - 45 = 135°$$

Equilateral triangle

each angle = 60°

$$165° + 135° + 60° = 360°$$

Therefore, it fit together exactly at x.

19. (a) 3.5 min

(b) $\text{acceleration} = \dfrac{1.5}{0.5} = 3 \text{ km / min}^2$

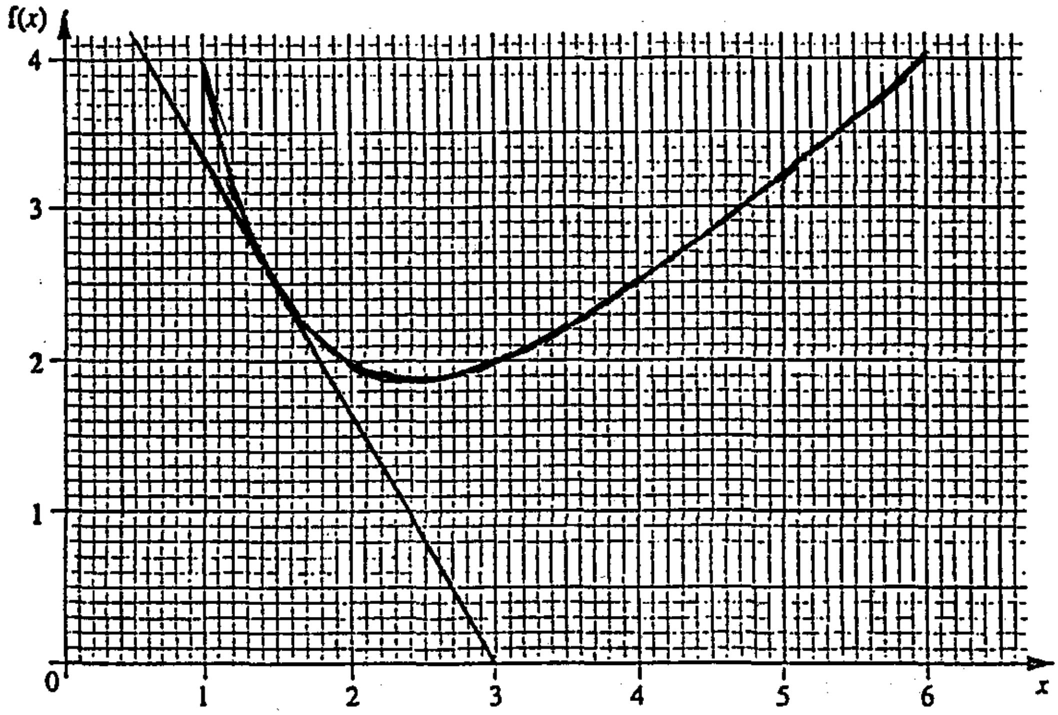
(c) (i) $\text{distance} = \text{Area} = \frac{1}{2} \times 0.5 \times 1.5 = 0.375 \text{ km}$

(ii) $\text{distance} = \text{total area} = \dfrac{3.5+5}{2} \times 1.5 = 6.375 \text{ km}$

20. (a)

x	1	1.2	1.5	2	3	4	5	6	*2.5*
f(x)	4 ·	3.2	2.5	2	2	*2.5*	*3.2*	*4*	*1.9*

(b)



(c) points on the tangent (1 , 3.3) and (3 , 0)

$$\text{gradient} = \frac{3.3-0}{1-3} = \frac{3.3}{-2} = -1.65$$

$$\cong -1.7$$

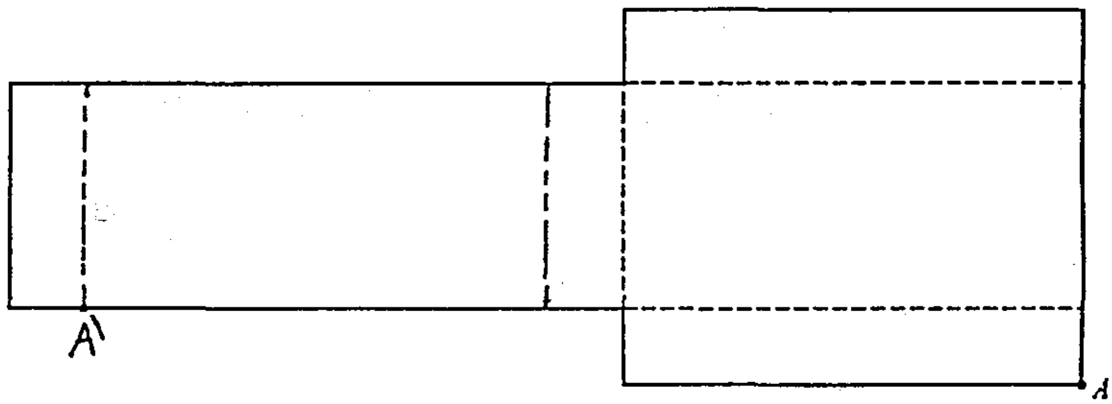
June 1997

Paper 2

1. New temperature = – 34.8 + 81.9 = 47.1°   *Answer* 47.1°C

2. Using calculator   8.347   *Answer* 8.347

3.



4. (a) $25 - \frac{1}{2} \times 5 = 22.5$ m ≤ length of the wall $< 25 + \frac{1}{2} \times 5 = 27.5$ m.

   (b) $2 - \frac{1}{2} \times 0.1 = 1.95$ m ≤ height of the wall $< 2 + \frac{1}{2} \times 0.1 = 2.05$ m.

5. $\frac{82}{99}$,   82%, $\sqrt{0.674}$

   0.828282   0.82   0.82097

   (a) *Answer (a)*   82%   < $\sqrt{0.674}$ < $\frac{82}{99}$

   (b) *Answer (b)*   0.0083 .

6. $3x + 4y = 3$
$x + 6y = 8 \quad (x - 3)$
$3x + 4y = 3$
$\underline{-3x - 18y = -24}$
$-14y = -21$
$y = 1.5$
$x + 6 \times 1.5 = 8$
$x + 9 = 8 \Rightarrow x = -1$

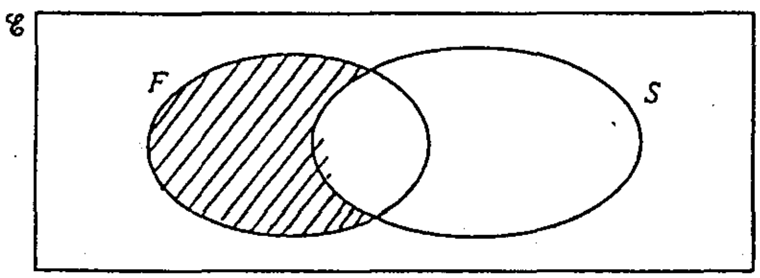
Answer $x = -1$
$y = 1.5$

7. (a) 135 000 miles per hour $= \dfrac{135000 \times 1580}{60 \times 60} = 59250$

Answer (a) 59250 m/s

(b) *Answer (b)* 5.925×10^4 m/s

8. (a)



(b) (i) $30 - 5 = 25$

Answer (b) (i) 25

(ii) $10 + 18 - 25 = 3$ study both.
Number study French but not Spanish $= 10 - 3 = 7$

Answer (b) (ii) 7

9. (a) 10000 francs $= \dfrac{10000}{5.05} = 1980.2$ \$
dollars spent $= 1980 - 190 = 1790$

Answer (a) \$ 1790.

(b) rate $= \dfrac{1000}{190} = 5.26$
\$ 1 = 5.26 francs

Answer (b) \$ 1 = 5.26 francs

10.

$$T = 3 + \frac{5}{v}$$

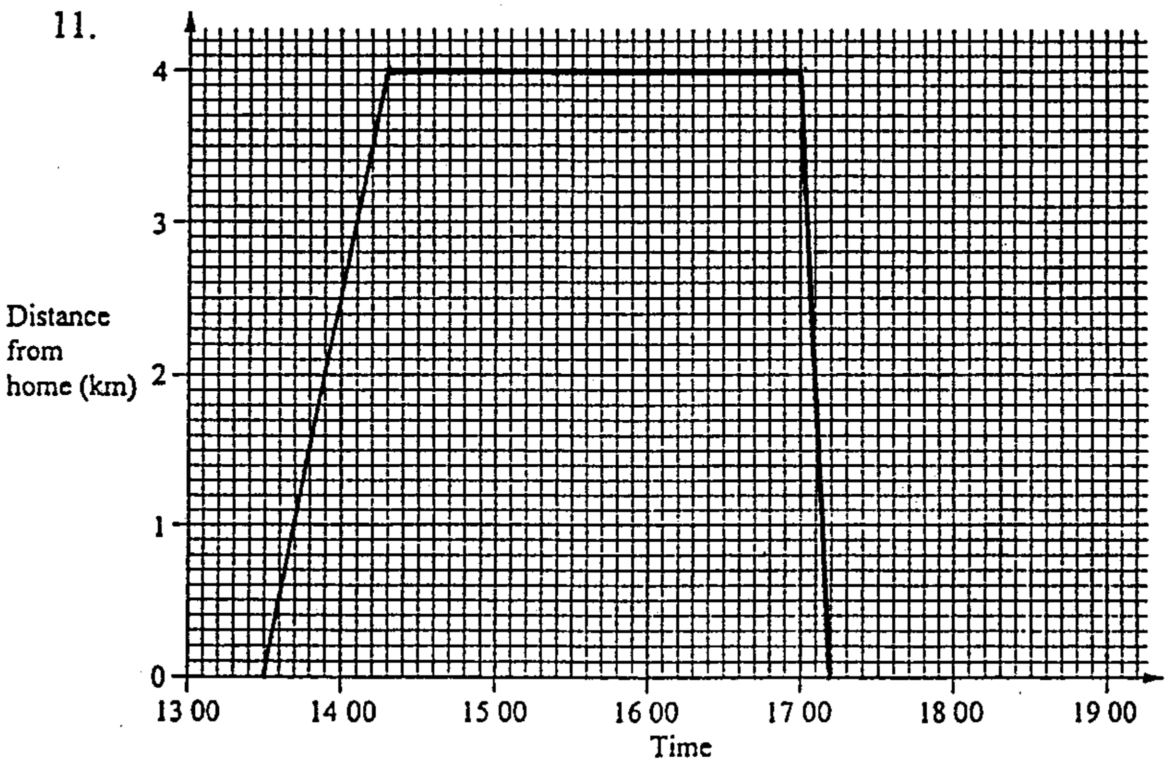
$$T - 3 = \frac{5}{v}$$

$$v(T-3) = 5$$

$$v = \frac{5}{T-3}$$

Answer $V = \dfrac{5}{T-3}$

11.



(a) $\frac{4}{20} = \frac{1}{5}$ h = 12 min.

time of arrival 17 12.

(b) 14 18 – 13 30 = 0 48 min = 0.8 h.

walking speed = $\frac{4}{0.8}$ = 5 km/h.

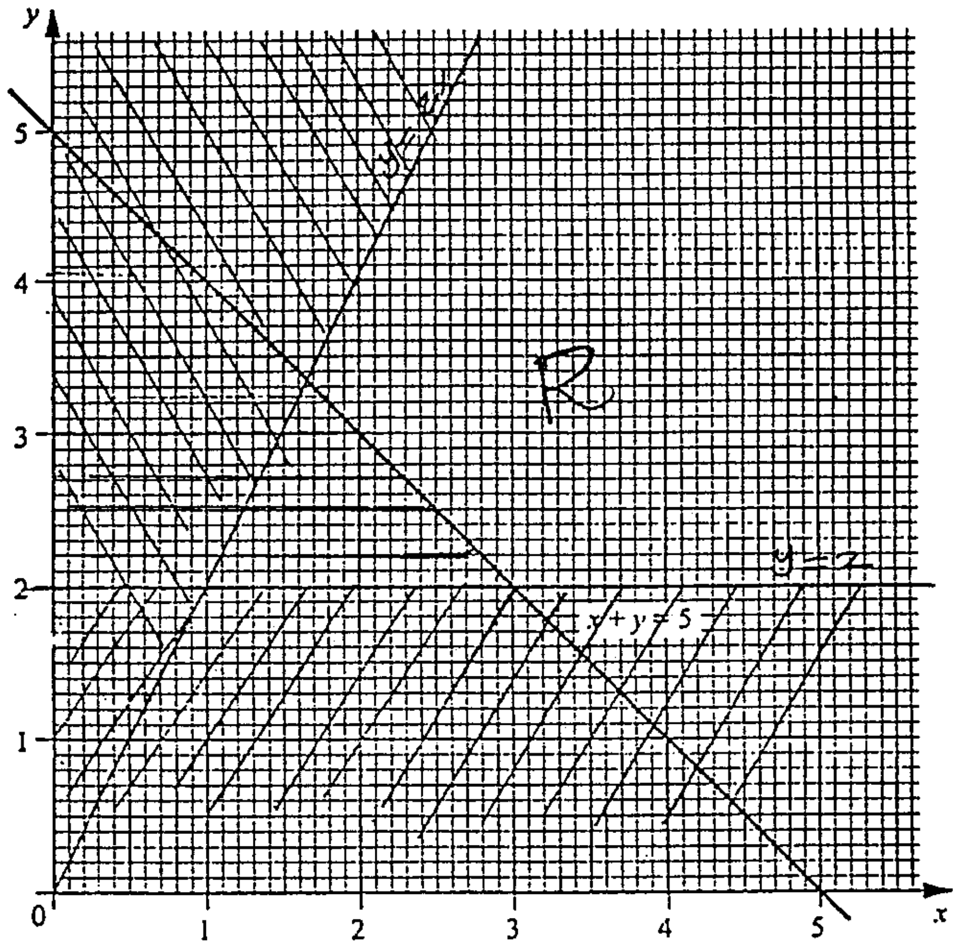
Answer (b) 5 km/h.

12. $\text{Cos } 35° = \frac{GA}{GP}$ $\text{Cos } 35° = \frac{21.3}{GP}$

$GP = 26$

Answer $GP = 26$

13.



14. (a) 5 : 4

? 48

New length $= 48 \times \frac{5}{4} = 60$ cm

3 : 4

36

New width $= \frac{36 \times 3}{4} = 27$ cm

Answer (a) length = 60 cm.

width = 27 cm.

(b) $\frac{\text{new area}}{\text{old area}} = \frac{60 \times 27}{48 \times 36}\ \frac{45}{48} = \frac{15}{16}$

15.(a) original reduction sale price

original	reduction	sale price
100	20	80
488		?

Sale price = $\frac{488 \times 80}{100} = 390.4$

original	reduction	sale price
1	$\frac{1}{3}$	$\frac{2}{3}$
579		?

Sale price = $597 \times \frac{2}{3} = 386$

Answer (a) \$ 390.4
\$ 386

(b)

Cost price	Profit	Selling price
100	52.5	152.5
?		488

Cost price = 320

Answer (b) \$ 320

16. $w = \frac{180 - 68}{2} = \frac{112}{2} = 56°$ *Answer* $w = 56°$
$x = B = 68°$ $x = 68°$
$y = 90 - x = 90 - 68 = 22°$ $y = 22°$
$z = 180 - 2 \times 68 = 44°$ $z = 44°$
Since ∠ TAD = ∠ TAO = 68°

17. (a) (i) $g(2) = 9 - 2 \times 2 = 5$
(ii) $fg(2) = f(5) = 5 \times 5 + 1 = 26$

(b) $gf(x) = 9 - 2(5x + 1) = 9 - 10x - 2 = 7 - 10x$

18. (a) Each exterior angle = $\frac{360}{10} = 36°$

Each interior angle = $180 - 36 = 144°$

(b) all interior angles = $(2 \times 10 - 4) \times 90 = 1440$

Less 7 angles each 156°

$1440 - 156 \times 7 = 348$

3 : 4 : 5 total 12

smallest angle = $\frac{3}{12} \times 348 = 87°$

19. (a) $AB = \sqrt{[3-(-4)]^2 + (2-26)^2} = \sqrt{7^2 + 24^2} = 25$

Answer (a) AB = 25

(b) Vectors $\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ vector $\overrightarrow{OB} = \begin{pmatrix} -4 \\ 26 \end{pmatrix}$

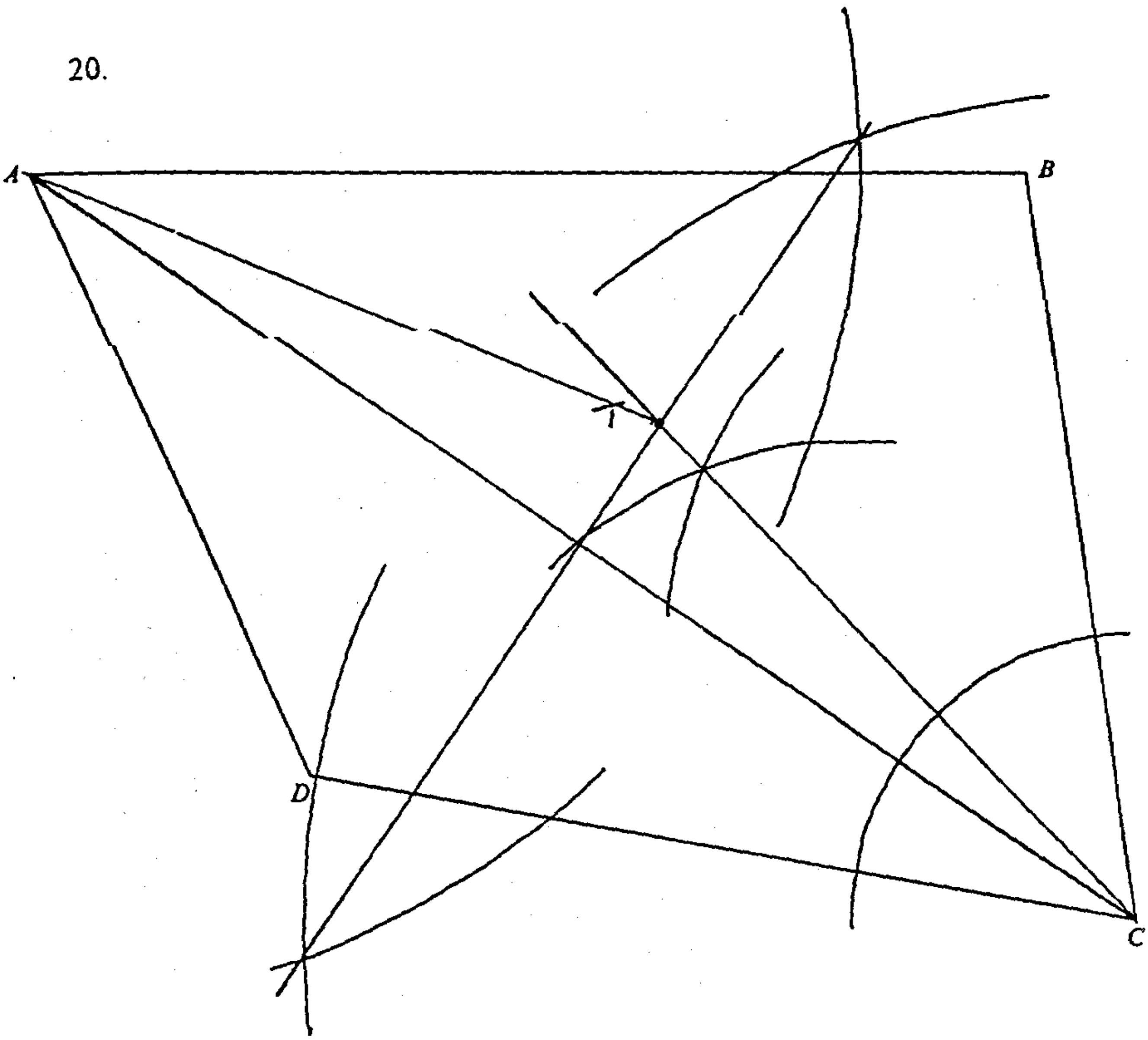
vector $AB = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -7 \\ 24 \end{pmatrix}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix} -7 \\ 24 \end{pmatrix} + \begin{pmatrix} 1 \\ -20 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

Answer (b) $\overrightarrow{AC} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

20.



Scale: 1 centimetre represents 2 metres

Distance TA = $10.6 \times 2 = 21.2$ m

21. (a) $\sin 34° = \frac{CB}{20}$

$CB = 20 \times \sin 34°$

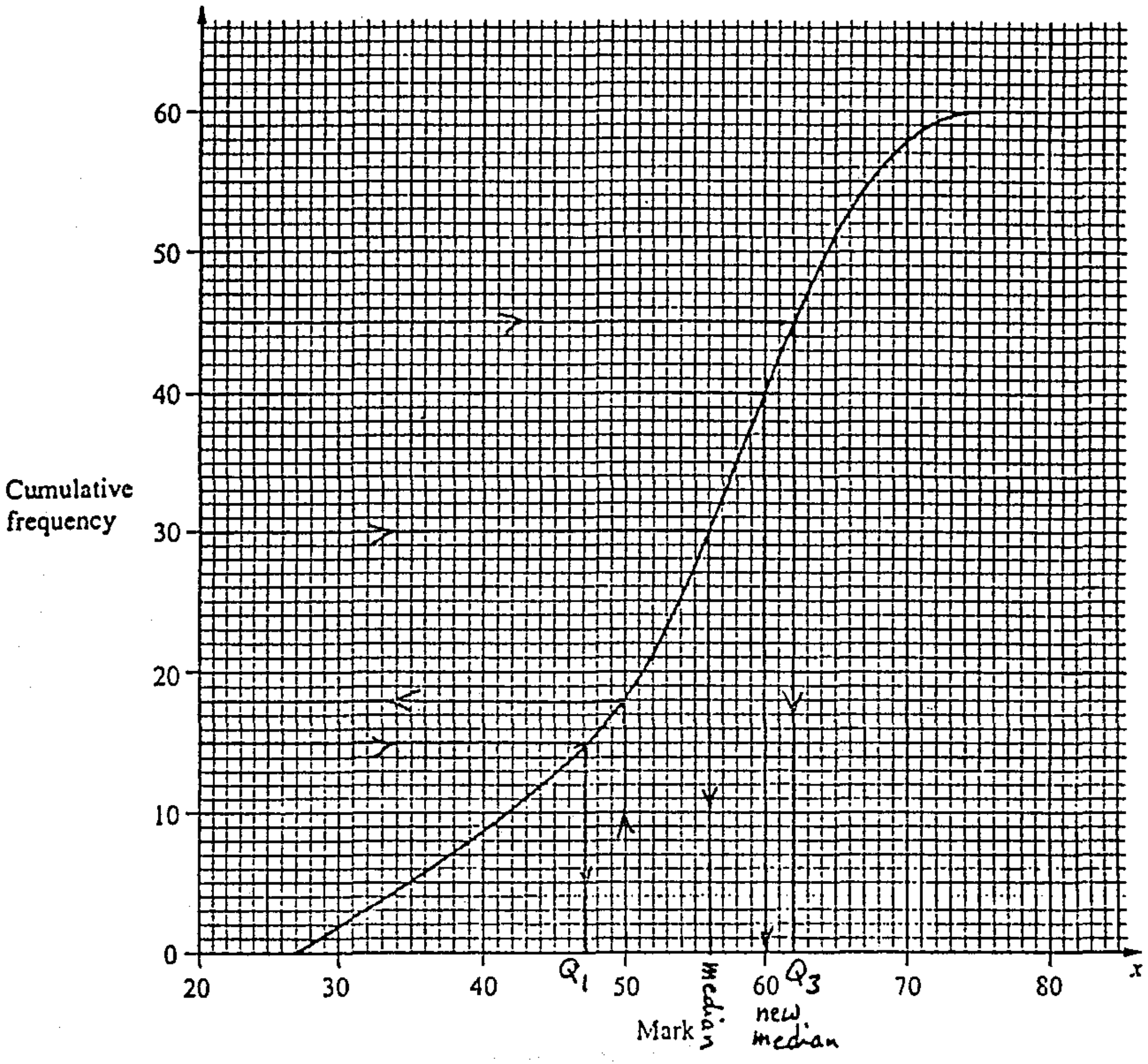
$= 11.2$ cm.

(b) $\angle COB = 2 \times 34 = 68°$

angle COB is twice angle CAB.

(c) Length of arc $= \frac{68}{360} \times 2 \times 3.142 \times 10 = 11.9$ cm.

22.



(a) (i) 56
(ii) 62
The lower quartile = 47
(iii) 62 – 47 = 15

(b) (i) 60
(ii) Number of candidates scoring less than 50 = 18

Percentage failed = $\frac{18}{80} \times 100 = 22.5\ \%$

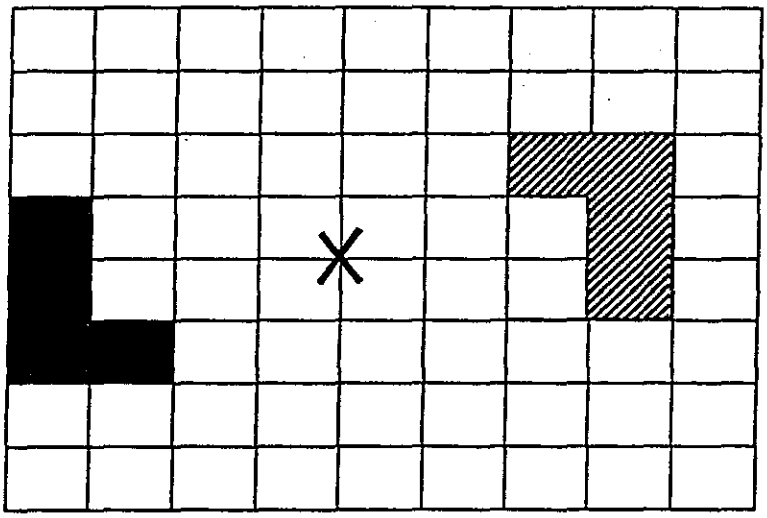
November 1997
Paper 2

1- $A = \{2,3,5,7,11,13,17,19\}$

$B = \{3,5,7,9,11,13,15,17,19\}$

Answer : $A \cap B = \{3,5,7,11,13,17,19\}$

2-



3- (a) $15 - (-1) = 16$

Answer (a) 16 C°

(b) The temperature decreased and then increased.

4- $X > 4$

$\frac{4}{X}$ is less than one (and positive i.e. > 0)

$\frac{X}{4}$ is more than one

$4 - X$ is negative.

Answer: $4\text{-}X < \frac{4}{X} < \frac{X}{4}$

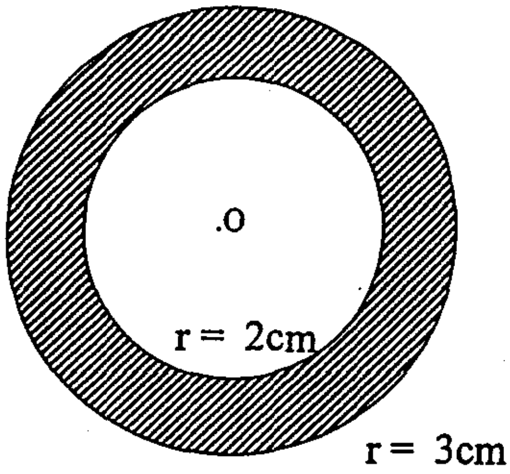
5- 5.5 d < 6.5

$r = \frac{d}{2}$

$\therefore \frac{5.5}{2} \le r < \frac{6.5}{2}$

Answer: 2.75 r < 3.25

6-



7- Amount received $= \frac{2000}{81.50} = 24.54$

Answer \$ 24.54

8- (a) $0.0013 = 1.3 \times 10^{-3}$

(b) $1.3 \times 10^{-3} \times 100 \times 100 \times 100 = 1.3 \times 10^{3} = 1300$

Answer: $1300 g \backslash m^3$

9- (a) $2x^2 \times 3x^3 = 6x^{2+3} = 6x^5$

Answer: (a) $6x^5$

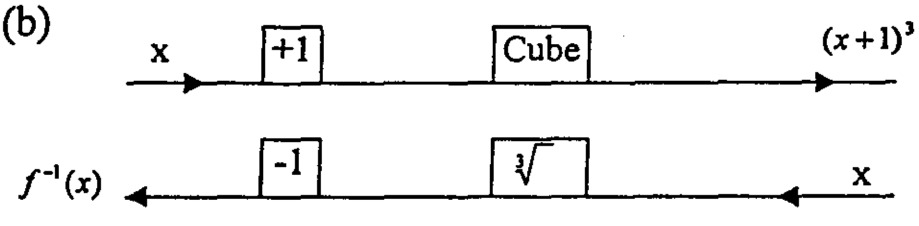
(b) $a^{\frac{5}{6}} \div a^{\frac{1}{2}} = a^{\frac{5}{6} - \frac{1}{2}} = a^{\frac{1}{3}}$

Answer: (b) $a^{\frac{1}{3}}$

10- (a) $f(-3) = (-3+1)^3 = (-2)^3 = -8$

Answer: (a) f(-3) = -8

(b)



Answer: (b) $f^{-1}(x) = \sqrt[3]{x} - 1$

11- (a) $\angle ACD = 25$ alternate angles.

(b) $\angle ABC = 90\text{-}25 = 65°$
(since $\angle C = 90°$)

(c) $\angle ABD = \angle ACD = 25$ same arc
Bearing of D from B is 025°
$\therefore$ Bearing of B from D = 180 + 25 = 205°

12- (a) $\tan A = \frac{5}{12}$

(b) $$\frac{2\tan A}{(1-\tan A)(1+\tan A)} = \frac{2\tan A}{1-(\tan A)^2} = \frac{2\times\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} = \frac{\frac{10}{12}}{1-\frac{25}{144}} = \frac{10/12}{199/144} = \frac{10}{12}\times\frac{144}{119} = \frac{120}{119}$$

Answer: (b) $\frac{120}{119}$

13- Similar figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{\left(\frac{1}{4}\right)}{A_2} = \left(\frac{80}{120}\right)^2$$

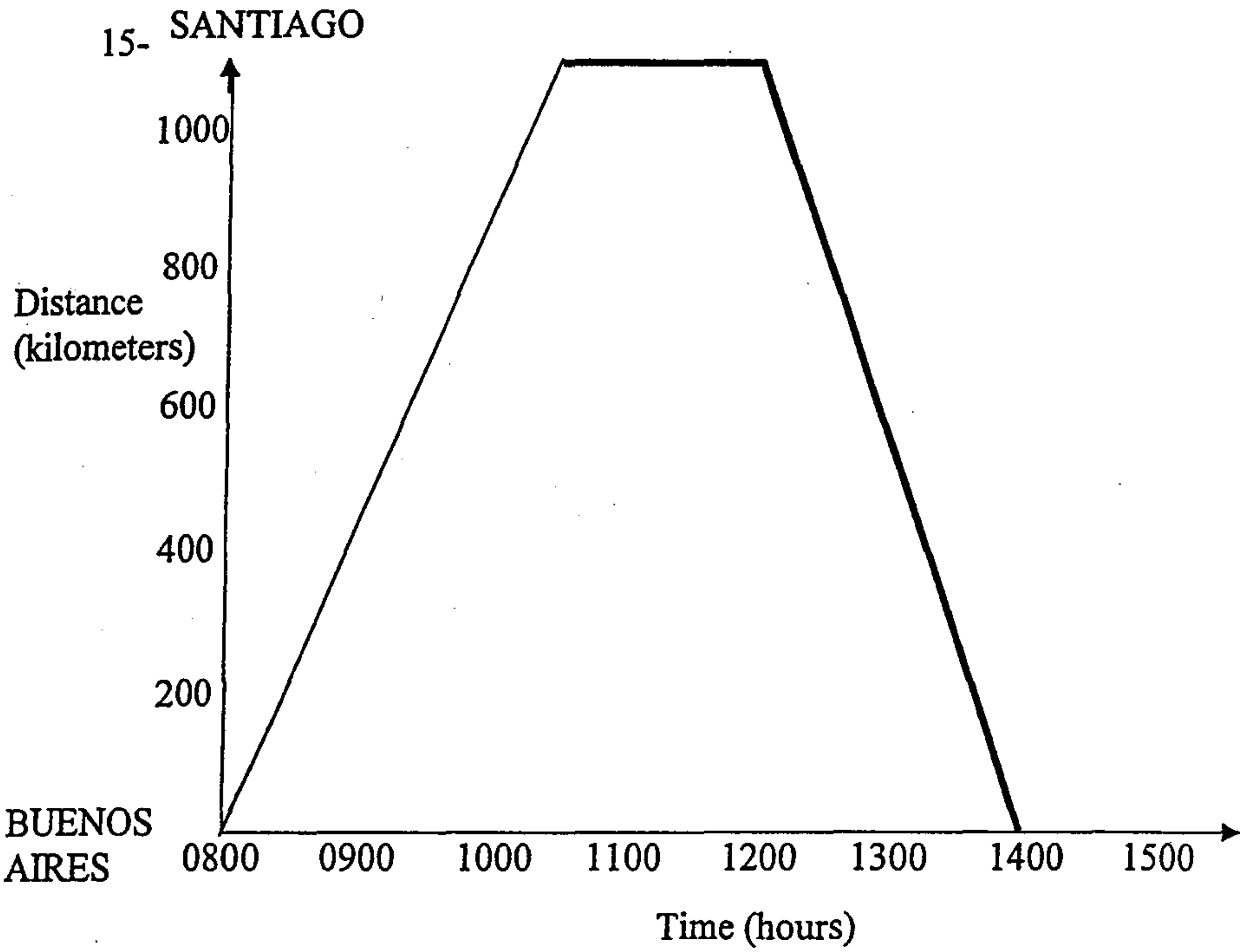
$$A_2 = \frac{\frac{1}{4}}{\left(\frac{80}{120}\right)^2} = \frac{1}{4}\times\left(\frac{120}{80}\right)^2 = \frac{1}{4}\times\frac{9}{4} = \frac{9}{16}$$

Answer: $\frac{9}{16}m^3$

14-(a) $x^2 - 7x + 10 = (x-2)(x-5)$

(b) 3ax – 6x – ay + 2y = 3x(a-2) – y(a-2) = (a-2)(3x-y)

15-



(a) Average speed $= \dfrac{\text{distance}}{\text{time}}$

Distance = 1100 km Time = $1030 - 0800 = 230 = 2\frac{1}{2}h$

Average speed $= \dfrac{1100}{2\frac{1}{2}} = 440\ km/h$

(b) time $= \dfrac{1100}{550} = 2\,h$

16- (a) $\cos 50 = \frac{AB}{100}$

AB = 100 cos 50
$= 64.279 \approx 64.3$ m

(b) $\sin 65° = \frac{BE}{AB}$

BE = AB sin 65 = 58.3 m

17- $\frac{x}{x+2} - \frac{x-2}{x} = \frac{x^2-(x+2)(x-2)}{x(x+2)} = \frac{x^2-(x^2-4)}{x(x+2)} = \frac{4}{x(x+2)}$

18- (a) $\cos \angle ROT = \frac{20^2+20^2-32^2}{2 \times 20 \times 20} = \frac{400+400-1024}{800} = \frac{-224}{800}$

$\therefore \angle ROT = 106.26°$

(b) Length of arc RST = $\frac{\theta}{360} \times 2\Pi r = \frac{106.26}{360} \times 2 \times \Pi \times 20 = 37.1$

Answer (b) Arc RST = 37.1 cm

19- (a) $A = 800\left(1+\frac{6}{100}\right)^5 = 800 \times (1.06)^5 = 1070.58$

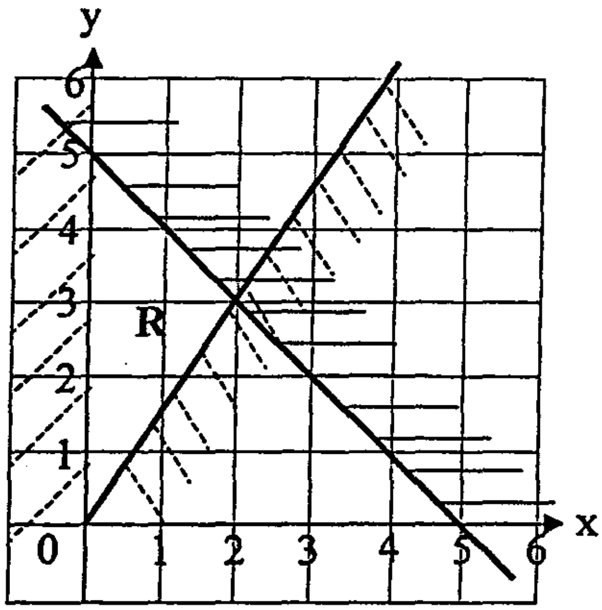
(b) $A = p\left(1+\frac{r}{100}\right)$

$\frac{A}{p} = 1 + \frac{r}{100}$

$\frac{r}{100} = \frac{A}{p} - 1 = \left(\frac{A-p}{p}\right)$

$r = \frac{100(A-p)}{p}$

20-



(a) $x + y = 5$ Line joining (5,0) and (0,5)

$2y = 3x$ through the origin and $x = 2$ $2y = 6$

$y = 3$

(2,3)

21- (a) $2x^2 - 3x = 0$

$x(2x - 3) = 0$

$x = 0$ $2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

Answer (a) $x = 0$ or $= \frac{3}{2}$

(b) $2x^2 - 3x - 1 = 0$

$a = 2$ $b = -3$ $c = -1$

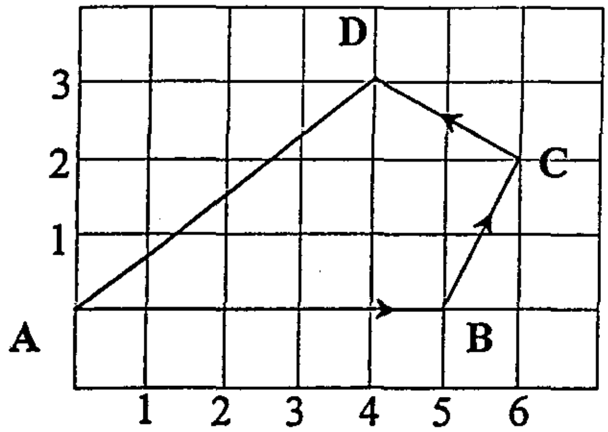
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{9 - 4 \times 2 \times -1}}{4}$$

$$x = \frac{3 \pm \sqrt{17}}{4} = 1.78, -0.28$$

Answer (b) $x = 1.78$ or -0.28

22- (a)



(b) $\left|\overrightarrow{BC}\right| = \sqrt{1^2 + 2^2} = \sqrt{5} = 2.24$

(c) From A draw an arc of radius 5 (length of AB) and from C draw an arc of radius equal length of CB
The point of intersection is D
D is the point (4,3)

Answer (c) $\overrightarrow{AD} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$\overrightarrow{DC} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

June 98

Paper 2

1- $52 - 3\,(4.1 - 1.8) = 52 - 3\,(2.3) = 45.1$ Answer: 45.1

2- (a) $3\text{ cm / min} = \dfrac{3}{100\times1000}\times60 = 0.0018$ Answer: 0.0018 Km / h

(b) $0.0018 = 1.8\times10^{-3}\,km/h$ Answer: $1.8\times10^{-3}\,km/h$

3- (a) $\angle ABT = \dfrac{1}{2}\angle AOT = \dfrac{1}{2}\times64 = 32°$ Answer: Angle ABT = 32°

(b) AB perpendicular to OT

$\angle OTB = 90 - \angle ABT = 90 - 32 = 58°$ Answer: Angle OTB = 58°

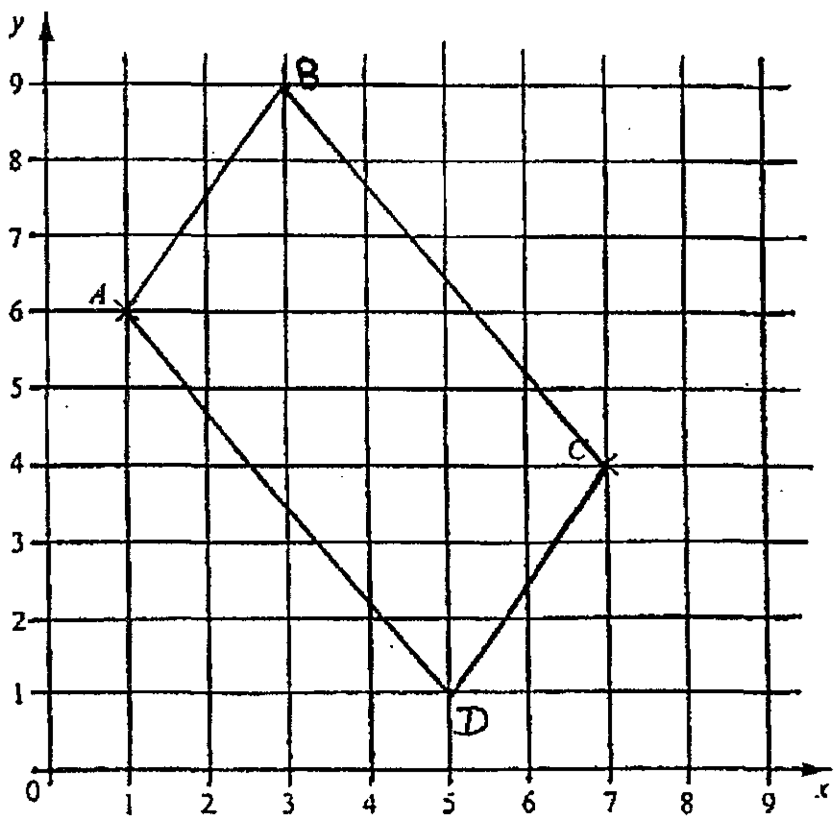
4- $I = \dfrac{PRT}{100}$

$50 = \dfrac{250\times R\times5}{100}$

$5000 = 1250\,\text{R}$

$R = 4$ Answer: R = 4

5- (a)



(b) $\overrightarrow{AD} = \overrightarrow{BC} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ Answer: $\overrightarrow{AD} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

6- $\frac{a}{6} + \frac{b}{21} = \frac{17}{42}$ all times 42

7a + 2b = 17

now we can select any positive integer for a and then find b

try a = 1

7 x 1 + 2b = 17

2b = 10 b = 5

as b obtained is positive integer, it is correct Answer: a = 1 b = 5

7- 2 min 23 sec $= 2\frac{23}{60} = 2.383$

(or use calculator 2 [°'''] 23 [°'''] = 2.383).

2.3 is 2.3

$2\frac{1}{3} = 2.333$

2.23 = 2.23 Answer: $2.23 < 2.3 < 2\frac{1}{3} <$ 2 min 23 sec

8- 3x – y = 4 (1)

x – y = 8 (2)

-x + y = -8 (2) x –1

3x – y = 4 (1)

2x = -4

x = -2

substitute to get y

3 (-2) – y = 4

-6 - y = 4

-y = 10 y = -10 Answer: x = -2 , y = -10

9- (a) Look at the graph, Locate where the gradient of the graph is Largest.

It is in the part of the graph after 18 sec

You will find that the gradient is largest at t = 19

(b) Total distance travelled = 2 d

Average speed = $\frac{Total distance}{Total time}$

$1.5 = \frac{2d}{24}$

2d = 24 x 1.5 = 36 d = 18 m

10- (a) $\angle OAM = 180 - 83 = 97°$

$\angle AOM = 180 - (97 + 58) = 25°$

OR $\angle AOM = 83 - 58 = 25°$

(b) Given AM : MB = 1 : 2

AM : AB = 1 : 3

Area of parallelogram = 96 cm^2

Area of $\Delta AOB = \frac{1}{2} \times 96 = 48 cm^2$

$\Delta's$ AOM and AOB have the same height but different base

base AM = $\frac{1}{3}$ base AB

Area of $\Delta AOM = \frac{1}{3}$ area of ΔAOB

$= \frac{1}{3} \times 48 = 16 cm^2$ Answer: $16 cm^2$

11- sin x = -0.866 cos x = -0.5 $0 \le x \le 360°$
The quadrant in which sine and cosine are both negative is the 3rd. Quad
Using calculator the angle whose sine = 0.866 (or its cosine 0.5) is 60°
∴ x = 180 + 60 = 240° Answer: x = 240°

12- (a) $3x - 2 < 15$

$3x < 17$ $x < \frac{17}{3}$ $x < 5\frac{2}{3}$

∴ A = { 1,2,3,4,5 } n (A) = 5 Answer: n (A) = 5

(b) $4x + 1 \ge 13$ $4x \ge 12$ $x \ge 3$

B = { 3,4,5, ... etc }

Answer: A ∩ B = { 3,4,5 }

13- (a) $\frac{360}{20} = 18$ 180 – 18 = 162° Answer: Angle ABC = 162°

(b) $\angle ACB = \frac{180 - 162}{2} = 9°$ Answer: Angle ACB = 9°

14- (a) $50 - \frac{5}{2} \leq \text{mass} < 50 + \frac{5}{2}$

(i) $47.5 \text{ g} \leq \text{mass} < 52.5 \text{ g}$

(ii) $8.5\ cm^3 \leq \text{volume} < 9.5\ cm^3$

(b) Least possible density $= \frac{Least mass}{Largest volume}$

$= \frac{47.5}{9.5} = 5$ Answer: $5\ g / cm^3$

15- (a) $\sqrt{x^{36}} = (x^{36})^{1/2} = x^{18}$ Answer: P = 18

(b) $10^q = 1$ $q = 0$ Answer: q = 0

(c) $r^{-\frac{1}{2}} = \frac{1}{4}$

$r = \left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^{2} = 16$ Answer: 16

16- (a) Answer Angle OBC = 90 – 50 = 40°

(b) (i) $\angle OAB = 50°$

Bearing of B from A is 180 – 50 = 130° Answer: 130°

(ii) $\angle OCB = \angle OBC = 40°$

Bearing of B from C is 040°

Bearing of C from B is 180 + 40 = 220° Answer: 220°

17- $\frac{T}{W+3} = V$

(W + 3) V = T

$W + 3 = \frac{T}{V}$

$W = \frac{T}{V} - 3$ Answer: $W = \frac{T}{V} - 3$

18- (a) Number of boys $= \frac{5}{12} \times 480 = 200$

Number of girls = 480 – 200 = 280 Answer: 280

(b) Number of students aged 15 or over $= \frac{3}{10} \times 480 = 144$

Number of students aged under 15 = 480 – 144 = 336 Answer: 336

(c) Number of girls under 15 = $\frac{7}{16} \times 480 = 210$

Number of girls 15 or above = $280 - 210 = 70$

Number of boys aged 15 or over = $144 - 70 = 74$ Answer: 74

Boys	Girls	
	70	15 or over 15, Total 144
	210	Under 15, Total 336
200	280	Total

19- (a) $fg(5) = f(2 \times 5 + 1) = f(11) = 11^2 = 121$ Answer: 121

(b) $y = 2x + 1$

$2x = y - 1$

$x = \frac{y-1}{2}$

$g^{-1}(x) = \frac{x-1}{2}$

Answer: $g^{-1}(x) = \frac{x-1}{2}$

20- Answer: $x \geq 1$

$y \leq 5$

$y \leq x + 2$

21- (a) (i) Ratio of areas is K^2

$K^2 = 36$ $K = 6$ Answer: 6 : 1

(ii) Length = 6 x 0.7 = 4.2 m Answer: 4.2 m

(b) Ratio of volumes = $K^3 = 6^3 = 216$

$\frac{RealVolume}{ModelVolume} = K^3$

$\frac{0.54}{ModelVolume} = 216$

$ModelVolume = \frac{0.54}{216} = 2.5 \times 10^{-3} m^3$

Answer: $2.5 \times 10^{-3} m^3$

22- (a) $Sin \angle AOC = \frac{12}{13}$

$\angle AOC = 67.4°$

(b) (i) Area of sector = $\frac{\theta}{360} \times \pi \times R^2$

$= \frac{67.4}{360} \times \pi \times 13^2 = 99.4 cm^2$ Answer: $99.4 cm^2$

(ii) shaded area = area of sector – area of triangle

Third side of the triangle = $\sqrt{13^2 - 12^2} = 5$

area of triangle = $\frac{1}{2} \times 12 \times 5 = 30$

Shaded area = $99.4 - 30$

$= 69.4 \ cm^2$ Answer: $69.4 \ cm^2$

23- (a) Pyramid

(b) By measurement

Length of one side of the square base = 6 cm

Height of each of the triangular faces is = 5.2 cm

Total surface area = $6 \times 6 + 4 \times \frac{1}{2} \times 6 \times 5.2$

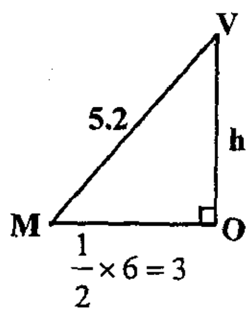
$= 98.4 \ cm^2$ Answer: $98.4 \ cm^2$

(c) VM is the height of one of the triangular faces = 5.2

h is the height of pyramid

height h = $\sqrt{5.2^2 - 3^2}$

= 4.25 cm



24- (a) (i) x (x – 1) (x + 1) = 40 (x + x – 1 + x + 1)

x (x – 1) (x + 1) = 40 (3x)

(ii) $x(x^2 - 1) = 120x$

$x^3 - x = 120x$

$x^3 - 121x = 0$

(b) $x^3 - 121x = x(x^2 - 121) = x(x+11)(x-11)$

(c) x (x+11) (x-11) = 0

x = 0, -11, 11

Possible answer is 11

Three positive integers are 10, 11, 12

Math 0580

NOV. 1998

Paper 2

1- Using calculator Angle A = 22.5°

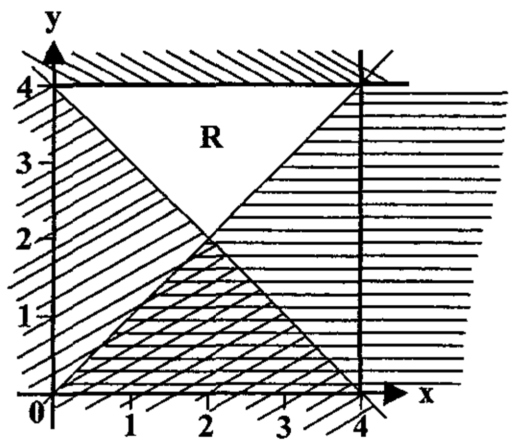
2- Sugar fruit

3 $2\frac{1}{2}$

? 4

Quantity of sugar $= \dfrac{4 \times 3}{2\frac{1}{2}} = 4.8\,kg$

3-



4- x - 4, x, 2x, 2x + 12

Median is the average of x and 2x,

the two middle numbers, therefore $\dfrac{x + 2x}{2} = 9$

3x = 18 x = 6

5- (a) $\dfrac{20}{2} = 10$ 220 - 10 ≤ r < 220 + 10

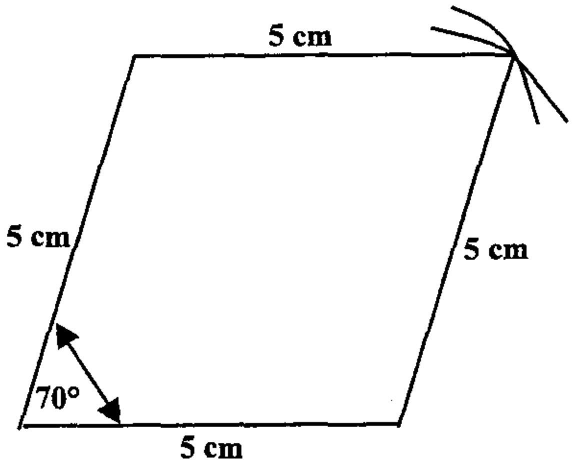
210 ≤ r < 230

(b) Circumference $= 2\pi r = 2 \times 3.142 \times 210$

$= 1319.64 \approx 1320$ cm

6- (a) Trapezium.

(b)



7- Time difference between 2034 and 1634 is 4 hours

The new train journey time is 80% $= \frac{80}{100} \times 4 = 3.2$ hours

Using calculator 16 [°’’’] 34 [°’’’] [+] 3.2 [=] shift [°’’’] 1946

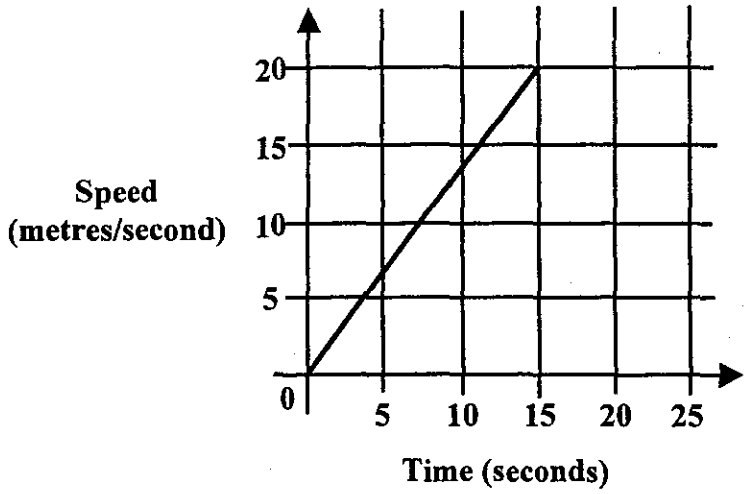
8- (a) $2x^2 - 5x - 3 = (2x + 1)(x - 3)$

(b) $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}$ or $x = 3$

9-(a)



(b) Acceleration $= \frac{20}{15} = \frac{4}{3} m / s^2$

(c) Distance = area under the graph.

$= \frac{1}{2} \times 15 \times 20 = 150 \, m$

10- $\frac{x+3}{2} - \frac{x-4}{5} = \frac{5(x+3) - 2(x-4)}{10} = \frac{5x + 15 - 2x + 8}{10} = \frac{3x + 23}{10}$

11- (a) (i) $x = 4\cos(180t)°$

$t = 0.4 \qquad x = 4\cos(180 \times 0.4)$

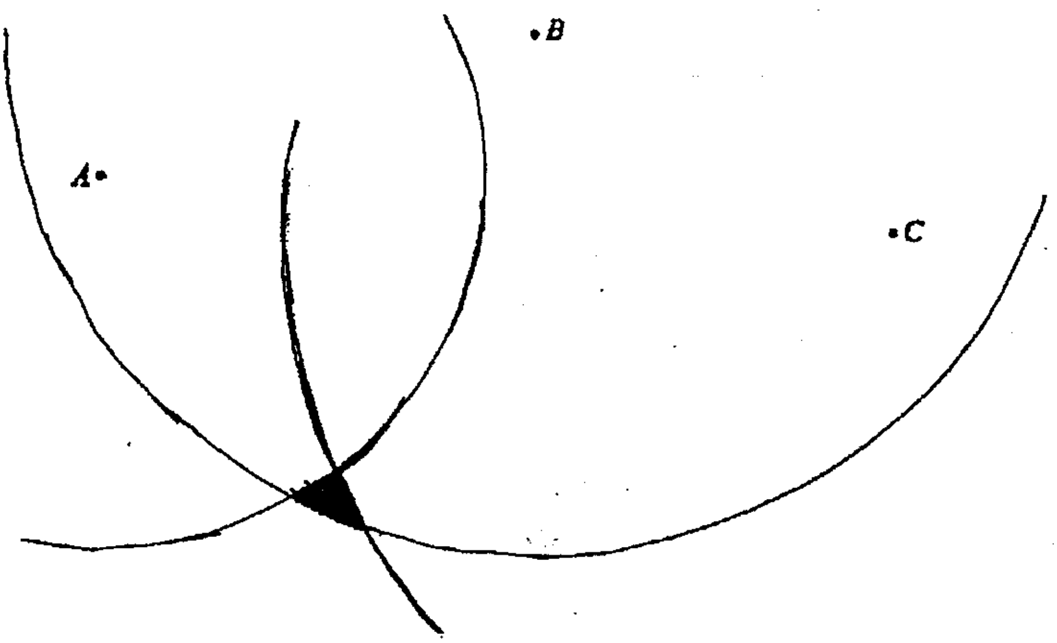
$= 4\cos 72° = 1.236 \approx 1.24$

(ii) $x = 4\cos(180 \times 1.3)$

$= 4\cos 234° = -2.351$

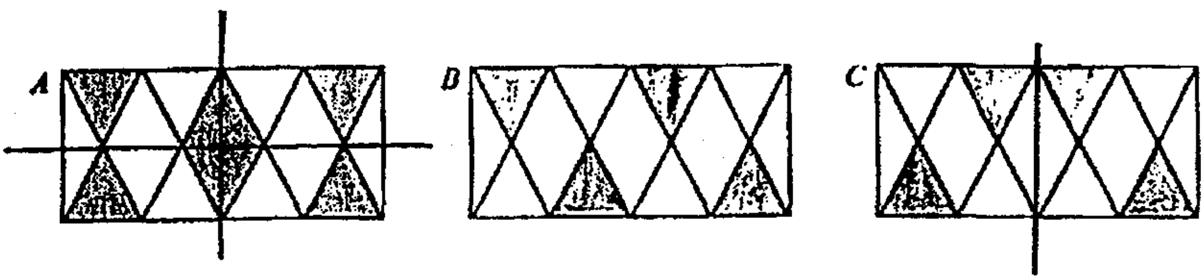
(b) negative x means to the left of the vertical line (or on the other side).

12-



13- (a) Answer: C

(b)



14- (a) (i) $\frac{L}{100} = (0.9)^{5d} = (0.9)^{5\times1.4} = (0.9)^7$

$L = 100 \times 0.4783 = 47.83\ \%$

(ii) $\frac{L}{100} = (0.9)^{5\times2.7} = (0.9)^{13.5} = 0.2411$

$L = 24.1\%$

(b) $\frac{81}{100} = (0.9)^{5d}$ $\quad$ $0.81 = (0.9)^2$

$\therefore 5d = 2$ $\quad$ $d = 2/5 = 0.4$

15- $\angle x = 180 - (135 + 27) = 180 - 162 = 18°$

$\frac{12}{\sin 135} = \frac{YZ}{\sin 18°}$ $\quad$ $YZ = \frac{12 \sin 18°}{\sin 135°} = 5.24 cm$

16- (a) gradient $m = \frac{8-2}{8-0} = \frac{6}{8} = \frac{3}{4}$

y intercept c is 2

(b) AB $= \sqrt{(8-0)^2 + (8-2)^2} = \sqrt{64+36} = \sqrt{100} = 10$

17- (a) Cost for 5 days = 5 x 23 = 115

Free kilometres = 5 x 40 = 200 Km

Extra distance charge = (350 – 200) x 0.25 = 37.5

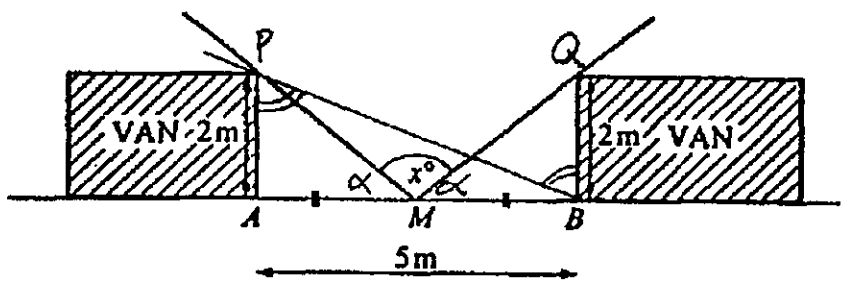
Total cost = 115 + 37.5 = 152.5 $

(b) Cost for p days = 23 p

Extra distance charge = (q – 40 p) x 0.25 = $\frac{1}{4}q$ - 10 p

Total cost = 23 p + $\frac{1}{4}q$ - 10 p = 13 p + $\frac{1}{4}q$ $

18-



(a) $\tan\alpha = \frac{2}{2.5}$ $\quad$ $\alpha = 38.66°$

Angle x = 180 - 2x = 102.68 = 102.7°

(b) Angle of view now is angle PBQ = angle APB

$\tan\theta = \frac{5}{2} = 2.5$ $\quad$ Angle = 68.2

19-(a) $h \alpha v^2$ $\quad$ $\therefore h = kv^2$

v = 4 $\quad$ h = 80

$\therefore 80 = k4^2 = 16k$ $\qquad$ $k = \frac{80}{16} = 5$

$\therefore h = 5v^2$

(b) (i) $h = 5v^2 = 5(6)^2 = 180\,cm$

(ii) h = 20 m = 20 x 100 = 2000 cm

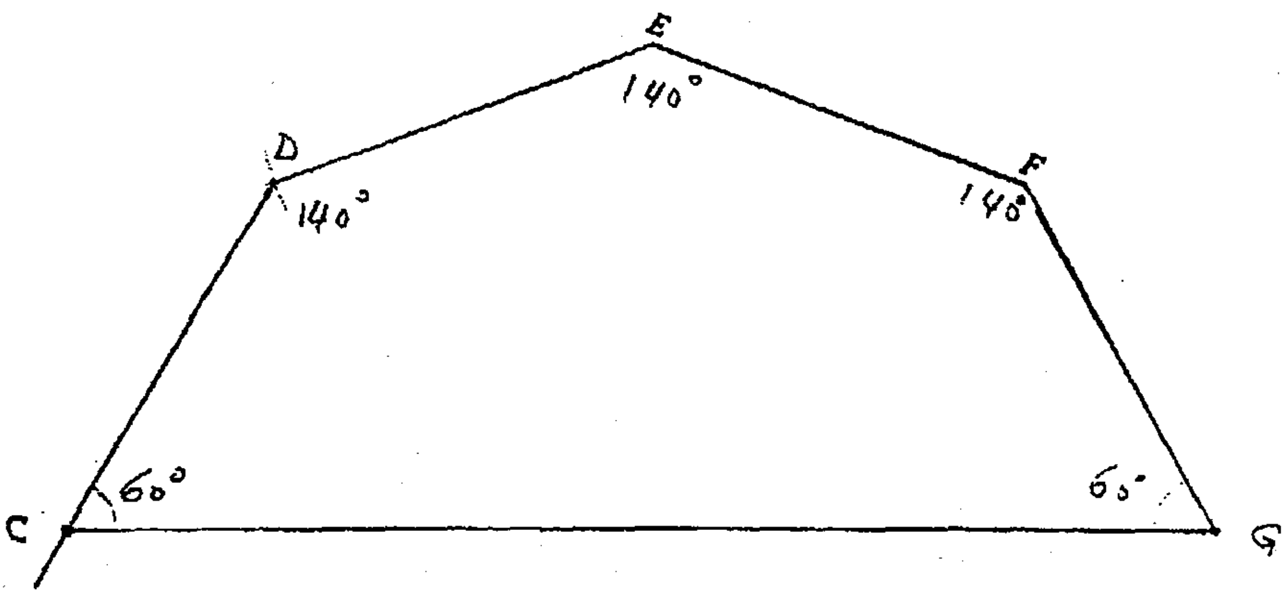
$2000 = 5v^2$ $\qquad$ $v^2 = \frac{2000}{5} = 400$

v = 20 m/s

20- (a) Each Exterior angle = $\frac{360}{9} = 40°$

Each Interior angle = 180 – 40 = 140°

(b) (i)



(ii) Angle DCG = 60° $\qquad$ Angle FGC = 60°

(iii) The shape CDEFG is a 5 sided polygon (pentagon)

The sum of all its interior angles = (2n - 4) x 90 = (2 x 5 – 4) x 90 = 540°

Three of its angles are each 140

140 x 3 = 420

(Sum of the other two angles) = 540 – 420 = 120°

Value of each angle = $\frac{120}{2} = 60$

Math 0580

June 1999

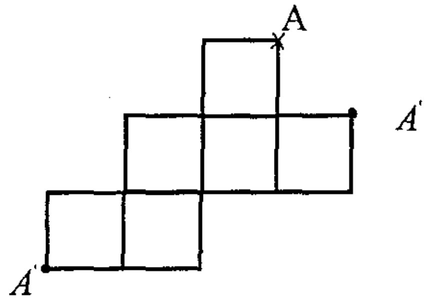
Paper 2

1. $\left(\frac{1}{8}+\frac{1}{2}\right)\div\frac{5}{6}=\frac{3}{4}$

2. (a) $300\ 000 = 3\times10^5$

   (b) $4.2\times3\times10^5\times365\times24\times60\times60=3.97\times10^{13}\,Km$

3.



4. (a) 3 min 58.2 sec - 0.9 sec = 3 min 57.3 sec

   (Using calculator: 0 [°'"] 3 [°'"] 58.2 [°'"] - 0 [°'"] 0 [°'"] 0.9 = shift [°'"] 0 3 57.3)

   or just 58.2 - 0.9 = 57.3

   (b) 3 min 58.2 sec + 3.1 sec = 4 min 1.3 sec

   (Similar way to (a))

5. (a) 1 mm = 0.1 cm

   $\frac{0.1}{2}=0.05$

   $5.2 - 0.05 \le AC < 5.2 + 0.05$

   $5.15 \le AC < 5.25$

   (b) The least value of AD is $\sqrt{(5.15)^2-(2.35)^2}$ cm.

6. 10 % on administration

   90 % on charitable work

   90 % of income is 234000

   income $= \frac{234000\times100}{90} = \$260000$

7. Ratio of volumes is 64 : 1

   Ratio of diameters (or radii) = $\sqrt[3]{64}$: 1 = 4 : 1

   Ratio of surface areas = $(4)^2$: 1 = 16 : 1

8. $x = \sqrt{y^3 + 3}$

$x^2 = y^3 + 3$

$y^3 = x^2 - 3$

$y = \sqrt[3]{x^2 - 3}$

9. Angle ACD = 90° angle of a semicircle

x = 90 - 40 = 50°

y = x alternate angle

y = 50°

$Z = \frac{1}{2}y = \frac{1}{2} \times 50 = 25°$

Angle at centre double angle at circumference

10. Method A: \$ 1 = 4.15 F

? = 1000 F

$\frac{1000 \times 1}{4.15} = \$240.96$

Method B: 1000 - 20 = 980

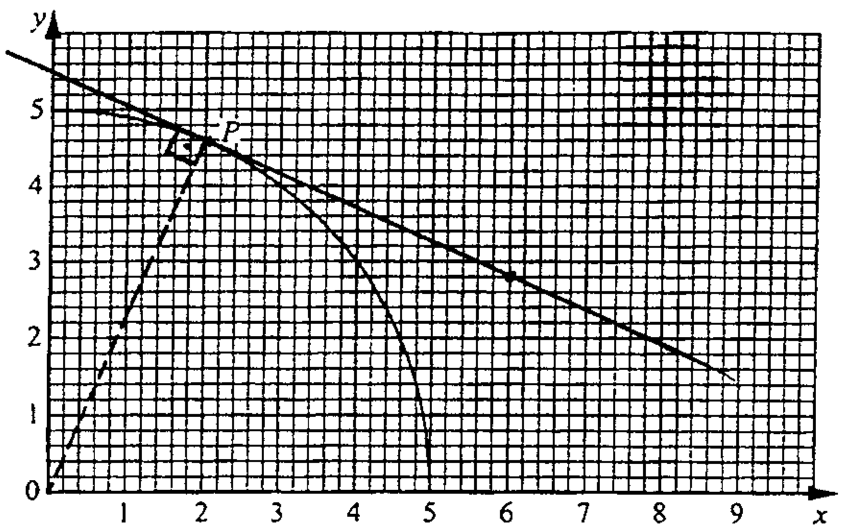
\$ 1 = 4 F

? = 980

$980 \times \frac{1}{4} = \$245$

Method B gives more by 245 - 240.96 = \$ 4.04

11.



Take two points on the tangent (2, 4.6) and (6, 2.8)

Gradient = $\frac{4.6 - 2.8}{2 - 6} = -0.45$ (any answer from -0.4 to -0.46)

12. (a) 9 litres = 9 x 1000 = 9000 cm^3
0.0009 m^3 = 0.0009 x 100 x 100 x 100 = 900 cm^3
0.0009 m^3 < 7000 cm^3 < 9 litres.

(b) 3 litres = 3000 cm^3
3000 - 900 = 2100
7000 - 3000 = 4000
9000 - 3000 = 6000
closest is 900 cm^3 i.e 0.0009 m^3

13. $\frac{n}{2}\times 150+\frac{n}{2}\times 170=(2n-4)\times 90$
75 n + 85 n = 180 n - 360
360 = 20 n
n = $\frac{360}{20} = 18$

14. (a) (i) f (-5) = 2 (-5) + 1 = -9
(ii) $gf(-5) = g(-9) = (-9)^2 + 3 = 84$
(b) gf (x) = g (2x + 1) = $(2x+1)^2 + 3$
= $4x^2 + 4x + 4$

15. $2x^2 + 4x - 3 = 0$
a = 2 b = 4 c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(2)(-3)}}{4}$$

$$x = \frac{-4 \pm \sqrt{40}}{4}$$

x = 0.58 or -2.58

16. (a) Shaded area = large sector - small sector.

$$= \frac{60}{360}\pi R^2 - \frac{60}{360}\pi r^2$$

$$= \frac{\pi}{6}(R^2 - r^2)$$

(b) shaded area = $\frac{\pi}{6}(R+r)(R-r)$

17. (a) AM is shorter because the opposite angle is smaller.

(b) 180 - (63 + 65) = 52°

$$\frac{100}{\sin 52°} = \frac{BM}{\sin 65°}$$

BM = 115 cm.

18. (a) T = K h

-5 = K . 500 $\qquad K = \frac{-5}{500} = -0.01$

T = -0.01 h

(b) (i) T = -18 $\qquad$ -18 = -0.01 h

$$h = \frac{-18}{-0.01} = 1800$$

heght above see level = 1800 + 2500 = 4300 m.

(ii) at sea level h = -2500

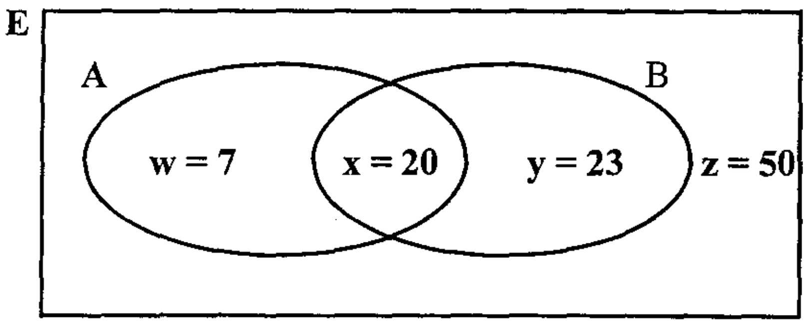
T = -0.01 x -2500 = 25°C

19. (a) 27 + 43 = 70

70 - 50 = 20

people reading both magazines = 20

(b)



(c) $Z = n(A \cup B)'$

20. (a) $(64x^8)^{\frac{1}{2}} = (64)^{\frac{1}{2}}(x^8)^{\frac{1}{2}} = 8x^4$

(b) $\frac{3x^2}{x^2 + 3x} = \frac{3x^2}{x(x+3)} = \frac{3x}{x+3}$

21. (a) AB = C

$$\begin{pmatrix} 4 & x \\ -3 & 6 \end{pmatrix}\begin{pmatrix} 5 & -3 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ y & 21 \end{pmatrix}$$

4 x 5 + x (-2) = 6

20 - 2x = 6

2x = 14 x = 7

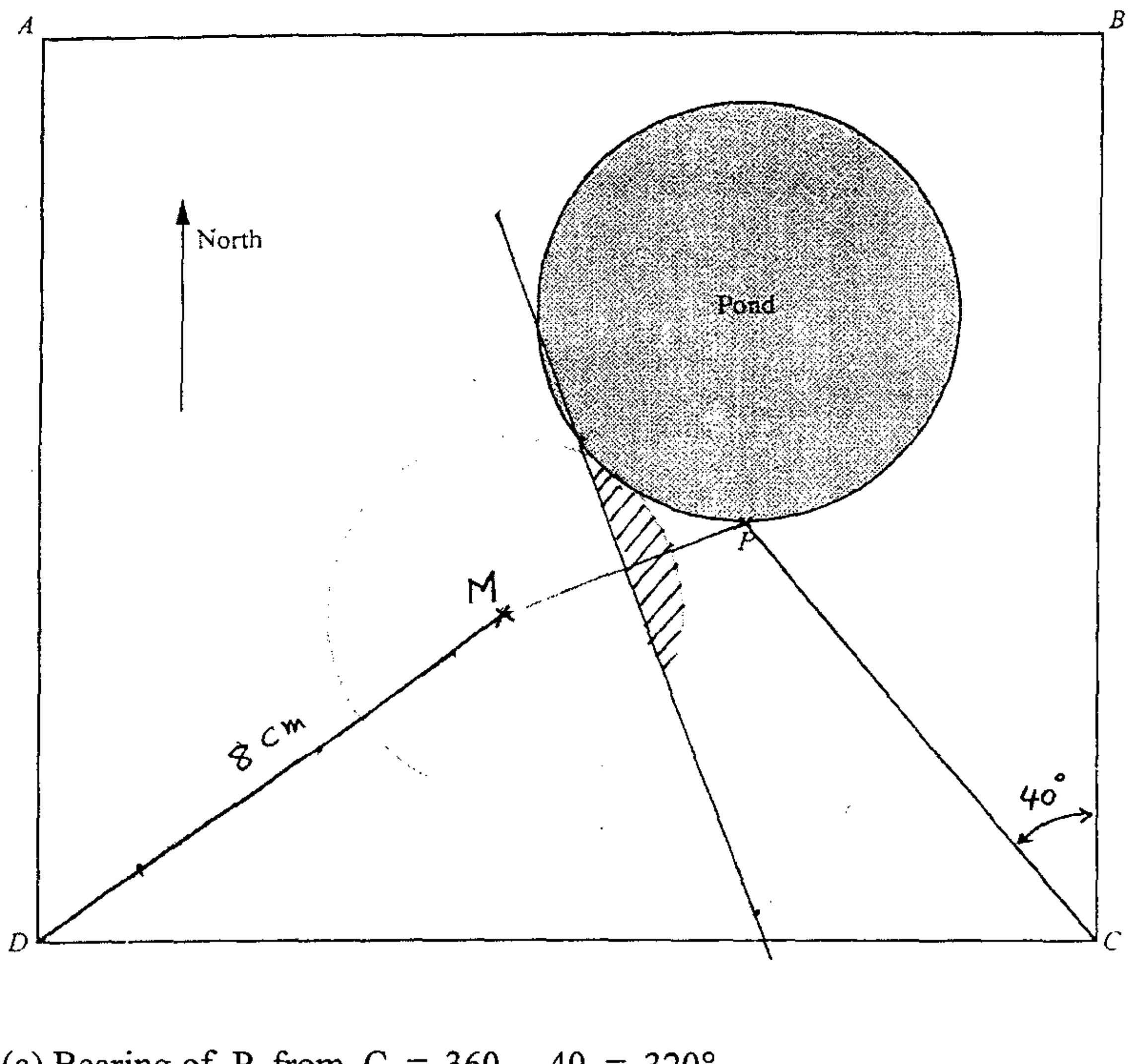
-3 (5) + 6 (-2) = y ∴ y = -27

(b) $B = \begin{pmatrix} 5 & -3 \\ -2 & 2 \end{pmatrix}$

$|B| = 5(2) - (-3)(-2) = 10 - 6 = 4$

$B^{-1} = \frac{1}{4}\begin{pmatrix} 2 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{5}{4} \end{pmatrix}$

22.



(a) Bearing of P from C = 360 - 40 = 320°

(b) 80 m = $\frac{80}{10}$ = 8 cm

November 99

Paper 2

1. Sea level $= -2.40 + 1.97$
$= -0.43$

2. $3(x+1) \geq 5 - x$
$3x + 3 \geq 5 - x$
$3x + x \geq 5 - 3$
$4x \geq 2$
$x \geq \frac{1}{2}$

3. $I = \frac{PRT}{100}$
$P = 560 \quad R = 5.5 \quad I = 123.20$
$123.20 = \frac{560 \times 5.5 \times T}{100}$
$T = \frac{123.20 \times 100}{560 \times 5.5} = 4$ years

4. $x = 0.083 \qquad y = \frac{84}{991} = 0.08476$
$z = 8.4 \times 10^{-3} = 0.0084$
$z \prec x \prec y$

5. $\dfrac{478 \times 49.82}{0.1248}$

Writing each number correct to two significant figures

478 approximated to 480

49.82 approximated to 50

0.1248 approximated to 0.12

$$\frac{480 \times 50}{0.12} = 200\ 000$$

6. Cost in Paris = 1600 French francs

Cost in London = Ł 170 (pounds).

= 170 × 9.30 = 1581 French francs

The cycle cost less in London than Paris

<u>OR</u> cost in London = Ł170 (pounds).

cost in Paris $= \dfrac{1600\ francs}{9.30}$

= 172.04 (pounds).

The cycle cost less in London than Paris

7. Perimeter P is 65cm to the nearest centimeter

$64.5 \le P < 65.5$

$P = 3L$ where L is the length of one side $L = \dfrac{P}{3}$

smallest possible length of one side

$= \dfrac{64.5}{3} = 21.5$ cm

8. $3x - y = -3$ (1)

$9x + 2y = 1$ (2)

(1)x2 $6x - 2y = -6$

(2) $9x + 2y = 1$

adding $15x = -5$

$$x = \frac{-5}{15} = -\frac{1}{3}$$

substituting in (1)

$3\left(-\frac{1}{3}\right) - y = -3$

$-1 - y = -3$

$1 + y = 3$

$y = 2$

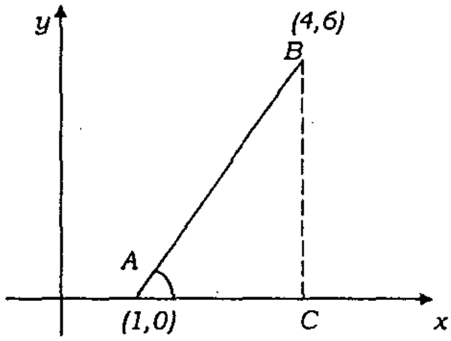
$x = -\frac{1}{3}$ $y = 2$

9.

Distance AC = 4 – 1 = 3
Distance BC = 6 – 0 = 6

$\tan A = \frac{6}{3} = 2$

$A = 63.4°$



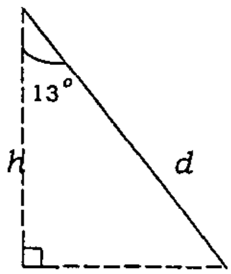
10. (a) angle BCD = 180 – (55 +26) = 180 – 81 = 99 °
(b) angle ACD = angle ABD = 55 ° (same arc)
angle BAC = angle ACD = 55 ° (alternate)
angle BXC = angle BAX + angle ABX
= 55 + 55 = 110 (exterior)
(c) angle ACB = 180 – (26 + 110) = 44 °
angle ADB = angle ACB = 44 °

11.

$\cos 13° = \frac{h}{d}$

$h = d \cos 13°$
$= 1800 \cos 13°$
$= 1753.87$
$= 1754$ m

vertical distance = 1754 m



12. $\frac{ax - ay}{px - py + qx - qy} = \frac{a(x-y)}{p(x-y) + q(x-y)}$

$= \frac{a(x-y)}{(x-y)(p+q)} = \frac{a}{p+q}$

13. $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

$\frac{24}{3} = \left(\frac{h_1}{15.5}\right)^3$

$8 = \left(\frac{h_1}{15.5}\right)^3$

$\frac{h_1}{15.5} = \sqrt[3]{8} = 2$

$h = 15.5 \times 2 = 31$ cm

14. $F = K V^2$

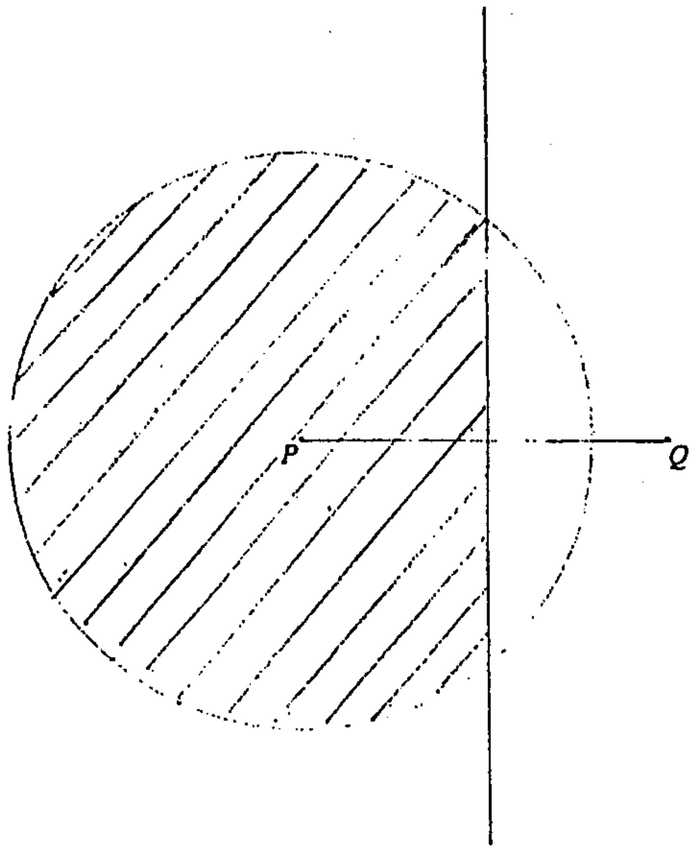
$180 = K(6)^2 = 36 K$

$K = \frac{180}{36} = 5$

$F = 5 V^2$

$F = 5 (3)^2 = 5 \times 9 = 45$

15.



16. (a) $2x^4 \times 5x = 10x^5$

(b) $x^2 \div x^{\frac{1}{2}} = x^{2-\frac{1}{2}} = x^{1\frac{1}{2}}$

(c) $\left(\sqrt{2x}\right)^6 = \left[(2x)^{\frac{1}{2}}\right]^6 = (2x)^3 = 8x^3$

17. $x^2 - 2x - 5 = 0$

$a = 1 \qquad b = -2 \qquad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(-5)}}{2}$$

$$= \frac{2 \pm \sqrt{24}}{2} = \frac{2 \pm 4.899}{2}$$

$= 3.45 \quad or -1.45$

18. $M = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix} \qquad N = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

(a) $MN = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -11 \\ -17 \end{pmatrix}$

(b) $|M| = 2(-5) - (-3)(4) = 2$

$$M^{-1} = \frac{1}{2}\begin{pmatrix} -5 & 3 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} & \frac{3}{2} \\ -2 & 1 \end{pmatrix}$$

19. $f : x \rightarrow 2x - 7 \qquad g : x \rightarrow \frac{x+1}{x}$

(a) $fg(2) = f\left(\frac{2+1}{2}\right) = f\left(\frac{3}{2}\right)$

$= 2\left(\frac{3}{2}\right) - 7 = 3 - 7 = -4$

(b) $fg(x) = 2\left(\frac{x+1}{x}\right) - 7$

$$= \frac{2x+2}{x} - 7 = \frac{2x+2-7x}{x}$$

$$= \frac{2-5x}{x}$$

20. (a) Area $= \frac{4.6+5}{2} \times \left(\frac{8}{10}\right) = \frac{9.6}{2} \times 0.8$

$= 3.84\ cm^2$

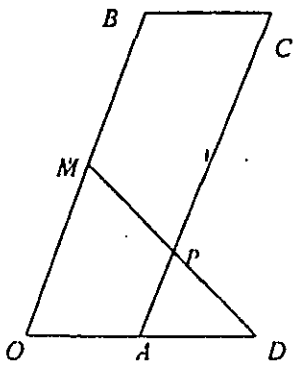
(b) Volume $=$ Area $\times$ length

$= 3.84 \times 9.5 \quad = 36.48$

$= 36\ cm^3$

(c) Two planes of symmetry .

21.



(a) $\overline{DM} = \overline{DO} + \overline{OM}$

$= -2a + \frac{1}{2}b$

(b) similar triangles

(c) A is mid OD

AP is parallel to OB

$\overline{AP} = \frac{1}{2}\overline{OM} = \frac{1}{2}\left(\frac{1}{2}b\right) = \frac{1}{4}b$

$\overline{OP} = \overline{OA} + \overline{AP}$

$= a + \frac{1}{4}b$

22. (a) deceleration $= \frac{\text{change in velocity}}{\text{time}}$

$= \frac{8-3}{2} = \frac{5}{2} = 2.5 m/s^2$

(b) distance $=$ Area under the graph

$= \frac{8+3}{2} \times 2 = 11m$

(c) speed when $t = 0$ is 10 m/s

$10m/s = \frac{10}{1000} \times 60 \times 60$

$= 36\ Km/h$

Math 0580

June 2000

Paper 2

1- Least possible Length = $4810 - \frac{10}{2} = 4805\,m$

2- Next two prime numbers after 29 are 31 and 37

Mean = $\frac{31+37}{2} = 34$

3- Length of a "Life time" = $\frac{650 \times 10^6}{1000}$

= 650000 hours

= $\frac{650000}{24 \times 365} = 74.2$ year

= 74 to the nearest year.

4- $X = 3.4 \times 10^{-3} = 0.0034$

$y = 1.2 \times 10^{-1} = 0.12$

$z = 4.6 \times 10^{-4} = 0.00046$

(a) X < Y

(b) x + y > z

5- (a) Bearing of Z from C = 024°

Therefore bearing of C from Z = 180 + 24 = 204°

(b) Angle AZ makes with the south direction = 90 + 24 = 114

Bearing of Z from A = 114° (alternate angles)

6- 1 min 31.649 sec = 1 x 60 + 31.649 = 91.649 sec

208.303 Km / h = $\frac{208.303 \times 1000}{3600}$ m/s = 57.862 m/s

Length of Lap = 57.862 x 91.649

= 5302.99 = 5303 m

7- (a) $(8x^4y)^2 = 64x^8y^2$

(b) $(8x^4y)^2 \div x^2y^{-1} = \frac{64x^8y^2}{x^2y^{-1}} = 64x^6y^3$

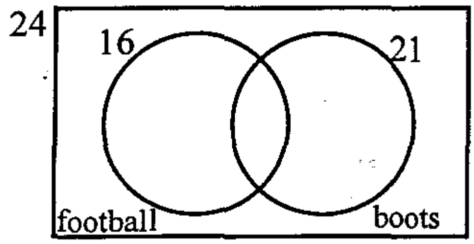
8- (a)(i) $\overrightarrow{QM} = -\frac{1}{2}r$

(ii) $\overrightarrow{RM} = P - \frac{1}{2}r$

(b) $\overrightarrow{OS} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

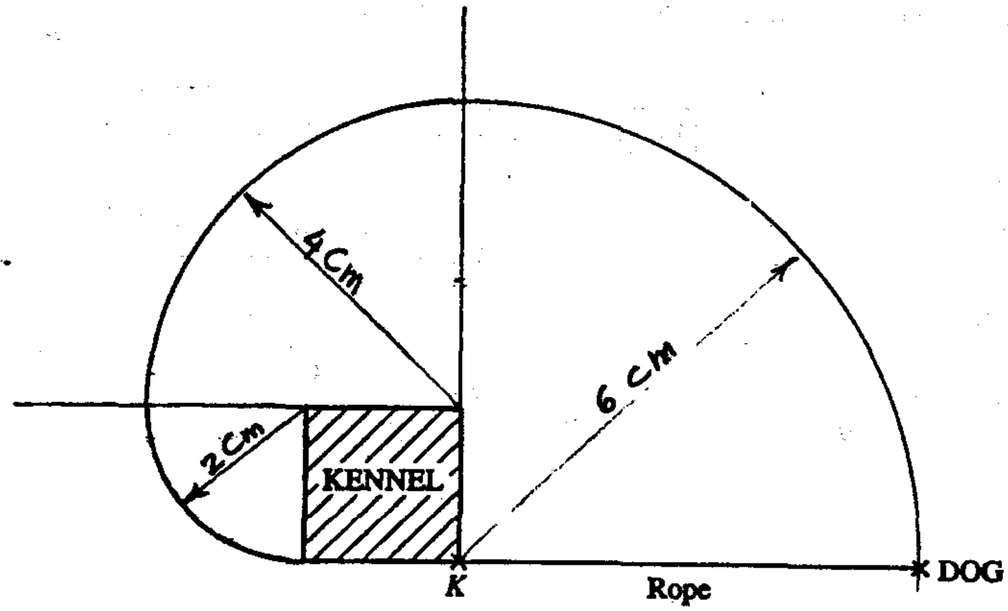
9- (a) The number of students should be exactly divisible by 3 and 8 , Therefore the number of students = 24

(b)



16 + 21 - 24 = 13

10-



11- Monday and Tuesday temperatures are - 2.4°
Wednesday (the median) is -1.3°
Friday is 4.5°, the maximum
Let Thursday temperature be x

$$\therefore \frac{(-2.4)+(-2.4)+(-1.3)+x+4.5}{5} = 0$$

x - 1.6 = 0
x = 1.6

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Temperature (°C)	-2.4	-2.4	-1.3	1.6	4.5

12- (a) x = 0

(b) $10^y = 0.01 = \frac{1}{100} = 10^{-2}$ y = -2

(c) $16^z = 2$ $(2^4)^z = 2$ $2^{4z} = 2^1$

$4^z = 1$ $z = \frac{1}{4}$

13- (a)

Cost	Loss	Selling
100	22.5	77.5
8400		?

Selling price = $\frac{8400 \times 77.5}{100}$ = \$6510

(b)

Cost	Profit	Selling
100	40	140
?		8400

Amount paid for the car = $\frac{8400 \times 100}{140}$ = \$6000

14- $y = 3 + \sqrt{2x}$

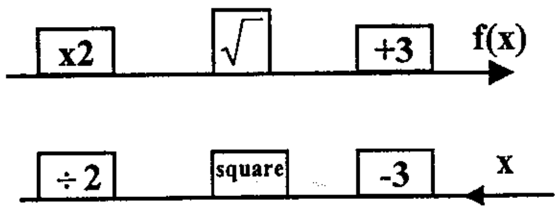
$y - 3 = \sqrt{2x}$

$2x = (y-3)^2$

$x = \frac{1}{2}(y-3)^2$

$f^{-1}(x) = \frac{1}{2}(x-3)^2$

OR



$f^{-1}(x) = \frac{(x-3)^2}{2}$

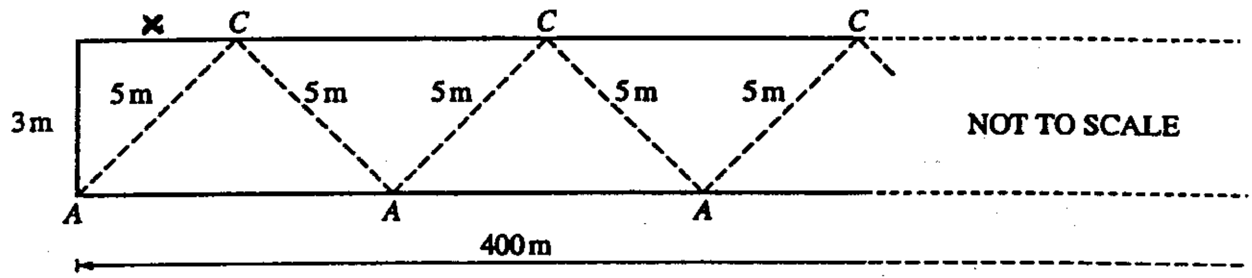
15- (a) $B = \frac{K}{d^2}$

$2 = \frac{K}{(12)^2}$ ∴ K = 2 x 144 = 288

$B = \frac{288}{d^2}$

(b) d = 3 $B = \frac{288}{3^2} = 32$

16-



(a) distance x = $\sqrt{5^2 - 3^2}$ = 4 m

distance between two apple trees = 8 m

number of apple trees = $\frac{400}{8} + 1 = 51$

(b) number of cherry trees is one less than apple trees i.e 50

17- (a) x = $\frac{1}{2} \times 100 = 50°$

y = angle OBA = 90 - 50 = 40°

z = angle ABT = 40°

(b) No, angle z = 40° and angle OAT = 50° alternate angles are not equal.

18- (a)

	Alex		Bukki		chris
	2	:		:	1
	3	:	4	:	
	6		$\frac{6 \times 4}{3} = 8$		$\frac{6 \times 1}{2} = 3$
Ratio	6	:	8	:	3

(b) Sum of shares = 6 + 8 + 3 = 17

Cost of present for Bukki = $\frac{8}{17} \times 21.25 = \$10$

19- (a) Similar triangles $\frac{h}{1.59} = \frac{8}{3}$

$h = \frac{8 \times 1.59}{3} = 4.24m$

(b) $\frac{x}{4.24} = \frac{2x}{2x + 5}$

$\frac{x}{2x} = \frac{4.24}{2x + 5}$

$\frac{1}{2} = \frac{4.24}{2x + 5}$

2x + 5 = 2 x 4.24 = 8.48

2x = 3.48 x = 1.74

20- (a) (i) Acceleration $= \frac{27}{3} = 9m/s^2$

(ii) distance = area $= \frac{1}{2} \times 3 \times 27 = 40.5m$

(b) 1000 - 112 = 888 m

$\frac{888}{2} = 444$ sec

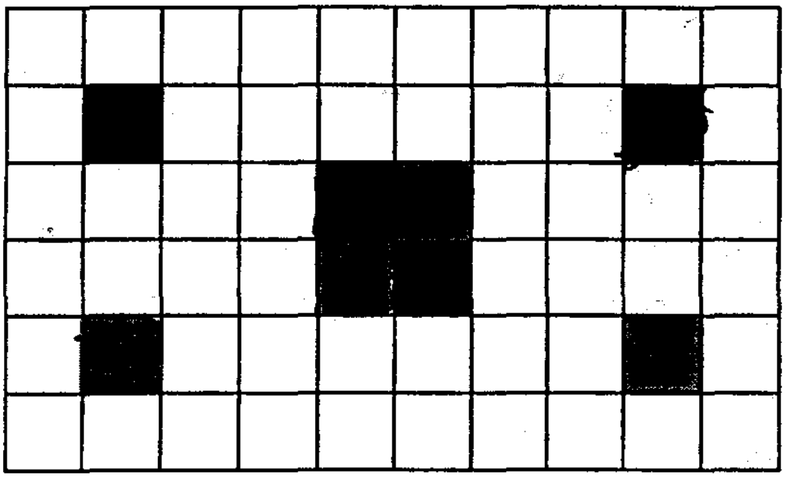
total time = 444 + 12 = 456 sec
= 7.6 min = 7 min 36 sec

21- (a) $\frac{1}{x-3} - \frac{1}{x} = \frac{x-(x-3)}{x(x-3)} = \frac{3}{x(x-3)}$

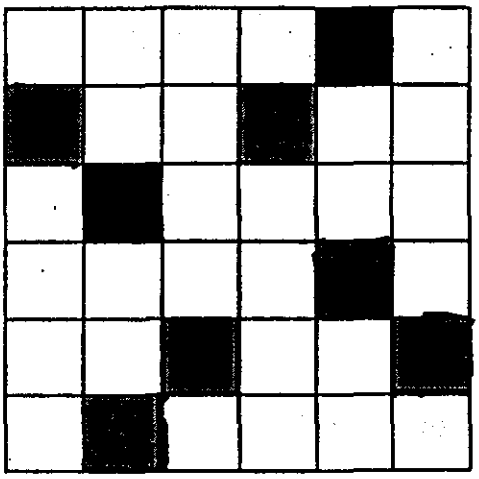
(b) $\frac{1}{y} = \frac{1}{x-3} - \frac{1}{x} = \frac{3}{x(x-3)}$

$y = \frac{x(x-3)}{3} = \frac{x^2 - 3x}{3} = \frac{1}{3}x^2 - x$

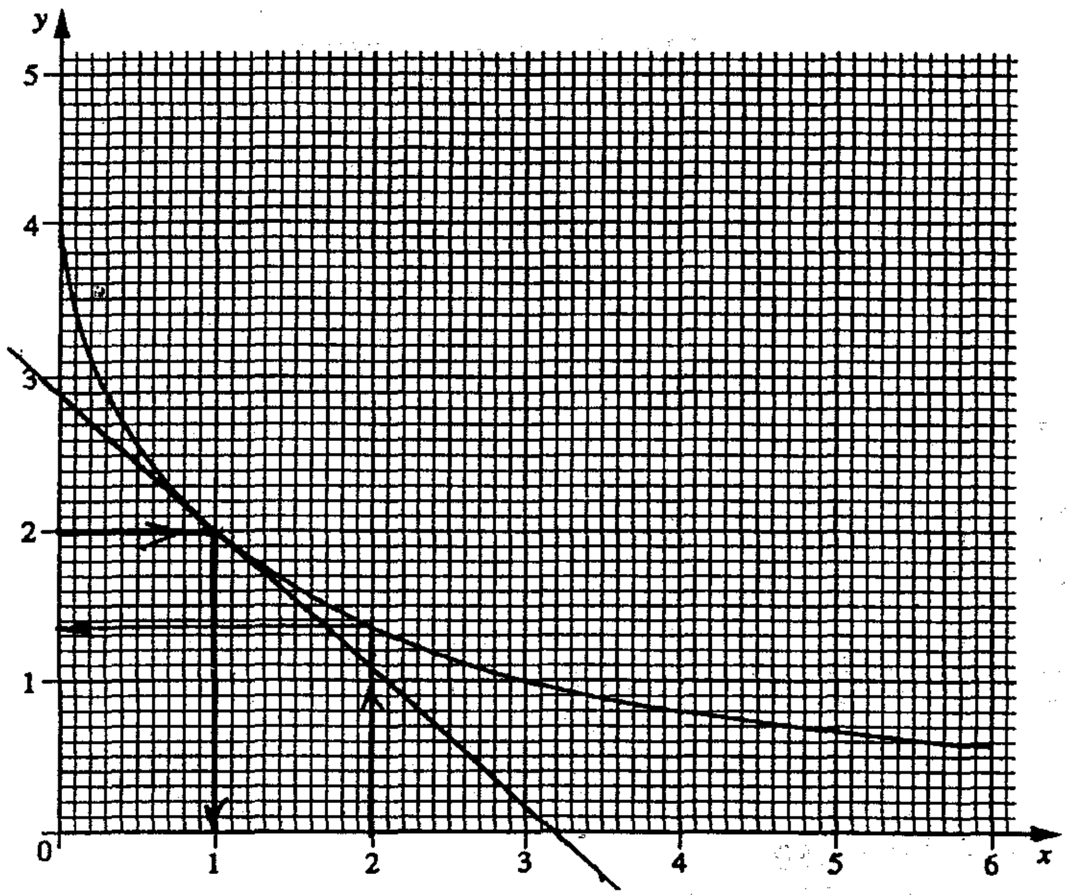
22- (a)



(b)



23-



(a)(i) f(2) means to find y when x = 2

from graph y = 1.35 f(2) = 1.35

(ii) y = f(x) $x = f^{-1}(y)$ so y = 2

from graph x = 1 $f^{-1}(2) = 1$

(b) Tangent is drawn on graph

Two points on the tangent are (0, 2.9) (3.2 , 0)

Gradient $= \dfrac{2.9-0}{0-3.2} = \dfrac{2.9}{-3.2} = -0.906 = -0.91$

Mathematics 0580

November 2000

Paper 2

1- 7 - 5 (6 − 1) = 7 - 5 (5) = 7 - 25 = -18

2- $g = \sqrt{h+i}$ $\qquad g^2 = h + i \qquad h = g^2 - i$

3- $\left(\frac{9}{4}\right)^{-\frac{3}{2}} = \left(\frac{4}{9}\right)^{\frac{3}{2}} = \frac{8}{27}$

4- $1\frac{1}{4} \div \frac{2}{3} - 1\frac{1}{3} = \frac{5}{4} \div \frac{2}{3} - \frac{4}{3}$

$= \frac{5}{4} \times \frac{3}{2} - \frac{4}{3} = \frac{15}{8} - \frac{4}{3} = \frac{45-32}{24} = \frac{13}{24}$

5- 36 - x + 7 x = 180

6 x = 180 - 36 = 144

$x = \frac{144}{6} = 24$

6- 21.65 ≤ Perimeter < 21.75

7.65 ≤ One side < 7.75

Smallest possible third side

= 21.65 - 2 (7.75)

= 6.15

7- 2 x - y = 81 (1)

x + 2 y = 23 (2)

(1) x 2 4 x - 2 y = 162

(2) x + 2 y = 23

5 x = 185

x = 37

from (2) 37 + 2 y = 23

2 y = - 14

y = -7

x = 37 y = -7

8- (a) time $= \dfrac{43.4}{2.8} = 15.5$ h

(b) Using calculator

20 [,,,] 40 [,,,] + 15.5 - 24 = shift [,,,] 1210

9- Sum of all interior angles $= (n-2) \times 180$

$= (5-2) \times 180 = 540$

$2t + 91 + 104 + 117 = 540$

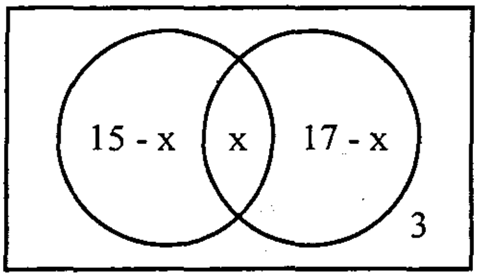
$2t = 228 \qquad t = 114°$

10-
$$7 - 5x \geq -17$$
$$-5x \geq -17 - 7$$
$$-5x \geq -24$$
$$x \geq \frac{-24}{-5}$$
$$x \geq 4.8$$
$$x \in \{1,2,3,4\}$$

11- (a) Number of stndents who study both physics and chemistry $= 15 + 17 + 3 - 22$

$= 13$

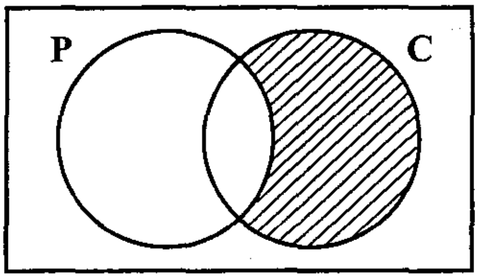
OR



$(15 - x) + x + (17 - x) + 3 = 22$

$35 - x = 22 \qquad x = 13$

(b)



$P' \cap C$

12- (a) Acceleration $= \dfrac{\textbf{Change in Velocity}}{\textbf{time}}$

$= \dfrac{14 - 10}{7 - 2} = \dfrac{4}{5} = 0.8 m / s^2$

(b) Distance = Area under the graph

= Area of $\triangle$ + Area of trapezium

$= \dfrac{1}{2} \times 2 \times 10 + \dfrac{10+14}{2} \times 5 = 10 + 60 = 70$ m.

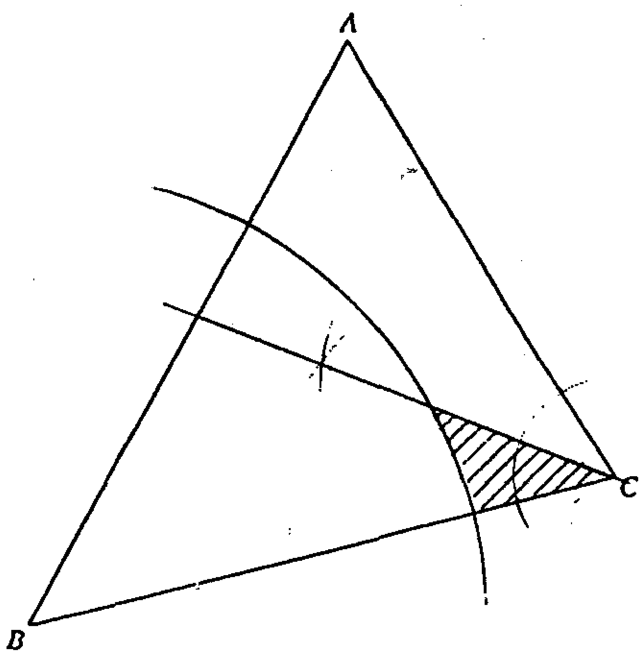
13- (a) (i) From graph at cumulative frequency of 25 height = 35 cm
Lower quartile = 35 cm
(ii) Upper quartile at cumulative frequency of 75
upper quartile = 52
Interquartile range = 52 – 35 = 17
(b) When height = 25 cm
Cumulative frequency = 10
number of plants with a height greater than 25 cm = 100 – 10 = 90

14- Bank charges = $\frac{1\frac{1}{2}}{100} \times 250 = 3.75$
number of Drachma received = (250 – 3.75) x 485 = 119431.25
= 119430 to the nearest 10

15- (a) $t^2 - 4 = (t+2)(t-2)$
(b) $at^2 - 4a + 2t^2 - 8$
$= a(t^2 - 4) + 2(t^2 - 4) = (t^2 - 4)(a + 2)$
$= (t+2)(t-2)(a+2)$

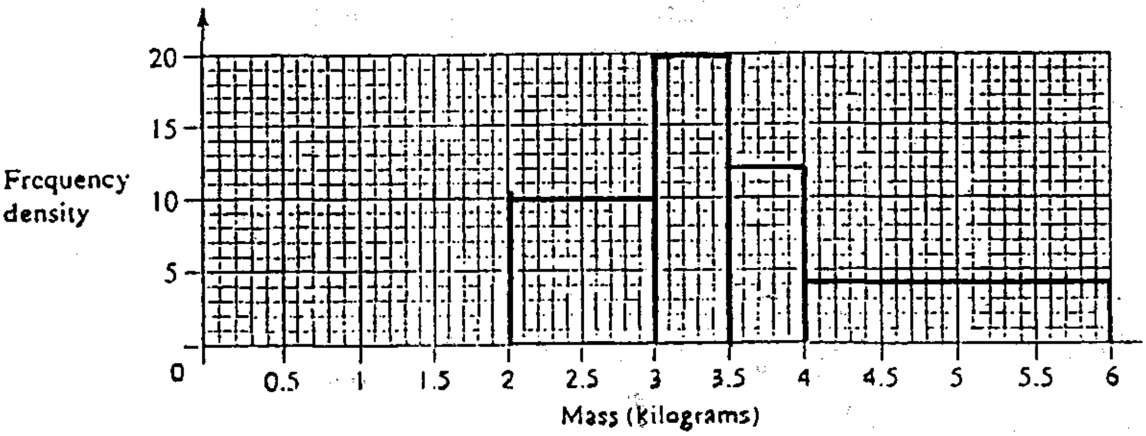
16- (a) $V \propto h^3$
$V = K h^3$
When h = 3 V = 6.75
$6.75 = K(3)^3$ $K = \frac{6.75}{27} = 0.25$
$V = 0.25 h^3$
(b) $V = 0.25 h^3 = 0.25 \times (2.5)^3 = 3.906\ cm^3$
$= 3.91\ cm^3$

17-



18-

Mass m	Frequency
2 $< m \leq 3$	10
3 $< m \leq 3.5$	$0.5 \times 20 = 10$
3.5 $< m \leq 4$	$0.5 \times 12 = 6$



19- (a) Reflection on the line $y = x$

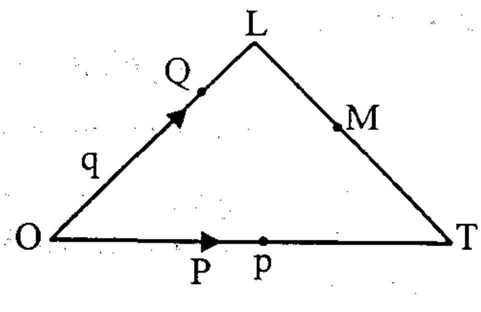
(b) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Matrix $= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

20- (a) (i) OQ : QL = 2 : 1

$\overrightarrow{OQ} = q$

$\overrightarrow{QL} = \frac{1}{2}q \qquad \overrightarrow{OL} = 1\frac{1}{2}q$



(ii) $\overrightarrow{OP} = \overrightarrow{PT}$

$\overrightarrow{OP} = P$

$\therefore \overrightarrow{OT} = 2P$

$\overrightarrow{LT} = -1\frac{1}{2}q + 2P = 2P - 1\frac{1}{2}q$

(b) $\overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM} = \overrightarrow{OL} + \frac{1}{2}\overrightarrow{LT}$

$= 1\frac{1}{2}q + \frac{1}{2}(2p - 1\frac{1}{2}q) = \frac{3}{2}q + P - \frac{3}{4}q = p + \frac{3}{4}q$

OR $\overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OL} + \overrightarrow{OT})$

$= \frac{1}{2}(\frac{3}{2}q + 2p) = p + \frac{3}{4}q$

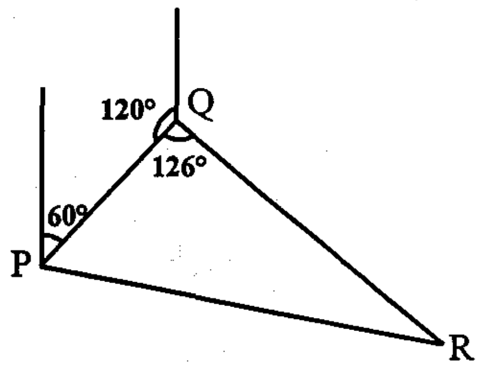
21- (a) $\dfrac{140}{\sin 31} = \dfrac{220}{\sin Q}$

$\text{Sin } Q = \dfrac{220 \sin 31}{140} = 0.8093$

$Q = 54° \quad \text{or} \quad 180 - 54$

$= 54° \quad \text{or} \quad 126°$

$\therefore \angle \text{ PQR } = 126° \text{ (obtuse)}$



(b) Bearing of R from Q

$= 360 - (120 + 126)$

$= 360 - 246$

$= 114°$

22- (a) $f\left(-\dfrac{3}{4}\right) = 3 - 2\left(-\dfrac{3}{4}\right) = 3 + \dfrac{3}{2} = 4\dfrac{1}{2}$

(b) $g(x) = \dfrac{x+1}{4}$

$y = \dfrac{x+1}{4}$ $\qquad\qquad 4y = x + 1$

$x = 4y - 1$

$g^{-1}(x) = 4x - 1$

(c) $fg(x) = f\left(\dfrac{x+1}{4}\right) = 3 - 2\left(\dfrac{x+1}{4}\right) = 3 - \dfrac{x+1}{2} = \dfrac{6-x-1}{2} = \dfrac{5-x}{2}$

23- (a) l_1 : $y = 1$

l_2 : using points (0, 0), (1, 2)

gradient $m = \dfrac{2-0}{1-0} = 2$

equation is $y = 2x$

l_3 : line joining (0, 5) and (5, 0) its equation is $x + y = 5$

(b) $y > 1$

$y \leq 2x$

$x + y \leq 5$

Mathematics 0580

June 2001

Paper 2

1- $\dfrac{7.7}{3+\sqrt{6.25}} = 1.4$

2- 1 cm = 1 x 250000 cm

$= \dfrac{250000}{100\times1000} = 2.5$ Km

3- (a) 12.6 Gigabytes = 12.6 x 10^9 bytes

= 1.26 x 10^{10} bytes

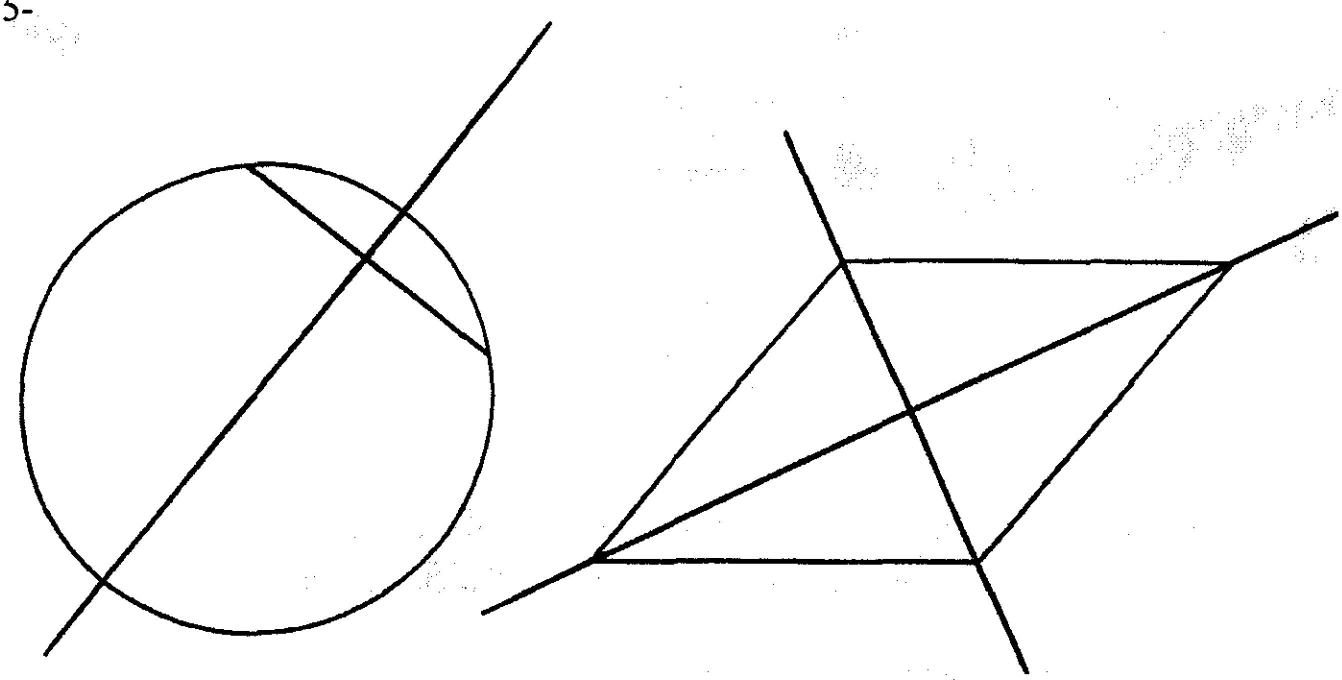
(b) 150 picoseconds = 150 x 10^{-12}

= 1.5 x 10^{-10} sec

4- $\tan\theta = \dfrac{30}{43}$

$\theta = 34.9°$

5-



6- 25 - 3x < 7

- 3x < 7 – 25

- 3x < -18

x > 6

7- Ratio of volume $= \left(\frac{100}{50}\right)^3 = 8$

Ratio 8 : 1

8- 2 hosepipes 2 h 30 min
3 hosepipes ?
It is inverse proportion so

The time $= \frac{2 \times 2h30\,\text{min}}{3}$

$= 1$ h 40 min

9- (a)

1999	2000
100	90
1320	?

tax paid in 2000 $= \frac{90 \times 1320}{100} = \$\ 1188$

(b)

1998	1999
100	110
?	1320

tax paid in 1998 = \$ 1200

10- $3x + 4y = 27$ (1)
$4x - 2y = 25$ (2)

(2) x 2 $8x - 4y = 50$
(1) $3x + 4y = 27$

adding $11x = 77$
$x = 7$

using $3x + 4y = 27$
$21 + 4y = 27$
$4y = 6$
$y = 1.5$

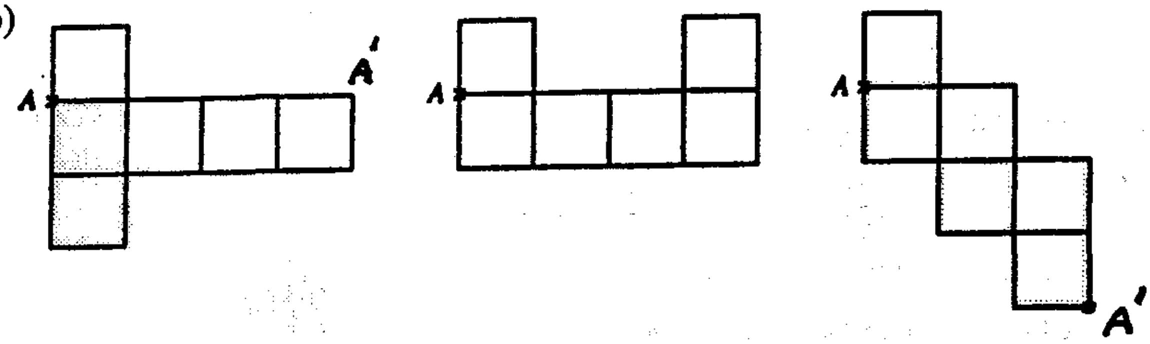
11- (a) (i) The minimum capacity of the jug is **3.45** litres.
(ii) The maximum capacity of the glass is **0.255** litres.

(b) Greatest number we can sure to fill $= \frac{3.45}{0.255} = 13.52$

Therefore the answer is 13

12- (a) Diagram 2

(b)



13- $x = \dfrac{4+\sqrt{y}}{3}$

$4 + \sqrt{y} = 3x$

$\sqrt{y} = 3x - 4$

$y = (3x-4)^2$

14- (a) angle 50 in the second quadrant

= 180 - 50 = 130

$\therefore$ x = 130°

(b) angle 50 in the third and fourth quadrant = 180 + 50 and 360 – 50

= 230° , 310°

x = 230 or x = 310

15- $\dfrac{4x-3}{8} - \dfrac{3x-4}{12}$

$= \dfrac{3(4x-3)-2(3x-4)}{24}$

$= \dfrac{12x-9-6x+8}{24} = \dfrac{6x-1}{24}$

16- (a) Time = 1116 - 0940 = 1.6 h

Length of the race = 30 x 1.6 = 48 Km

(b) Difference = 1.6 - 1 h 25 min 27 sec

= 10 min 33 sec

17- (a) The size of the interior angle of a regular hexayan $= 180 - \dfrac{360}{n}$

$= 180 - \dfrac{360}{6} = 120°$

Size of the interior angle of the n – sided polygon = 120 + 48 = 168

(b) exterior angle of the n – sided polygon = 180 - 168 = 12

$$n = \frac{360}{12} = 30$$

18- (a) area $= \frac{\theta}{360}\pi r^2$

$= \frac{40}{360} \times \pi \times 6^2 = 12.566\, cm^2$

answer area $= 12.6 cm^2$

(b) (i) area of one hole $= \pi r^2$

$= \pi(0.3)^2 = 0.2827 = 0.283 cm^2$

(ii) Area of the brooch = 12.566 - 4 x 0.2827

= 11.4 cm^2

19- (a) (i) $(x^2-1)(x^2+1) = x^4 - 1$

(ii) $x^2 - 1 = (x+1)(x-1)$

(b) $9999 = 3^2 \times 11 \times 101$

3	9999
3	3333
11	1111
101	101
	1

20- (a) $f(x) = \frac{x+1}{3x}$

(i) $f\left(\frac{3}{4}\right) = \frac{\frac{3}{4}+1}{3\times\left(\frac{3}{4}\right)} = \frac{\frac{7}{4}}{\frac{9}{4}} = \frac{7}{9}$

(ii) $gf\left(\frac{3}{4}\right) = g\left(\frac{7}{9}\right) = 3 - 3\left(\frac{7}{9}\right)$

$= 3 - \frac{21}{9} = \frac{6}{9} = \frac{2}{3}$

(b) g (x) = 3 – 3x

y = 3 – 3x

3x = 3 – y

$x = \frac{3-y}{3}$

$g^{-1}(x) = \frac{3-x}{3}$

$g^{-1}(18) = \frac{3-18}{3} = -5$

21- w = 90° (angle of a semicircle)

x = 20° (isosceles triangle)

y = 40° (exterior angle of a triangle)
z = 180° - $\angle D$
$\angle D$ = 90 – 40 = 50
z = 180 – 50 = 130°

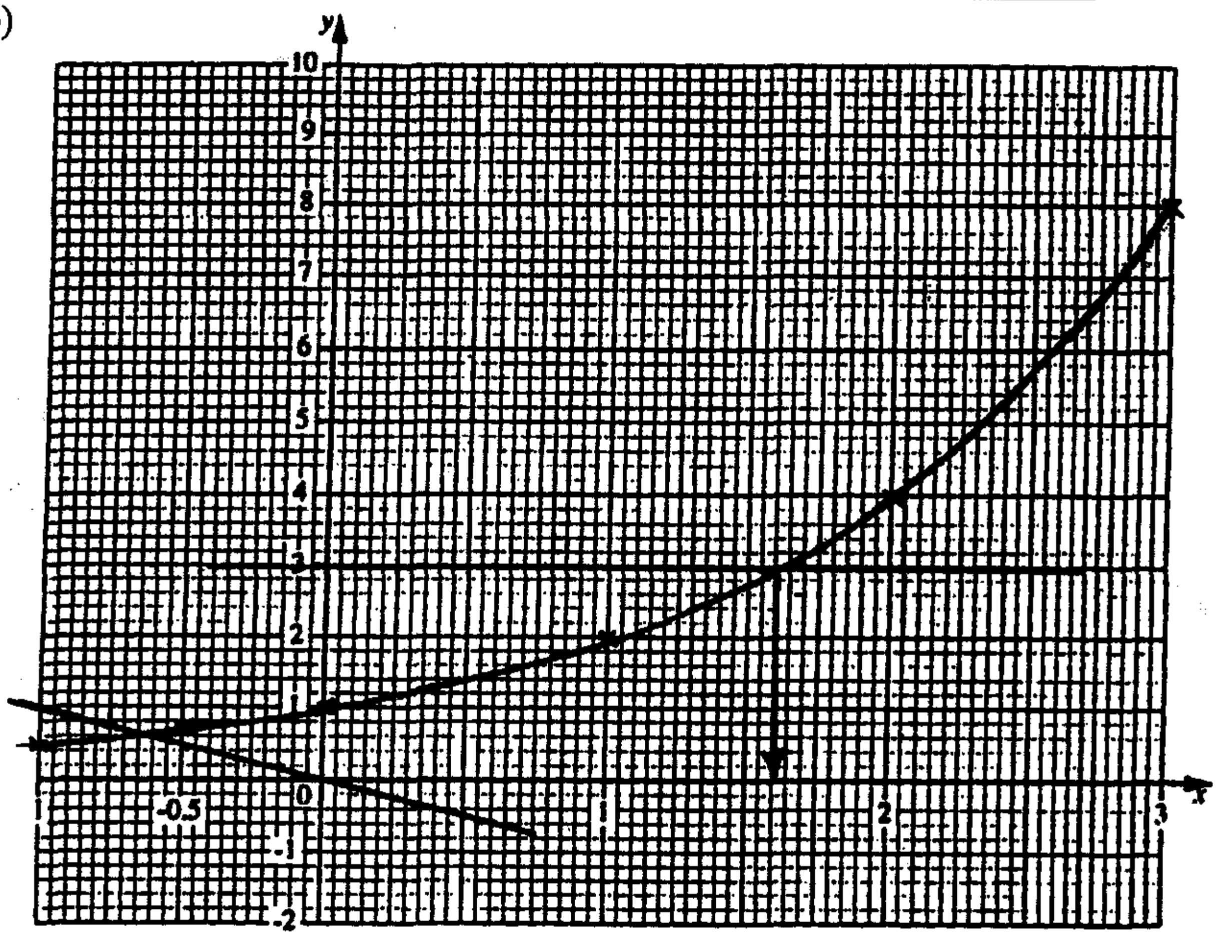
22- (a) (i) $\overrightarrow{WP} = \overrightarrow{WO} + \overrightarrow{OP} = -W + P = P - W$
(ii) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 3P + 3W$
(iii) $\overrightarrow{RV} = \overrightarrow{RW} + \overrightarrow{WV} = -3P + W = W - 3P$

(b) $\left|\overrightarrow{OB}\right| = \sqrt{(OA)^2 + (AB)^2} = \sqrt{15^2 + 15^2} = 21.2$

23- (a)

x	-1	-0.5	0	1	2	3
$f(x)$	***0.5***	***0.71***	***1.0***	***2***	***4***	***8***

(b)



(c) (i) $2^x = 3$
From graph y = 3 ∴ x = 1.6
(ii) Draw the line y = - x Using points (0 , 0) and (- 1 , 1)
The point of intersection with the curve is at x = - 0.6
Answer x = - 0.6

Mathematics 0580

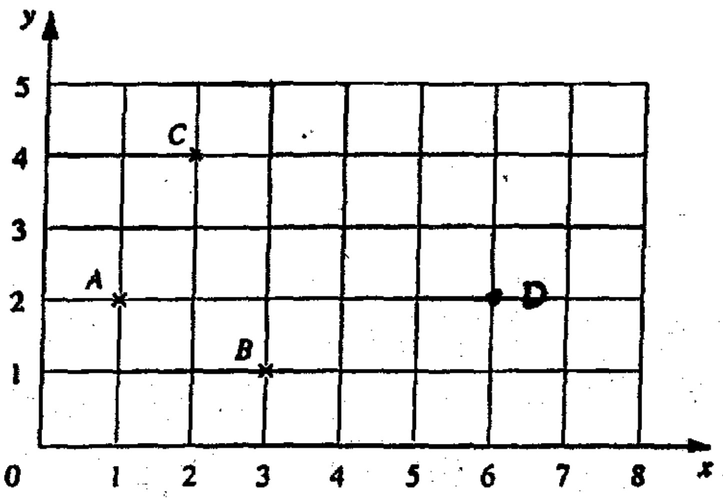
November 2001

Paper 2

1. 3.2 x 5 - 2 (4.1 – 2.9) = 13.6

2. 4 [°’’’] 39 [°’’’] + 17 [°’’’] 36 [°’’’] = 22° 15° ,:e 22 15 h

3.



$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$\therefore$ point D is (6 , 2)

$$\overrightarrow{CA} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

4. (a) Difference = $5.66 \times 10^{14} - 5.17 \times 10^{14}$
$= 4.9 \times 10^{14}$

(b) 530 nanometers = 530×10^{-9}
$= 5.3 \times 10^{-7}$

5. $4 - 4 \times \frac{1}{100} = 3.96$

3.96 [shift] [°’’’] 3° 57° 36 3 h 57 m 36 s

6. $I = \frac{PRT}{100}$

$I = 39 \qquad R = 4 \qquad T = \frac{9}{12}$

$$39 = \frac{P \times 4 \times \frac{9}{12}}{100} = \frac{3P}{100}$$

$$P = \frac{100 \times 39}{3} = 1300$$

7. 0.75 tonnes = 0.75 x 1000 = 750 kg
error = 750 - 650 = 100 kg
60 000 grams = 60 kg
error = 650 - 60 = 590 kg

8. (a) 1 kilobyte = 2^{10} bytes
= 8 x 2^{10} bits
= $2^3 \times 2^{10} = 2^{13}$ bits
x = 13

(b) 4 kilobytes = $4 \times 2^{13} = 2^2 \times 2^{13} = 2^{15}$
y = 15

9. $\angle x = 2 \times 70 = 140°$
$\angle y = 90 - 40 = 50°$
$\angle z = y - \left(\frac{180 - x}{2}\right)$
$= 50 - \left(\frac{180 - 140}{2}\right)$
$= 30°$

10. (a) $6x^2 + 6x = 6x(x + 1)$

(b) $6x^2 + 5x + 1 = (3x + 1)(2x + 1)$

11. Sam of inferior angles = $(n - 2) \times 180$
$= (5 - 2) \times 180 = 540$
$2x + 3x + 4x + 5x + 6x = 540$
$20x = 540$
$x = 27$
Smallest angle = 2x = 54°

12. (a) $\cos x = \frac{1}{2}$

shift $\cos \frac{1}{2} = 60°$

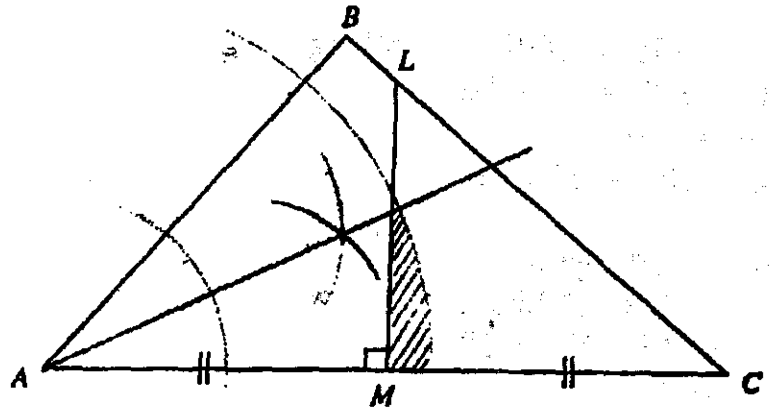
cosine is positive in the first and fourth quadrant

$\therefore$ x = 60° or 360 - 60

x = 60° or 300°

(b) from graph $270 < x < 300$

13.



14. $I = K\sqrt{P}$

$4 = K\sqrt{100}$ $K = 0.4$

(a) $I = 0.4\sqrt{P}$

(b) P = 144

$I = 0.4\sqrt{144} = 0.4 \times 12$

$= 4.8$

15. (a) smallest A C is 2.5 cm

(b) $\tan A = \frac{BC}{AC}$

Largest tan A = $\frac{\text{Largest BC}}{\text{Smallest AC}}$

$= \frac{10.5}{2.5} = 4.2$

Largest angle A = 76.6°

16. (a) Number of rands $= \frac{x}{24}$

(b) $\frac{x}{24} = 500 + 800$

$x = 24 \times 1300 = 31200$

17. (a)

Mass kg	$0 < x \le 2$	$2 < x \le 5$	$5 < x \le 9$	$9 < x \le 15$
Frequency	10	$3 \times 4 = 12$	$4 \times 3.5 = 14$	12

Frequency is the area

(b) Bar height $= \frac{12}{(15-9)} = \frac{12}{6} = 2$

A bar of height 2 units is drawn from x = 9 to x = 15

18. $y = \frac{3x}{2} + 5$

$y - 5 = \frac{3x}{2}$

$3x = 2(y-5)$

$x = \frac{2(y-5)}{3}$

19. (a) $\overrightarrow{OP} = 6P$

$\therefore \overrightarrow{OA} = \overrightarrow{AB} = \overrightarrow{BP} = 2P$

$\overrightarrow{OB} = 4P$

(b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= 2q - 4P$

(c) $\overrightarrow{AQ} = -2P + q = q - 2P$

(d) $\overrightarrow{BC} = 2\overrightarrow{AQ}$

$\therefore$ BC is parallel to AQ

20. (a) $\angle KTP = 70°$

(b) $\frac{KT}{\sin 35} = \frac{25}{\sin 70}$

$KT = \frac{25 \sin 35}{\sin 70} = 15.3$ m

21. (a) Diagonals bisect each other for figures A, B, C, D

Probability $= \frac{4}{5}$

(b) A, C, E

(c) Parallelogram (B)
Rhombus (C)
Rectangle (D)

22. (a) d = 42°
e = 74°
f = 64°

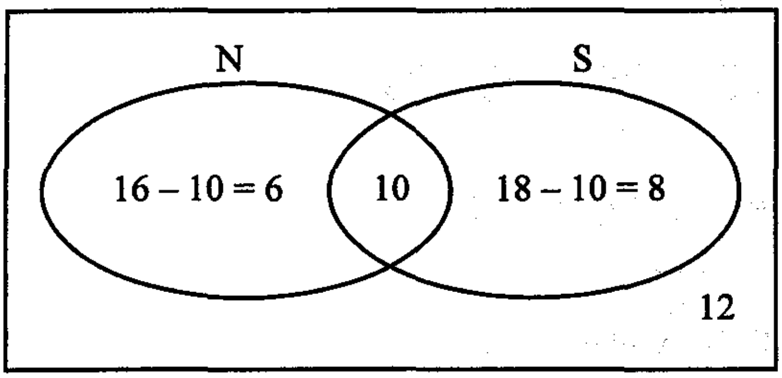
(b) 252°

23. (a) $\frac{1}{2} \times 36 = 18$

$\frac{4}{9} \times 36 = 16$

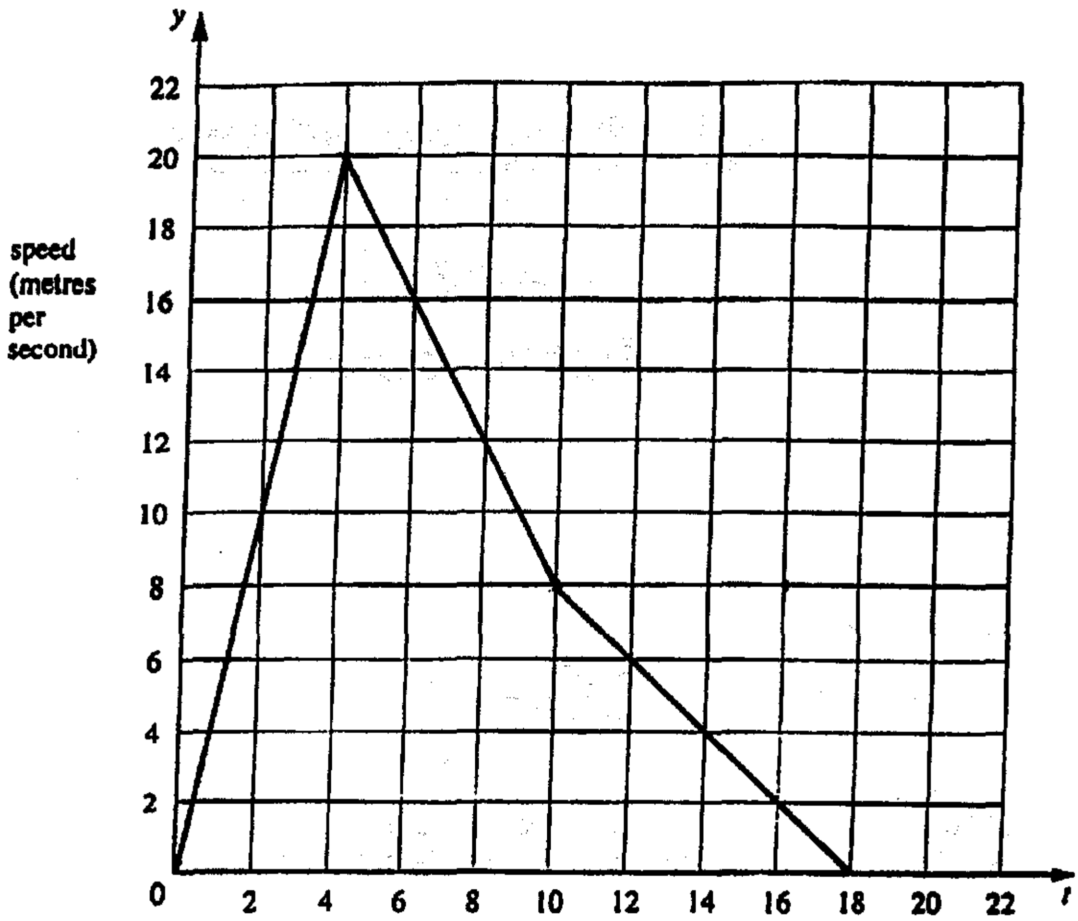
$\frac{1}{3} \times 36 = 12$

intersection of two sets $= 18 + 16 - (36 - 12) = 10$



(b) Probability $= \frac{10}{36} = \frac{5}{18}$

24. (a)



(b) Deceleration $d = \dfrac{\text{Change in speed}}{\text{Time}} = \dfrac{8}{8} = 1$

$d = 1$

(c) Distance = area of triangle = $\frac{1}{2} \times 4 \times 20 = 40$ m

Mathematics

Paper 2
June 2002

1. a) 4.15×10^8

   b) Average number = $\dfrac{4.15 \times 10^8}{5.26 \times 10^5 \times 79} = 9.987$

   $= 10$ times

2. a) 2008 (Luis is 19 and Hans 23)

   b) 1993 (nine years before, Hans is 8 and Luis is 4)

3.

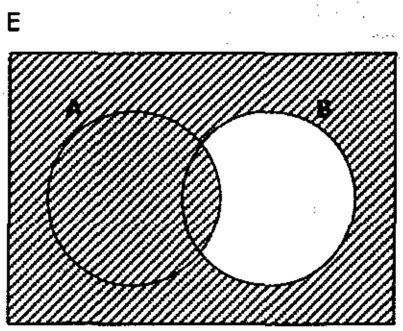


Diagram 1

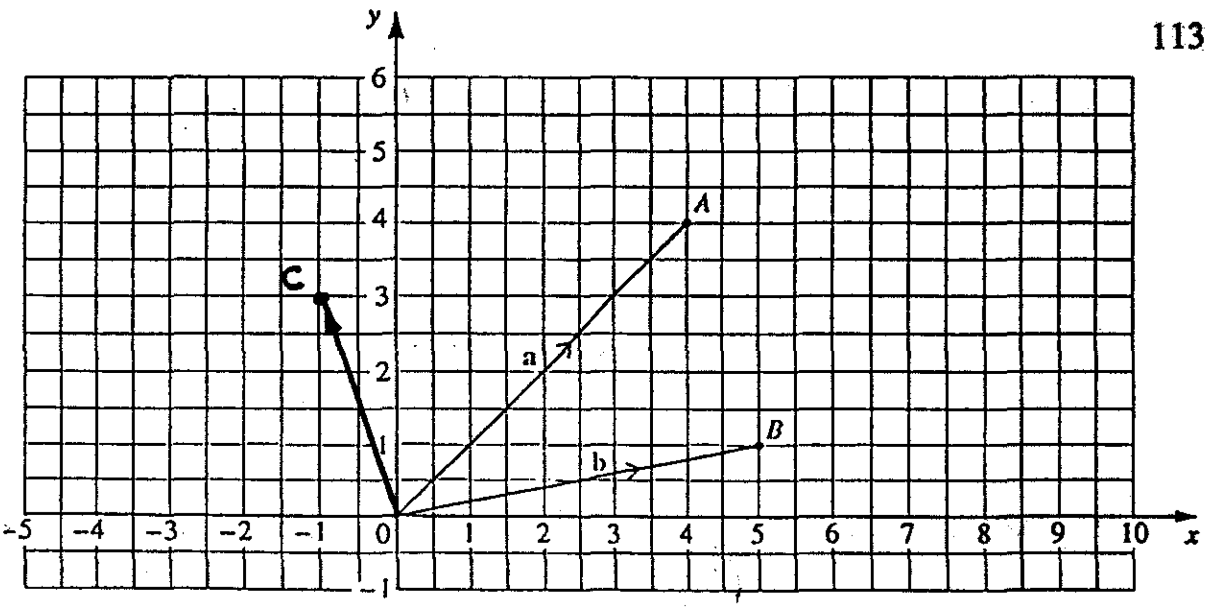
   b) $(C \cup D)'$ or $C' \cap D'$

4. $x = 544 - 20 = 34°$

   $y = 180 - 54 = 126°$

5. Length = $\sqrt{(5+1)^2 + (-4-4)^2} = 10$

6.



a) $\overrightarrow{OC} = a - b = \begin{pmatrix}4\\4\end{pmatrix} - \begin{pmatrix}5\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$

b) $\overrightarrow{AB} = b - a$

7. Decrease = 25 – 22 = 3

Percentage decrease $= \dfrac{3}{25} \times 100$

$= 12\%$

8. $3(x + 7) < 5x - 9$

$3x + 21 < 5x - 9$

$3x - 5x < -21 - 9$

$-2x < -30$

$x > 15$

9. a) Length of one rod ℓ

$9.5 \le \ell < 10.5$

Minimum Length = $3 \times 9.5 = 28.5$ cm

b) Minimum Area $= 28.5 \times 9.5$

$= 270.75$

$= 271\ cm^2$

10. a) $\dfrac{\text{Actual Area}}{\text{Model Area}} = (\text{Scale})^2$

$\dfrac{300}{\text{Model Area}} = (20)^2 = 400$

Model area $= \dfrac{300}{400} = 0.75\ m^2$

b) $0.75\ m^2 = 0.75 \times 100 \times cm^2$
$= 7500\ cm^2$

11. $T = \dfrac{5}{V+1}$

$TV + T = 5$

$TV = 5 - T$

$V = \dfrac{5-T}{T}$

$V = \dfrac{5}{T} - 1$

12. Sum of interior angles

$= (n-2) \times 180$
$= (7-2) \times 180$
$= 900$

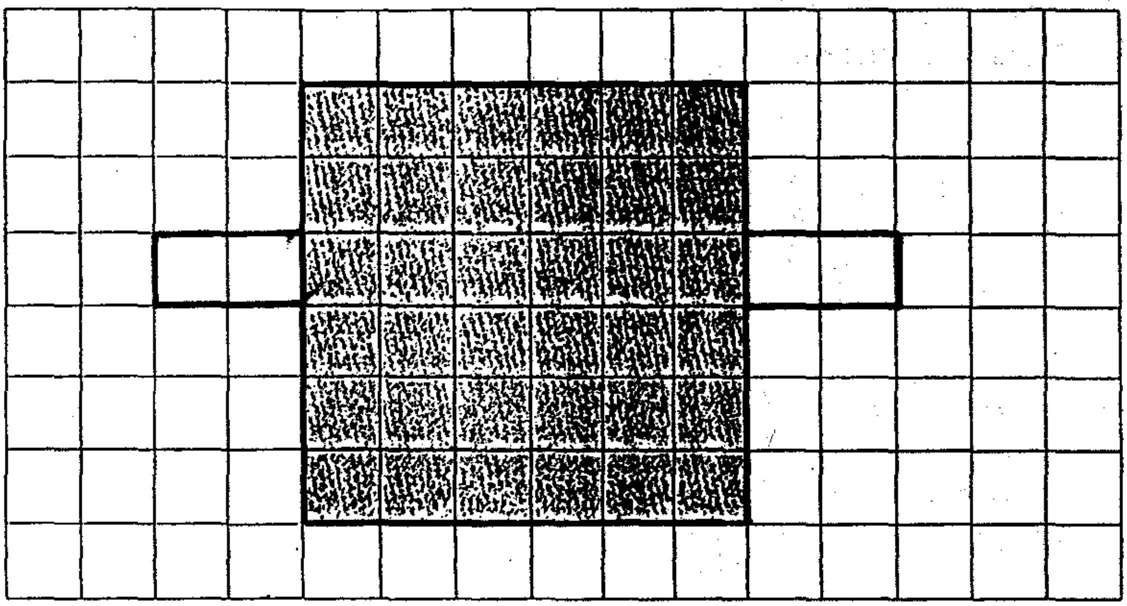
Total of all six equal angles

$= 900 - 90 = 810$

Size of one angle

$= \dfrac{810}{6} = 135°$

13.



b) $6 \times 1 \times 2 = 12\ cm^3$

c) $6 \times 6 + 2 \times 2 = 40\ cm^2$

14. a) $x^{-1} = \frac{1}{x} = \frac{1}{1/4} = 4$

$x^0 = 1$

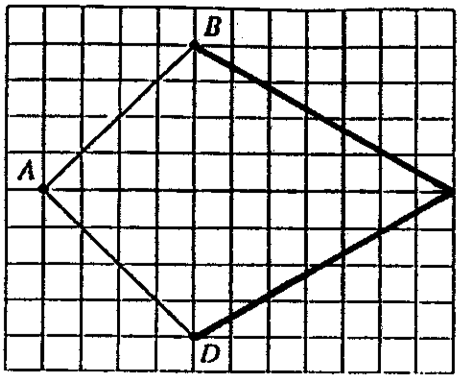
$x^{1/2} = \left(\frac{1}{4}\right)^{1/2} = \frac{1}{2}$

$x^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

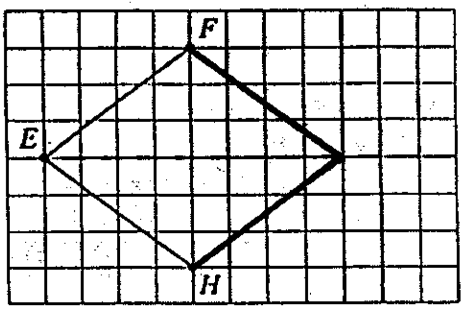
b) $y^3 < y^{-1} < y^0 < y^2$

15. a) ii) Kite
    b) ii) Two

(a)



(b)



16. a) i) $g(4) = 2(4)^2 - 5 = 27$

ii) $fg(4) = 27^{1/3} = 3$

b) $gf(x) = 2(x^{1/3}) - 5$

$= 2x^{2/3} - 5$

c) $y = x^{1/3}$

$y^3 = x$

$f^{-1}(x) = x^3$

17. a) $\Pi(4r)^2 = 16\,\Pi r^2$

b) $16\,\Pi r^2 - \Pi r^2 = 15\,\Pi r^2$

c) $2\,\Pi r + \Pi(4r) = 10\,\Pi r$

18. a) i) $\frac{800}{6.25} = \$\,128$

ii) $128 \times 6.45 = 825.6$

$825.6 - 800 = 25.6$

b) $I = \frac{PRT}{100} = \frac{800 \times 12 \times \frac{3}{12}}{100} = 24$

19. a) i) $-3 + 6x = -15$

ii) $6x = -12$

$x = -2$

b) Because $|c| = 0$

c) $A^{-1} = \frac{1}{(2)(5)-(-3)(-2)}\begin{pmatrix} 5 & 3 \\ 2 & 2 \end{pmatrix}$

$= \frac{1}{4}\begin{pmatrix} 5 & 3 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

20. a) i) $x^2 - 5x = x(x-5)$

ii) $2x^2 - 11x + 5$

$= (2x-1)(x-5)$

b) $\frac{x^2-5x}{2x^2-11x+5} = \frac{x(x-5)}{(2x-1)(x-5)} = \frac{x}{2x-1}$

21. a) $AC^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \cos 95$

$= 126.41$

$AC = 11.24 = 11.2m$

b) Area $= \frac{1}{2} \times 9 \times 6 \sin 95$

$= 26.9\ m^2$

22. a) Line ℓ, $C = 3$

Using points (0, 3) and (8, 7)

$m = \frac{7-3}{8-0} = \frac{1}{2}$

Equation is $y = \frac{1}{2}x + 3$

b) $y \geq \frac{1}{2}x + 3$

$x > 2$

$y \leq 7$

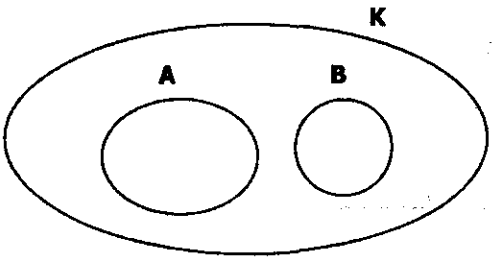
Mathematics

Paper 2
November 2002

1. a) $-1.5 - (2.5) = -4^\circ$ c
   b) Greatest is 3° C, least is -1.5° C
   Difference $= 3 - (-1.5) = 4.5^\circ$ C

2. Her loss $= 62 - 46 = 16$
   Percentage loss $= \frac{16}{62} \times 100 = 25.8\%$

3.



4. 1 Euro — 0.975 Pesos
   ? — 500
   Amount received $= \frac{500 \times 1}{0.975} = 512.82$ euros

5. $\frac{1}{1000} = 0.001,\ \frac{11}{1000} = 0.011,\ 0.11\% = \frac{11}{100}\% = 0.0011$
   $0.0108 = 0.0108$
   $\frac{1}{1000} < 0.011\% < 0.0108 < \frac{11}{1000}$

6. $2x - \frac{10x}{5-x} = \frac{2x(5-x)-10x}{5-x}$

$= \frac{10x - 2x^2 - 10x}{5-x} = \frac{-2x^2}{5-x}$

OR $\frac{-2x^2}{x-5}$

7. a) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

b) $\left(1\frac{7}{9}\right)^{½} = \left(\frac{16}{9}\right)^{½} = \frac{4}{3}$

8. Distance = 380 correct to the nearest 10m ($\frac{10}{2} = 5$)

$375 \leq$ Distance < 385

Speed is 3.9 m/s to 1 d.p.

$3.85 \leq$ speed ≤ 3.95.

Time = $\frac{\text{Distance}}{\text{Speed}}$

Greatest Time = $\frac{\text{Greatest distance}}{\text{Least speed}}$

$= \frac{385}{3.85} = 100\text{sec}$

9. a)

b) Rectangle

10. a) Numbers are 3 5 7 8 8 8

Mode = 8

b) Median = $\frac{7+8}{2} = 7.5$

c) Mean = $\frac{\Sigma x}{n} = \frac{39}{6} = 6.5$

11. Circumference $= 2\pi r = 2 \times 6.4 \times 10^6$
$= 4.02 \times 10^7$
$= 4.0 \times 10^7$ m

12. a) Angle EDC = 180 – 109 = 71°
b) Angle C = 180 – 7 1= 109
Total sum of angles $(6-2) \times 180 = 720$
95 + 109 + 71 + 109 + 2x = 720
($\angle A = \angle B = x$)
384 + 2x = 720
336 = 2x
x = 168°
$\angle FAB = 168°$

13. a) Deceleration $= \dfrac{\text{Change in velocity}}{\text{Time}}$

$= \dfrac{4-0}{2.5} = 1.6 \text{ m/s}^2$

b) Total distance = Area under the graph

$= \dfrac{3.5+6}{2} \times 4 = 19\text{m}$

14. a) $\angle PQS = \angle PRS = 80°$
b) In ΔPQX, sum of angles equal to 180°
$80 + 33 + \angle QPX = 180$
$\angle QPX = 67°$
c) $\angle PSQ = 33 - 21 = 12°$

15. First equation × 3
I.e. (4x + 5y = 0) × 3
12x + 15y = 0
Second equation 8x – 15y = 5
Adding 20x = 5

$x = \dfrac{1}{4}$

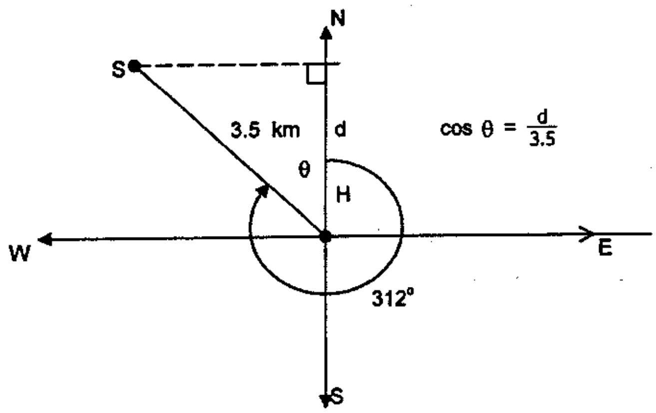
4x + 5y = 0

$4\left(\dfrac{1}{4}\right) + 5y = 0$

5y = -1

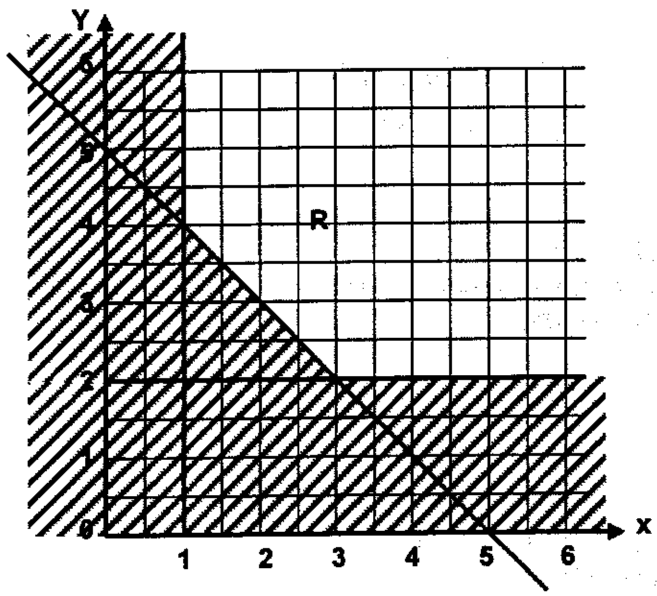
$y = -\dfrac{1}{5}$

16. a)

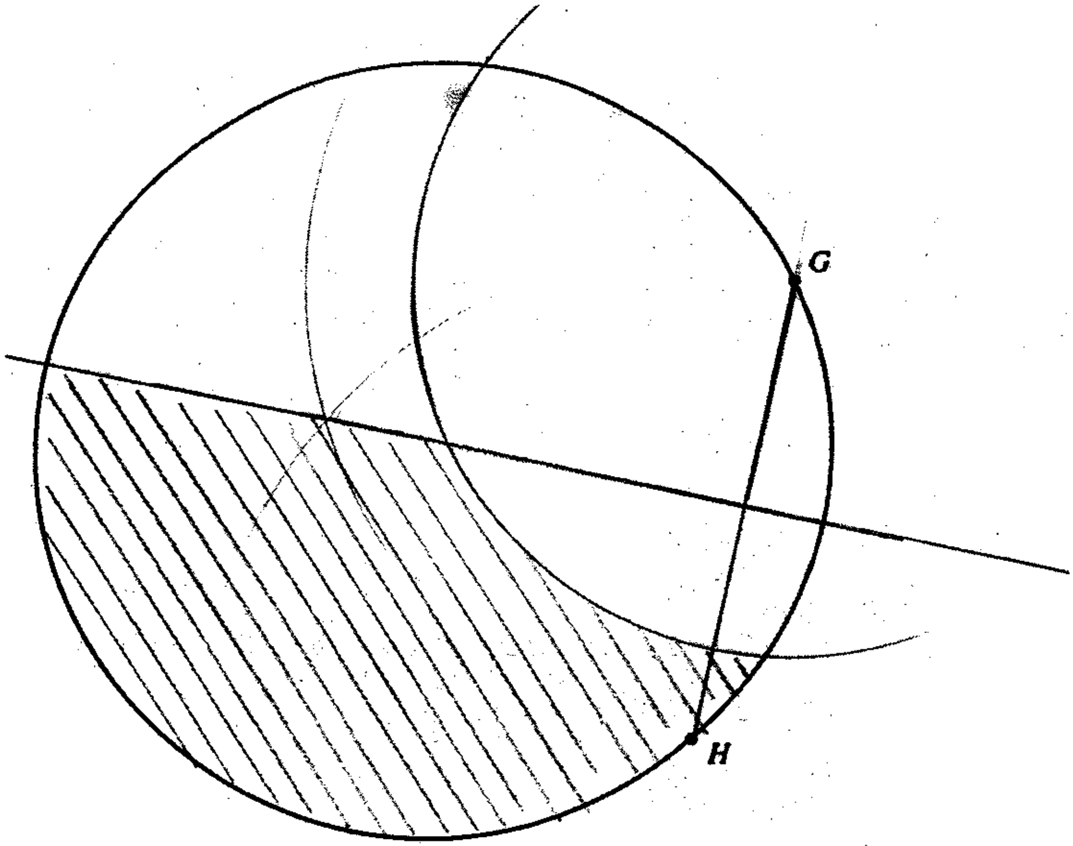


b) $\theta = 360 - 312 = 48^{o}$
Distance the ship is north of the harbor $H = 3.5 \cos \theta$
$= 2.34$ km

17.



18.



19. a) $5 - \frac{2x}{3} > \frac{1}{2} + \frac{x}{4}$

All by 12

$60 - 8x > 6 + 3x$

$60 - 6 > 8x + 3x$

$54 > 11x$

$x < \frac{54}{11}$

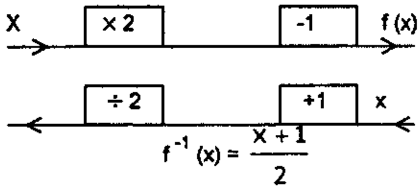
$\therefore x < 4\frac{10}{11}$

b) Positive integers of x are {1, 2, 3, 4}

20. a) $y = 2x - 1$ OR $y + 1 = 2x$

$x = \frac{y+1}{2}$

$f^{-1}(x) = \frac{x+1}{2}$



b) $g\,f(x) = (2x-1)^2 - 1$

$= 4x^2 - 4x + 1 - 1$

$= 4x^2 - 4x$

21. a) $A + P = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$P = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -1 & -1 \end{pmatrix}$$

b) Q is the inverse of A

$$Q = A^{-1} = \frac{1}{2 \times 1 - (-1)(1)} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 & 1/3 \\ -1/3 & 2/3 \end{pmatrix}$$

22. a) KM = ½ KL = 4cm

$OK^2 = 4^2 + 3^2 = 25$

$\therefore$ OK = 5cm

b) Since chord PQ is also 8cm, therefore the distance OR = OM = 3cm.

In Δ MOR, $\cos \angle ROM = \dfrac{3^2 + 3^2 - (5.5)^2}{2 \times 3 \times 3}$

Cos $\angle ROM = -0.6806$

$\angle ROM = 132.9^\circ$

Mathematics 0580

June 2003

Paper 2

1- $\frac{5}{98} = 0.051 \quad \& \quad 0.049 \quad \& \quad 0.05$

$$0.049 < 5\% < \frac{5}{98}$$

2- (a) From graph ⟶ 5 £ ⟶ 7.9 to 8 €

(b) € 9 ⟶ £ 5.65
€ 90 ⟶ £ 56.5

3- $S = 54 \frac{m}{sec} = \frac{54 \times 60 \times 60}{1000} = 194.4 \frac{Km}{hour}$ or 194 Km / h

---

4- $3a - 2b = 3 \begin{pmatrix} 2 \\ -3 \end{pmatrix} - 2 \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \end{pmatrix} - \begin{pmatrix} 10 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -7 \end{pmatrix}$

5- 2 : 17 : 18
students = 665
Teachers $= \frac{2}{(17+18)} \times 665 = 38$

6- $18 - 0.5 \leq \ell < 18 + 0.5$
$17.5 \leq \ell < 18.5$ ⟶ (1)
$12 - 0.5 \leq W < 12 + 0.5$
$11.5 \leq W < 12.5$ ⟶ (2)
smallest area $= 17.5 \times 11.5 = 201.25\ m^2$

7- $3 < 2x - 5 < 7$
$8 < 2x < 12 \qquad 4 < x < 6$

8-

3	9	27
11	121	1331
14	196	2744
- 7	49	- 343

9- A = (-1, 1) , B = (5, 2)

(a) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-1}{5-(-1)} = \dfrac{1}{6}$

(b) Angle = θ $\tan\theta = \dfrac{1}{6}$ $\theta = \tan^{-1}\dfrac{1}{6}$

$$\boxed{\theta = 9.5°}$$

10- $\dfrac{2}{x-3} - \dfrac{1}{x+4} = \dfrac{2(x+4)}{(x-3)(x+4)} - \dfrac{(x-3)}{(x-3)(x+4)}$

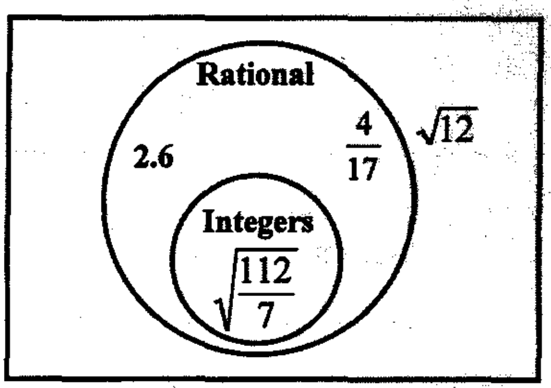
$= \dfrac{2x+8-x+3}{(x-3)(x+4)} = \dfrac{x+11}{(x-3)(x+4)}$

11- $2.6 = \dfrac{26}{10}$ rational

$\dfrac{4}{17}$ rational

$\sqrt{12}$ irrational

$\sqrt{\dfrac{112}{7}} = 4$ integer



12- (a) $\angle ABC = 90°$

$\therefore 2P + 3P = 90$

$5P = 90$

$P = \dfrac{90}{5}$ $\therefore P = 18$

(b) $q + 5q = 180°$

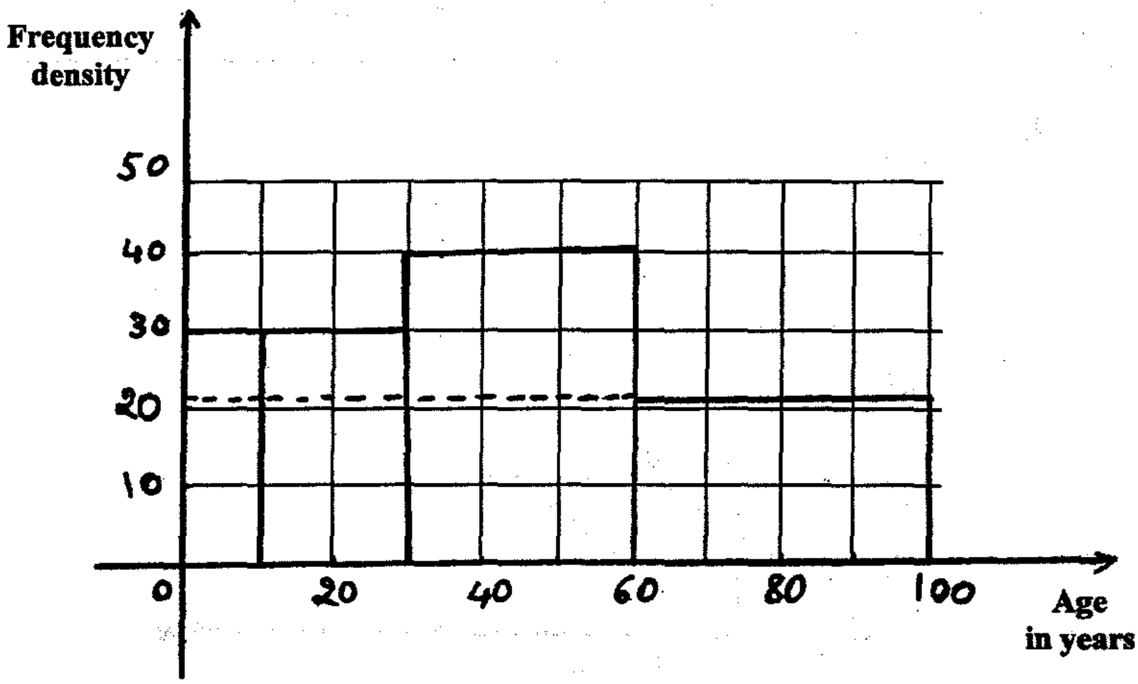
$6q = 180$

$q = 30°$

13- (a) $3\ cm^2 \longrightarrow 300$ patients
$1\ cm^2 \longrightarrow 100$ patients

(b) $30 \le x < 60 \longrightarrow 12$ squares
patients $= 12 \times 100 = 1200$

(c) $10 \le x < 30 \longrightarrow$ patients $= 600$ (6 squares)
$60 \le x < 100 \longrightarrow$ patients $= 880$ (8.8 squares)
$\frac{8.8}{4} = 2.2$ (height)



14- (a) $$\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix} = \begin{pmatrix} 10 & 17 & 4 \\ -6 & -9 & 0 \end{pmatrix}$$

(b) $A = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$ $\qquad A^{-1} = \frac{1}{|A|}\begin{pmatrix} -2 & -4 \\ 3 & 5 \end{pmatrix}$

$|A| = -10 - (-12) = 2$

$$A^{-1} = \frac{1}{2}\begin{pmatrix} -2 & -4 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 3/2 & 5/2 \end{pmatrix}$$

15- (a) % Increase $= \frac{7087000 - 4714900}{4714900} \times 100$

$= 50.3\ \%$

(b) (i) 4714900 ⟶ 4710000

(ii) $7087000 = 7.087 \times 10^6$

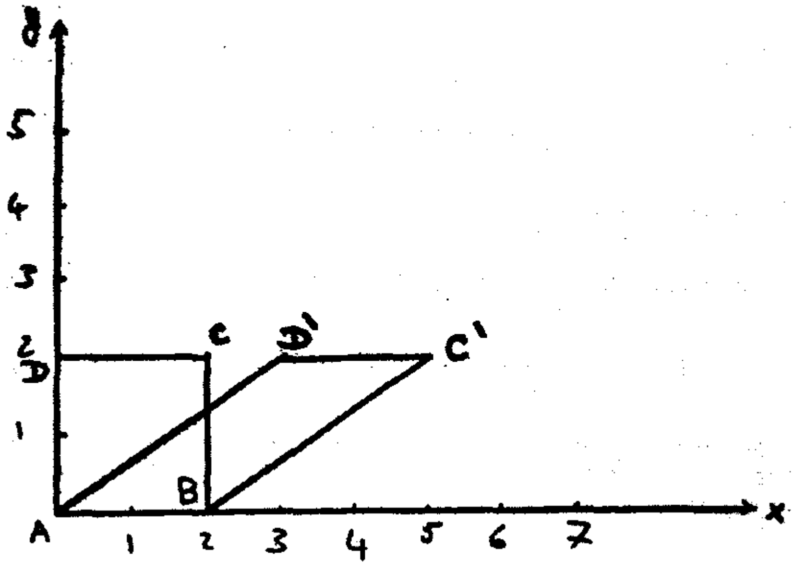
16- (a) BC = ? $\sin 18 = \frac{BC}{80}$

$BC = 80 \sin 18 = 24.7$ m

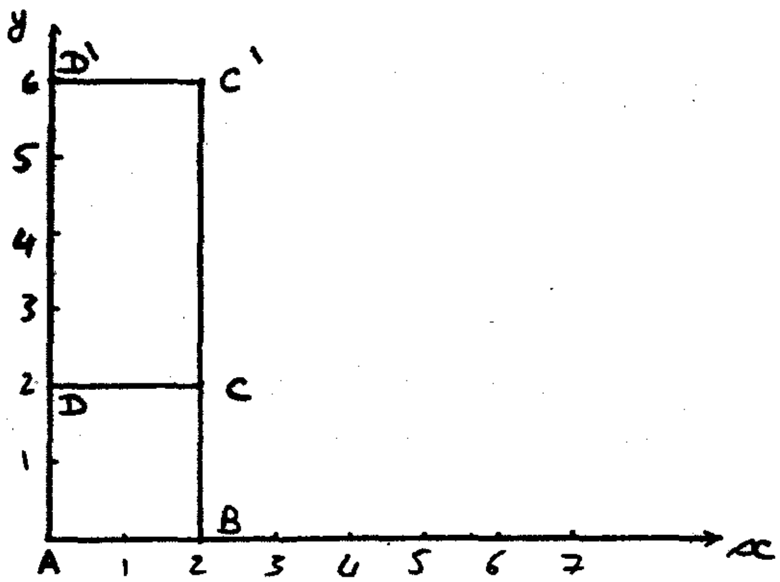
(b) $s = t(p + qt)$

$s = 3(4 + (3.8 \times 3))$ $s = 46.2$ m

17- (a)



(b) (i)



(ii) Matrix of stretch :

$K = 3$

$$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

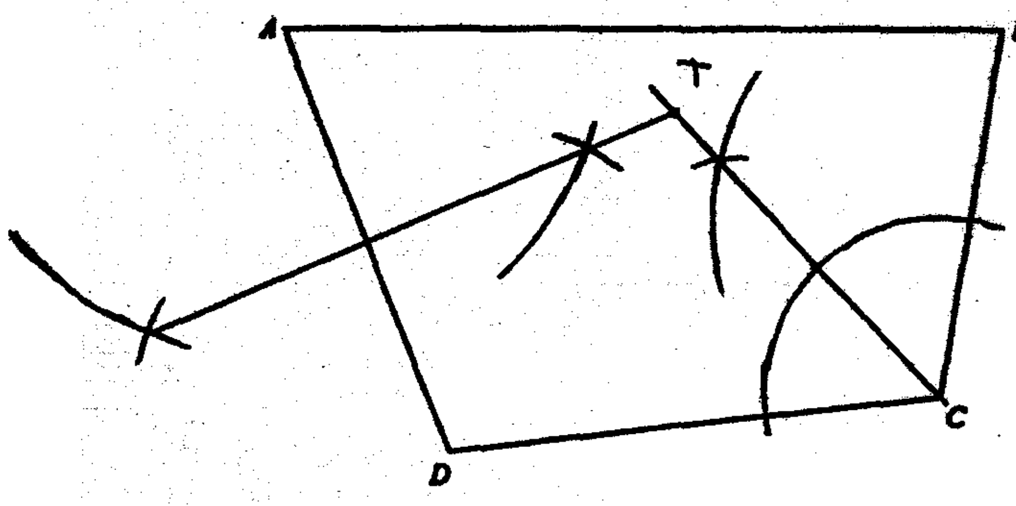
18- (a) AB = 9.6 cm

9.6 cm 100 m = 10 000 cm

$$1 \text{ cm} \quad \frac{10000}{9.6} = 1041.6 \approx 1042$$

m = 1042

(b)



19- cars = x

trucks = y

(a) $20x + 80y \leq 3600$

$x + 4y \leq 180$

(b) $25x + 50y = 3000$

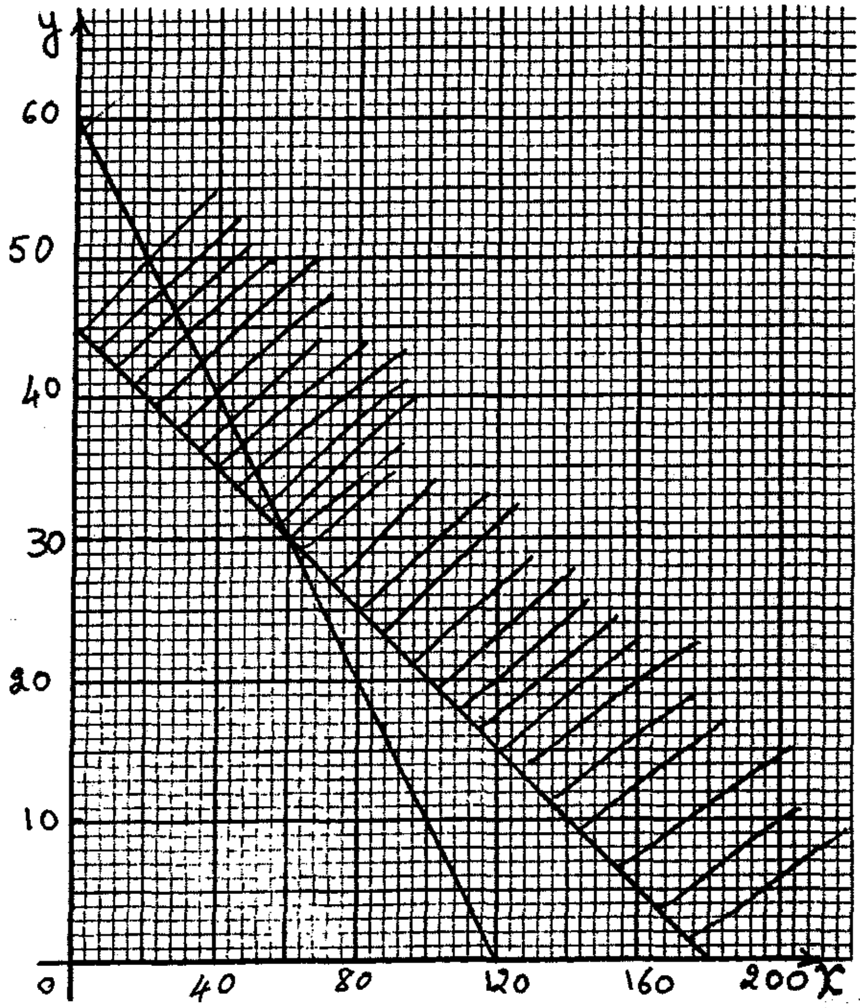
$x + 2y = 120$

(c) (i) $x + 2y = 120$ $\quad$ $x = 0$ $\quad$ $2y = 120$

$y = 60$

point (0 , 60)

$y = 0$ $\quad$ $x = 120$ $\quad$ point (120 , 0)



(ii) $x + 2y = 120$ & $x + 4y \leq 180$

to find point of intersection of the two lines

$x + 2y = 120$ (1)

$x + 4y = 180$ (2)

$-x - 2y = -120$

multiply (1) by (- 1)

adding $2y = 60$

$y = 30$ trucks

and $x = 60$ cars

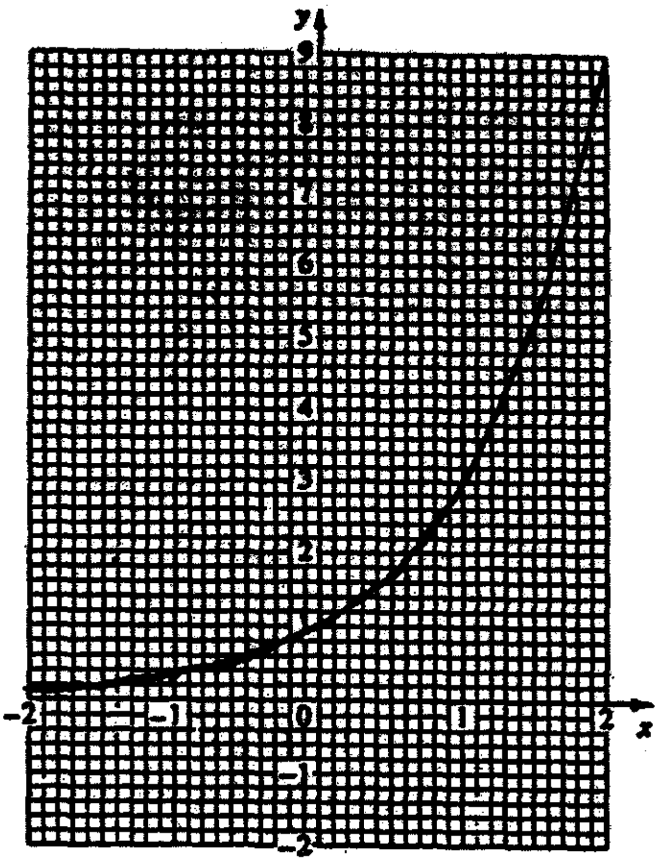
other possible points on the line $x + 2y = 120$

80 , 20 and 100 , 10 and 120 , 0

20- (a)

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	**0.1**	0.2	**0.3**	**0.6**	**1**	**1.7**	**3**	5.2	9

(b)



(c) Horizontal line at $y = 6$ intersect the graph at $x = 1.6$

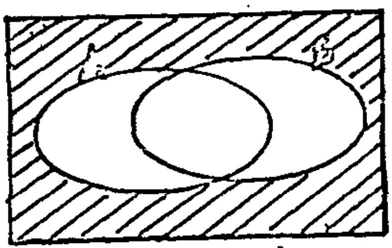
Answers to Examination Paper

4

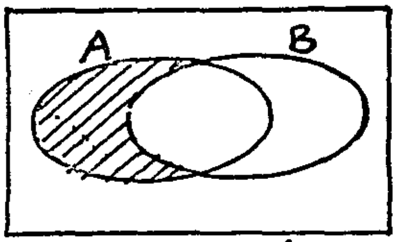
June 1993

Paper 4

1- (a)



$(A \cup B)'$



$A \cap B'$

(b) $P \cup Q'$ or $(P' \cap Q)'$

(c) (i) 50

(ii) $35 + 25 = 60$

(iii) Probability =

$$\frac{\text{Number of students speaking more than one language}}{\text{Total Number of students}}$$

$$= \frac{40 + 35}{150} = \frac{1}{2}$$

(iv) Probability =

$$\frac{\text{Number of girls speaking more than one language}}{\text{Total number of girls}}$$

$$= \frac{40}{90} = \frac{4}{9}$$

(v) Probability =

$$\frac{\text{Number of girls speaking more than one language}}{\text{Number of students speaking more than one language}}$$

$$= \frac{40}{40 + 35} = \frac{40}{75} = \frac{8}{15}$$

(vi) Number of boys = 35 + 25 = 60

Probability = $\frac{60}{150} \times \frac{59}{149} = \frac{118}{745}$

2- (a) (i) Area = $\frac{71+110}{2} \times 110 = 9955$

Area = 9960 m^2 correct to 3.S F

(ii) $\cos 50° = \frac{AD}{150}$

AD = 150 cos 50° 96.4 m

(iii) $\sin 50° = \frac{AB}{150}$

AB = 150 sin 50° = 115 m

(iv) $\frac{BC}{\sin 50} = \frac{150}{\sin 100}$

$BC = \frac{150 \sin 50}{\sin 100} = 117$ m

(b) Area of Δ ADB = ½ x AD x BD sin 50°

= ½ x 96.4 x 150 sin 50 = 5539

OR ½ AD x AB = ½ x 96.4 x 115 = 5543

Area of Δ BDC = ½ x BD x BC sin B

< DBC = 180 − (100 · 50) = 30°

Area of Δ BDC = ½ x 150 x 117 sin 30 · 4388

Total area = 9960 − 5539 · 1388 1988

· 20000 correct to 2 S.F

3- (a) (i) Volume = 150 x 100 x 80 = 1200000 cm^3 1200 Litres

(ii) Volume of water per sec · 2.1 x 35 = 73.5 cm^3/s

time = $\frac{1200000}{73.5}$ 16326.53 sec

$= \frac{16326.53}{3600} = 4.535$ h = 4 h 32 min

(b) (i) $V = \frac{1}{3} \times 3.142 \times 8^2 \times 10 + \frac{2}{3} \times 3.142 \times 8^3 = 1740 \text{ cm}^3$

(ii) $V = \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$V = \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

$3V - 2\pi r^3 = \pi r^2 h \Rightarrow h = \frac{3V - 2\pi r^3}{\pi r^2}$

4- (a) (i) $p = 6$, $q = 2$, $r = 1.2$

(b) gradient $= -\frac{6}{4} = -1\frac{1}{2}$ (from diagram)

or using points (0 , 6) and (4 , 0) , gradient $= \frac{6-0}{0-4} = -1\frac{1}{2}$

(c) (i) $k = \frac{1}{5} = 0.2$, $l = \frac{9}{5} = 1.8$, $m = \frac{25}{5} = 5$

(d) (i) x coordinate is 3.1

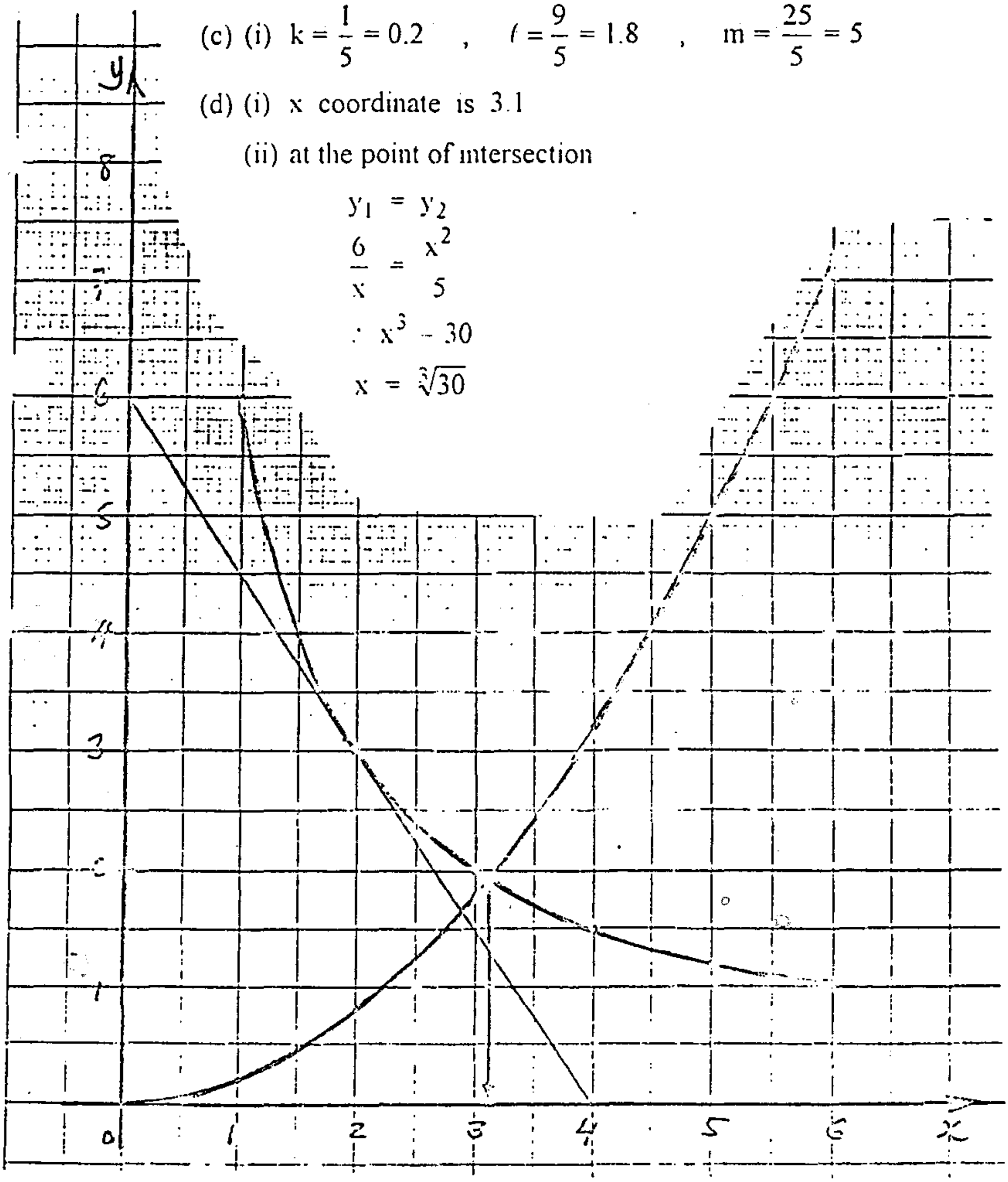
(ii) at the point of intersection

$$y_1 = y_2$$

$$\frac{6}{x} = \frac{x^2}{5}$$

$$\therefore x^3 = 30$$

$$x = \sqrt[3]{30}$$



5- (a) $\frac{1}{2} \times 100 = 50$

When $y = 50$ x is 44 for graph A and 56 for B

Median for city A is 44 and for city B is 56

(b) For city A Lower quartile (at $y = 25$) equal 26

Upper quartile (at $y = 75$) equal 58

interquantile range $= 58 - 26 = 32$

(c) (i) at $x = 20$, $y = 18000$

Number of people less than 20 years old are 18000

(ii) at $x = 20$, $y = 3000$

Number of people less than 20 years old are 3000

(d) (i) $x = 70$, $y = 91000$ (less than 70 years)

Number of people at least 70 $= 100000 - 91000$

$= 9000$

(ii) $x = 70$, $y = 76000$

Number of people at least 70 $= 100000 - 76000$

$= 24000$

(e) City A will have the larger population since :

(1) City B has more people over 70 years

<u>OR</u> (2) City A has more people of younger age (less than 20 years).

6- (a) (i) angle x is the same (verticaally opposite)

angle R equal angle P and angle S equal angle Q since SR is parallel to PQ

(ii) Δ's RSX and PQX are similar

$$\therefore \frac{RS}{PQ} = \frac{SX}{QX} = \frac{RX}{PX}$$

$$\frac{4}{7} = \frac{2}{QX} = \frac{3}{PX}$$

$$\therefore QX = \frac{2 \times 7}{4} = 3.5$$

$$PX = \frac{3 \times 7}{4} = 5.25$$

(iii) Since triangles are similar

Ratio of areas = square of ratio of sicles

$$\frac{2.9}{\text{area of } \Delta \text{ PQX}} = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$$

Area of triangle PQX $= \dfrac{2.9 \times 49}{16} = 8.9 \text{ cm}^2$

(iv) $\cos \angle R = \dfrac{3^2 + 4^2 - 2^2}{2 \times 3 \times 4} = \dfrac{21}{24}$

$\angle R = 29°$

OR Area of triangle $= 2.9 = \frac{1}{2} \times 4 \times 3 \sin \angle R$

$\sin R = \dfrac{2 \times 2.9}{12} \Rightarrow \angle R == 29°$

(b) (i) $\angle CDE = \angle ABC = 65°$

OR $\angle CDE = 180 - \angle CBE = 180 - (180 - 65) = 65°$

(ii) $\angle BED = 65 + 19 = 84$ exterior angle of Δ

$\angle BCD = 180 - 84 = 96$

$\angle CBD = 180 - (80 + 65) = 35°$

$\angle CDB = 180 - (96 + 35) = 49°$

(iii) $\angle BEC = \angle BDC = 49°$

$\angle BCE = 65 - 49 = 16°$

7- (a) (i) $(x+5)^2 = x^2 + (x+2)^2$

$$x^2 + 10x + 25 = x^2 + x^2 + 4x + 4$$

$$-x^2 + 6x + 21 = 0$$

$$x^2 - 6x - 21 = 0$$

(ii) $x = \dfrac{6 \pm \sqrt{36+84}}{2} = \dfrac{6 \pm 10.95}{2}$

$$= 8.84 \qquad -2.48$$

(iii) hypotenuse $= x + 5$

$$= 8.48 + 5 = 13.48$$

(using only the positive root of x)

(b). (i) time $= \dfrac{y}{3}$

(ii) total time $= \dfrac{y}{3} + \dfrac{y+3}{4}$

$$\therefore \frac{y}{3} + \frac{y+3}{4} = 4 \text{ h } 50 \text{ min} = 4\frac{50}{60}$$

$$\frac{4y + 3y + 9}{12} = 4\frac{5}{6} = \frac{29}{6} = \frac{58}{12}$$

$$\therefore 7y = 58 - 9 = 49 \Rightarrow y =$$

(iii) total distance $= y + y + 3$

$$= 7 + 7 + 3 = 17 \text{ km}$$

8- (b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -3 \\ 2 & 5 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 & -3 \\ -2 & -5 & -5 \end{pmatrix}$

(d) Reflection on x axis

point $(1, 0) \rightarrow (1, 0)$

$(0, 1) \rightarrow (0, -1)$

Matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(e) $NM = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

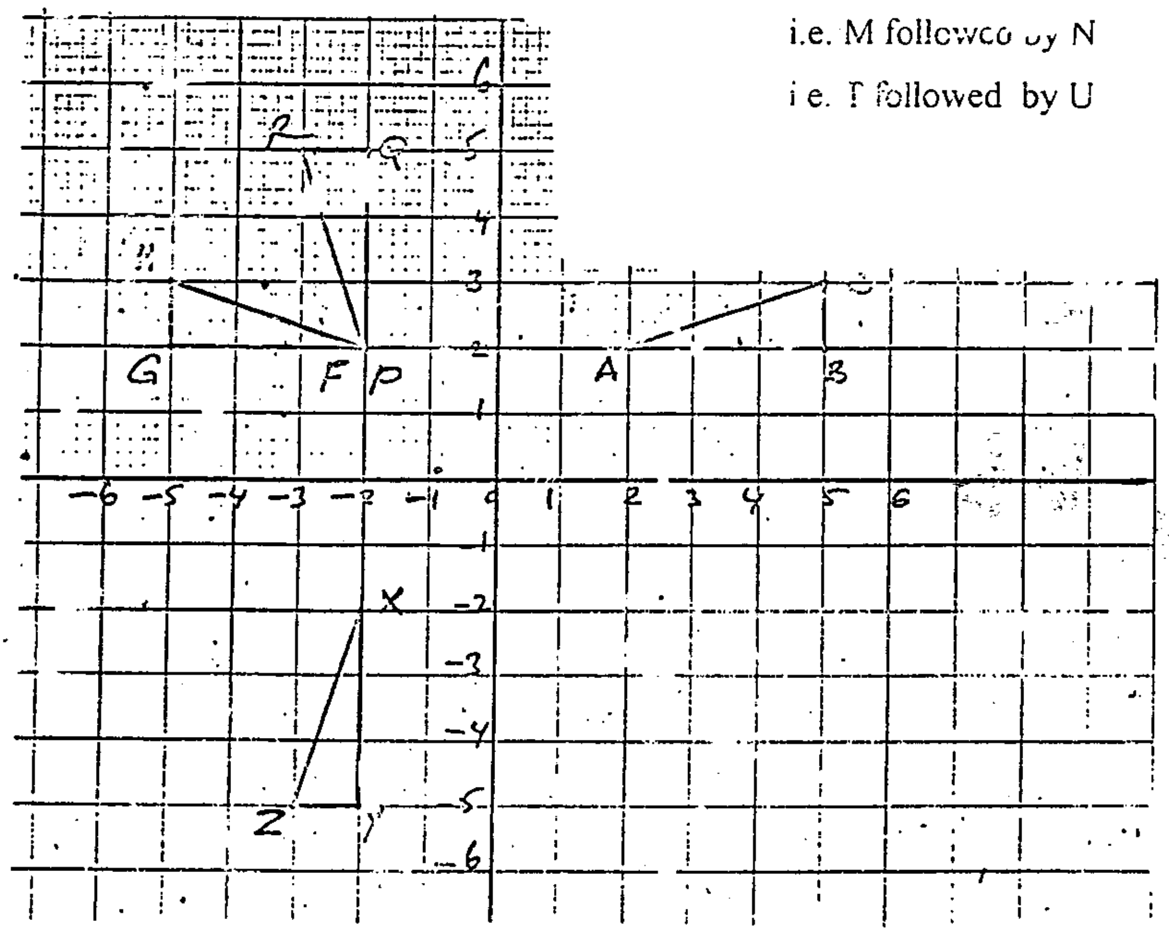
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 \\ 2 & 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} -2 & -5 & -5 \\ 2 & 2 & 3 \end{pmatrix}$

V = NM

i.e. M followed by N

i.e. T followed by U



9- (a) (i) $p = \frac{2.3125}{2} + 1 = 2.15625$

$q = \frac{2.15625}{2} + 1 = 2.078125$

(ii)

$0.5 \to 1.25$

$1.25 \to 1.625$

$1.625 \to 1.8125$

$1.8125 \to 1.90625$

$1.90625 \to 1.953125$

$1.953125 \to 1.9765625$

(iii) Limit is 2

(by continuing the procedure few more times in each case).

(b)

$8 \to 2.6$

$2.6 \to 1.52$

$1.52 \to 1.304$

$1.304 \to 1.2608$

$1.2608 \to 1.25216$

$1.25216 \to 1.250432$

$1.250432 \to 1.2500864$

Limit is 1.25 or $1\frac{1}{4}$ i.e. $\frac{5}{4}$

(c) Start with 8 again

$8 \to 3$

$3 \to 1.75$

$1.75 \to 1.4375$

$1.4375 \to 1.359375$

$1.359375 \to 1.3398438$

$1.3398438 \to 1.3349609$

Limit will reach 1.333 i.e. $\frac{4}{3}$

<u>OR</u> Start with x

$$x \rightarrow \frac{x}{4}+1$$

$$\left(\frac{x}{4}+1\right) \rightarrow \frac{x}{16}+\frac{1}{4}+1=\frac{x}{16}+\frac{5}{4}$$

$$\left(\frac{x}{16}+\frac{5}{4}\right) \rightarrow \frac{x}{64}+\frac{5}{16}+1=\frac{x}{64}+1\frac{5}{16}$$

$$\left(\frac{x}{64}+1\frac{5}{16}\right) \rightarrow \frac{x}{256}+\frac{21}{64}+1=\frac{x}{256}+1\frac{21}{64}$$

$$\left(\frac{x}{256}+1\frac{21}{64}\right) \rightarrow \frac{x}{1024}+1\frac{85}{256}$$

as we go on $\frac{x}{1024}$ gets smaller and its value is negligible . The fraction $1\frac{85}{256}$ will approach $1\frac{1}{3}=\frac{4}{3}$

(d) by inspection :

dividing by 2 Limit is $\frac{2}{1}$

dividing by 5 Limit is $\frac{5}{4}$

dividing by 4 Limit is $\frac{4}{3}$

dividing by n Limit is $\frac{n}{n-1}$

<u>By Algebra</u>

When the limit is reached then dividing by n and adding one will result in getting the same number (limit), and that is why it is called Limit

Let the limit is x

$$\frac{x}{n}+1 = x$$

$$x + n = nx$$

$$n = nx - x = (n-1)x$$

$$x = \frac{n}{n-1}$$

Nov. 1993

Paper 4

1- (a) amount received $= \frac{800}{1.68} \times \frac{99}{100} = 471.4$

answer is 471 dollars

(b) (i) amount $= p + \frac{PRT}{100}$

$$= 800 + \frac{800 \times 9 \times \frac{6}{12}}{100} = 836 \text{ DM}$$

(ii) amount in dollars $= \frac{836}{1.87} = 447.1$

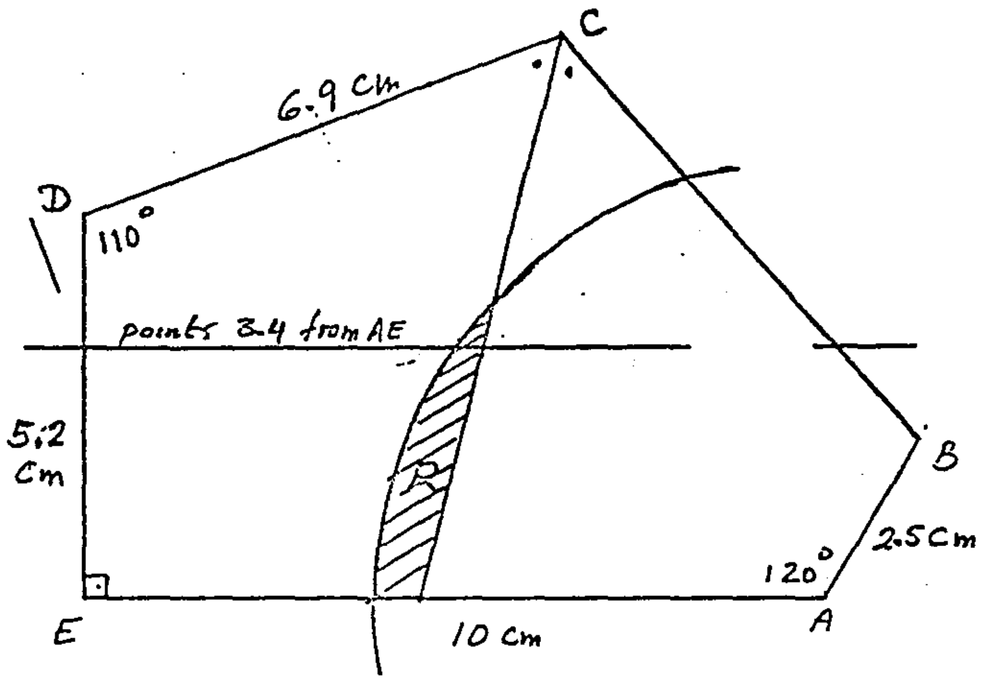
answer is 447 dollars

(c) K Laus , he got the larger amount

(d) amount $= 120 \times 1.72 = 206.4$

$= \text{DM } 206$

(2)



(b) (iii) Yes

3- (a) Four.

(b) A is (4 , 6) and H is (0 , 2)

$$\begin{pmatrix} 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$$

Vector of translation is $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$

(c) The line through (0 , 6) and (6 , 0) its equation is $x + y = 6$

(d) Reflection on the line $x = 3$

(e) Rotation clockwise by 90° centre point (3 , 3)

(f) $\begin{pmatrix} 1 & 3 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$ i.e. point B

4- (a) $\overline{OS}^2 = 400^2 + 850^2 - 2 \times 400 \times 850 \cos 110$

$= 1115073.7$

OS = 1056 km

(b) $\dfrac{850}{\sin < SOT} = \dfrac{1056}{\sin 110}$

$\sin < SOT = 0.7564$

$< SOT = 49°$

(c) The bearing of Tokyo from Osaka

= 30 + 49 = 79

∴ The bearing of Osaka from Tokyo

= 180 + 79 = 259°

(d) Time of journey $= \dfrac{850}{500} = 1.7$ h = 1 h 42 min

time of arrival = 9 h 30 min + 1 h 42 mmin

= 11 h 12 min i.e 11 12

5- (a) amount = $10000 \times 15 + 20000 \times 8$ = \$ 310000

(b) number of standing places replaced

$= 20000 - 4000 = 16000$

number of the extra seats $= \frac{16000}{2} = 8000$

(i) number of seats now $= 10000 + 8000 = 18000$

(ii) amount $= 18000 \times 15 + 4000 \times 8$ = \$ 302000

(iii) number of seats $= \frac{200000-(4000 \times 8)}{15} = 11200$

(c) (i) x standing places remain

$(20000 - x)$ standing places replaced to $\frac{20000-x}{2}$ seats

number of seats now $= 10000 + \frac{20000-x}{2}$

$= \frac{20000+20000-x}{2} = 20000 - \frac{x}{2}$

(ii) $20000 - \frac{x}{2} = 2x$

$40000 = 5x \Rightarrow x = 8000$

Total number of places $= 8000 + \left(20000 - \frac{8000}{2}\right)$

$= 24000$

maximum number of spectators = 24000

6- (a) Area of the circle radius 10 cm $= \pi \times 10^2 = 100\pi$

Area of the circle radius 20 cm $= \pi \times (20)^2 = 400\pi$

Area of the circle radius 30 cm $= \pi \times (30)^2 = 900\pi$

Area of bull $= 100\pi$

Area of inner $= 400\pi - 100\pi = 300\pi$

Area of outer $= 900\pi - 400\pi = 500\pi$

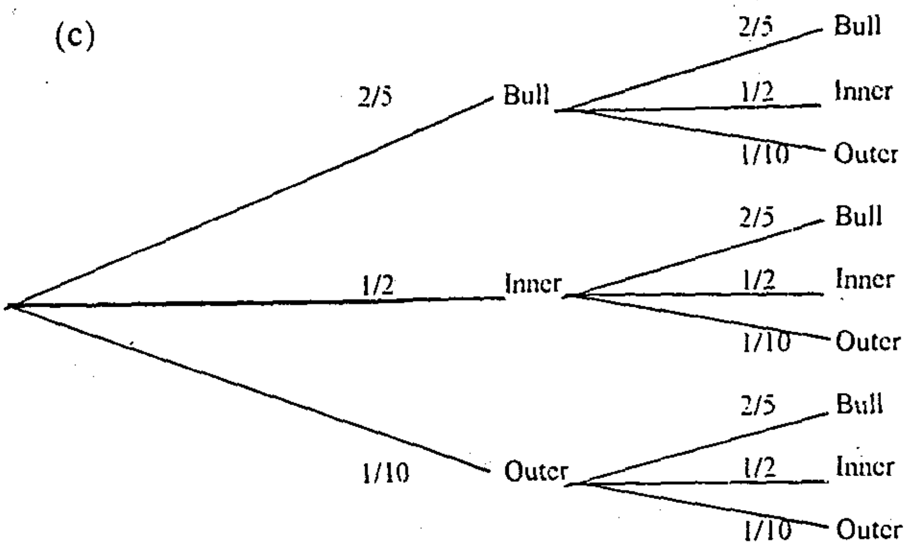
∴ ratio of the areas of bull : inner : outer

is $100\pi : 300\pi : 500\pi$

i.e. $1 : 3 : 5$

(b) Probability $= \dfrac{\text{area of bull}}{\text{total area}} = \dfrac{1}{1+3+5} = \dfrac{1}{9}$

(c)



(i) Probability $= \dfrac{1}{10} \times \dfrac{1}{10} = \dfrac{1}{100}$

(ii) to win \$ 12 means to get one bull and one inner.

Probability $= \dfrac{2}{5} \times \dfrac{1}{2} + \dfrac{1}{2} \times \dfrac{2}{5} = \dfrac{2}{5}$

(iii) To win \$ ·30 means hitting the bull all three times,

therefore,

Probability $= \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} = \dfrac{8}{125}$

7- (a) (i) < VTO = 90 , since VB is targent to the circle and OT is a radius.

(ii) < TOV = 90 − 20 = 70

< TOS = 2 x 70 = 140°

(iii) < TPS = ½ < TOS = 70° (theorem)

(b) (i) $\sin 20 = \dfrac{OT}{VO} = \dfrac{10}{VO}$

$VO = \dfrac{10}{\sin 20^\circ} = 29.2$ cm

(ii) $VP = VO + OP = 29.2 + 10 = 39.2$ cm

$\therefore$ height of the cone $= 39.2$ cm

(iii) $\tan 20^\circ = \dfrac{AP}{VP} = \dfrac{AP}{39.2}$

$AP = 39.2 \times \tan 20^\circ = 14.3$

$\therefore \quad R = 14.3$ cm

(c) (i) Volume of the cone $= \dfrac{1}{3} \pi R^2 h$

$= \dfrac{1}{3} \times 3.142 \times (14.3)^2 \times 39.2$

$= 8395.4 = 8400 \text{ cm}^3$

(ii) Volume of the sphere $= \dfrac{4}{3} \pi r^3$

$= \dfrac{4}{3} \times 3.142 \times (10)^3 = 4189.3$

$= 4190 \text{ cm}^3$

(iii) Volume of empty space

$= 8395.4 - 4189.3 = 4206.1$

percentage of the volume of the cone

$= \dfrac{4206.1}{8395.4} \times 100 = 50.1\ \%$

8- $y = 4 + 2x - x^2$

(a) At A $y = 1$

$\therefore \quad 4 + 2x - x^2 = 1$

$x^2 - 2x - 3 = 0$

(b) $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\therefore\ x = 3$ or $x = -1$ (rejected)

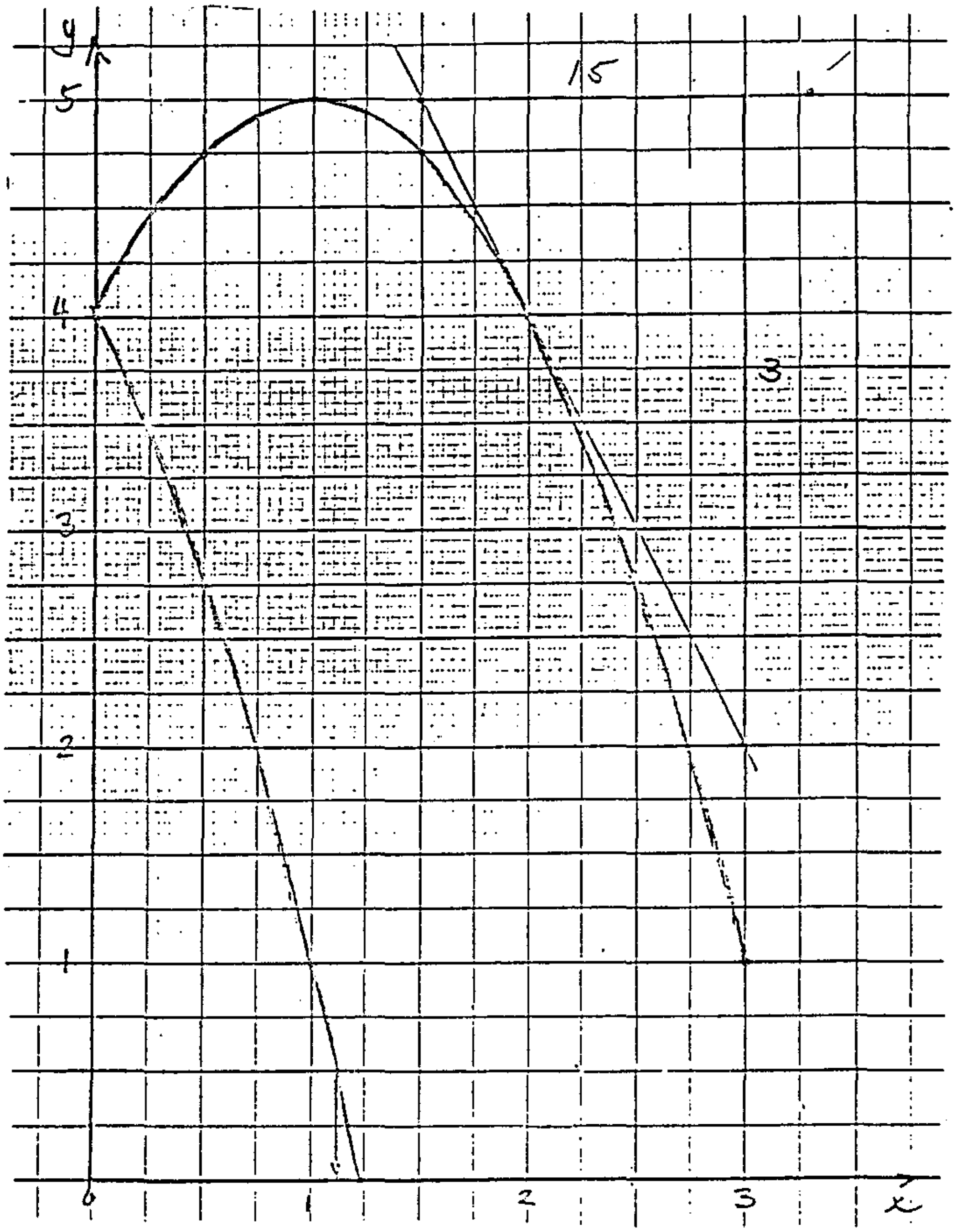
x coordinate of A is 3

(c) $x^2 + 2x - 4 = 0$

$$x = \frac{-2 \pm \sqrt{4+16}}{2} = \frac{-2 \pm \sqrt{20}}{2}$$

$= 1.24$ (only positive root)

8-



(e) 1.12 m

(f) gradient = − 2 or gradient using point (3 , 7) , (1½ , 5)

$$\text{gradient} = \frac{5-2}{1\tfrac{1}{2}-3} = -2$$

9- (a) $\frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$

(b)

n	$\frac{1}{n} - \frac{1}{n+1}$	$\frac{1}{n(n+1)}$
1	$\frac{1}{1} - \frac{1}{2}$	$\frac{1}{1 \times 2}$
2	$\frac{1}{2} - \frac{1}{3}$	$\frac{1}{2 \times 3}$
3	$\frac{1}{3} - \frac{1}{4}$	$\frac{1}{3 \times 4}$
4	$\frac{1}{4} - \frac{1}{5}$	$\frac{1}{4 \times 5}$
↓	↓	↓
99	$\frac{1}{99} - \frac{1}{100}$	$\frac{1}{99 \times 100}$
100	$\frac{1}{100} - \frac{1}{101}$	$\frac{1}{100 \times 101}$

(c) adding all terms of column 2 and column 3

$$\therefore \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} \cdots\cdots\cdots - \frac{1}{100} + \frac{1}{100} - \frac{1}{101}$$

$$= \frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} \cdots\cdots\cdots + \frac{1}{100x101}$$

$$\therefore \frac{1}{1} - \frac{1}{101} = \frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \cdots\cdots\cdots + \frac{1}{100x101}$$

$$\therefore \frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} + \cdots\cdots + \frac{1}{100x101} = 1 - \frac{1}{101} = \frac{100}{101}$$

June 1994

Paper 4

1- (a) Each exterior angle $= \frac{360}{5} = 72$

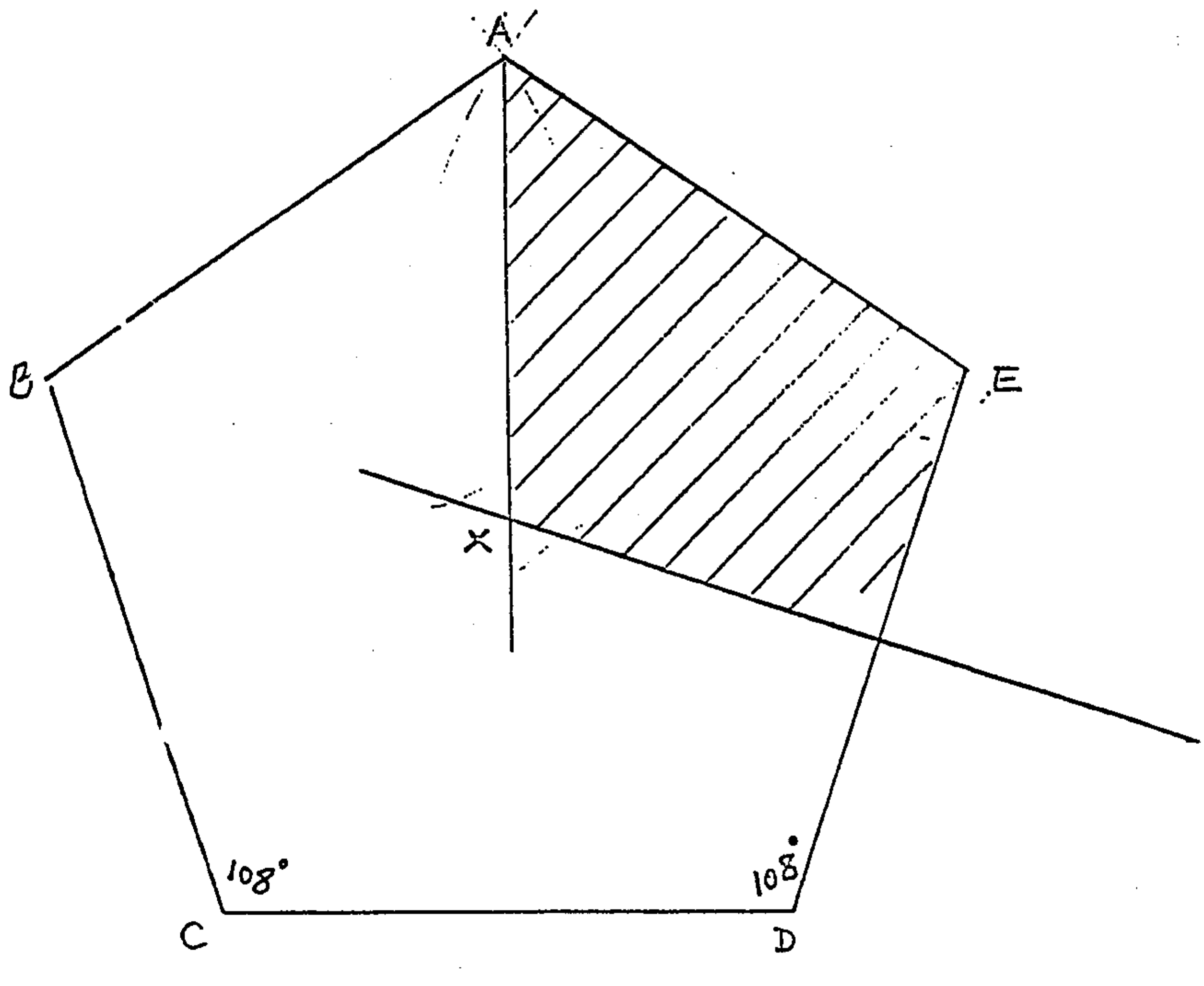
Each interior angle $= 180 - 72 = 108°$

<u>OR</u> Sum of all angles $= (2n - 4) \times 90$

$= (10 - 4) \times 90 = 540$

each interior angle $= \frac{540}{5} = 108°$

(b)



(iii) AX = 6.6 cm

2- (a) (i) speed $= \dfrac{80}{10} = 8$ m/s

(ii) speed $= \dfrac{8 \times 3600}{1000} = 28.8$ km/h

(b) total time $= 10.5 + 13 + \dfrac{120}{8.5}$

$= 37.6$ s

overall average speed $= \dfrac{80+100+120}{37.6}$

$= 7.98$ m/s

(c) $\text{Cos BAC} = \dfrac{80^2 + 120^2 - 100^2}{2 \times 80 \times 120} = 0.5625$

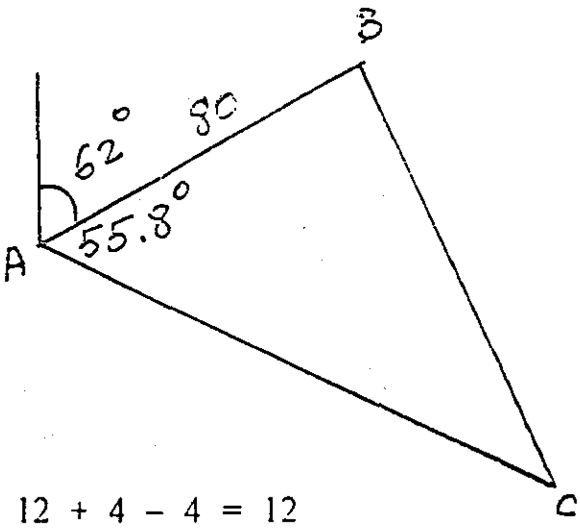
$\angle BAC = 55.8°$

(d) Bearing of C from A

$= 55.8 + 62 = 117.8° = 118°$

Bearing of A from C

$= 180 + 118 = 298°$



3- (a) $f(x) = 3x^2 - 2x - 4$

$f(-2) = 3(4) - 2(-2) - 4 = 12 + 4 - 4 = 12$

(b) $f(x) = -3$

$3x^2 - 2x - 4 = -3$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -\dfrac{1}{3}$ or $x = 1$

(c) $f(x) = 0$ $\qquad 3x^2 - 2x - 4 = 0$

$x = \dfrac{2 \pm \sqrt{4 - 4 \times 3(-4)}}{6}$

$\dfrac{2 \pm \sqrt{52}}{6} = 1.54$ or -0.87

(d) $g(x) = 2g(x) - 1$

$4 - 3x = 2(4 - 3x) - 1$

$4 - 3x = 8 - 6x - 1$

$4 - 3x - 8 + 6x + 1 = 0$

$3x - 3 = 0$

$x = 1$

(e) $g(x) = 4 - 3x$

$y = 4 - 3x$

$3x = 4 - y$

$x = \frac{4-y}{3}$

$g^{-1}(x) = \frac{4-x}{3}$

(f) (i) $y = f(x)$ graph B

(ii) $y = g(x)$ graph C

4- (a) No of teachers x , No of students y

$\therefore \quad 24x + 20y \geq 240$

$\div 4 \quad 6x + 5y \geq 60$

(b) $x + y \leq 13$, $x \geq 4$, $y \geq 3$

(c) $6x + 5y = 60$ $x = 0$ $y = 12$, $y = 0$ $x = 10$

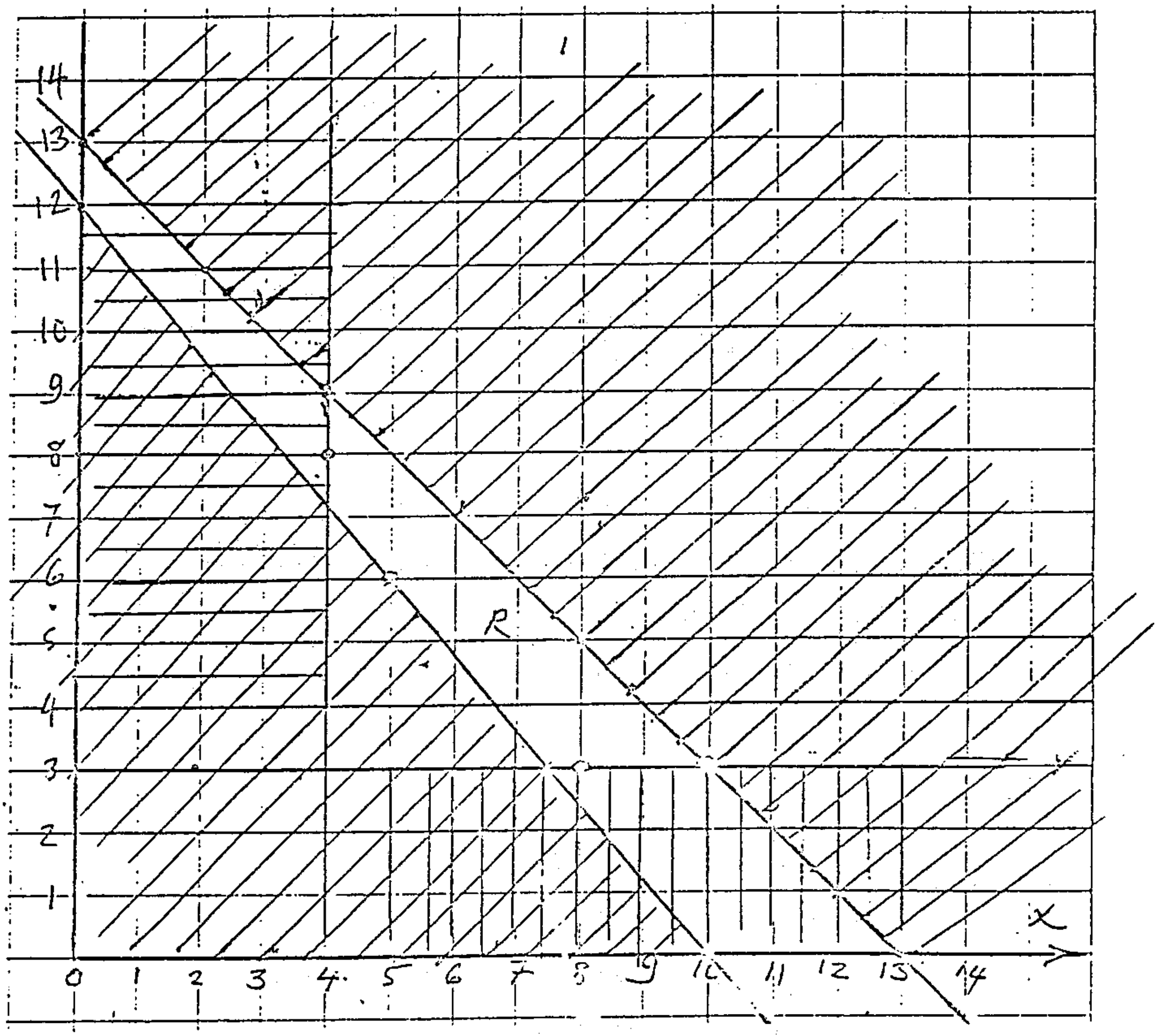
$x + y = 13$ $x = 0$ $y = 13$, $y = 0$ $x = 13$

(d) The region satisfying all inequelities are marked R

To find the answers to (i) and (ii) we test the corners of the quad R or points closer to corners, so we check points (4 , 8) , (4 , 9) , (5 , 6) , (8 , 3) , (10 , 3) .

(i) Least x + y is for (5 , 6) and (8 , 3) which is equal 11.

(ii) Greatest value of 24x + 20y is for (10 , 3) and equal 300 kg.



14
13
12
11
10
9
8
7
6
5
4
3
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1
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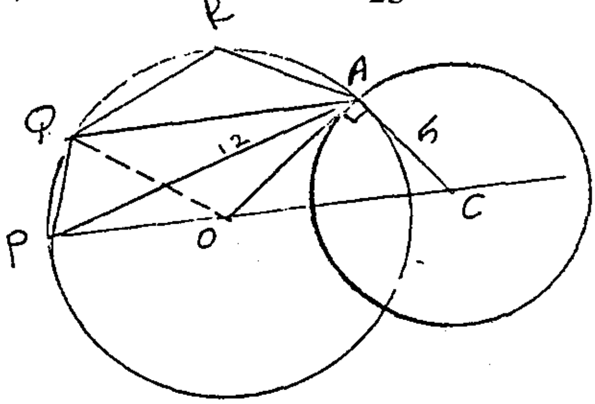
13
14
R
x



5- (a) $\overline{OC}^2 = 12^2 + 5^2 = 169$

$\therefore$ OC = 13

(b) circle through O , A and C has OC as diameter = 13

its radius = $\frac{13}{2} = 6\frac{1}{2}$ cm

(c) Sin < AOC = $\frac{5}{13}$ or tan < ADC = $\frac{5}{12}$

< AOC = 22.6°

(d) < APC = ½ < AOC = $\frac{22.6}{2}$ = 11.3°

(e) < OAQ = < AOC = 22.6°

OA = OQ

$\therefore$ < OQA = < OAQ

$\therefore$ < AOQ = 180 − 2 (22.6) = 134.8°

(f) < APQ = ½ < AOQ = ½ x 134.8 = 67.4°

(g) APQR is a cyclic quad

< QRA = 180 − < APQ

= 180 − 67.4 = 112.6°

6- (b) (i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 \\ 1 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 5 \\ 2 & 5 & 5 \end{pmatrix}$

(ii) transformation is the reflection on the line y = x

(c) (ii) For reflection on the line y = − x

point (1 , 0) is reflected into (0 , − 1)

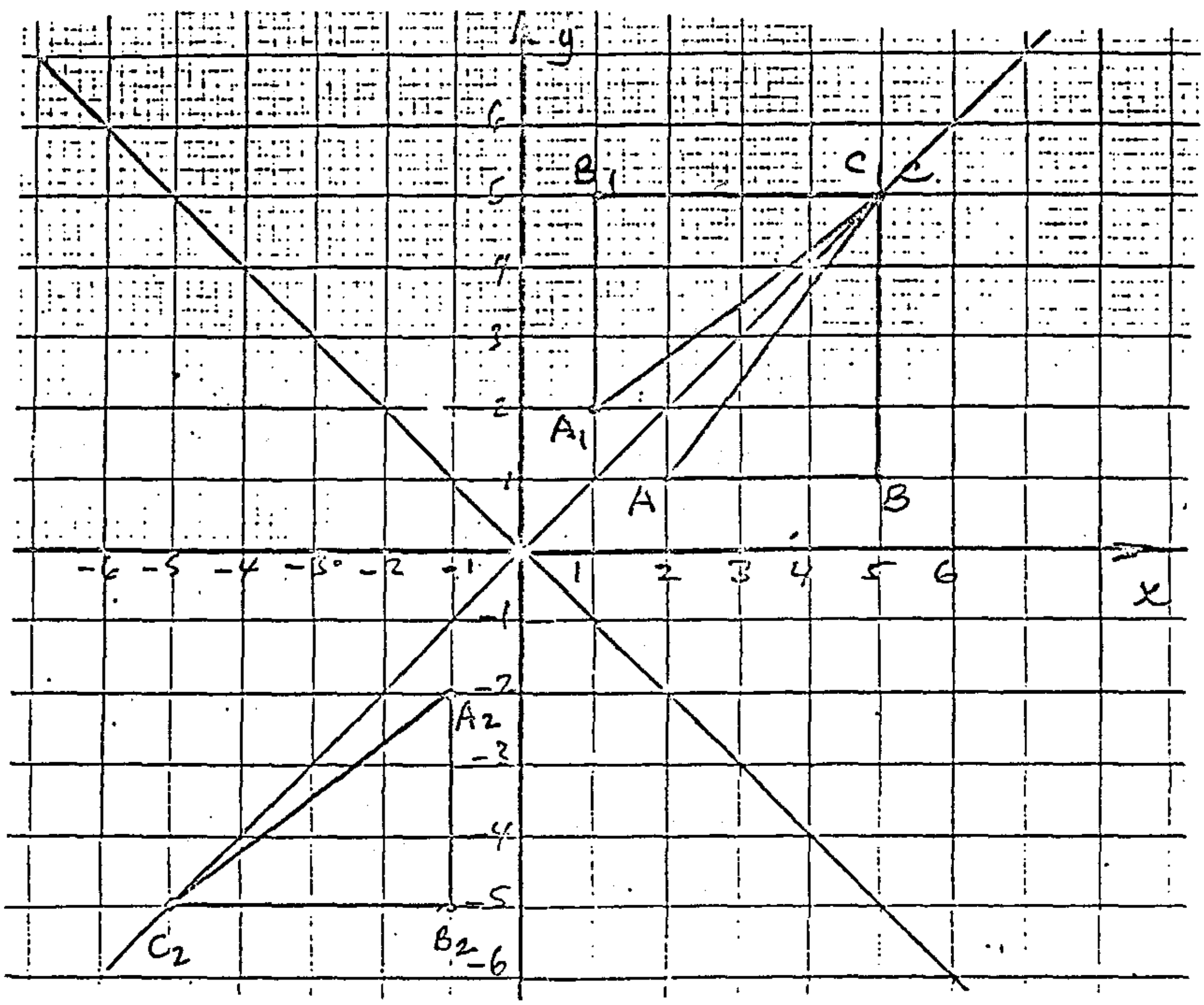
point (0 , 1) is reflected into (− 1 , 0)

Matrix of trasformation is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(d) (i) transformation which maps $A_1B_1C_1$ to $A_2B_2C_2$ is a rotation by 180° centre origin or enlargement by − 1 centre origin .

point (1 , 0) → (− 1 , 0)

(0 , 1) → (0 , − 1) matrix is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$



7- (a)

Height	Frequency
$0 < h \le 5$	20
$5 < h \le 10$	40
$10 < h \le 15$	60
$15 < h \le 25$	80
$25 < h \le 50$	50

(b)

Mid Interval x	Frequency f	fx
2.5	20	50
7 5	40	300
12.5	60	750
20	80	1600
37.5	50	1875
	250	4575

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{4575}{250} = 18.3$$

(c)

height h	Cummulative frequency
≤ 5	20
≤ 10	60
≤ 15	120
≤ 25	200
≤ 50	250

(d) (i) class interval $15 < h \le 25$

$$\text{Median} = 15 + \frac{125-120}{200-120} \times (25 - 15)$$

$$= 15 + \frac{5}{80} \times 10 = 15\frac{5}{8} = 15.6$$

(e) $\text{probability} = \frac{250-60}{250} = \frac{190}{250} = \frac{19}{25}$

(f) probability $= \dfrac{190}{250} \times \dfrac{189}{249} = 0.577$

8- (a) Length $= \dfrac{7.56}{0.42} = 18$

(b) mass $= 7.56 \times 0.88 = 6.65$ g

(c) $0.5\ m^3 = 0.5 \times 10^6\ cm^3$

no. of prisms $= \dfrac{75}{100} \times \dfrac{0.5 \times 10^6}{7.56}$

$= 49603$

$= 50000$ to the nearest thousand

(d) (i) area of Δ OAB $= \dfrac{1}{6} \times 0.42 = 0.07\ cm^2$

(ii) Equilateral.

(iii) Area $= \dfrac{1}{2}x^2 \sin 60$

$\dfrac{1}{2}x^2 \times 0.866 = 0.07$

$x^2 = \dfrac{0.14}{0.866}$

$x = 0.402$ cm

$= 4$ mm

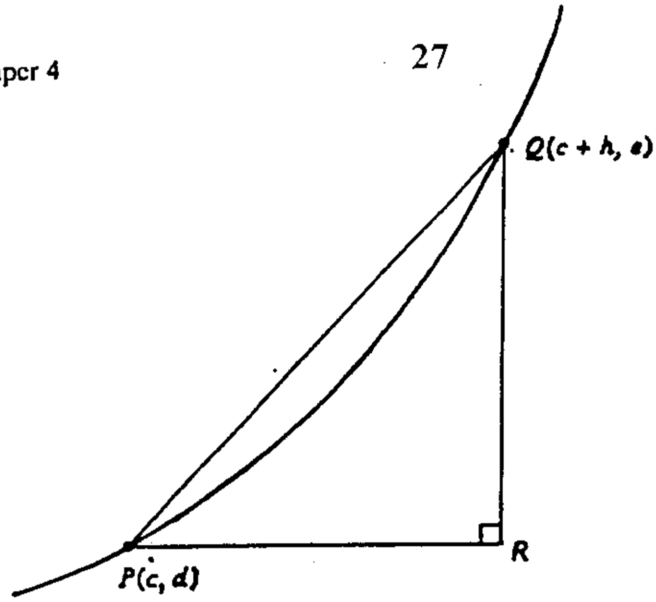
9- (a) gradient $= \dfrac{24.5 - 18}{3.5 - 3} = \dfrac{6.5}{0.5} = 13$

(b) (i) $y = 2x^2$ P is (c, d)

$d = 2c^2$

(ii) Q is $(c + h, e)$

$e = 2(c + h)^2$



(iii) PR = h

QR = e − d

= $2(c + h)^2 - 2c^2$

= $2(c^2 + 2ch + h^2) - 2c^2$

(iv) gradient = $\frac{QR}{PR} = \frac{4ch + 2h^2}{h}$

= $4c + 2h$

(v) P is (c , d) which is (3 , 18)

i.e. c = 3

Q is (c + h , e) which is (3.5 , 24.5)

∴ h = 0.5

gradient = 4c + 2h

= 4 x 3 + 2 (0.5) = 13

(vi) c = 3 h = 0.1

gradient = 4 x 3 + 2 x 0.1

= 12.2

(vii) (a) h approaches zero

(b) gradient = 4c = 4 x 3 = 12

* * * * * * *

Nov. 1994

Paper 4

1. (a) (i) Amount divided between them $= \frac{40}{100} \times 9000 = 3600$

Amount Alexis receives $= \frac{5}{9} \times 3600 = \$\,2000$

Amount Biatriz receives $= \frac{3}{9} \times 3600 = \$\,1200$

Amount Carlos receives $= \frac{1}{9} \times 3600 = \$\,400$

(ii) Carlos receives $\frac{1}{9}$ of the Amount

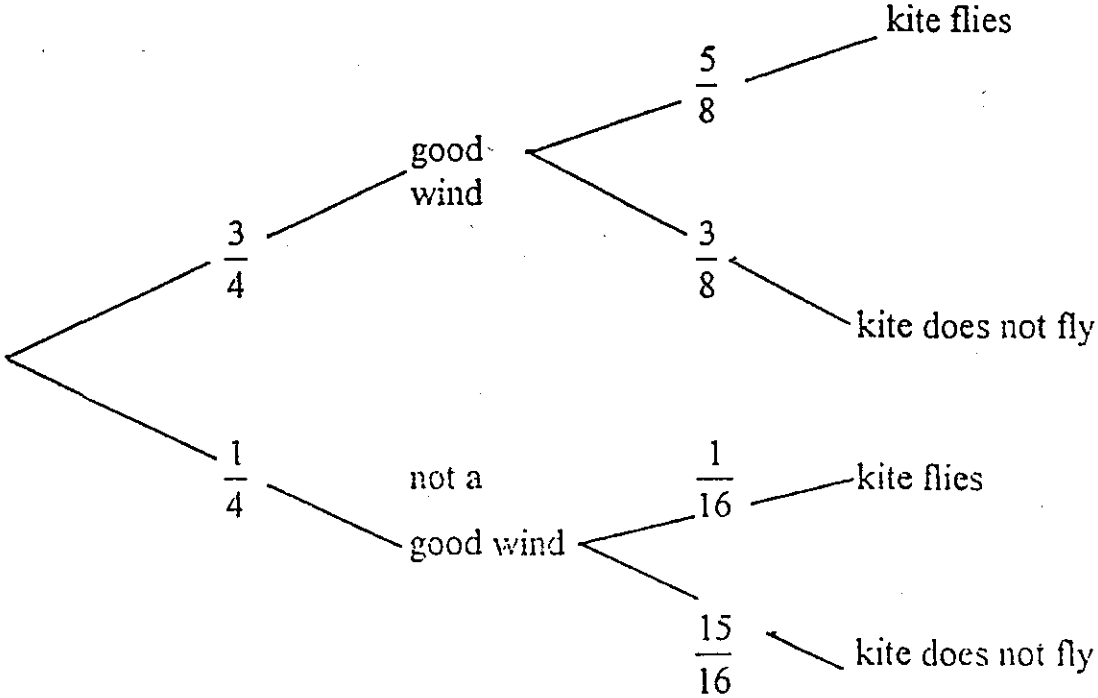
Amount divided $= 9 \times 420 = 3780$

Income $= \frac{3780 \times 100}{40} = \$\,9450$

(b) Interest $I = \frac{PRT}{100}$

$$I = \frac{16000 \times 12 \times \frac{6}{12}}{100} = \$\,960$$

2. (a) (i)



(ii) Prob. of a good wind and the kite flying

$$= \frac{3}{4} x \frac{5}{8} = \frac{15}{32}$$

(iii) Prob. that the kite does not fly

$$= \frac{3}{4} x \frac{3}{8} + \frac{1}{4} x \frac{15}{16} = \frac{9}{32} + \frac{15}{64} = \frac{33}{64}$$

(b) Prob. that the kite stick in a tree

$$= \frac{3}{4} x \frac{5}{8} x \frac{1}{2} + \frac{1}{4} x \frac{1}{16} x \frac{1}{2}$$

$$= \frac{15}{64} + \frac{1}{128} = \frac{31}{128}$$

(c) (i) mode is the most frequent wind strength, therefore the mode is 7. To find the median construct the following table :

wind strength	1	2	3	4	5	6	7	8	9
frequency	3	5	6	8	6	7	9	5	1
cummulative freq.	3	8	14	22	28	35	44	49	50

order of median $= \frac{50}{2} = 25$ (or $\frac{50+1}{2} = 25.5$)

from the above table this term number 25 (or 25.5) lies within the group of wind strength of 5.

Therefore the median is 5.

(ii) Mean = (1x3+2x5+3x6+4x8+5x6+6x7+7x9+8x5+9x1) ÷ 50

= 247 ÷ 50 = 4.94

(iii) Number of days for which the wind strength x given by $3 \le x \le 7$ is equal to $6+8+6+7+9 = 36$

Prbability of a good wind $= \frac{36}{50} = \frac{18}{25}$

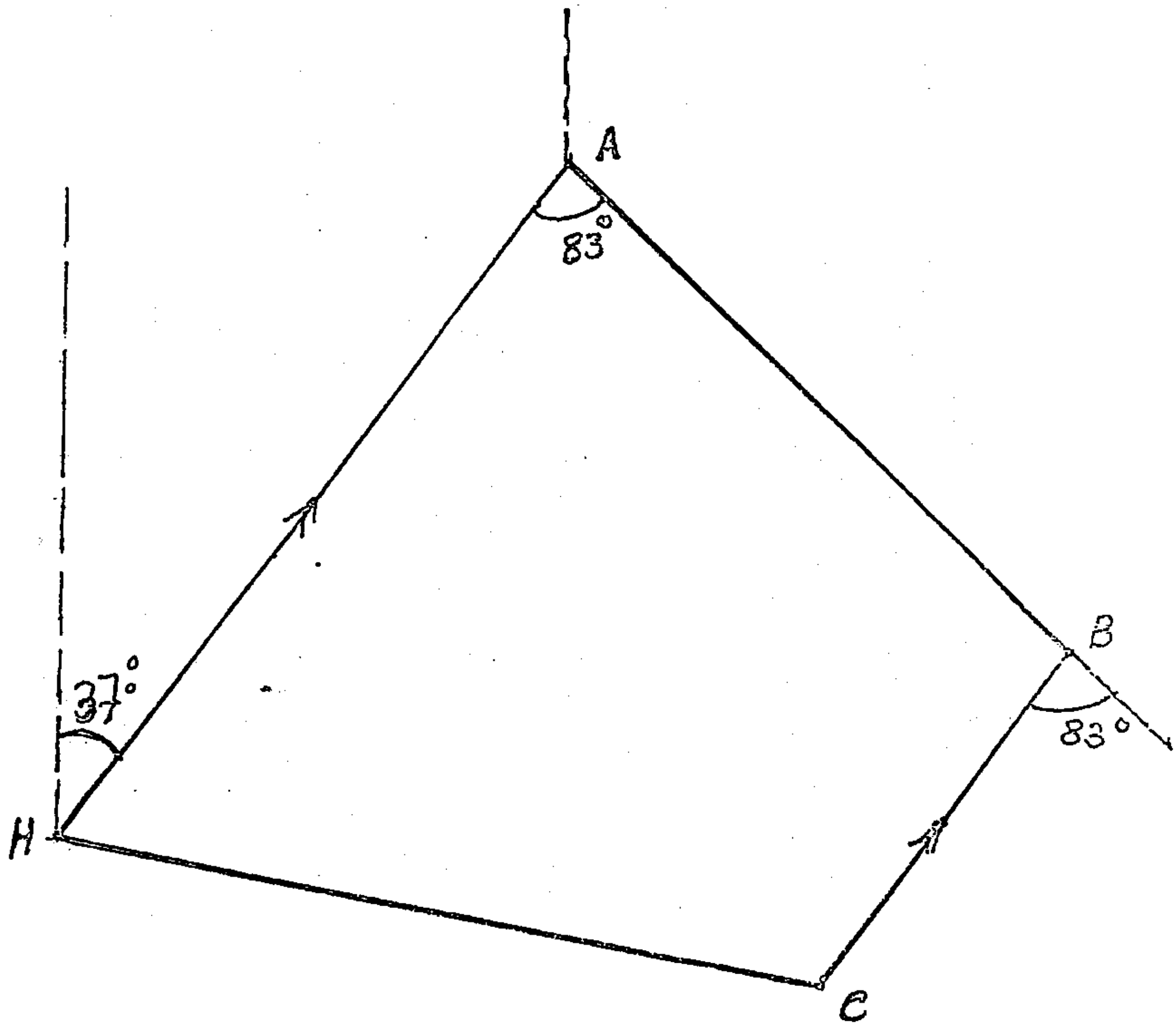
3. (a) (i) Bearing of B from A = 360 − (180 − 37) − 83 = 134°

(ii) Bearing of C from B = 134 + 83 = 217°

(b) Using a scale of 1 cm = 10 km

120 km ⇒ 12 cm.

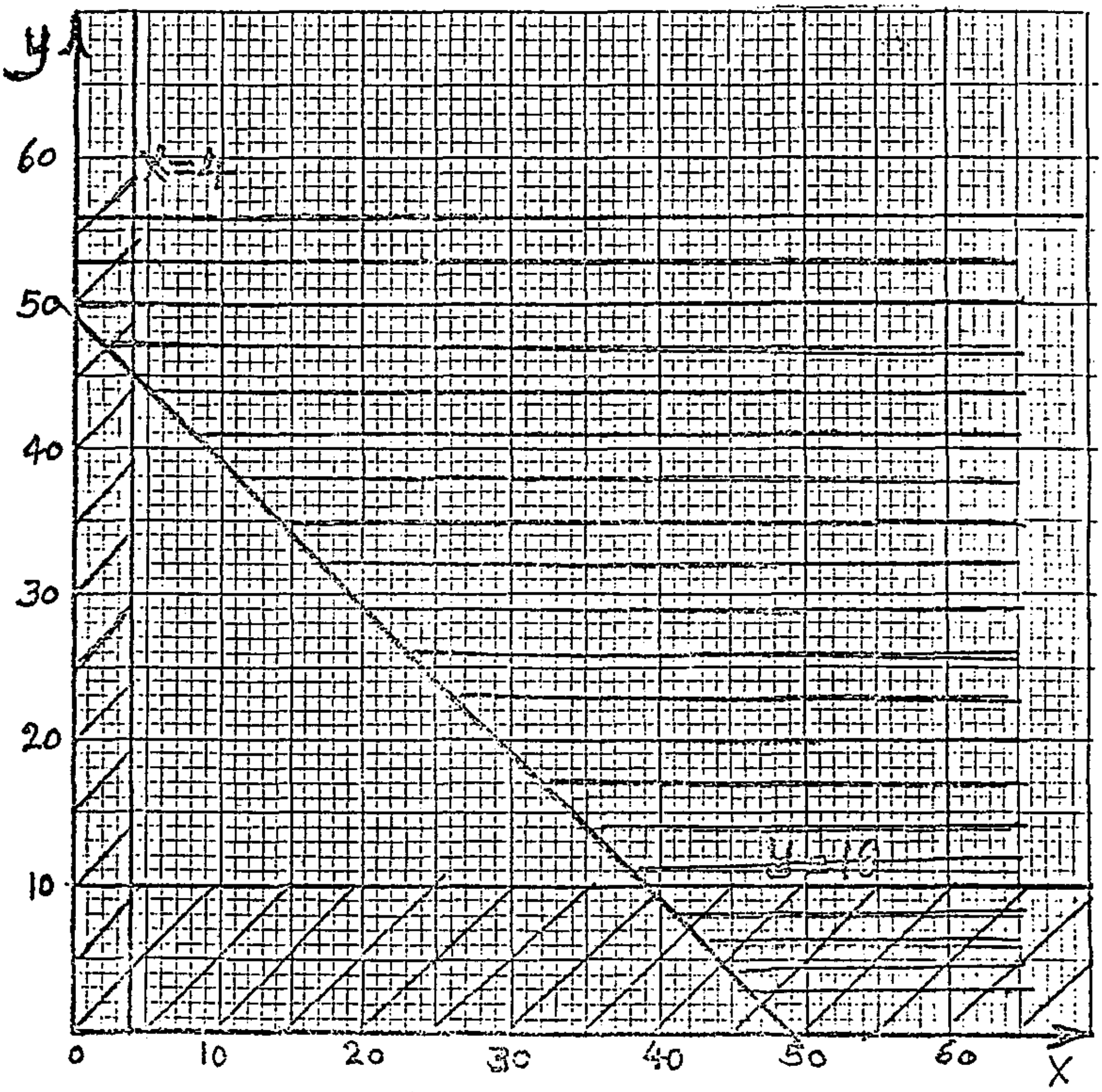
100 km ⇒ 10 cm.



Distance CH = 11 x 10 = 110 km

4. (b) $x + Y \leq 49$

$x = 0 \quad y = 49 \quad , \quad y = 0 \quad x = 49$



(d) Profit = 100 x + 50 y

for the corner points

(39 , 10) profit = 3900 + 500 = 4400

(10 , 39) profit = 1000 + 1950 = 2950

Maximum profit = $ 4400

5. (a) (i) $3xa + 6xb - 9xc = 3x(a + 2b - 3c)$

(ii) $x^2 - 10x - 24 = (x + 2)(x - 12)$

(iii) $10x^2 - 7x + 1 = (2x - 1)(5x - 1)$

(b) $y = \frac{a}{x} + bx$

(i) $x = 1, y = 2$ $\quad \therefore \quad 2 = a + b$

$x = 2, y = -5$ $\quad \therefore \quad -5 = \frac{a}{2} + 2b \quad x-2$

$$\begin{aligned} 10 &= -a - 4b \\ 2 &= \ \ a + \ b \\ \hline 12 &= -3b \qquad \Rightarrow \qquad b = -4 \end{aligned}$$

$\therefore\ a = 2 - b = 2 + 4 = 6$

$a = 6$ and $b = -4$

(ii) $y = 16$ $\qquad 16 = \frac{6}{x} - 4x$

$16x = 6 - 4x^2$

$4x^2 + 16x - 6 = 0$

$2x^2 + 8x - 3 = 0$

$x = \frac{-8 \pm \sqrt{8^2 - 4x2x-3}}{2x2}$

$= \frac{-8 \pm \sqrt{88}}{4} = \frac{-8 \pm 9.3808}{4}$

$= 0.35$ or -4.35

6. (a) (i) $BC^2 = BA^2 + AC^2 - 2\,BA \times AC \cos A$

$= 7^2 + 9^2 - 2 \cdot 7 \cdot 9 \times \cos 120°$

$= 193$

$BC = 13.9$ cm

(ii) $\frac{BC}{\sin A} = \frac{CA}{\sin B}$ sine rule

$\frac{13.9}{\sin 120} = \frac{9}{\sin B}$

$\sin B = \frac{9 \times \sin 120}{13.9}$

$B = 34.1°$

(b) (i) angle OAS $= \frac{120}{2} = 60°$

from (a) above, angle B $= 34.1°$

angle OBS $= \frac{34.1}{2} = 17.1°$

(ii) $\tan 60° = \frac{r}{AS} \Rightarrow AS = \frac{r}{\tan 60°}$

(iii) $\tan 17.1° = \frac{r}{BS} \Rightarrow BS = \frac{r}{\tan 17.1°}$

(iv) $AS + BS = AB = 7$

$$\frac{r}{\tan 60°} + \frac{r}{\tan 17.1°} = 7$$

$$\frac{r}{1.732} + \frac{r}{0.307} = 7$$

$$r\left(\frac{1}{1.732} + \frac{1}{0.307}\right) = 7$$

$$r \times 3.835 = 7$$

$$r = 1.83 \text{ cm}$$

7. (a) (i) $13^2 - 5^2 = 169 - 25 = 144$

$\sqrt{144} = 12 \qquad \therefore \; CD = 2 \times 12 = 24$ cm

(ii) $\cos x = \frac{5}{13} \qquad x = 67.4$

$< COD = 2 \times 67.4 = 135°$

(iii) arc CBD $= \frac{135}{360} \times 2 \times 3.142 \times 13 = 30.6$ cm

(iv) distance CD round the semicircle

$= \pi r = \pi \times 12 = 37.7$ cm

(b) (i) Area above the water level $= 2\pi r(r - h)$

$= 2\pi \times 13\,(13 - 5) = 654$ cm

(ii) total surface area $= 4\pi r^2 = 4 \times 3.142 \times 13^2 = 2124$

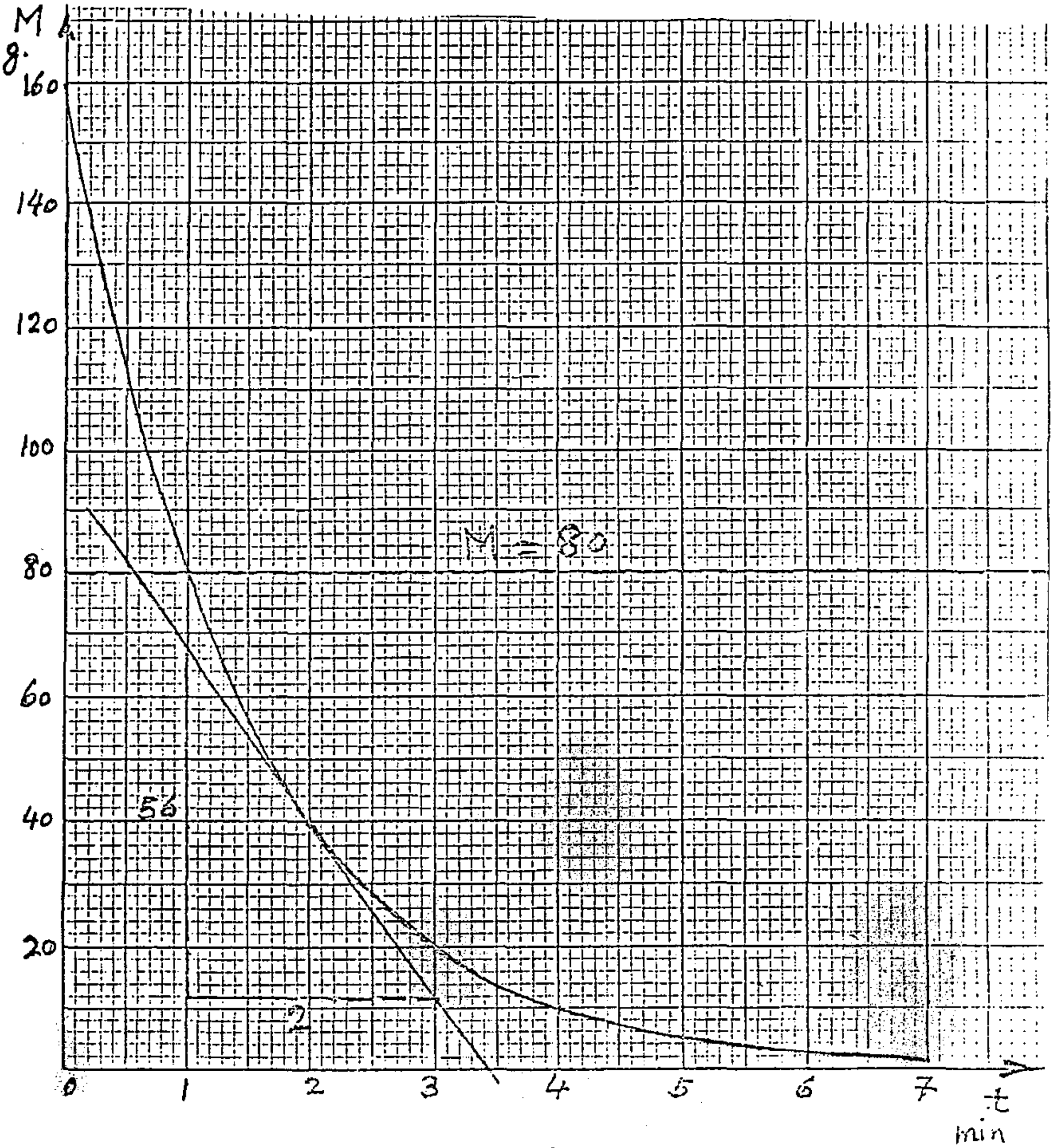
percentage $= \frac{654}{2124} \times 100 = 30.8$ %

8. (a) $M = 160 \times 2^{-t}$

$t = 0 \qquad M = 160 \times 2^{0} = 160 \qquad , p = 160$

$t = 4 \qquad M = 160 \times 2^{-4} = \frac{160}{16} = 10 \;, q = 10$

$t = 6 \qquad M = 160 \times 2^{-6} = \frac{160}{64} = 2.5 \;, r = 2.5$



rate of change = gradient = $\frac{-56}{2} = -28$ grams per min.

(b) (i) $m = 160 - M$

when $m = M$ $\therefore 2M = 160$

$M = 80$

from graph $t = 1$ min.

(ii) reflection on the line $M = 80$

9. (a) (i) Translation of $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

(ii) Enlargement by factor 3 Centre the origin.

(iii) Rotation by 90° anticlockwise centre the origin.

(iv) Stretch along the y-axis factor 4.

(v) Shear parallel to the x-axis.

(b) Shapes B , D and E

(c) matrix of stretch $= \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$

(d) matrix which trasnform F onto A is the inverse of

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \text{ i.e. } \frac{1}{1}\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$$

10. (a) $a = \dfrac{6.8+6.9+7+7.1+7.2}{5} = 7$

$b = 7 \times 1.8 = 12.6$

$c = \dfrac{4.7+4.9+5.1+5.1+5.2}{5} = 5$

$d = 5 \times 2.3 = 11.5$

(b) $\dfrac{7.3+7.6+7.7+8+x}{5} \times 2.2 = 16.5$

$30.6 + x = \dfrac{16.5 \times 5}{2.2} = 37.5$

$x = 37.5 - 30.6 = 6.9$

(c) Since the mean is 7.2 and all the known marks are less than 7.2, this means y and z an greater than 7.2.
Assuring that $y < z$, this means z is the largest and 7 is the least.
Deleting these two marks

$$\therefore \frac{7.1+7.1+7.1+7.1+y}{5} = 7.2$$

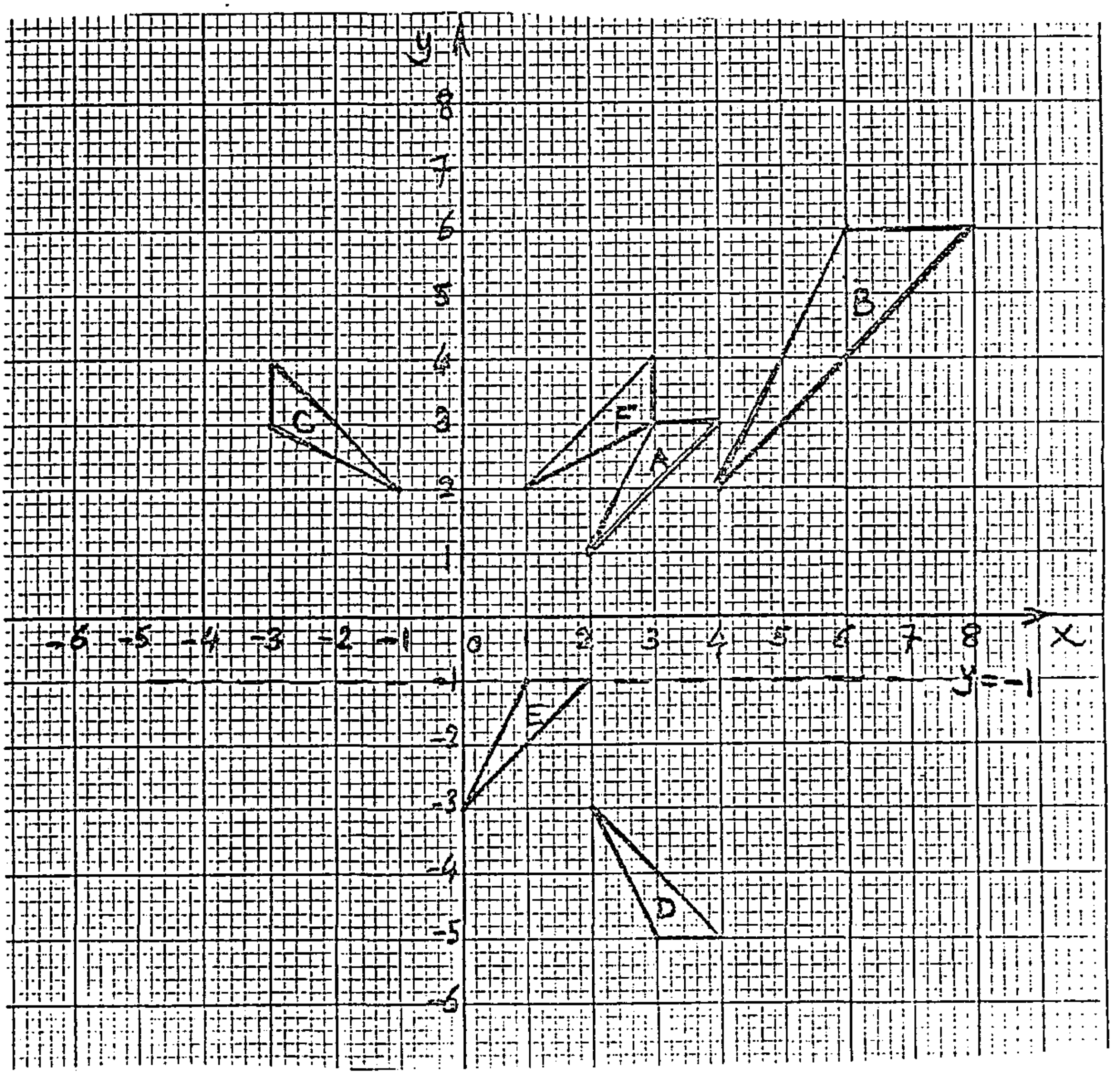
$$y = 5 \times 7.2 - 4 \times 7.1$$
$$= 36 - 28.4 = 7.6$$

and z is any value greater than 7.6
A possible pair of values for y and z is 7.6, 7.7
(7.6 and any number greater than 7.6)

June 1995

Paper 4

1.



(f) (i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \\ 1 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 3 \\ 2 & 3 & 4 \end{pmatrix}$

(ii) reflection on the line $y = x$

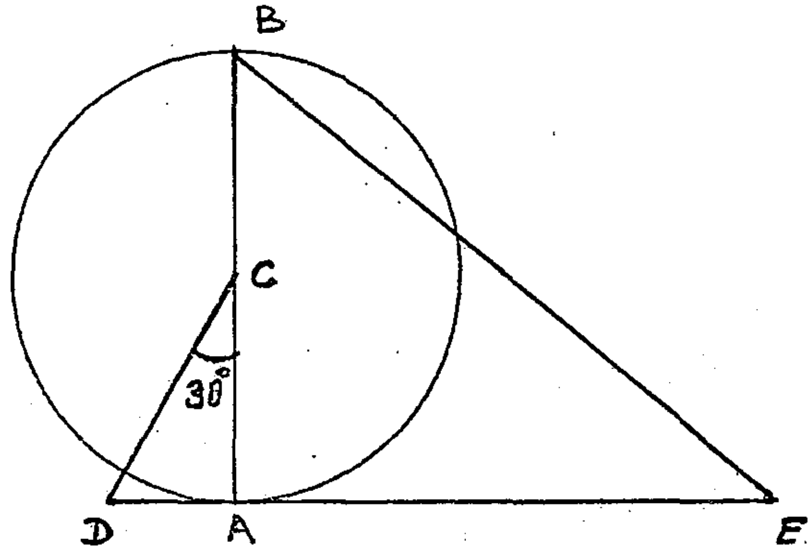
(g) (i) reflection on the y axis

(ii) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Matrix of transformation is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

2. (a) (i)



(ii) BE = 9.4 cm

(iii) semicircular arc BA = πr = 3.142 x 3 = 9.43 cm

(b) (i) r = 10

$\tan 30° = \frac{DA}{10} \Rightarrow DA = 5.77$ cm

(ii) AE = DE − DA = 3 x 10 − 5.77 = 24.2

(iii) $BE^2 = (20)^2 + (24.2)^2 = 985.64$

BE = 31.4

(iv) Semi circular arc BA = π x 10 = 31.4 cm

(v) Length BE equal to the semicircular arc BA.

3. (a) taxable income = 20 000 − 3000 = 17 000

tax paid = $\frac{25}{100}$ x 17 000 = \$ 4250

(b) taxable income = 20 000 − 4000 = 16 000

tax paid = $\frac{30}{100}$ x 16 000 = \$ 4800

(c) taxable income $= 20\,000 - x$

(i) tax paid $= \dfrac{30}{100}(20\,000 - x) = 4950$

$(20\,000 - x) = \dfrac{4950 \times 100}{30} = 16\,500$

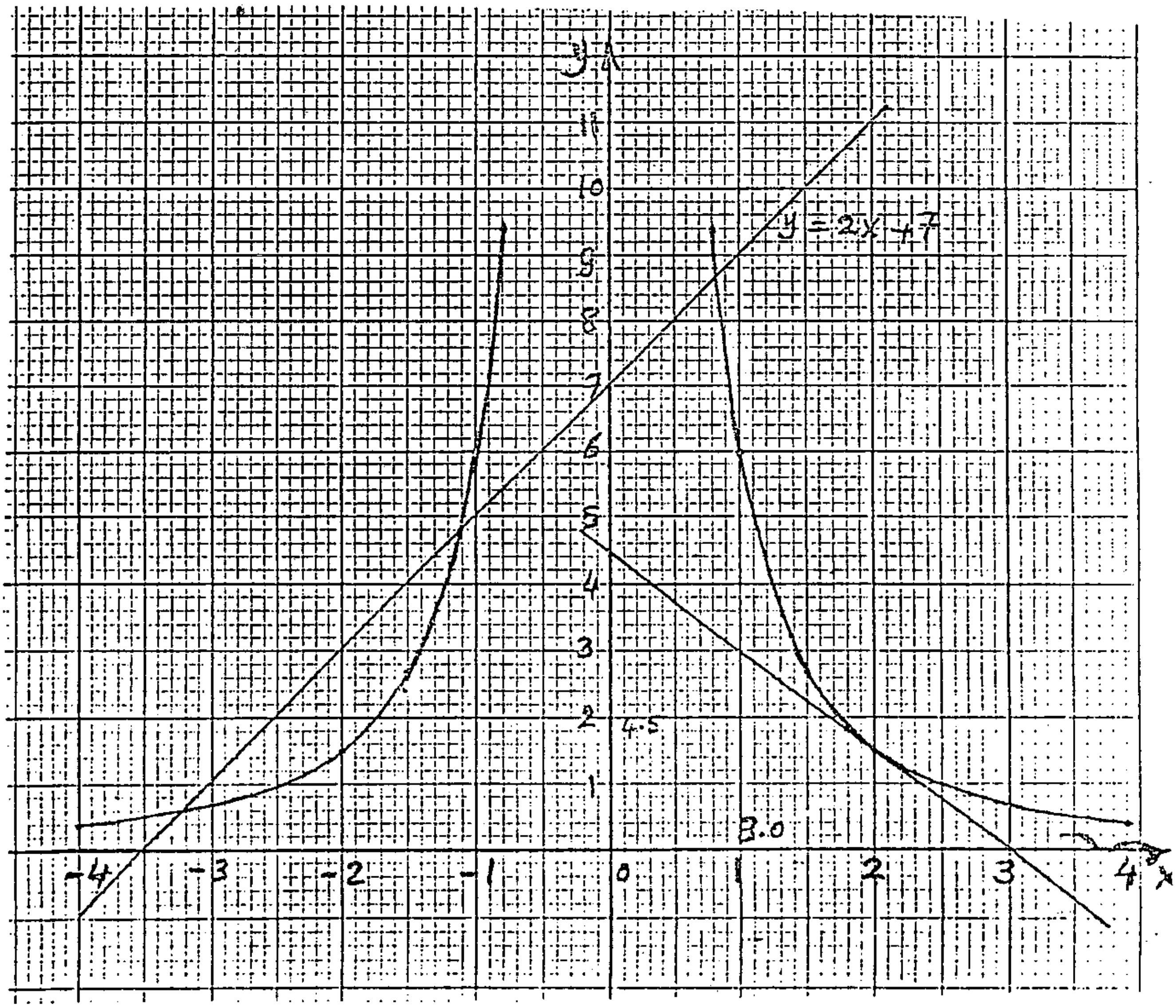
$x = 20\,000 - 16\,500 = \$\,3500$

4. (a) $p = \dfrac{6}{(-4)^2} = \dfrac{6}{(4)^2} = \dfrac{6}{16} = 0.4$

$q = \dfrac{6}{(-1)^2} = \dfrac{6}{(1)^2} = 6$

$r = \dfrac{6}{(-0.8)^2} = \dfrac{6}{(0.8)^2} = \dfrac{6}{0.64} = 9.4$

(b)



(c) $y = 2x + 7$

$x = 0 \qquad y = 7$

$x = 2 \qquad y = 11$

(d) solutions of the equation $2x + 7 = \dfrac{6}{x^2}$ are

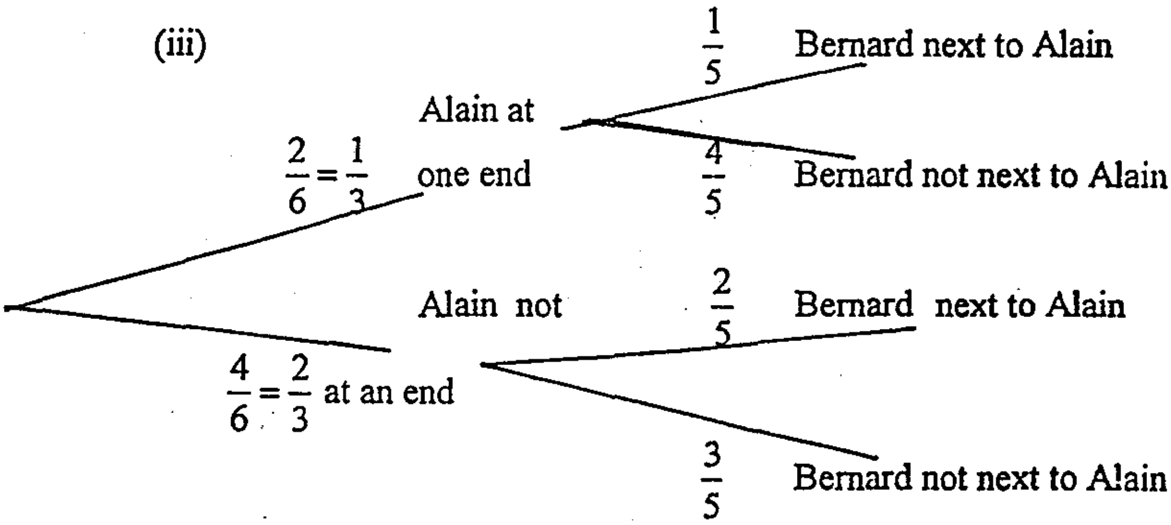
$x = -3.3\,, -1.1\,, 0.9$

(e) gradient $= -\dfrac{4.5}{3} = -1.5$

5. (a) (i) $\dfrac{2}{6} = \dfrac{1}{3}$

(ii) (a) $\dfrac{1}{5}$ (b) $\dfrac{2}{5}$

(iii)



(iv) $\dfrac{1}{3} \times \dfrac{1}{5} + \dfrac{2}{3} \times \dfrac{2}{5}$

$= \dfrac{1}{15} + \dfrac{4}{15} = \dfrac{5}{15} = \dfrac{1}{3}$

(b) $\dfrac{2}{5}$

(c) $\dfrac{2}{n-1} = \dfrac{1}{4}$ $\qquad n - 1 = 8$

$n = 9$

6. (a) (i) (a) $OC = \sqrt{18^2 - 3^2} = 17.7$ cm

(b) $\text{Sin} < AOC = \frac{3}{18}$

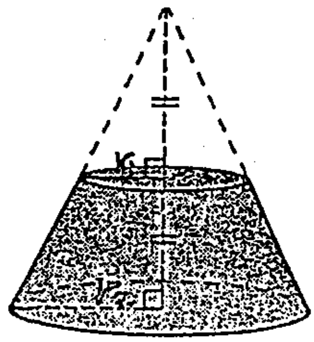
$< AOC = 9.6°$

(c) Circumference $= 2\pi r = 2 \times 3.142 \times 3 = 18.9$ cm

(ii) (a) Circumference of circle of radius 18 cm

$= 2\pi r = 2 \times 3.142 \times 18 = 113$

(b) $< AOÀ = \frac{18.9}{113} \times 360 = 60°$



(b) (i) (a) Ratio $= \frac{r_1}{r_2} = \frac{1}{2}$

(b) Ratio $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(c) Ratio $= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

(ii) curved surface area of the cone removed = ¼ of the curved surface area of the original cone.

∴ the curved surface area of the remaining solid = ¾ of the curved surface area of the original cone = ¾ x 24 π = 18 π

(iii) Volume of the cone removed $= \frac{1}{8}$ of the volume of the original cone.

Volume of the remaining solid $= \frac{7}{8}$ of the volume of the original cone $= \frac{7}{8}$ V

7. (a) arranging the numbers in order

61 , 62 , 64 , 65 , 67 , 68 , 69 , 70 , 73 , 74 , 74

(i) the median is the $\frac{11+1}{2}$ term

median is the 6th term

median is 68

(ii) mode is the most frequent number

mode is 74

(iii) mean $= \frac{\sum x}{n}$

$= \frac{61+62+64+65+67+68+69+70+73+74+74}{11}$

$= 67.9$

(b) median 17 i.e. middle term is 17
numbers are x x 17 x x
mode is 19 i.e. there is more than one number of 19 and it can only be two 19

$x, y, 17, 19, 19$

mean = 14 $\frac{x+y+17+19+19}{5} = 14$

$x + y = 15$

we can take any two integers their sum is 15 say 7 and 8
A possible set of numbers is 7 , 8 , 17 , 19 , 19

(c)

Height	Mid value x	frequencey f	fx
1 – 4	2.5	8	20
5 – 8	6.5	7	45.5
9 – 12	10.5	5	52.5
		20	118.0

Mean height $= \frac{\sum fx}{\sum f} = \frac{118}{20} = 5.9$

(d) (i) In histograms areas represent frequency, area in the histogram corresponding to height of 1 – 4 is 4 x 8 = 32
Area in the histogram corresponding to height of 13 – 20 is 8 x 6 = 48

frequency of height of 13 – 20 is $\frac{48}{32} \times 8 = 12$

(ii) total frequency = 8 + 7 + 5 + 12 = 32

median order is $\frac{32+1}{2} = 16\frac{1}{2}$

median class interval is 9 – 12

8. (a) (i) area of $\Delta = \frac{1}{2} x (x + 1) = 5$

$x^2 + x = 10$

$x^2 + x - 10 = 0$

(ii) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1 - 4x1x - 10}}{2} = \frac{-1 \pm \sqrt{41}}{2}$

$= 2.7$ or -3.7 (invalid as x should be positive)

Length of PR $= 2.7$ cm

(b) (i) $(AB)^2 = (AC)^2 + (BC)^2 - 2.\ AC\ .\ BC\ .\ \text{Cos } 120°$

$AB^2 = y^2 - 2.\ y (y + 2) x - \frac{1}{2}$

$= y^2 + y^2 + 4y + 4 + y^2 + 2y$

$= 3y^2 + 6y + 4$

(ii) $AB = 7$

$\therefore\ 3y^2 + 6y + 4 = 7^2 = 49$

$3y^2 + 6y - 45 = 0$

$y^2 + 2y - 15 = 0$

(iii) $y^2 + 2y - 15 = (y + 5)(y - 3)$

(iv) $y^2 + 2y - 15 = 0$

$(y + 5)(y - 3) = 0$

$y = 3$ ($y = -5$ is invalid)

AC = 3 cm CB = 5 cm

9. $1^2 + 2^2 + 3^2$ .... $+ k^2 = \frac{k(k + 1)(k + 1)}{6}$

(a) $k = 100$

$1^2 + 2^2 + 3^2$ .... $+ 100^2 = \frac{100 (101) (201)}{6}$

$= 338\ 350$

(b) (i) $2^2 + 4^2 + 6^2 + \ldots\ldots + 100^2 = 2^2 (1^2 + 2^2 + 3^2 + \ldots + n^2)$

$= 2^2 + 4^2 + 6^2 + \ldots\ldots + (2n)^2$

$2n = 100$ $n = 50$

(ii) $2^2+4^2+6^2+ \ldots + 100^2 = 2^2(1^2+2^2+ \ldots + 50^2)$

$= 4 \times \dfrac{50 \times 51 \times 101}{6}$ (substituting K = 50)

$= 171\ 700$

(c) $1^2+2^2+3^2+ \ldots + 99^2+100^2 = 338\ 350$

$(1^2+3^2+5^2+ \ldots + 99^2)+(2^2+4^2+6^2+ \ldots + 100^2) = 338\ 350$

$1^2+3^2+5^2+ \ldots +99^2+171\ 700 = 338\ 350$

$1^2+3^2+5^2+ \ldots +99^2 = 338\ 350 - 171\ 700 = 166\ 650$

(d) $1^2-2^2+3^2-4^2+5^2-6^2 \ldots 99^2-100^2$

$= (1^2+3^2+5^2+ \ldots +99^2)-(2^2+4^2+6^2+ \ldots +100^2)$

$= 166\ 650 - 171\ 700$

$= -\ 5050$

Nov. 1995

Paper 4

1. (a) Volume = Cross sectional area of trapezium × Length.

$= \frac{1.1+0.8}{2} \times 0.7 \times 500 = 332.5 \ m^3$

$\underline{= 333 \ m^3}$

(b) mass = $1.8 \times 332.5 = 598.5$ tonnes

$= 599$ tonnes

(c) area of the circular pipe $= \pi r^2 = 3.142 \times (0.25)^2$

$= 0.196 \ m^2$

area of trapezium $= \frac{1.1+0.8}{2} \times 0.7 = 0.665 \ m^2$

percentage of the earth which is not replaced

$= \frac{0.196}{0.665} \times 100 = 29.5 \ \%$

(d) Volume of water flowing in 1 hour

= Area × speed × time

$= 3.142 \times (0.25)^2 \times 0.8 \times 60 \times 60 \ m^3$

$= 565.56 \ m^3 = 565.56 \times 1000$

$= 5.66 \times 10^5$ Litres

2. (a) $h = 1.6$ $\quad t = \pi \sqrt{\frac{h}{9.81}} = 3.142 \sqrt{\frac{1.6}{9.81}} = 1.27$ sec.

(b) $t = 1$ $\quad 1 = \pi \sqrt{\frac{h}{9.81}} \quad \frac{1}{\pi} = \sqrt{\frac{h}{9.81}} \quad \left(\frac{1}{\pi}\right)^2 = \frac{h}{9.81}$

$h = \frac{9.81}{\pi^2} = 0.994 \ m$

(c) $t = \pi \sqrt{\frac{h}{9.81}} \quad t^2 = \pi^2 \frac{h}{9.81} \quad \Rightarrow \quad h = \frac{9.81 t^2}{\pi^2}$

(d) (i) Length of arc $= \frac{\theta}{360} \times 2\pi r$

$$1 = \frac{\theta}{360} \times 2\pi \times 1$$

$$\theta = \frac{360}{2\pi} = 57.3°\ , \text{ angle } AOB = 57.3°$$

(ii) Area of sector $= \frac{\theta}{360}\, \pi r^2$

$$= \frac{\theta}{360} \times \pi \times 1^2 = 0.5\ m^2$$

3. (a) $f(x) = 6 - x - x^2$

$f(-4) = 6 - (-4) - (-4)^2 = -6$ $\quad \therefore\ p = -6$

$f(3) = 6 - 3 - 3^2 = -6$ $\quad \therefore\ q = -6$

(b) $g(x) = x^3$

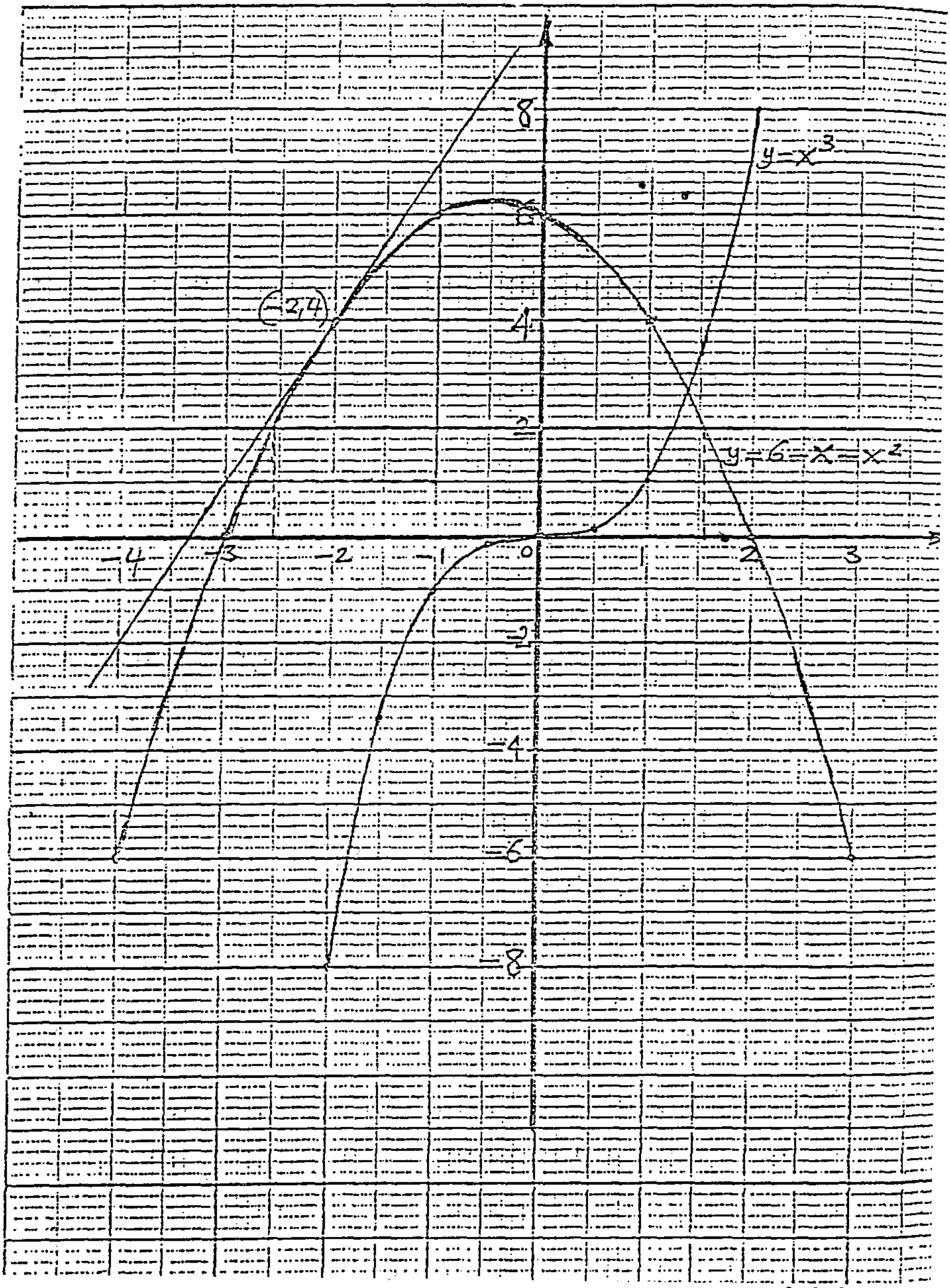
$g(-1) = (-1)^3 = -1$ $\quad r = -1$

$g(2) = 2^3 = 8$ $\quad s = 8$

(e) (i) $6 - x - x^2 = 2$ $\quad$ i.e. $f(x) = 2$

from graph $\quad x = -2.56$ or $x = 1.56$

(ii) point of intersection is $(1.4\ ,\ 2.7)$


8
y = x^3
(-2,4)
4
2
y = 6 - x - x^2
-4
-3
-2
-1
0
1
2
3
-2
-4
-6
-8

(f) (i) gradient $= \frac{7-1}{2} = \frac{6}{2} = 3$

(ii) gradient at (1 , 4) is -3
due to the symmetry of the graph.

4. (a) (i) Bearing of B from C = 21 + 41 = 062°

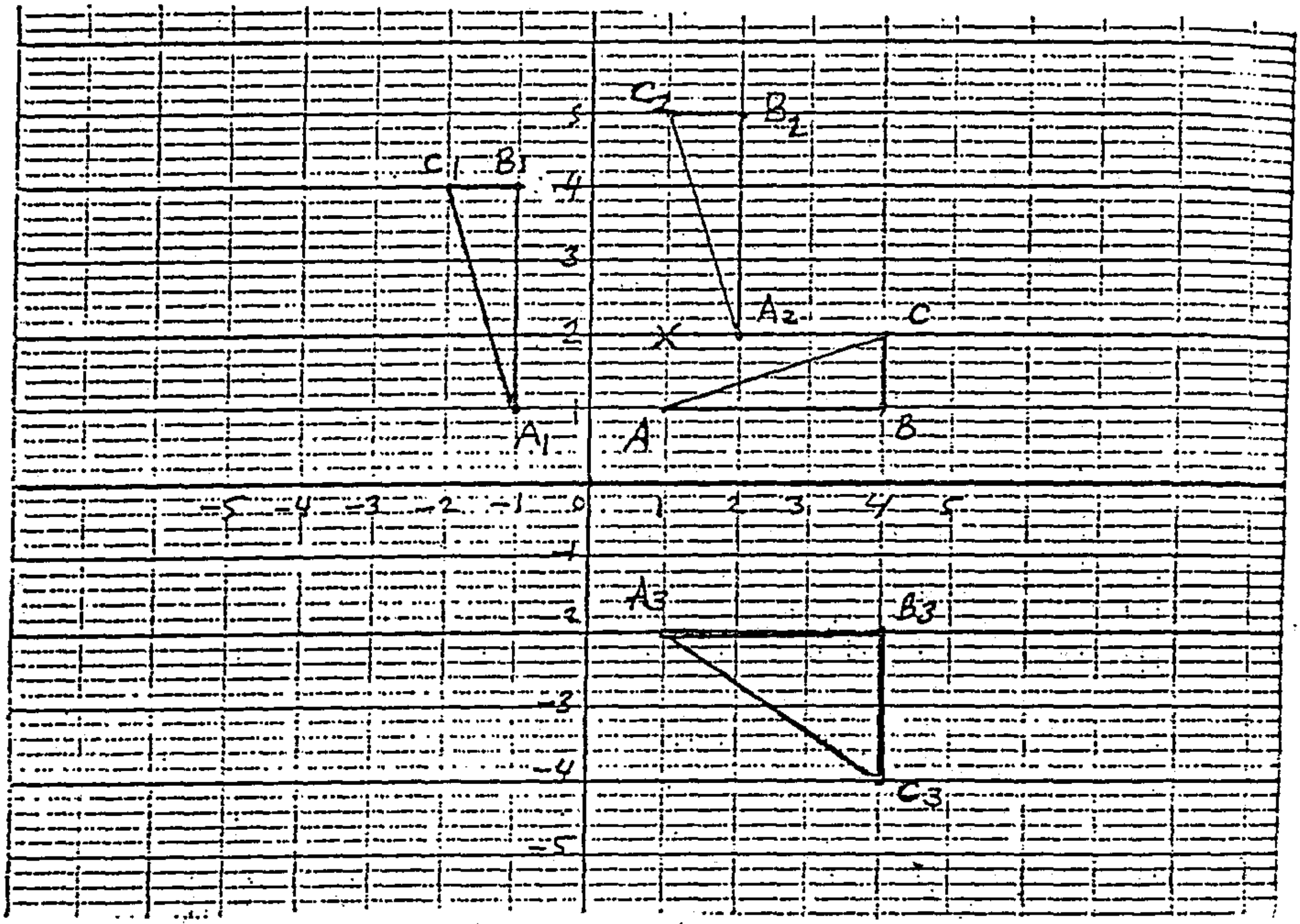
(ii) Bearing of A from C = 021°
Therefore, the bearing of C from A = 180 + 21 = 201°

(b) The distance A north of
C is CD
$\text{Cos } 21° = \frac{CD}{450}$
CD = 450 × Cos. 21°
= 420 m
A is 420 m north of C.

(c) $AB^2 = 450^2 + 600^2 - 2 \times 450 \times 600 \times \cos 41°$
AB = 394 m

(d) Area of triangle ABC = $\frac{1}{2} \times 450 \times 600 \times \sin 41°$
$= 88568 \text{ m}^2$
$= \frac{88568}{10000} = 8.8569$ hectare
Average number of people per hectare $= \frac{374}{8.8568} = 42$

5. (a) & (b) (i) ; (ii)



(b) (iii) rotation 90° anticlockwise about the point (1 , 2).

(c) (i) $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 4 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 4 \\ -2 & -2 & -4 \end{pmatrix}$

(ii) Stretch parallel to the y axis by a scale factor − 2

6. (a) (i) perimeter of the triangle

$p = x + x - 3 + x - 5 = 3x - 8$

(ii) $p = 2\frac{1}{2}x$

$2\frac{1}{2}x = 3x - 8$

$8 = 3x - 2\frac{1}{2}x = \frac{1}{2}x \Rightarrow x = 16$ cm

AB = 16 cm

(iii) the smallest angle is opposite the smallest side i.e. angle A. using the sine rule

$\frac{16}{\sin 83.2} = \frac{11}{\sin A} \Rightarrow A = 43°$

(b) (i) If the triangle is right angled then

$x^2 = (x-3)^2 + (x-5)^2$

$x^2 = x^2 - 6x + 9 + x^2 - 10x + 25$

$\therefore \quad x^2 - 16x + 34 = 0$

(ii) $x = \frac{16 \pm \sqrt{16^2 - 4 \times 1 \times 34}}{2} = \frac{16 \pm \sqrt{120}}{2}$

$= 13.48 \quad \text{or} \quad 2.52$

2.52 rejected as x should be greater than 3 and 5.

(iii) AB = 13.48 , AC = 10.48 , BC = 8.48

7. (a) Probability $= \frac{120+128}{720} = \frac{248}{720} = \frac{31}{90}$

(b) Probability $= \frac{152+200+120}{720} \times \frac{3}{4} = \frac{472}{720} \times \frac{3}{4} = \frac{59}{120} = 0.492$

(c)

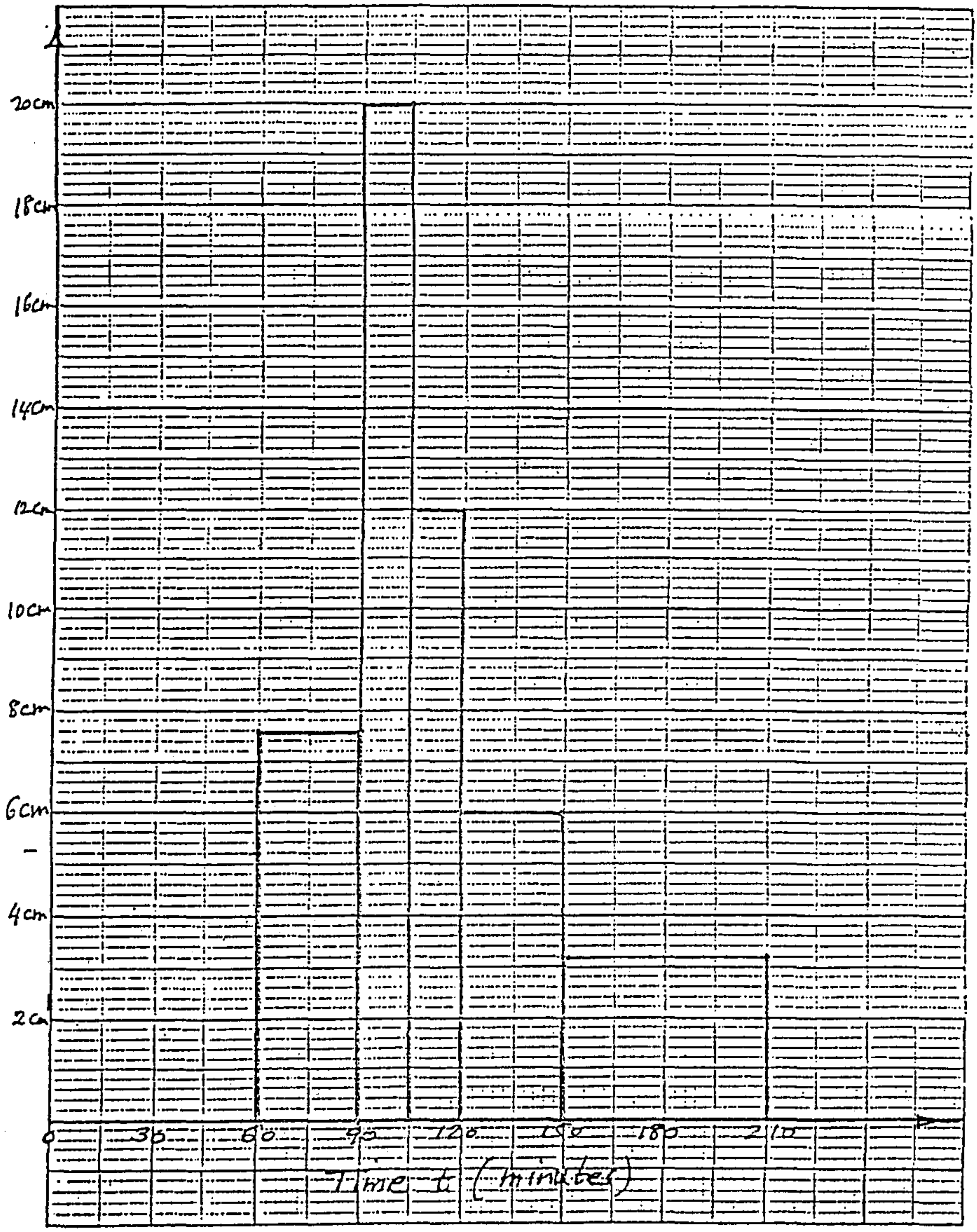
Time (mid interval) x	75	97.5	112.5	135	180
frequency f	152	200	120	120	128
fx	11400	19500	13500	16200	23040

$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{11400+19500+13500+16200+23040}{720}$

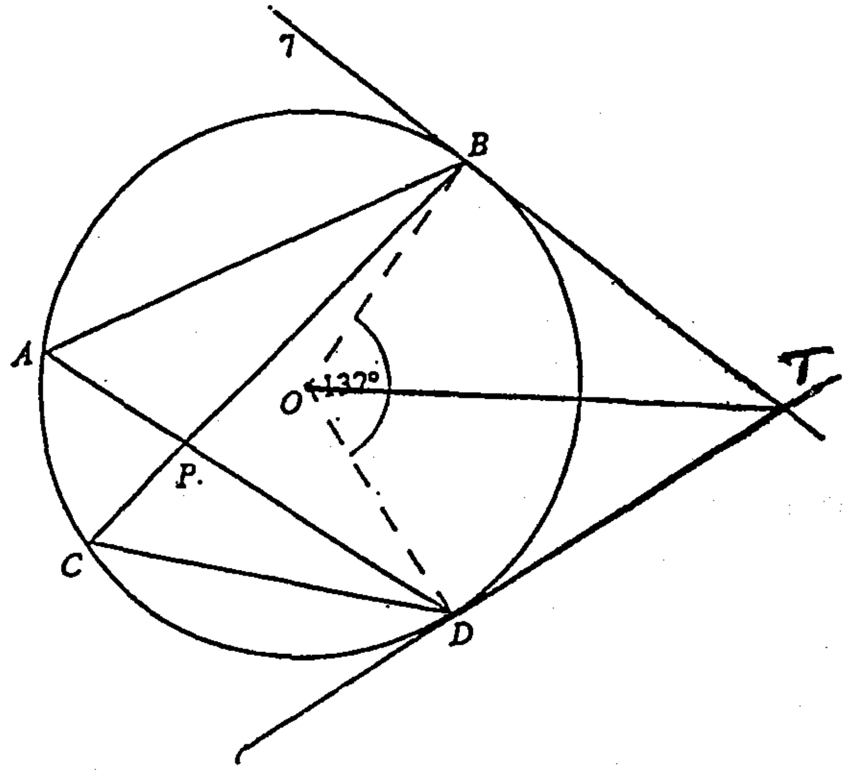
$= 116 \text{ min}$

(d)

Time	$60 < t \le 90$	$90 < t \le 105$	$105 < t \le 120$	$120 < t \le 150$	$150 < t \le 210$
Width	30	15	15	30	60
Width in cm	2	1	1	2	4
Frequecy	152	200	120	120	128
Area $= \frac{f}{10} cm^2$	15.2	20	12	12	12.8
height $= \frac{Area}{Width}$	7.6	20	12	6	3.2


20cm
18cm
16cm
14cm
12cm
10cm
8cm
6cm
4cm
2cm
0
30
60
90
120
150
180
210
Time t (minutes)

8.



(a) (i) $<BAD = <BCD = \frac{132}{2} = 66°$

(ii) Δ's ABP and CDP are equiangular
$<A = <C$, $<B = <D$ angles on the same arc
$<APB = <CPD$ vertically opposite
∴ Δ's are similar.

(iii) Since Δ's ABP and CDP are similar
$\frac{AB}{CD} = \frac{BP}{DP} = \frac{AP}{CP}$ $\qquad$ $\frac{17.5}{CD} = \frac{BP}{8} = \frac{6}{3}$
∴ $BP = \frac{6 \times 8}{3} = 16$ cm $\qquad$ $CD = \frac{3 \times 17.5}{6} = 8.75$ cm

(iv)
$\frac{\text{area of } \Delta\text{ CDP}}{\text{area of } \Delta\text{ ABP}} = \left(\frac{CP}{AP}\right)^2$

$\frac{\text{area of } \Delta\text{ CDP}}{n} = \left(\frac{3}{6}\right)^2 = \frac{1}{4}$

area of Δ CDP $= \frac{1}{4}n$

(b) (i) $<BTD = 180 - 132 = 18°$

(ii) OT is the diameter as angle OBT = 90°

$\frac{132}{2} = 66°$ $\cos 66 = \frac{OB}{OT}$

$OT = \frac{9.5}{\cos 66} = 23.4$ cm ∴ diameter = 23 cm

9. (a) (i) $\overrightarrow{MA} = \frac{1}{2}a$

(ii) $\overrightarrow{AB} = b - a$

(iii) $\overrightarrow{AC} = 3\overrightarrow{AB} = 3(b - a)$

(iv) $\overrightarrow{MC} = \overrightarrow{MA} + \overrightarrow{AC}$

$= \frac{1}{2}a + 3b - 3a = 3b - 2\frac{1}{2}a$

(v) Position vector of C

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = a + 3b - 3a = 3b - 2a$

(b) (i) $\overrightarrow{MN} = \frac{1}{5}\overrightarrow{MC} = \frac{1}{5}(3b - 2\frac{1}{2}a) = \frac{3}{5}b - \frac{1}{2}a$

$\overrightarrow{ON} = \overrightarrow{OM} + \overrightarrow{MN} = \frac{1}{2}a + \frac{3}{5}b - \frac{1}{2}a = \frac{3}{5}b$

(ii) $\overrightarrow{ON} = \frac{3}{5}b = \frac{3}{5}\overrightarrow{OB}$

ON : OB = 3 : 5

∴ ON : NB = 3 : 2

June 1996

Paper 4

1. (a) amount paid for the car = $\frac{3}{5} \times 12000 = \$\ 7200$

(b)

Cost	Loss	Selling price
100	30	70
7200		?

Selling price = $\frac{70}{100} \times 7200 = 5040$

(c) Amount invested in the bank = $\frac{2}{5} \times 12000 = \$\ 4800$

Interest $= \frac{PRT}{100} = \frac{4800 \times 8 \times \frac{18}{12}}{100} = \$\ 576$

Total amount he took from bank = 4800 + 576 = \$ 5376

(d) Amount left = 5040 + 5376 = \$ 10416

percentage = $\frac{10416}{12000} \times 100 = 86.8\ \%$

2. (a) (i) $AB^2 = 60^2 + 100^2 - 2 \times 60 \times 100\ \text{Cos}\ 80$

$AB = 107.3 = 107$ km

(ii) $\frac{100}{\text{Sin A}} = \frac{107.3}{\text{Sin } 80}$

$A = 66.6°$

(iii) Bearing of B from A

$360 - [(180 - 30) + 66.6] = 143.4°$

(b) faster ship is the one sailed to B.

(i) $t = \frac{100}{20} = 5$ h

(ii) speed of the slower ship = $\frac{60}{5} = 12$ km/h

3. (a) $\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3\times -3+2\times 2 \\ -1\times -3+6\times 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 15 \end{pmatrix} = \begin{pmatrix} x \\ 4 \end{pmatrix}$

$x = -5 \qquad y = 15$

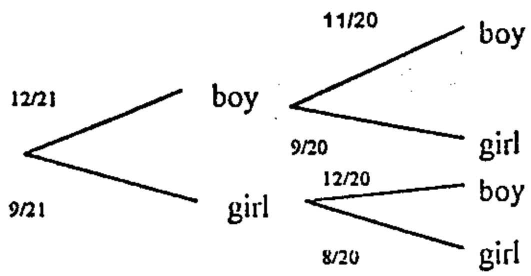
(b) inverse of $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix} = \dfrac{1}{6-(-4)} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} = \dfrac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 3t & u \\ -t & 3u \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ -10 \end{pmatrix}$

$3t + 2u = 10$
$-t + 6u = -10 \quad \times 3$
$-3t + 18u = -30$
adding $\quad 20u = -20$
$\underline{u = -1}$
$3t - 2 = 10 \quad \Rightarrow \quad 3t = 12$
$\underline{t = 4}$

4. (a) If the first student chosen is a boy, the number of boys then is $12 - 1 = 11$ and the total number of students is $11 + 9 = 20$, therefore, probability that the second student is also a boy $= \dfrac{11}{20}$

(b)



(c) (i) $P(BB) = \dfrac{12}{21} \times \dfrac{11}{20} = \dfrac{11}{35}$

(ii) $P(GG) = \dfrac{9}{21} \times \dfrac{8}{20} = \dfrac{6}{35}$

(iii) $P(BG\,,GB) = \frac{12}{21}\times\frac{9}{20}+\frac{9}{21}\times\frac{12}{20}=\frac{18}{35}$

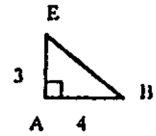
OR $1-\left(\frac{11}{35}+\frac{6}{35}\right)=\frac{18}{35}$

(d) (i) $P(BBB) = \frac{12}{21}\times\frac{11}{20}\times\frac{10}{19}=\frac{22}{133}$

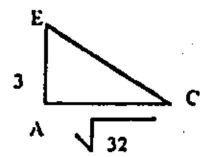
(ii) At least one of the three is a girl = 1 − all are boys

$= 1 - \frac{22}{133} = \frac{111}{133}$

5. (a) (i) $EB = \sqrt{3^2+4^2} = 5$ cm.

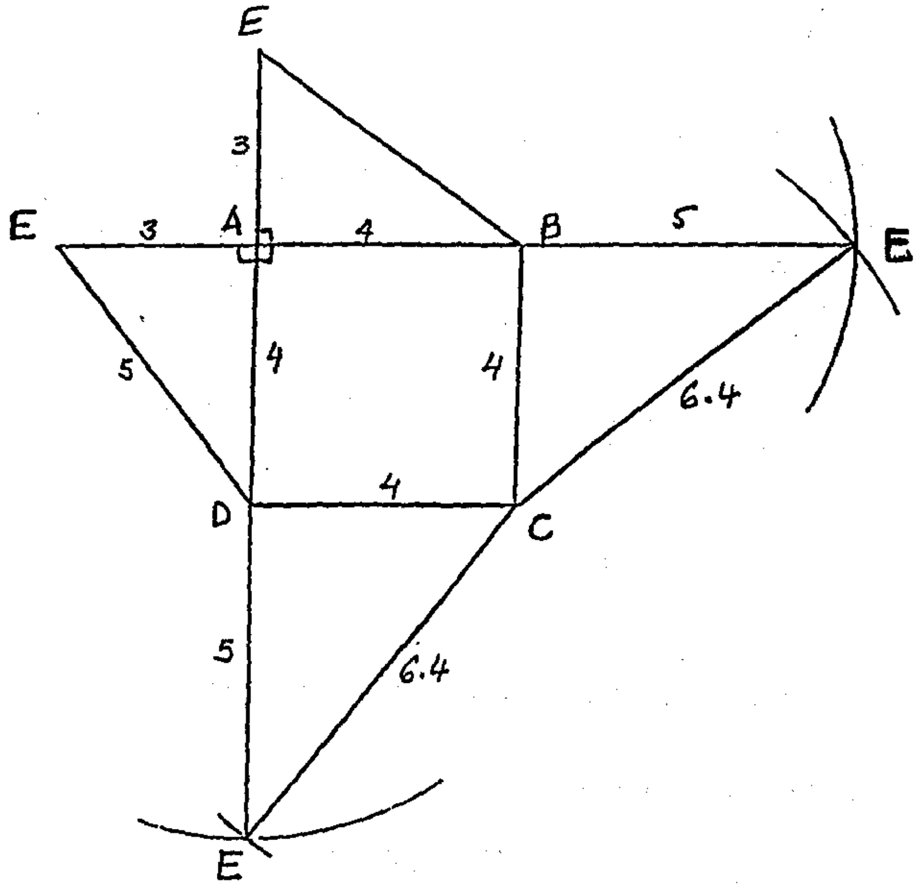


(ii) $AC = \sqrt{4^2+4^2} = \sqrt{32} = 5.66$ cm



(iii) $EC = \sqrt{3^2+\left(\sqrt{32}\right)^2} = 6.40$ cm

(b)



6. (a) perimeter $= 2(x + 3) + 2y = 20$

$$2x + 6 + 2y = 20$$
$$2y = 14 - 2x$$
$$y = 7 - x$$

(b) Area $= (x + 3)\,y = (x + 3)(7 - x) = 19$

$$7x - x^2 + 21 - 3x = 19$$
$$x^2 - 4x - 2 = 0$$

(c) $x = \dfrac{4 \pm \sqrt{4^2 - 4 \times 1 \times (-2)}}{2}$

$= \dfrac{4 \pm \sqrt{24}}{2} = 4.45 \quad \text{or} \quad -0.45$

(d) $x = 4.45$, Length $= 4.45 + 3 = 7.45$ cm

Width $= 7 - 0.45 = 2.55$ cm

or $x = 0.45$ Length $= -0.45 + 3 = 2.55$

Width $= 7 - (-0.45) = 7.45$

Same answers are obtained for any value of x.

7. (a)

Time	0–1	1–2	2–4	4–6	6–8	8–10	10–15
mid value (x)	½	1½	3	5	7	9	12½
frequency(f)	12	14	20	14	12	18	10
fx	6	21	60	70	84	162	125

(i) mean $= \frac{\sum fx}{\sum f} = \frac{6+21+60+70+84+162+125}{100}$

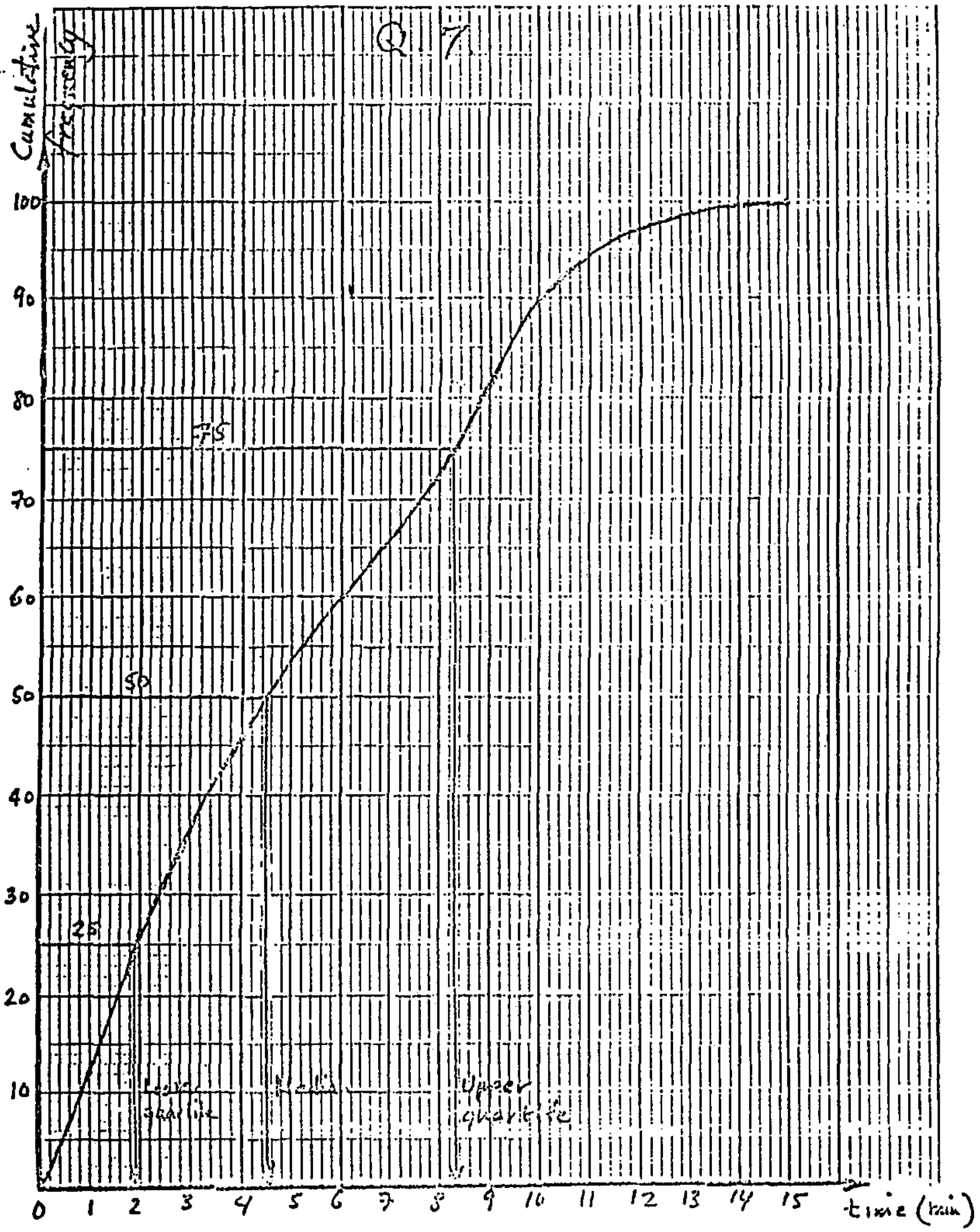
$= \frac{528}{100} = 5.28$ minutes

(ii) mean = 5 min 17 sec to the nearest sec.

(b) Commulative frequency table

Time	Commulative frequency
0	0
≤ 1	12
≤ 2	26
≤ 4	46
≤ 6	60
≤ 8	72
≤ 10	90
≤ 15	100

(c)


Q 7
Cumulative frequency
100
90
80
70
60
50
40
30
20
10
75
50
25
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
time (min)
Upper quartile

(d) (i) From the graph media = 4.5 min.
(ii) Upper quartile = 8.3 min.
(iii) Interquartile range = Upper − Lower = 8.3 − 1.9 = 6.4 min.

8. (a) (i) $\text{Arc} = \dfrac{\text{angle}}{360} \times 2\pi r$

$$\frac{4\pi}{3} = \frac{x}{360} \times 2\pi \times 4$$

$$x = \frac{4 \times 360}{3 \times 8} = 60°$$

(ii) $\text{Area of sector} = \dfrac{60}{360} \times \pi r^2$

$$= \tfrac{1}{6} \times 3.142 \times 4^2 = 8.38 \text{ cm}^2$$

(iii) Area of triangle ACB $= \tfrac{1}{2} \times 4 \times 4 \sin 60 = 6.93 \text{ cm}^2$

(iv) Shaded area = sector − triangle

$$= 8.38 - 6.93 = 1.45 \text{ cm}^2$$

(b) (i) $\text{Cos } \theta = \dfrac{10^2 + 10^2 - 4^2}{2 \times 10 \times 10} = 0.92$

$\theta = 23.07$ i.e. $23.1°$

(ii) Shaded area = sector − triangle

$$= \frac{23.1}{360} \times \pi \times 10^2 - \frac{1}{2} \times 10 \times 10 \sin 23.1°$$

$$= 20.16 - 19.62 = 0.542 \text{ cm}^2$$

(c) Shaded area enclosed = Sum of the previous two shaded area

$$= 1.45 + 0.542 = 1.99 \text{ cm}^2$$

9. (a) $p = \frac{5}{12} = 2.4$

$q = \frac{12}{6} = 2$

$r = \frac{12}{8} = 1.5$

$s = (3 - 8)(2 - 3) = 5$

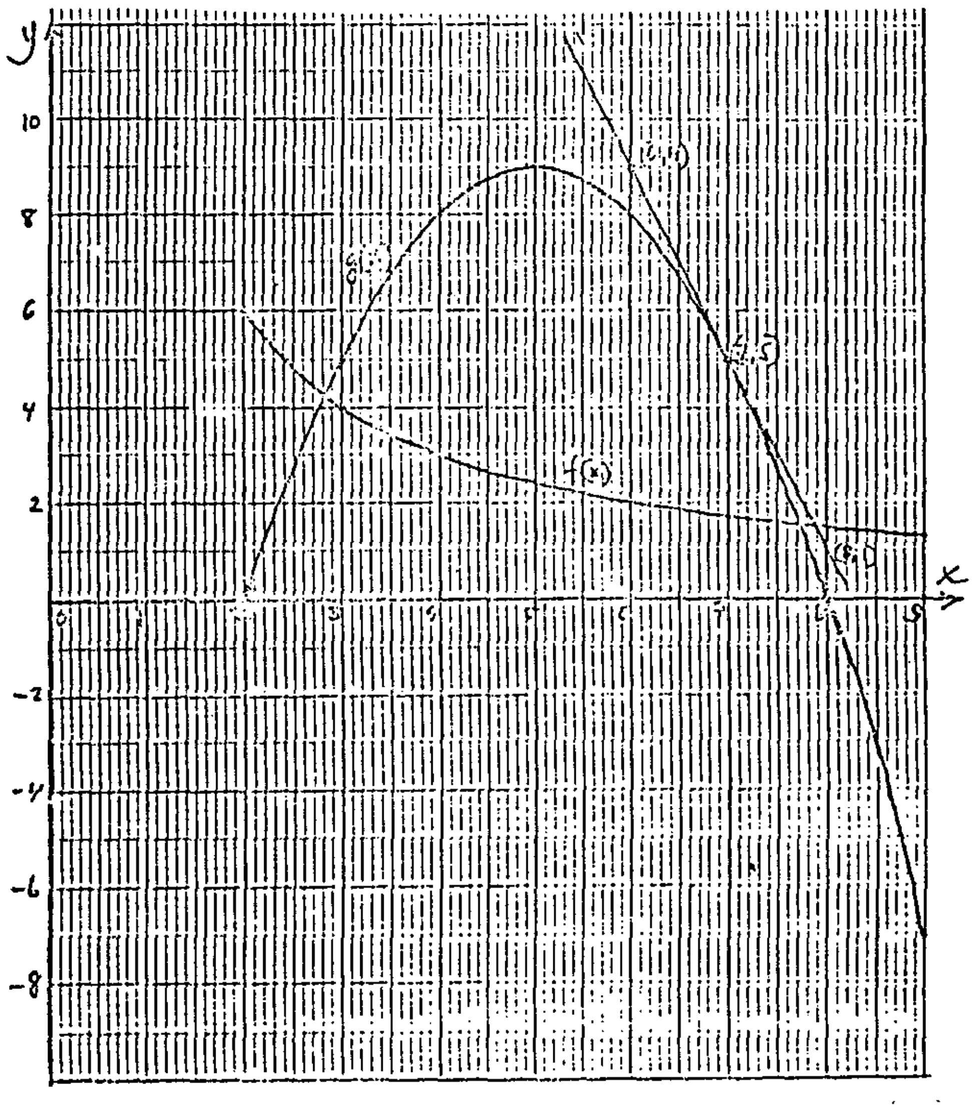
$t = (5 - 8)(2 - 5) = 9$

$u = (9 - 8)(2 - 9) = 7$

x	2	3	4	5	6	7	8	9
f(x)	6	4	3	2.4	2	1.7	1.5	1.3

x	2	3	4	5	6	7	8	9	2 ½	7 ½
g (x)	0	5	8	9	8	5	0	−7	2.75	2.75

(b)



(c) (i) At the points of intersection $F(x) = g(x)$

$$\frac{12}{x} = (x-8)(2-x) = -x^2 + 10x - 16$$

$$12 = -x^3 + 10x^2 - 16x$$

$$x^3 - 10x^2 + 16x + 12 = 0$$

(ii) Solutions are $x = 2.8$, $x = 7.7$

(d) gradient of tangent $= \frac{9-1}{6-8} = -4$

10. (a) (i) The next two terms are

$$\frac{13}{13+8} = \frac{13}{21} \quad \text{and} \quad \frac{21}{21+13} = \frac{21}{34}$$

(ii) $\frac{a}{b}, \frac{b}{a+b}, \frac{a+b}{a+2b}, \frac{a+2b}{2a+3b}, \frac{2a+3b}{3a+5b}$

(b) (i) $\frac{1}{x}, \frac{2}{x+1}, \frac{3}{x+2}, \frac{4}{x+3}, \frac{5}{x+4}, \frac{6}{x+5}$

(ii) 100<u>th</u> term is $\frac{100}{x+99}$

(iii) $\frac{10}{x+9} = \frac{1}{2}$

$x + 9 = 20$ $\qquad$ $x = 11$

(c) $$\frac{2}{x+1} - \frac{3}{x+2} = \frac{2(x+2) - 3(x+1)}{(x+1)(x+2)}$$

$$= \frac{2x + 4 - 3x - 3}{(x+1)(x+2)} = \frac{1-x}{(x+1)(x+2)}$$

Nov. 96

Paper 4

1. (a) (i) $<BDE = \frac{1}{2} \times 50 = 25°$

(ii) $<OED = <ODE = 25°$

$<OEC = 180 - 25° = 155°$

(iii) $<BCE = 90 - 25 = 65°$

(b) $<OBE = \frac{180-50}{2} = 65°$

$<DFE = 180 - <OBE$

$= 180 - 65° = 115°$

2. (a) (i) $1 - 0.15 = 0.85$

(ii) $0.15 \times 0.15 = 0.0225$

(b) (i) GBB BGB BBG

GGB GBG BGG

(ii) (a) P (at least one girl) = 1 – P (all boys)

$= 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8}$

(b) P (Two girls) = P (GGB, GBG, BGG)

$= 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$

(c) P (BGB *or* GBG)

$= 2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

3. (a) Volume = volume of cylinder + volume of hemisphere

$= \pi r^2 h + \frac{1}{2} + \frac{4}{3}\pi r^3$

$= \pi \times 3^2 \times 4 + \frac{2}{3} \times \pi \times 3^3 = 54\pi = 170 \text{ cm}^3$

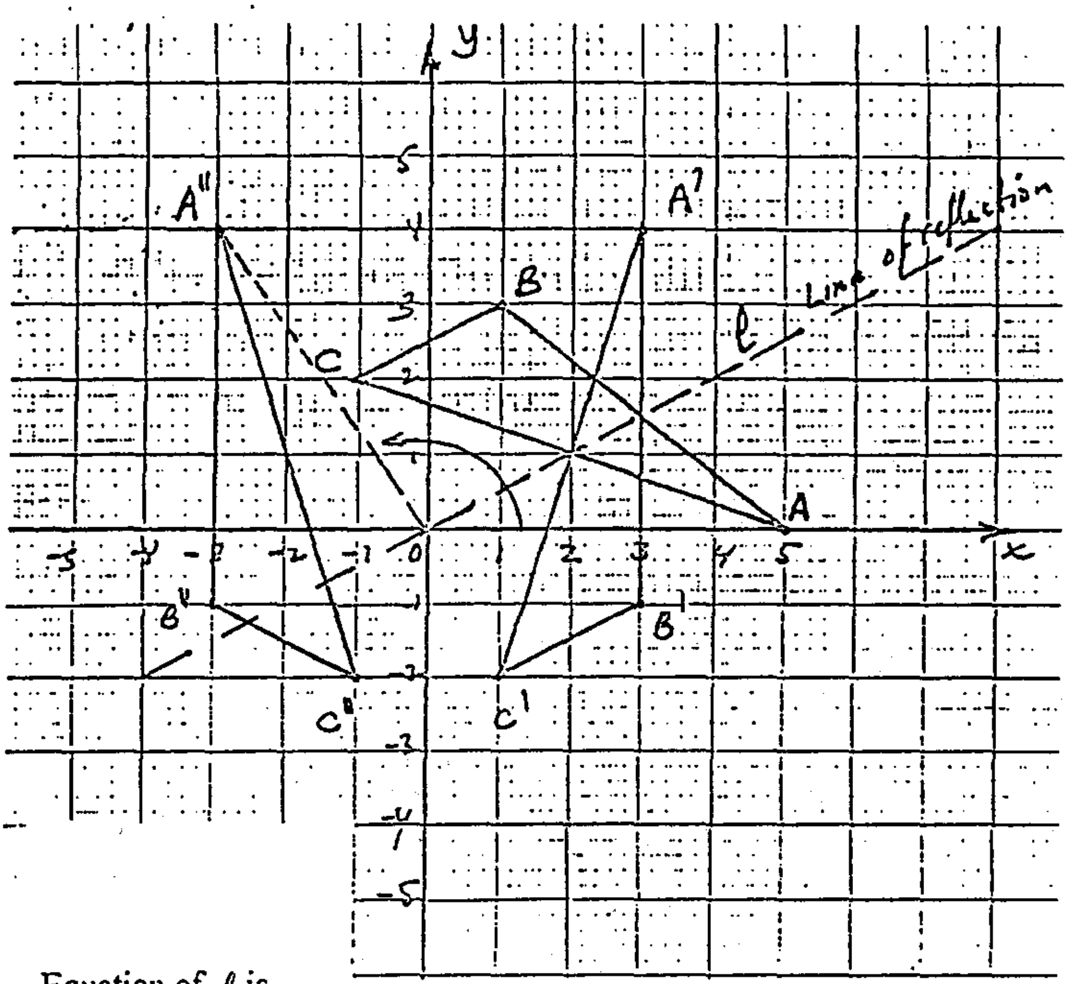
(b) (i) 0.7 litre $= 0.7 \times 1000 = 700 \text{ cm}^3$

no. of glasses $= \frac{700}{170} = 4.13$

no. of full glasses = 4

(ii) How much left $= 700 - 4 \times 170 = 20 \text{ cm}^3$

4. (a) and (b)



Equation of ℓ is
$y = \frac{1}{2}x$

(b) (iii) $\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} 5 & 1 & -1 \\ 0 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 1 \\ 4 & -1 & -2 \end{pmatrix}$

$5p + q \times 0 = 3$ $\qquad p = \frac{3}{5}$

$5r + 5 \times 0 = 4$ $\qquad r = \frac{4}{5}$

$p + 3q = 3$ $\qquad 3q = 3 - \frac{3}{5} = \frac{12}{5} \Rightarrow q = \frac{4}{5}$

$r + 3s = -1$ $\qquad 3s = -1 - \frac{4}{5} = -\frac{9}{5} \Rightarrow s = -\frac{3}{5}$

(iv) Matrix represent reflection on the lime $y = \frac{1}{2}x$

(c) on the diagram.

(d) Angle of rotation = < $AOA^{11} = 180° - \tan^{-1}\frac{4}{3} = 127°$

5. (a) (i) $\text{Cos} < BAC = \dfrac{160^2 + 100^2 - 120^2}{2 \times 160 \times 100}$

$< BAC = 48.5°$

(ii) Bearing of C from A = 048.5°

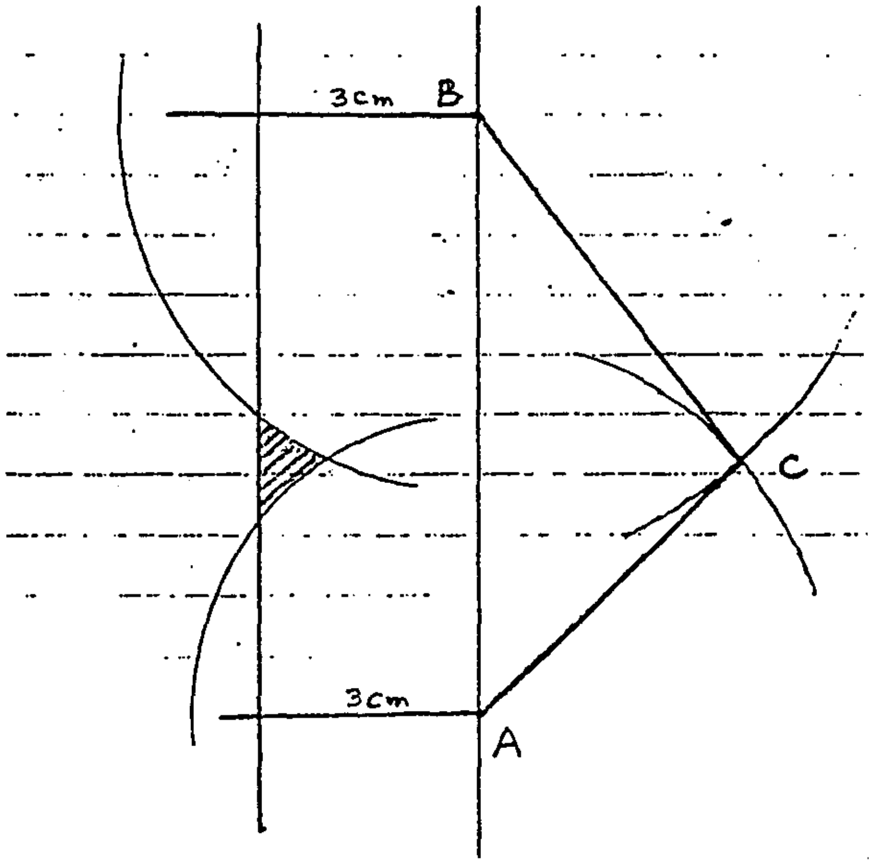
Bearing of A from C = 180 + 48.5 = 228.5°

(iii) Shortest distance = d

$\text{Sin } 48.5° = \dfrac{d}{100} \Rightarrow d = 74.9 \text{ m}$

(b) $160 \text{ m} \rightarrow \dfrac{160}{20} = 8 \text{ cm}$

120 m → 6 cm   100 m → 5 cm
80 m → 4 cm   60 m → 3 cm



6. (a) first $= x$

Second $= (x+3)$

Third $= (x+3)^2$

(b) (i) $x + (x+3) + (x+3)^2 = 77$

(ii) $x + x + 3 + x^2 + 6x + 9 = 77$

$x^2 + 8x - 65 = 0$

(iii) $(x+13)(x-5) = 0$

$x = 5$

(iv) numbers are 5, 8, 64

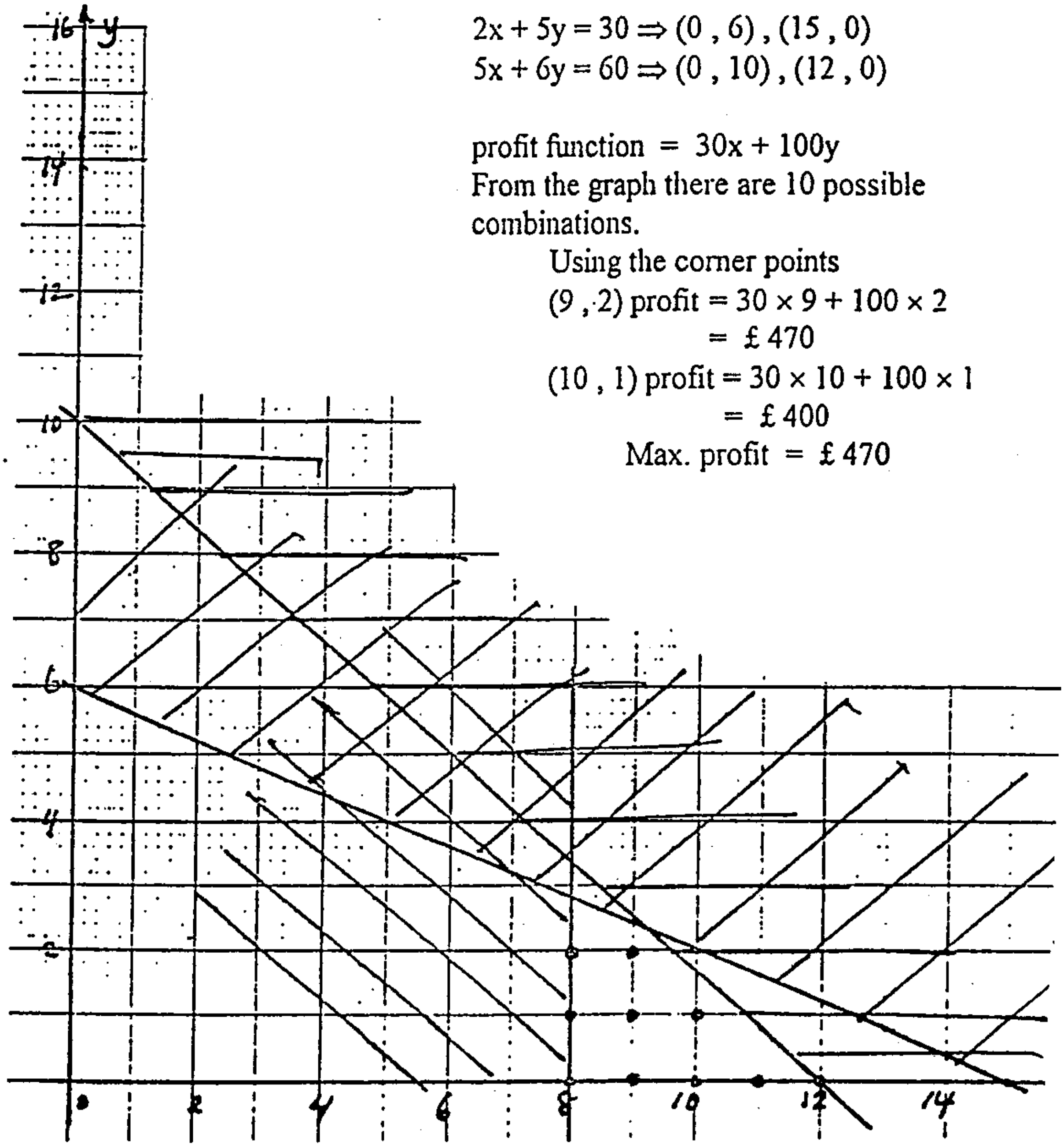
7.

	Cutting	Sewing
x jacket	5	4
y Suit	6	10

(a) $4x + 10y \leq 60 \Rightarrow 2x + 5y \leq 30$

(b) $5x + 6y \leq 60$

(c) $x \leq 8$



$2x + 5y = 30 \Rightarrow (0, 6), (15, 0)$

$5x + 6y = 60 \Rightarrow (0, 10), (12, 0)$

profit function $= 30x + 100y$

From the graph there are 10 possible combinations.

Using the corner points

$(9, 2)$ profit $= 30 \times 9 + 100 \times 2$
$= £\,470$

$(10, 1)$ profit $= 30 \times 10 + 100 \times 1$
$= £\,400$

Max. profit $= £\,470$

8.

Amount x	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	
mid. value x	5	15	25	35	45	55	
frequency f	0	4	8	12	11	5	$\sum f = 40$
fx	0	60	200	420	495	275	$\sum fx = 1450$

(a) $\text{Mean} = \dfrac{\sum fx}{\sum f} = \dfrac{1450}{40} = 36.25$

(b)

Amount x	$x \leq 10$	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60
Cumulative frequency	0	4	12	24	35	40

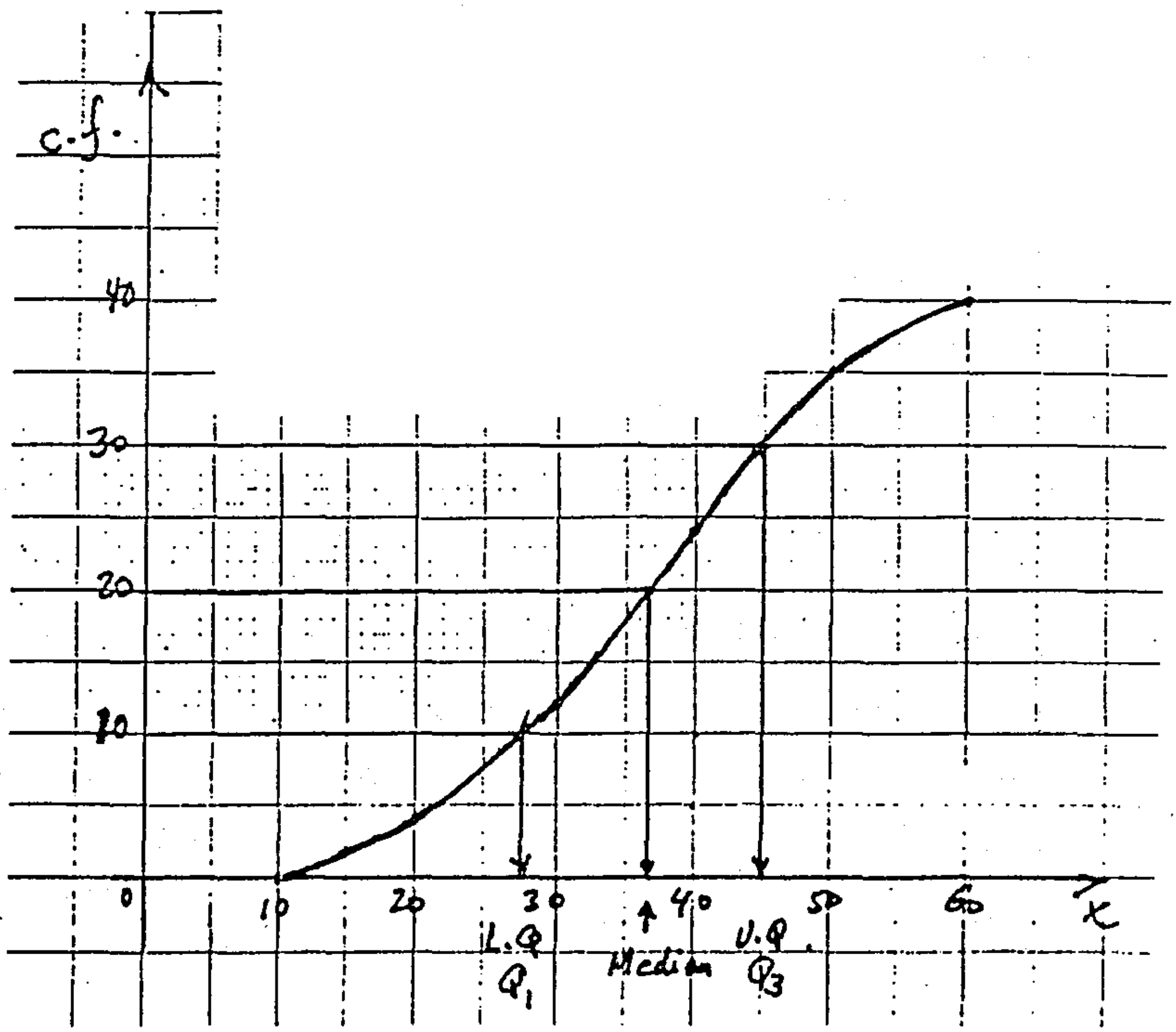
p = 12 , q = 24 , r = 35

(d) Median = 36.7

Upper quartile = 44.8

Lower quartile = 27.5

Inter quartile range = 44.8 – 27.5 = 17.3



9. (a) $\frac{2x+1}{3} - \frac{x-1}{2} = \frac{2(2x+1)-3(x-1)}{6} = \frac{4x+2-3x+3}{6} = \frac{x+5}{6}$

(b) (i) $x^2 - 5x + 6 = (x-2)(x-3)$

(ii) $\frac{x^2-5x+6}{x^2+x-6} = \frac{(x-2)(x-3)}{(x+3)(x-2)} = \frac{x-3}{x+3}$

(c) $3x^2 = 7x - 1$

$3x^2 - 7x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{7 \pm \sqrt{49 - 4 \times 3 \times 1}}{6} = \frac{7 \pm \sqrt{37}}{6}$

$= 2.18$ *or* 0.15

10. (a) $< OZM = \frac{1}{2} \times 60 = 30°$

(b) $\text{Cos } 30° = \frac{3}{OZ} \Rightarrow OZ = 3.464$

(c) $OW = \sqrt{6^2 - (3.464)^2} = \sqrt{24} = 4.9$

(d) base area $= \frac{1}{2} \times 6 \times 6 \sin 60 = 15.5885$

volume $= \frac{1}{3} \times 15.5885 \times 4.90 = 25.5 \text{ cm}^2$

(e) Cosine angle $= \frac{OZ}{6} = \frac{3.464}{6}$

angle $= 54.7°$

11. (a)

3	135		2	210		2	1120
3	45		3	105		2	560
3	15		5	35		2	280
3	5		7	7		2	140
	1			1		2	70
						5	35
						7	7
							1

$135 = 3 \times 3 \times 3 \times 5$
$210 = 2 \times 3 \times 5 \times 7$
$1120 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7$
$1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

(b)

a = 1	b = 9	c = 6	d = 3	e = 5	f = 4	g = 2	h = 7	i = 8

(i) 5
(ii) e ∴ <u>e = 5</u>
(iii) 210 & 1120
<u>h = 7</u>
$a \times b \times d \times e = 135$
$1 \times b \times d \times 5 = 135$
$bd = 27$
b and d are 3 and 9 —— (1)
$d \times e \times g \times h = 210$
$d \times 5 \times g \times 7 = 210$
$dg = 6$
d and g are 2 and 3 —— (2)
from (1) and (2) <u>d = 3</u> , <u>b = 9</u> , <u>g = 2</u>
$b \times c \times e \times f = 1080$
$g \times c \times 5 \times f = 1080$
$cf = 24$
one is 4 the other 6 —— (3)
$e \times f \times h \times i = 1120$ $5 \times f \times 7 \times i = 1120$
$fi = 32$ one is 4 the other 8 —— (4)
∴ from (3) and (4) <u>f = 4</u> , <u>c = 6</u> <u>i = 8</u>

June 1997

Paper 4

1. (a) $\frac{80}{100} \times 200 = 160$

$$\begin{aligned} &\text{20 children} = 20 \times 2.50 = 50 \\ &\text{140 adult} = 140 \times 5 = 700 \\ &\text{total} \quad 700 + 50 = 750 \end{aligned}$$

(b) Sale of children tickets $= 2.5\,x$
Sale of adult tickets $= (200 - x)\,5$

$$\begin{aligned} 2.5x + (200 - x)\,5 &= 905 \\ 2.5x + 1000 - 5x &= 905 \\ -2.5x &= -95 \\ x &= 38 \end{aligned}$$

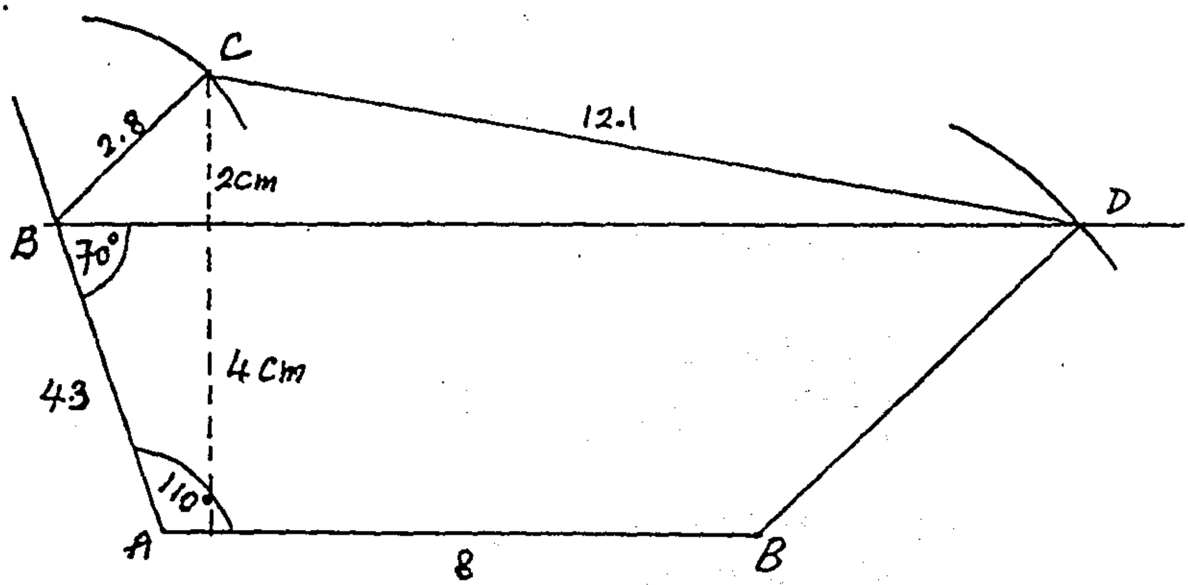
(c) (i)

					total
2	:	3	:	7	12
				?	10800

$$\text{profit} = \frac{7 \times 10800}{12} = 6300$$

(ii) $I = \frac{PRT}{100} = \frac{6300 \times 5 \times \frac{4}{12}}{100} = £\,105$

2.



(a) BD = 13.9

(b) Area of Δ BCD = ½ × 13.9 × 2 = 13.9

Area of trapezium = $\frac{8+13.9}{2} \times 4 = 43.8$

Total area of the pentagon = 13.9 + 43.8 = 57.7 cm^2

3. (a) (i) Square numbers are 1, 4, 9

Prob. $= \frac{3}{12} = \frac{1}{4}$

(ii) Prime numbers or numbers less than 6 are 1, 2, 3, 4, 5, 7, 11

Prob. $= \frac{7}{12}$

(b) (i) 12 + 9, 9 + 12 10 + 11, 11 + 10

(ii) $\frac{1}{12} \times \frac{1}{12} \times 4 = \frac{1}{36}$

(c)

Score	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	1	1	0	1	1	2	3	4	5	6	5
c.f	1	2	3	3	4	5	7	10	14	19	25	30

(i) the mode 11

(ii) the median

median order $\frac{30+1}{2} = 15\frac{1}{2}$

terms numbered 15, 16, 17, 18, 19 are all 10

Median is 10

(iii) the mean $\frac{\sum fx}{30} = 8.9$

(d) (i) area $= \frac{\theta}{360}\pi r^2 = \frac{30}{360} \times 3.142 \times 10^2 = 26.18\ cm^2$

$= 26.2\ cm^2$

(ii) Probability $= \frac{\text{Shaded area}}{\text{Area of square}} = \frac{26.18}{30 \times 30} = 0.0291$

4. (a) $\angle ABC = 90 + 25 = 115°$

(b) (i) $AC^2 = 12^2 \div 14^2 - 2 \times 12 \times 14 \text{ Cos } 115 = 21.95$

$= 22$ km

(ii) $\frac{AC}{\text{Sin B}} = \frac{BC}{\text{Sin A}}$

$\frac{21.95}{\text{Sin A}} = \frac{14}{\text{Sin A}}$

Sin A $= 0.5779$

A $= 35.3°$

(iii) Bearing of C from A $= 25 + 35.3 = 60.3°$

Bearing of A from C $= 180 + 60.3 = 240.3°$

5. (a) ABC and ADE are similar

(i) $\therefore \frac{AC}{AE} = \frac{BC}{DE}$ $\qquad \frac{5}{5+2x} = \frac{x+3}{4x+1}$

$(x+3)(5+2x) = 5(4x+1)$

$2x^2 + 11x + 15 = 20x + 5$

$2x^2 - 9x + 10 = 0$

(ii) $2x^2 - 9x + 10 = (2x - 5)(x - 2)$

(iii) $X = \frac{5}{2}, X = 2$

(iv) Ratio of sides $\frac{x+3}{4x+1} = \frac{2\frac{1}{2}+3}{4\times 2\frac{1}{2}+1} = \frac{5\frac{1}{2}}{11} = \frac{1}{2}$

ratio of areas $= \left(\frac{1}{2}\right)^2$

(b) (i) determinant of M $= (2y+1)(2y+3) - y(3y-4)$

$= 4y^2 + 8y + 3 - 3y^2 + 4y$

$= y^2 + 12y + 3 = 10$

$y^2 + 12y - 7 = 0$

(ii) $y^2 + 12y - 7 = 0$

$y = \frac{-12 \pm \sqrt{144 - 4\times 1\times(-7)}}{2} = \frac{-12 \pm \sqrt{172}}{2}$

$y = 0.557 \ , \ -12.557$

$y = 0.6 \ \text{or} \ -12.6$

6. (a) $OC = \sqrt{6^2 - (3.6)^2} = 4.8$

$VC = 6 + 4.8 = 10.8$

(b) (i) the volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times 3.142 \times 6^3 = 904.896$

$= 905$

(ii) the volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\times 3.142 \times 3.6^2 \times 10.8$

$= 146.59 = 147$

(iii) percentage of sphere occupied $= \frac{147}{905} = 16.2\%$

not occupied $= 100 - 16.20 = 83.8\%$

(c) (i) $2\pi r = 37.704$

(ii) $\frac{300}{37.628} = 7.957 = 7$

(iii) remaining part of revolution $= 1 - 0.957 = 0.043$

Angle $= 0.043 \times 36 = 15.5°$

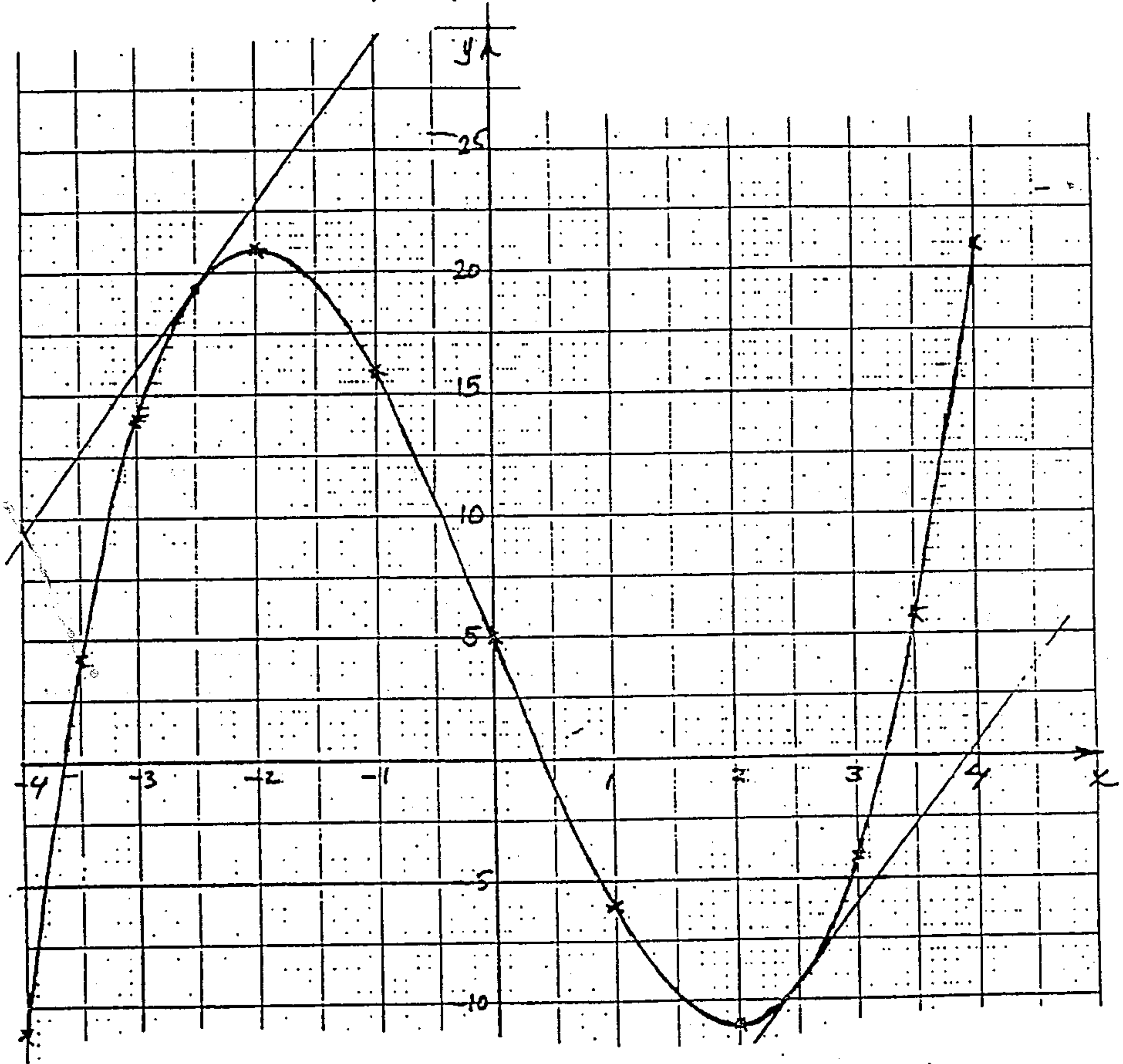
7. $f(x) = x^3 - 12x + 5$

(a) $a = (-1)^3 - 12(-1) + 5 = 16$ b = $(4)^3 - 12(4) + 5 = 21$

(c) (i) $f(x) = 0$ $y = 0$ $x = -3.65, 0.4, 3.25$

(ii) $x^3 - 12x + 10 = 0 \Rightarrow x^3 - 12x + 5 = -5$

$x = -3.8 , 0.9 , 2.9$



(d) Tangent passes through $(-3, 16)$ and $(-1, 29.5)$

gradient $= \frac{29.5-16}{(-1)-(-3)} = \frac{13.5}{2}$

$= 6.75$

(iii) To find another point the tangent at which is parallel to the tangent at $x = -2.5$

point is $x = 2.5$

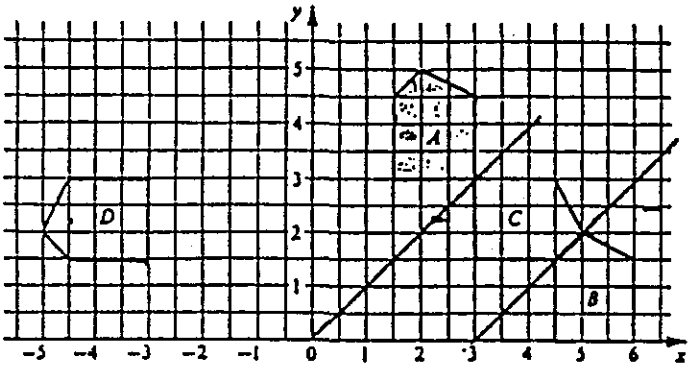
(tangent are parallel)

8. (a) (i) B, translation $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

(ii) C, reflection on the line $y = x$

(iii) D, rotation 90° anticlockwise centre origin

(b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



(c) reflection on the y axis.

(d) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(e) $y = x - 3$

9.

Birthday / Scheme	1st	2nd	3rd	4th	5th	6th	7th
A	\$10	\$20	\$30	\$40	\$50	\$60	\$70
B	\$1	\$2	\$3	\$8	\$16	\$32	\$64
C	\$1	\$4	\$9	\$16	\$25	\$36	\$49

(a) (i) his 7th birthday, 70, 64, 49.

(ii) his *n*th birthday, $10n$, 2^{n-1} , n^2

(b) (i) 550 , 1023 , 385.

(ii) 1710 , 262143 , 2109 A the smallest.

(iii) A and C an equal when

$$5n\ (n+1) = \frac{n(n+1)(2n+1)}{6}$$

$30 = 2n + 1$ $2n = 29$

$n = 14\frac{1}{2}$

i.e. up to 14th birthday C is smaller

& starting from 15th birthday A is smaller.

November 1997
Paper 4

1-(a) Volume = $\Pi r^2 h = 3.142 \times \left(\frac{8}{2}\right)^2 \times 11 = 552.992 = 553\ cm^3$

(b) (i) Length = $4 \times 8 = 32\ cm$
Width = $3 \times 8 = 24\ cm$

(ii) Volume of the box = $32 \times 24 \times 11 = 8448\ cm^3$
Volume occupied by the tins = $12 \times 552.992 = 6635.904$
Volume not occupied = 8448 - 6635.904 = 1812.096
Percentage not occupied = $\frac{1812.096}{8448} \times 100 = 21.45\% = 21.5\%$

(c)

Cost Price	Profit	Selling Price
100	25	125
?		0.60

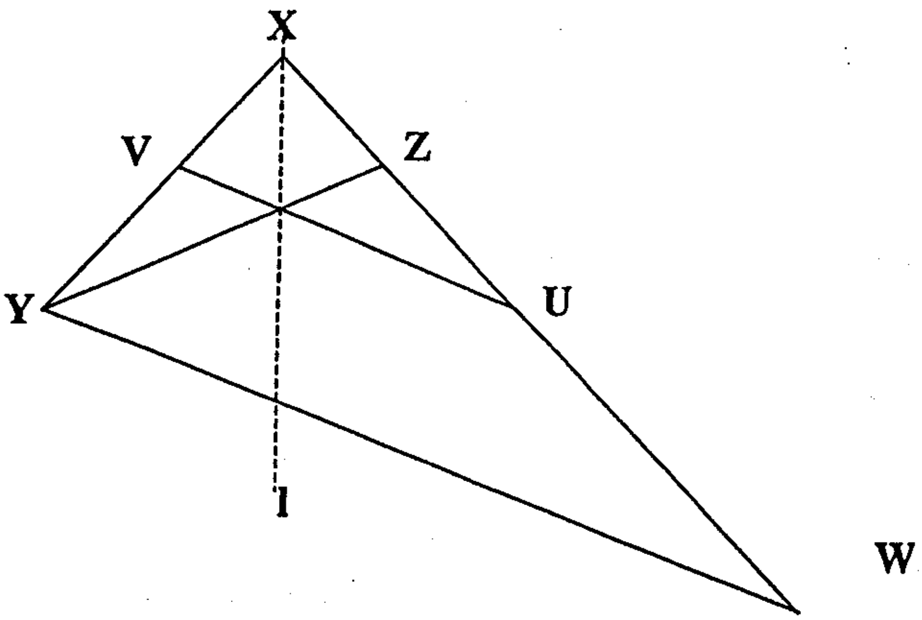
Cost price of one tin = $\frac{100 \times 0.60}{125} = 0.48$

Cost price of a box 12 tins = $12 \times 0.48 = \$5.76$

(d) (i) Selling price for a box = $12 \times 0.60 = \$7.20$
Saving per box = 7.20 - 6.49 = \$0.71

(ii) Cost price of a box = \$5.76
new selling price of a box = \$6.49
profit per box = 6.49 - 5.76 = 0.73
percentage profit = $\frac{0.73}{5.76} \times 100 = 12.7\%$

2-



(a) (i) M(Z) = V

(ii) XU = XY = 5 cm

XV = XZ = 2 cm

(iii) Scale factor of enlargement = $\frac{XY}{XV} = \frac{5}{2} = 2.5$

$XW = XU \times 2.5 = 5 \times 2.5 = 12.5cm$

VU = YZ = 6 cm

$YW = 6 \times 2.5 = 15cm$

(v) $\angle XYZ = \angle XUV = \angle XWY$

(b) $\cos \angle YXZ = \frac{5^2 + 2^2 - 6^2}{2 \times 5 \times 2} = \frac{-7}{20} = -0.35$

$\angle YXZ = 110.5°$

3-(a) (i) 11,12,13,14,21,22,23,24,31,32,33,34

(ii)(a) outcomes multiples of 4 are 12,24,32

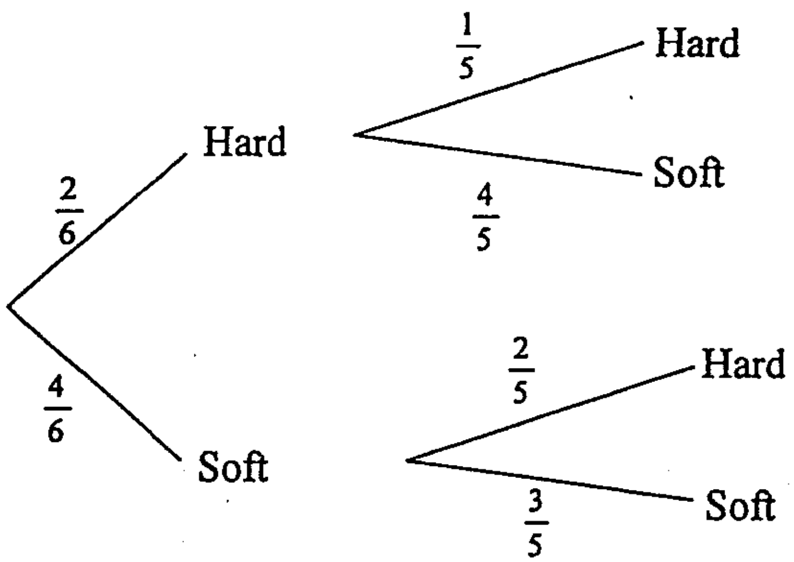
probability = $\frac{3}{12} = \frac{1}{4}$

(b) no outcome is a multiple of 5

$\therefore$ probability = Zero

(b) *Alice's choice* *Barbara's choice*

(i)



(ii)(a) $\frac{2}{6}\times\frac{1}{5}=\frac{1}{15}$

(b) Hard and Soft or Soft and Hard

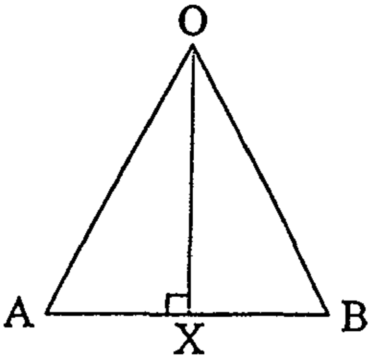
$= \frac{2}{6}\times\frac{4}{5}+\frac{4}{6}\times\frac{2}{5}=\frac{8}{15}$

(c) Hard and Hard or Soft and Hard

$=\frac{2}{6}\times\frac{1}{5}+\frac{4}{6}\times\frac{2}{5}=\frac{2}{30}+\frac{8}{30}=\frac{1}{3}$

4-(a) $\angle AOB=\frac{360}{7}=51.429$

$\angle OAB=\frac{180-51.429}{2}=64.29°$



(b)(i) $Sin\angle DAB=\frac{OX}{OA}$

$OX=1.5\sin 64.29°=1.35cm$

(ii) AB = 2 AX

$\cos 64.29=\frac{AX}{1.5}$

$AX=0.65$

$AB=2\times 0.65=1.30cm$

OR Use Cosine rule

$AB^2 = 1.5^2 + 1.5^2 - 2(1.5)(1.5)\cos 51.429$

(iii) area of $\Delta AOB = \frac{1}{2} base \times height$

$= \frac{1}{2} \times AB \times AX = \frac{1}{2} \times 1.30 \times 1.35$

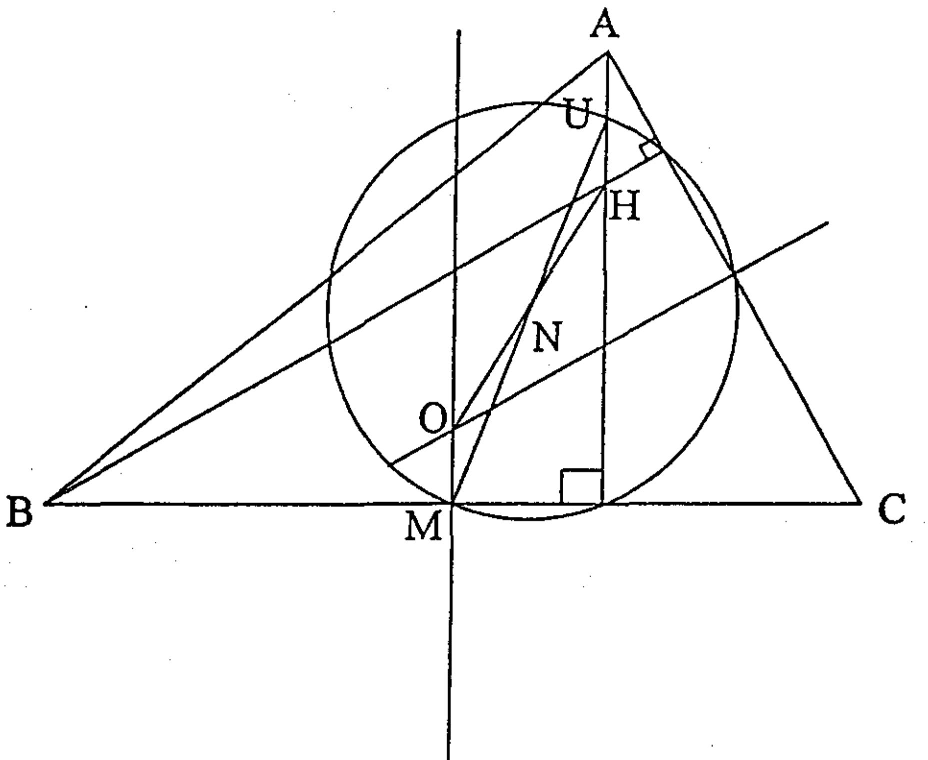
$= 0.8775 \approx 0.878 cm^2$

(iv) area of the whole face $= 7 \times 0.8775 = 6.14 cm^2$

(c) Volume = Area x thickness

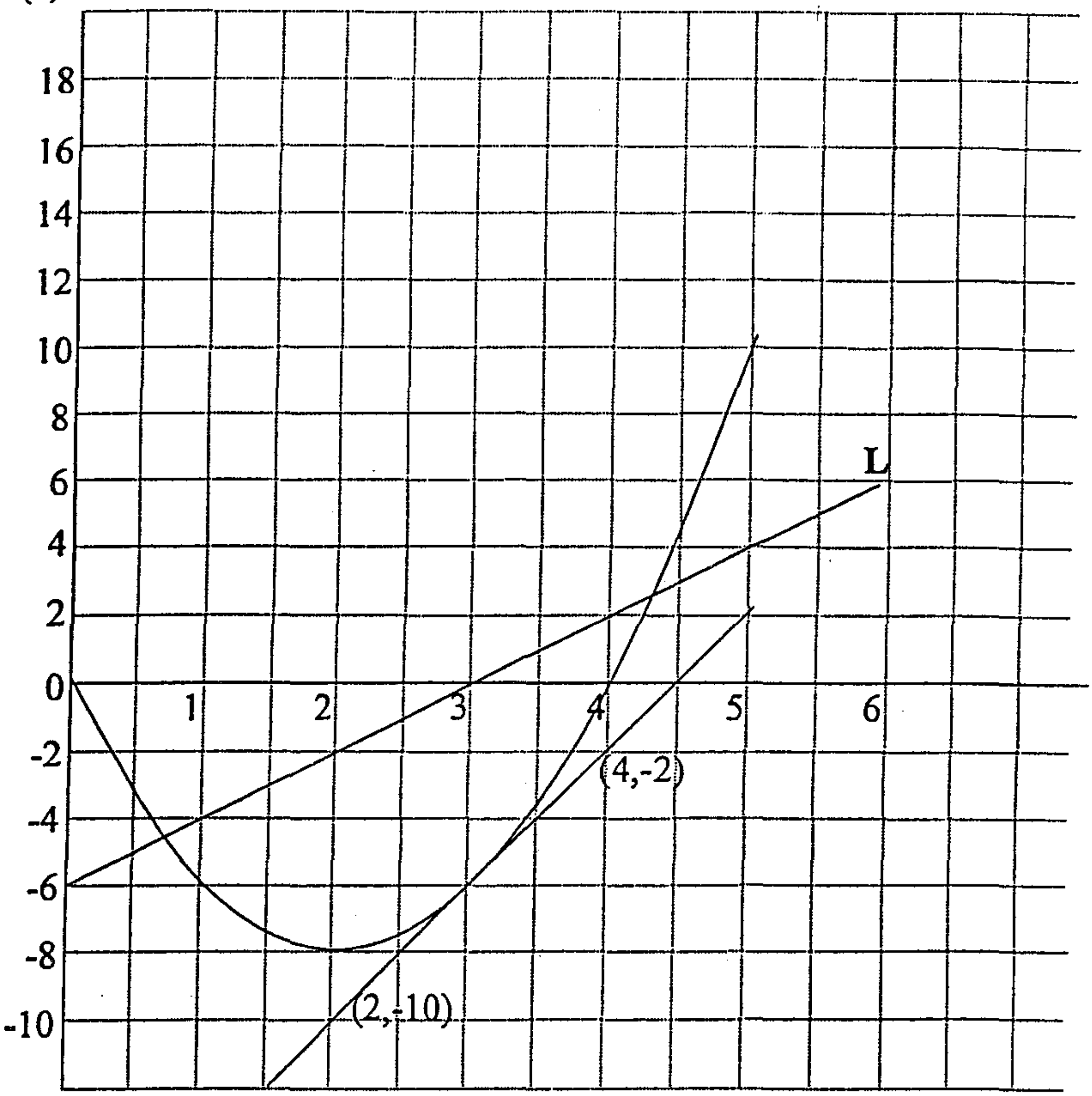
$= 6.14 \times \frac{3}{10} = 1.84 cm^3$

5-



(f) all equal.
(g) congruent
(h) radius = 2.6 cm

6- (a)



(b)(i) Line through (3,0) gradient 2, for each 1 unit along x y increases by 2 i.e. Line passes through (4,2) , (5,4) ....

(ii) Line intercepts y axis at –6 equation $y = 2x - 6$

(iii) (0.7 , -4.6) , (4.3 , 2.6)

(c) gradient of tangent = $\frac{-2-(-10)}{4-2} = 4$

(d) $y = ax^2 + bx$

$x = 4 \qquad y = 0$

$16a + 4b = 0$

$b = -4a$

$x = 1 \qquad y = -6$

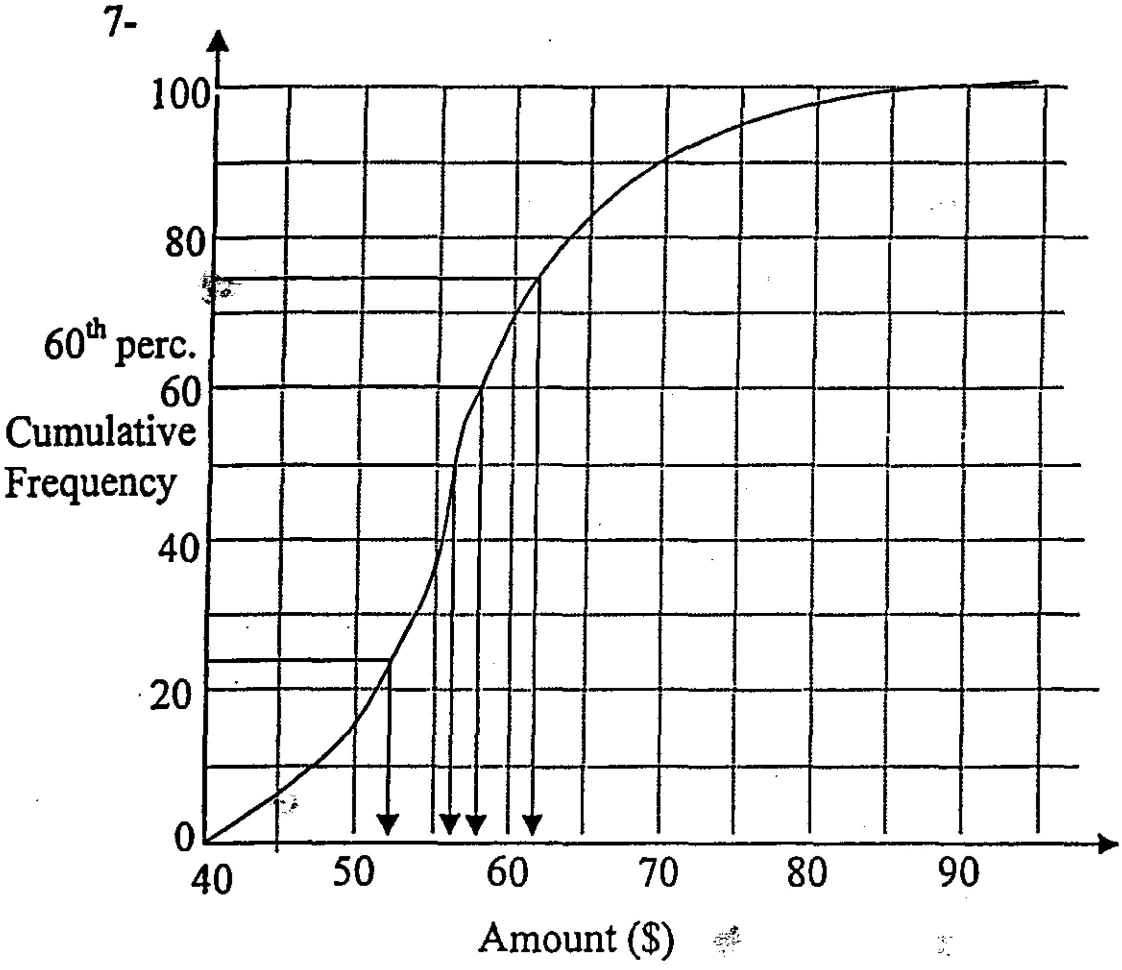
$-6 = a + b$

$-6 = a - 4a$

$-6 = -3a$

$a = 2 \qquad b = -8$

7-



(a)(i) Median = 56.5 ≈ $ 56 or $ 57 .

(ii) Upper quartile = $ 61

Lower quartile = $ 53

(iii) 60^{th} percentile = $ 58

(b)(i) Interquartile range = Upper quartile – Lower quartile
$= 61 - 53 = \$\ 8$

(ii) Percentage $= \frac{8}{50} \times 100 = 16\%$

(c)(i)

Weekly amount \$ x	Frequency	Midclass x	fx
$40 < x \le 50$	14	45	630
$50 < x \le 60$	$72 - 14 = 58$	55	3190
$60 < x \le 70$	$92 - 72 = 20$	65	1300
$70 < x \le 80$	$98 - 92 = 6$	75	450
$80 < x \le 90$	$100 - 98 = 2$	85	170
			5740

(ii) Modal class is $50 < x \le 60$

(iii) Mean $= \frac{\sum fx}{\sum f} = \frac{5740}{100} = 57.4$

(iv) Using smaller class intervals i.e. 40 – 42, 42 – 44, ...
Or 40 – 45, 45 – 50 etc.

8- (b)(i) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 & 6 \\ 2 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 6 \\ 6 & 8 & 16 \end{pmatrix}$

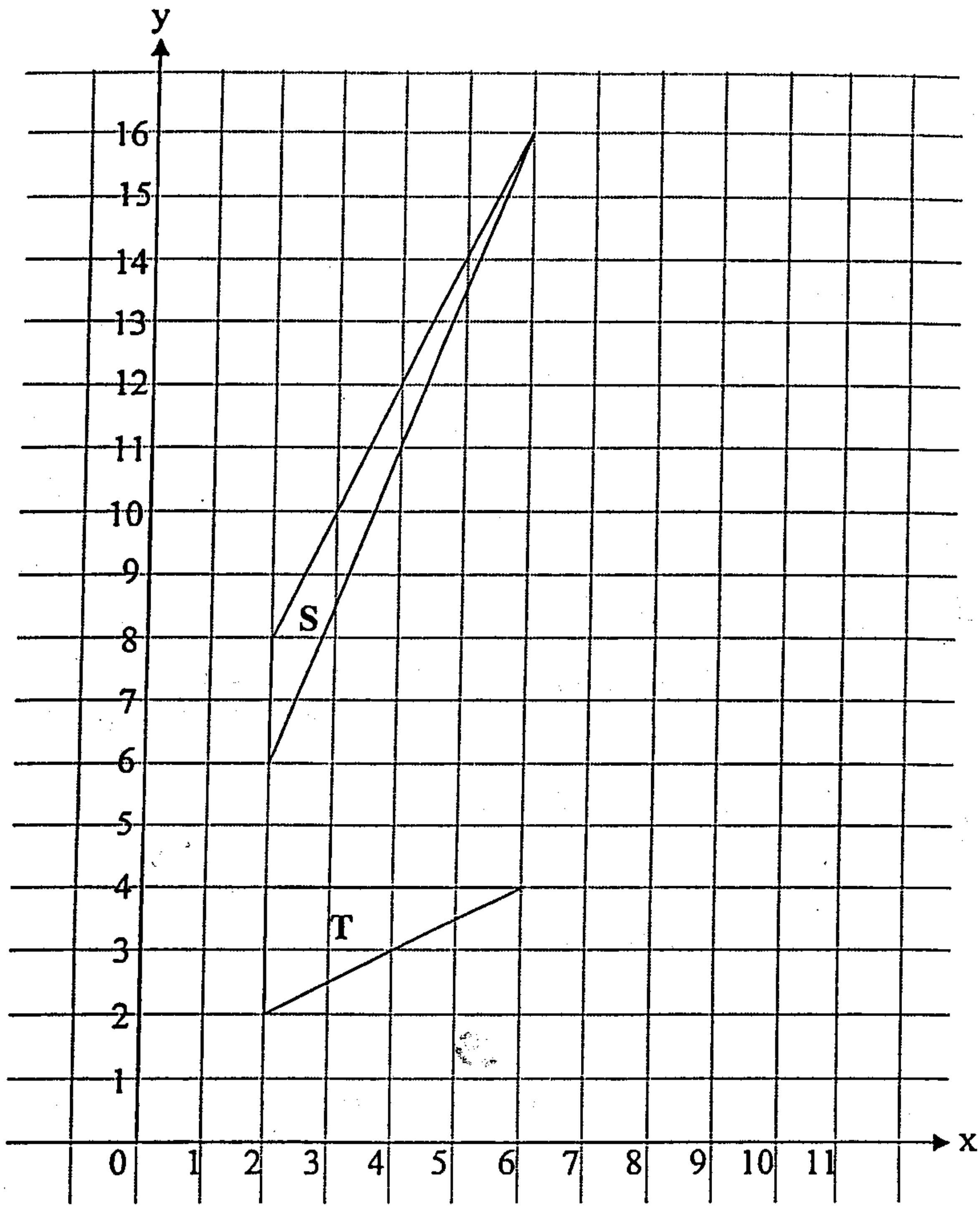
(ii) Area of S is the same as T
which equal $= \frac{1}{2} \times 2 \times 4 = 4$

(iii) transformation is a shear parallel to the y axis (y axis invariant)

(c)(i) $M = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

$M^{-1} = \frac{1}{1}\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$

(ii) Image of S under the transformation M^{-1} is T



9- (a) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

(b) $24^2 = 576$

$25^2 = 625$

$25^2 - 24^2 = 625 - 576 = 49 = 7^2$

Pythagorean triple is 7 , 24 , 25

(c) (i) $y^2 = x^2 - (x-2)^2$

$= x^2 - (x^2 - 4x + 4)$

$= 4x - 4$

$y = \sqrt{4x-4}$

(ii) $x = 50$ $\qquad$ $y = \sqrt{4 \times 50 - 4} = \sqrt{196} = 14$

$x - 2 = 50 - 2 = 48$

other two numbers are 48 , 14

(iii) $x = 101$ $\qquad$ $y = \sqrt{4 \times 101 - 4} = 20$

$101 - 2 = 99$

other two numbers are 99 , 20

(iv) since $\qquad$ $y = \sqrt{4x-4} = \sqrt{4(x-1)} = 2\sqrt{x-1}$

In order to get y whole number, x should be taken such that (x-1) is a perfect square.

Possible values of x are $9 + 1 = 10$ or $16 + 1 = 17$, $25 + 1 = 26$, $36 + 1 = 37$

for each x , $x - 2$ and y can be calculated

i.e.	$x = 10$	$x - 2 = 8$	$y = 2\sqrt{10-2} = 6$	{ 6,8,10 }
	$x = 17$	$x - 2 = 15$	$y = 2\sqrt{17-1} = 8$	{ 8,15,17 }
	$x = 26$	$x - 2 = 24$	$y = 2\sqrt{26-1} = 10$	{ 10,24,26 }
	$x = 37$	$x - 2 = 35$	$y = 2\sqrt{37-1} = 12$	{ 12,35,37 }

Any one set is a possible answer.

June 98

Paper 4

1- (a) Men women children total

6 : 7 : 3 16

42000

(i) number of children $= \dfrac{3 \times 42000}{7} = 18000$

(ii) Total number of people $= \dfrac{16 \times 42000}{7} = 96000$

(b) 10 years ago increase now

100 20 120

? 4200

number of women lived 10 years ago $= 4200 \times \dfrac{100}{120} = 35000$

(c) (i) number of boys $= \dfrac{48}{100} \times 12000 = 5760$

number of girls = 12000 – 5760 = 6240

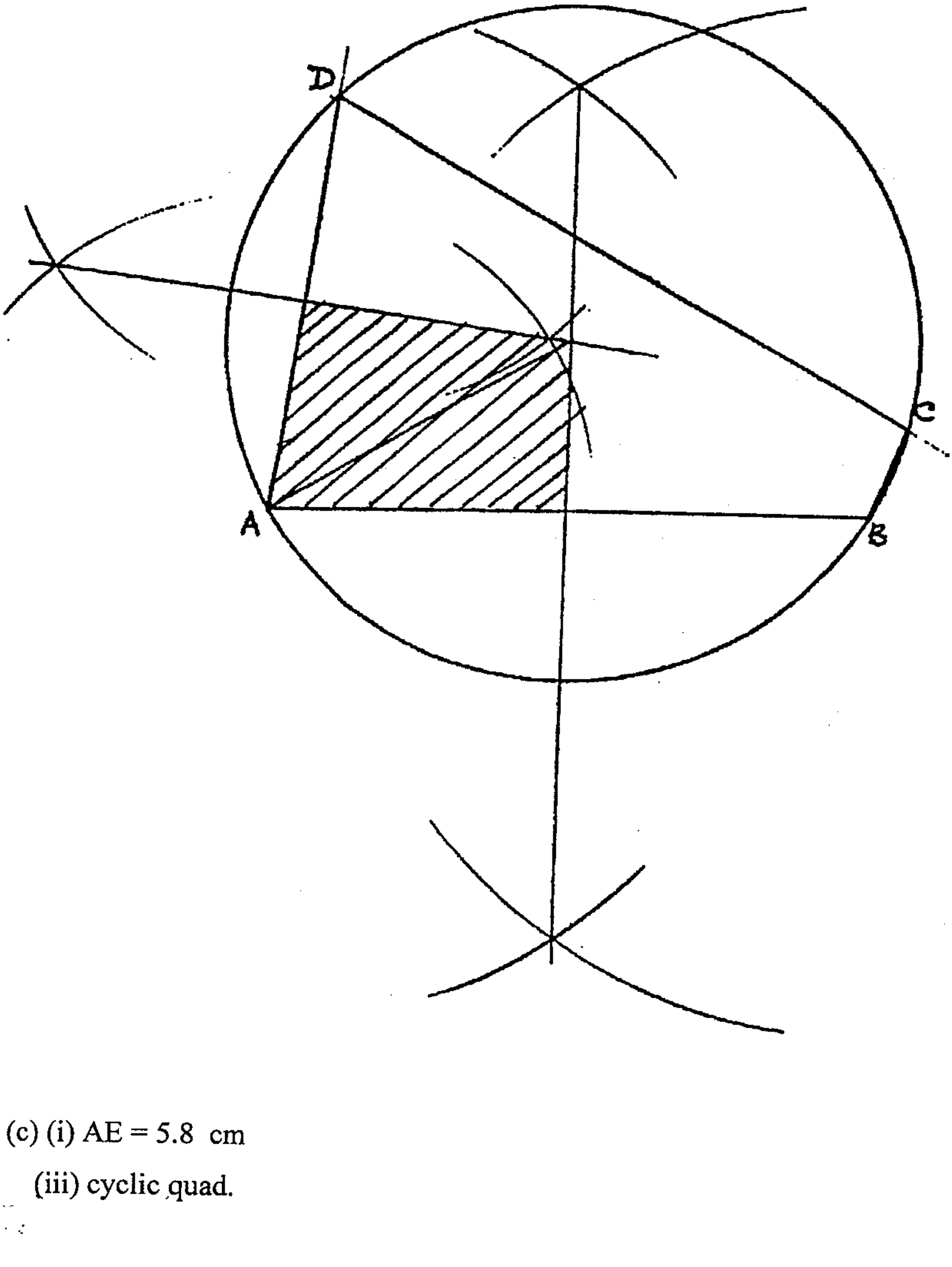
(ii) Total sum of ages of the 12000 children $= 12000 \times 10.54 = 126480$

Total sum of ages of the 5760 boys $= 5760 \times 10.35 = 59616$

Total sum of ages of the 6240 girls = 126480 – 59616 = 66864

Average age of the girls $= \dfrac{66864}{6240} = 10.72$

2- (a)



(c) (i) AE = 5.8 cm

(iii) cyclic quad.

3- (a) $\cos\angle ABC = \dfrac{10^2 + 9.2^2 - 11.6^2}{2 \times 10 \times 9.2} = 0.27217$

$\angle ABC = 74.2°$

(b) $\dfrac{9.2}{\sin A} = \dfrac{11.6}{\sin B}$

$\dfrac{9.2}{\sin A} = \dfrac{11.6}{\sin 74.2}$

Sin A = 0.76316 A = 49.7°

(c) Area of triangle= $\dfrac{1}{2} \times 10 \times 9.2 \sin B$

$= 44.263\, cm^2$

Area of car $= 12 \times 12 = 144$

Area remaining = 144 – 44.263

$= 99.7 cm^2$

4- y = x (x+2) (x-3)

x = -3 y = -3 (-1) (-6) = -18

x = -1 y = -1 (1) (-4) = 4

x = 2 y = 2 (4) (-1) = -8

(c) (i) x (x+2) (x-3) = 10

y = 10 x = 3.5

(ii) x (x+2) (x-3) + 15 = 0

x(x+2) (x-3) = -15

y = -15 x = -2.9

(d) y = 2x-6

x = 0 y = -6 (0,-6)

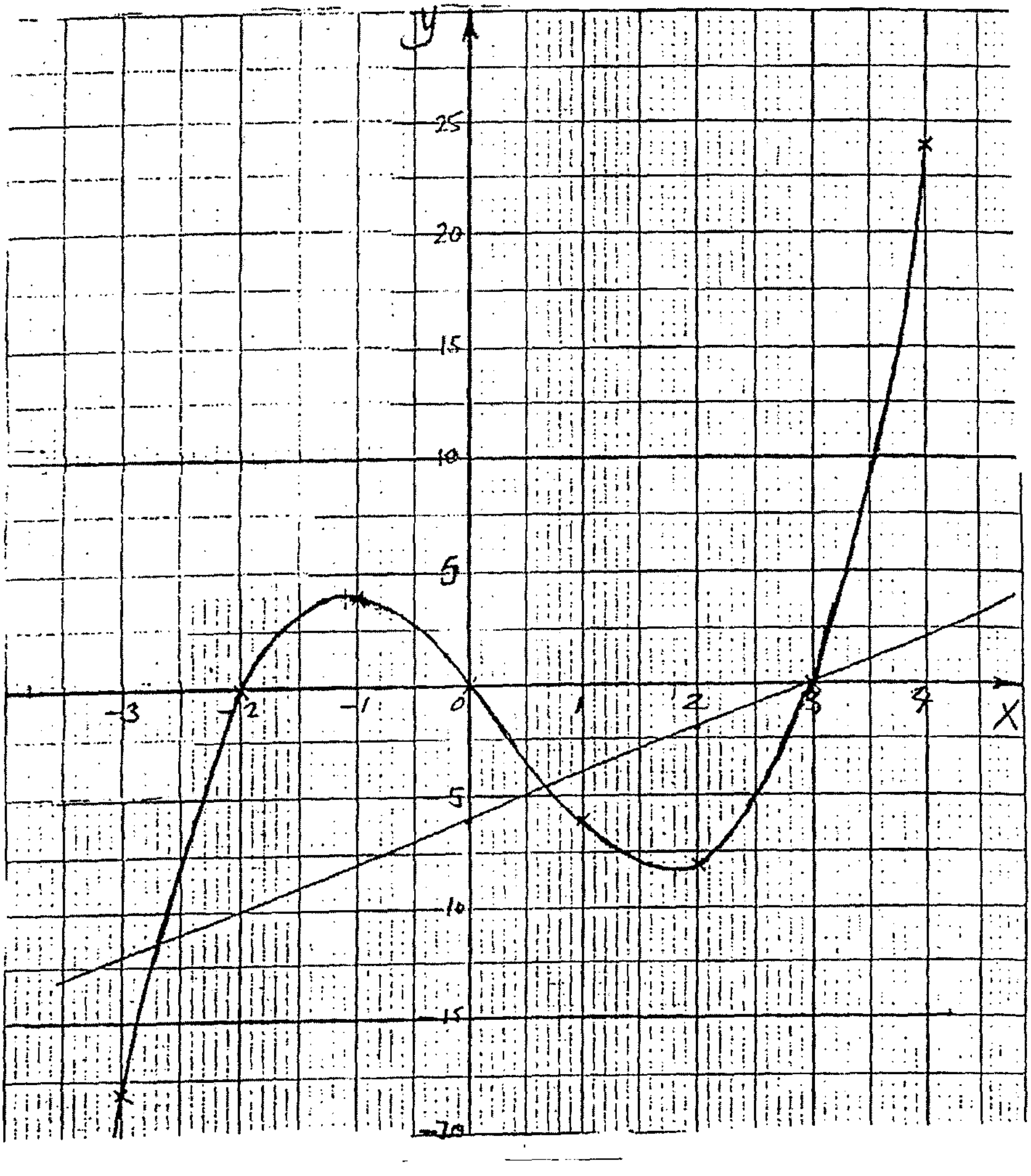
x = 2 y = -2 (2,-2)

(e) x (x+2) (x-3) = 2x-6

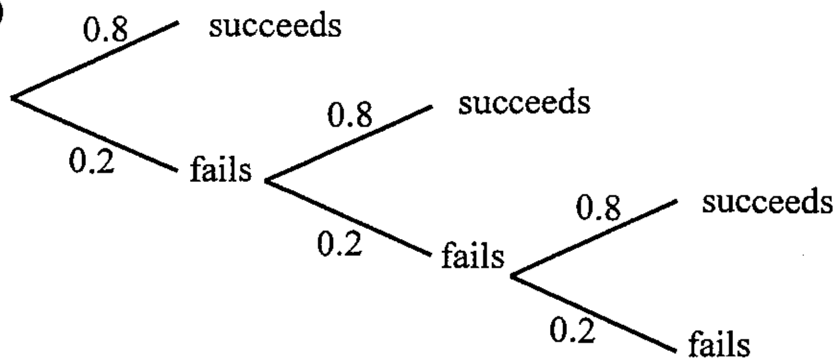
$$x(x^2 - x - 6) = 2x - 6$$
$$x^3 - x^2 - 6x = 2x - 6$$
$$x^3 - x^2 - 8x + 6 = 0$$

solutions are the value of x at the points of intersection

x = -2.7, 0.7 and 3



5- (a)



(b) (i) Prob for two tries to succeed

= Prob of failure and then succeed

$= 0.2 \times 0.8 = 0.16$

(ii) Prob for one, two or three tries to succeed

= Prob of one + Prob of two + Prob of three

$= 0.8 + 0.2 \times 0.8 + 0.2 \times 0.2 \times 0.8$

$= 0.992$

(iii) Prob of exactly five trials to succeed = Prob of 4 failure and then one succeed

$= (0.2)^4 \times 0.8 = 0.00128$

(c) Prob that he has not succeeded after n tries = Prob of failure in n tries = $(0.2)^n$

6- (a) $\dfrac{100}{x-2} - \dfrac{100}{x} = \dfrac{100x - 100(x-2)}{x(x-2)} = \dfrac{100x - 100x + 200}{x(x-2)} = \dfrac{200}{x(x-2)}$

(b) $\dfrac{100}{x}$

(c) $\dfrac{100}{x-2} - \dfrac{100}{x} = 5$

$\dfrac{200}{x(x-2)} = 5$

$5x(x-2) = 200$

$x(x-2) = 40$

$x^2 - 2x - 40 = 0$

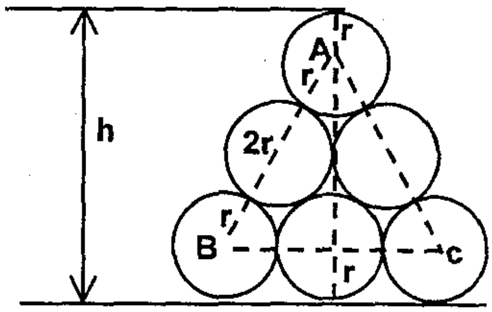
(d) (i) a = 1 b = -2 c = -40

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4 \times 1 \times (-40)}}{2}$$

$$= \frac{2 \pm \sqrt{164}}{2} = 7.40, -5.40$$

(ii) Original price of one kilogram of rice is 7.40 francs.



7- (a) (i) Equilateral

(ii) AB = 4r = 4 x 0.8 = 3.2 m

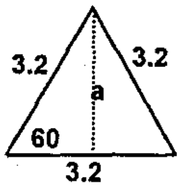
(iii) h = a + r

$\sin 60 = \frac{a}{3.2}$

a = 3.2 sin 60

= 2.7713

h = a + 2r = 2.7713 + 2 x 0.8 = 4.37 m



(b) Total Volume = 6 x volume of one cylinder

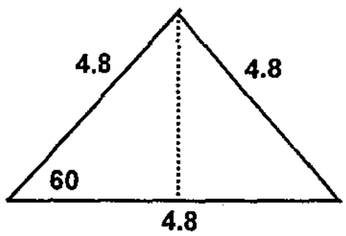
$$= 6 \times \pi r^2 \ell$$

$$= 6 \times \pi \times (0.8)^2 \times 1.5 = 18.1 m^3$$

(c) now the length of one side of the equilateral triangle is 6r = 6 x 0.8 = 4.8

h = 4.8 sin 60 + 2r

= 5.76 m



8- (a) (i) Reflection on the yaxis

(ii) Translation $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

(iii) Rotation by 180° centre the origin or

Enlargement by –1 centre the origin.

(b) (i) A and B respectively

(ii) E and D respectively.

(c) $M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(i) the point (0,1) is transformed to (0,1) and the point (0,1) is transformed to (-1,0)

Therefore the matrix represents rotation by 90° anticlockwise centre the origin.

(ii) The new position of G is E

The new position of H is F

(d) $N = \begin{pmatrix} 4 & -3 \\ -2 & 2 \end{pmatrix}$

(i)$N^{-1} = \frac{1}{(4\times 2)-(-3\times -2)}\begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3/2 \\ 1 & 2 \end{pmatrix}$

(ii) NN^{-1}= I so points don't change positions so G and H remain in their places after the transformation NN^{-1}

9- (a)

Cost	$0 < x \leq 5$	$5 < x \leq 10$	$10 < x \leq 15$	$15 < x \leq 25$	$25 < x \leq 35$
Mid value x	$\frac{0+5}{2} = 2.5$	$\frac{5+10}{2} = 7.5$	12.5	20	30
Number of people f	13	12	10	6	9
fx	32.5	90	125	120	270

Modal class is 0 – 5

Mean $= \frac{\sum fx}{\sum f} = \frac{32.5+90+125+120+270}{50} = 12.75$

(b) (i) From the diagram given, the interval 10 –15 is represented in the histogram by a rectangle of width 1cm and height 4cm, area = $4cm^2$ representing 20 people which confirms what is given in the question as a scale of $1cm^2$ representing 5 people.

For the interval 15 – 20, the area in the diagram is $2.4cm^2$ (2.4 x 1), so it represents 2.4 x 5 = 12 people $\therefore$ e = 12

For the interval 20 – 30, the area in the diagram is $2cm^2$ (2 x 1), so it represents 2 x 5 = 10 people $\therefore$ f = 10

For the interval 30 – T, the number of people is 24, so the area must be $\frac{24}{5} = 4.8cm^2$. The height is 1.6cm so the base is $\frac{4.8}{1.6} = 3cm$.

3cm on the horizontal axis represents 15 min so

T = 30 + 15 = 45 min

(ii) 18 people are represented by $\frac{18}{5} = 3.6cm^2$

The base is 1.2 cm, so the height is $\frac{3.6}{1.2}$ equal 3cm.

10- Hour hand completes one revolution in 12 hours so $1h = \frac{360}{12} = 30°$

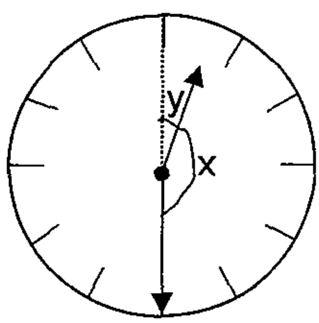
So each hour, the hour hand moves 30°

Minute hand completes one revolution every hour

$60\min = 360°$

$1\min = \frac{360}{60} = 6°$

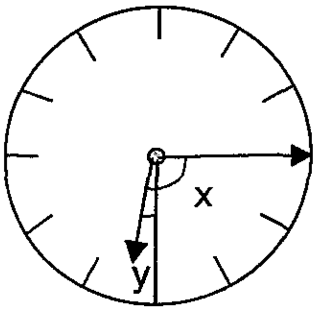
(a) 12:30



Angle $y = \frac{30}{2} = 15$

Angle $x = 180 - 15 = 165°$

06 : 15



15 min $= \frac{1}{4}h$,

Angle $y = \frac{1}{4} \times 30 = 7.5°$

Angle $x = 90 + 7.5 = 97.5°$

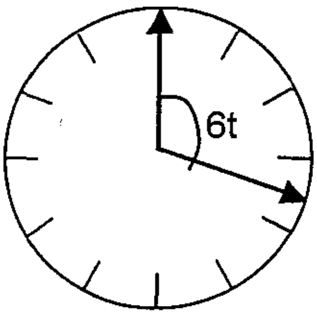
(b) (i) angle turned in one minute by the minute hand is 6°

(ii) angle turned in one minute by the hour hand is $\frac{30}{60} = \frac{1}{2} = 0.5°$

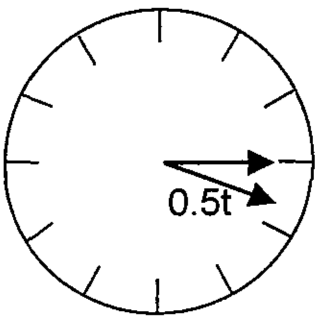
(c) (i) angle turned in degrees in t minutes by the minute hand = 6 t

(ii) angle turned in degrees in t minutes by the hour hand = 0.5 t

(d)



minute hand turned by 6t°
in t minutes.



Hour hand turned by 0.5t° in t minutes.

Therefore

$$6t - \frac{1}{2}t = 90°$$

$$5\frac{1}{2}t = 90$$

$$t = \frac{90}{5\frac{1}{2}} = \frac{90}{11/2} = 16\frac{4}{11}$$

16 min 22sec.

$$16\frac{4}{11} = 16 \text{ minutes}$$

$$\frac{4}{11} \times 60 = 21.8 \approx 22 \text{ sec.}$$

Math 0580

NOV. 1998

Paper 4

1- (a) (i) Tax paid $= \dfrac{28}{100} \times 24600 = 6888$

Amount Left after tax $= 24600 - 6888 = \$\ 17712$

(ii) Commission $= 24600 - 15000 = \$\ 9600$

Value of the furniture sold $= \dfrac{9600 \times 100}{6} = \$160000$

(b) Discount $= 560 - 392 = \$\ 168$

Percentage discount $= \dfrac{168}{560} \times 100 = 30\%$

2- (a) 149 million kilometers $= 149 \times 10^6 = 1.49 \times 10^8\ Km$

(b) Distance of Neptune from the sun $= 30 \times 1.49 \times 10^8 = 4.47 \times 10^9$

(c) Distance $= 2\pi r = 2 \times 3.142 \times 108 \times 10^6 = 6.78672 \times 10^8 = 6.79 \times 10^8\ Km$

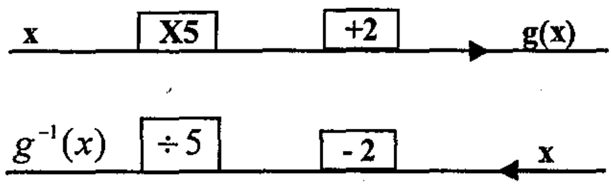
(d) Speed $= \dfrac{4.89 \times 10^9}{12 \times 365\frac{1}{4} \times 24} = 4.65 \times 10^4\ Km/h$

3- $f(x) = x^2 - 16$

(a) (i) $f(10) = 10^2 - 16 = 100 - 16 = 84$

$f(-2) = (-2)^2 - 16 = 4 - 16 = -12$

(b) g(x) = 5x + 2



$g^{-1}(x) = \dfrac{x-2}{5}$

(c) fg(x) = f(5x+2) = $(5x+2)^2 - 16$

$= 25x^2 + 20x + 4 - 16$

$= 25x^2 + 20x - 12$

(d) f(x) = g(x)

$$x^2 - 16 = 5x + 2$$

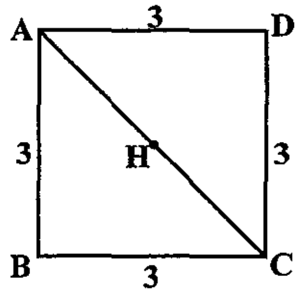
$$x^2 - 5x - 18 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4 \times 1 \times (-18)}}{2}$$

$$= \frac{5 \pm \sqrt{97}}{2}$$

= 7.42 , -2.42



4- (a) (i) number of faces = 8

(ii) number of vertices = 6

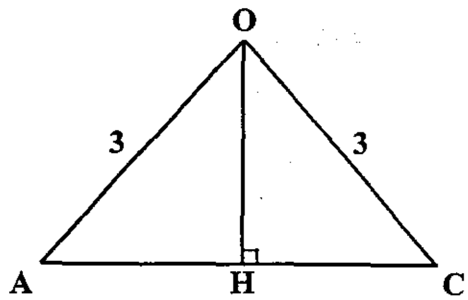
(iii) number of edges = 12

(b) (i) AC is the diagonal of the base

$$AC = \sqrt{3^2 + 3^2} = \sqrt{8} = 4.24$$

(ii) AH = HC = $\frac{1}{2}AC = \frac{4.24}{2} = 2.12$

OH = $\sqrt{3^2 - (2.12)^2} = 2.12$



(iii) Angle between OA and the base ABCD is angle OAH

since OH = AH

$\angle OAH = 45°$

(c) Area of the base ABCD = 3 x 3 = 9 cm^2

Volume of the pyramid = $\frac{1}{3} \times 9 \times 2.12 = 6.36$

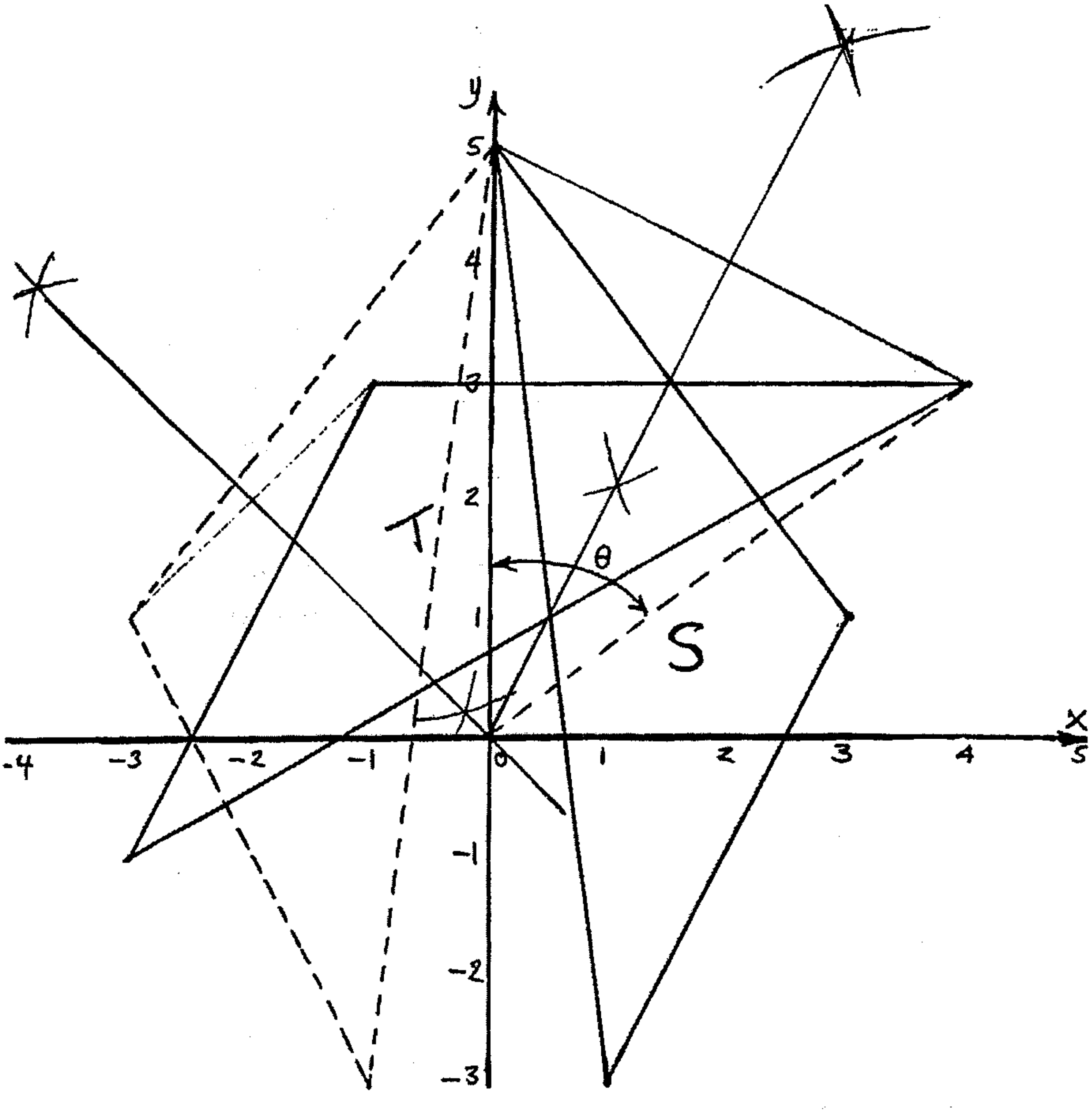
Volume of the octahedron = 2 x 6.36 = 12.7 cm^3

5-(b) $\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}\begin{pmatrix} -1 & 4 & -3 \\ 3 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 1 \\ 1 & 5 & -3 \end{pmatrix}$

(c) Determinant M = $\begin{vmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{vmatrix} = -0.36 - 0.64 = -1$

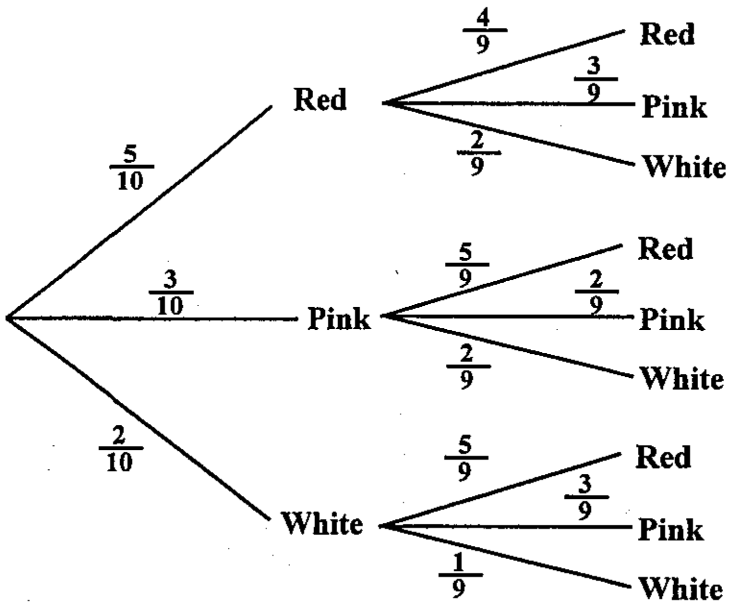
So there is a reflection (determinant = -1) the line of reflection passes through the points of intersection of the two drawn triangles. This line passes through the origin and the point (1,2) so its equation y = 2x. The single transformation which maps triangle T onto triangle S is a reflection on the line y = 2x.

Note: That also the transformation is a rotation centre the origin anticlockwise by 53° and a reflection on the y axis.


y
x
θ
S
T
5
4
3
2
1
0
-1
-2
-3
-4
1
2
3
4
5

6- (a) Probability of the first plant to flower pink $= \dfrac{3}{10}$

(b)



(c) (i) $\dfrac{5}{10} \times \dfrac{4}{9} = \dfrac{2}{9}$

(ii) $\dfrac{5}{10} \times \dfrac{3}{9} + \dfrac{3}{10} \times \dfrac{5}{9} = \dfrac{1}{3}$

(iii) 1 – both not pink

$= 1 - (RR + RW + WR + WW)$

$= 1 - (\dfrac{5}{10} \times \dfrac{4}{9} + \dfrac{5}{10} \times \dfrac{2}{9} + \dfrac{2}{10} \times \dfrac{5}{9} + \dfrac{2}{10} \times \dfrac{1}{9})$

$= 1 - \dfrac{42}{90} = \dfrac{8}{15}$

(d) P (WWW) = Zero

number that will flower white is two only.

7- (a) $P = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$|P| = \sqrt{(-3)^2 + (2)^2} = \sqrt{13} = 3.61$

(b) (i) $P + q + r = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

(ii) 10q - 2r

$$10\begin{pmatrix}1\\-1\end{pmatrix}-2\begin{pmatrix}-4\\1\end{pmatrix}=\begin{pmatrix}10+8\\-10-2\end{pmatrix}=\begin{pmatrix}18\\-12\end{pmatrix}$$

(c) $10q - 2r = \begin{pmatrix}18\\-12\end{pmatrix} = -6\begin{pmatrix}-3\\2\end{pmatrix} = -6p$

Therefore vector 10q - 2r is parallel to p

(d) ap + br = 5q

$$a\begin{pmatrix}-3\\2\end{pmatrix}+b\begin{pmatrix}-4\\1\end{pmatrix}=5\begin{pmatrix}1\\-1\end{pmatrix}$$

$-3a - 4b = 5$ (1)

$2a + b = -5$ (2)

(2)x4 $8a + 4b = -20$

$-3a - 4b = 5$

$5a = -15$

$a = -3$

$2a + b = -5$

$2(-3) + b = -5$

$b = 1$

$a = -3$ $b = 1$

8- (a) $x = -1$ $y = \frac{4}{1}+(-1)=3$ $\ell = 3$

$x = 1$ $y = \frac{4}{1} + 1 = 5$ $m = 5$

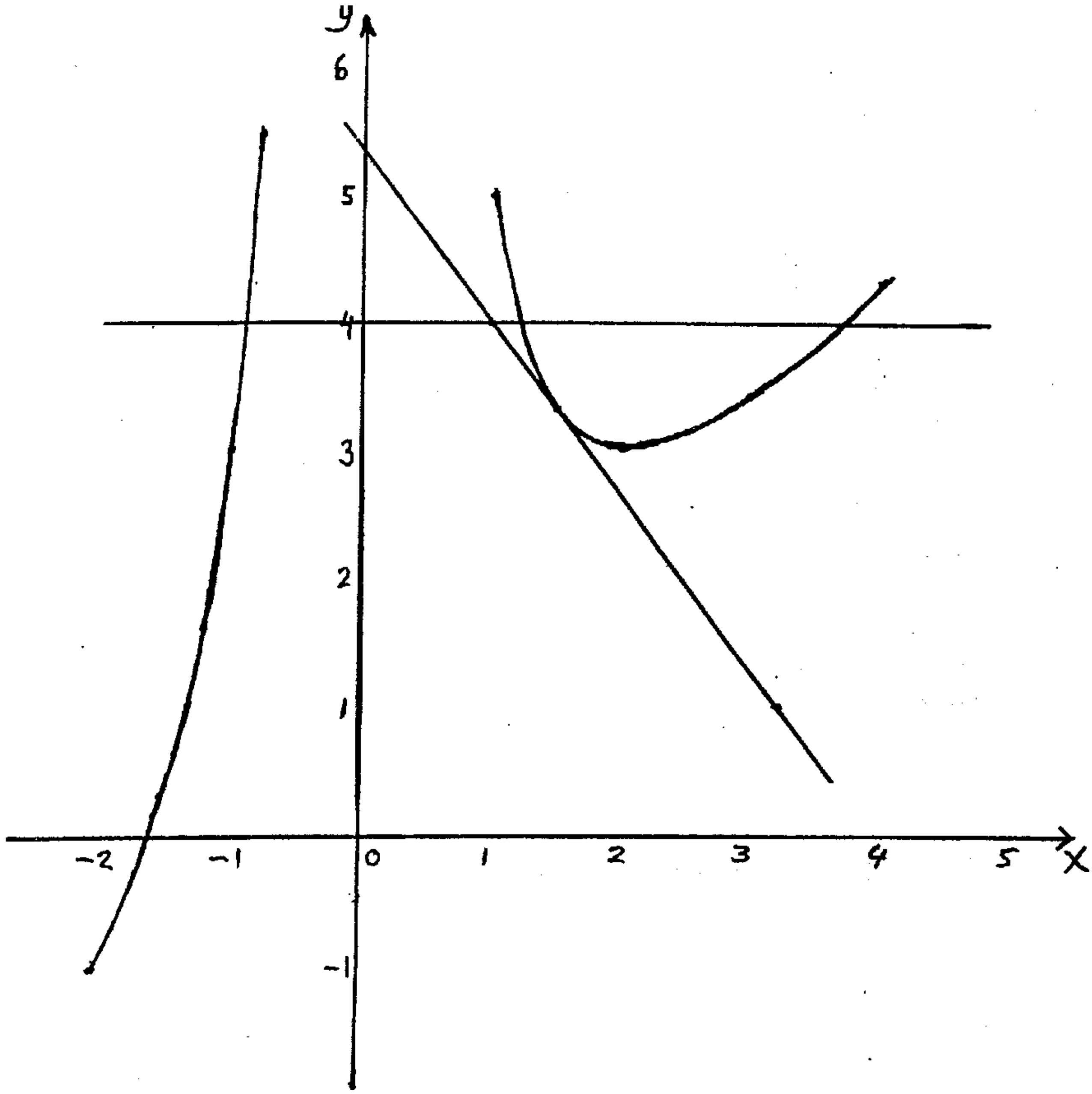
$x = 3$ $y = \frac{4}{9}+3=3.4$ $n = 3.4$

(c) (i) $\frac{4}{x^2}+x=0$ $y = 0$ $x = -1.6$

(ii) $\frac{4}{x^2}+x=4$ $y = 4$ $x = -0.9,\ 1.2,\ 3.7$

(d) Take two points on the tangent say (1, 4) and (3.2, 1) gradient

$$=\frac{4-1}{1-3.2}=\frac{3}{-2.2}=-1.4$$



9- (a) (i) Area of the semicircle $= \frac{1}{2}\pi r^2$

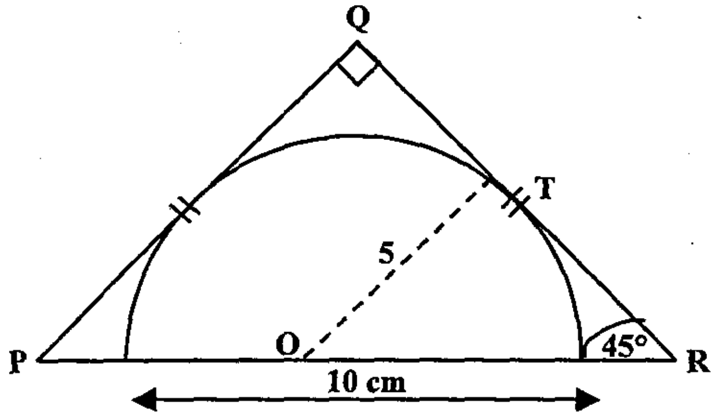
$= \frac{1}{2} \times 3.142 \times 5^2 = 39.275 = 39.3\ cm^2$

(ii) Area of ΔACE $= \frac{1}{2}$ base x height

$= \frac{1}{2} \times 10 \times 5 = 25\ cm^2$

(iii) Area of the segment ABC $= \frac{39.3 - 25}{2} = 7.14\ cm^2$

(b)



Since PQ = QR and $\angle Q = 90°$

$\therefore \angle P = \angle R = 45°$

$\angle OTR = 90°$

$\therefore \angle TOR = 45°$

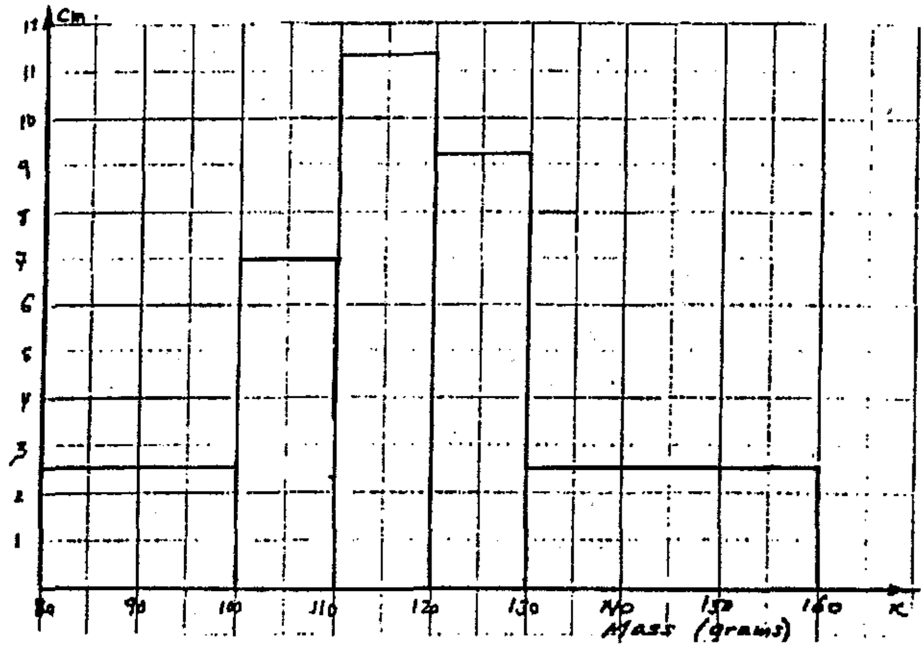
$\therefore OT = TR = 5cm$

$\therefore QR = PQ = 10cm$

Area of $\Delta PQR = \frac{1}{2} \times 10 \times 10 = 50cm^2$

10- (a)

Mass	80-100	100-110	110-120	120-130	130-160
Frequency	50	70	113	92	75
Class width (g)	20	10	10	10	30
Class width (cm)	4	2	2	2	6
Area in cm^2 $[freq \div 5]$	10	14	22.6	18.4	15
Height $[Area \div classwidth]$	2.5	7	11.3	9.2	2.5



(b) Mid class x	90	105	115	125	145	
frequency f	50	70	113	92	75	$\sum f = 400$
fx	4500	7350	12995	11500	10875	$\sum fx = 47220$

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{47220}{400} = 118$$

(c) Quantity of apples of mass greater than 110 grams = 113 + 92 + 75 = 280

$$\text{percentage} = \frac{280}{400} \times 100 = 70\%$$

11- (a) (i) 1 , $3 + 5 = 8$, $7 + 9 + 11 = 27$

$13 + 15 + 17 + 19 = 64$

(ii) cubes of natural numbers

(iii) $100^3 = 1000\,000$

(b) (i) $1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 + 9 + 11 = 36$

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$

(the answer is equal the square of the number of terms)

(ii) complete square numbers

(iii) Total number of odd numbers in the first ten rows

$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

Sum of all numbers in the first ten rows = $55^2 = 3025$

(c) Last numbers in the rows are 1, 5, 11, 19, ... etc

differences are 4, 6, 8, ... etc so the pattern can be continued.

1, 5, 11, 19, 29, 41, 55, 71, 89, 109, 131, 155, 181, 209, 239

Last number in the fifteenth row is 239

Math 0580

June 1999

Paper 4

1.

Yes	No	Total
7	5	12

(a) Total members voted = $\frac{48790 \times 12}{7} = 83640$

(b) Total membership = 83640 + 14760
= 98400

percentage did not vote = $\frac{14760}{98400} \times 100 = 15\%$

(c) 50 % of total number of members = $\frac{50}{100} \times 98400 = 49200$

members voted yes = 48790
Therefore the new stadium will not be built.

2. (a) c = 2.5 y　　　c = 100

100 = 2.5 y　　　$y = \frac{100}{2.5} = 40$ years.

(b) c = 2.5 y = 2.5 x 20 = 50 cm.
$c = 2\pi r$

$2\pi r = 50$　　　$r = \frac{50}{2 \times 3.142} = 7.96$ cm.

(c) Area = 1200　　　$\pi r^2 = 1200$

$r^2 = \frac{1200}{3.142}$　　　r = 19.543 = 19.5 cm.

$c = 2\pi r$ = 2 x 3.142 x 19.543 = 122.8

c = 2.5 y　　　$y = \frac{122.8}{2.5} = 49.1 = 49$ years old.

(d) diameter = 100 cm.
radius = 50 cm.
$C = 2\pi r$ = 2 x 3.142 x 50 = 314.2 cm.
C = 2.5 y

$y = \frac{314.2}{2.5} = 125.68 = 126$

1971 + 126 - 3 = 2094

The year in which the diameter will be one meter is 2094.

3. (a) $\cos 70° = \dfrac{5}{AC}$

$AC = \dfrac{5}{\cos 70} = 14619 = 14.62$ cm.

(b) (i) $BA^2 = 7^2 + 14.62^2 - 2 \times 7 \times 14.62 \cos 20$

BA = 8.39

(ii) Area of $\Delta ABC = \dfrac{1}{2} \times 7 \times 14.62 \sin 20$

$= 17.5 \quad cm^2$

(c) Area of $\Delta CAE = \dfrac{1}{2} AE \times CA \sin 70°$

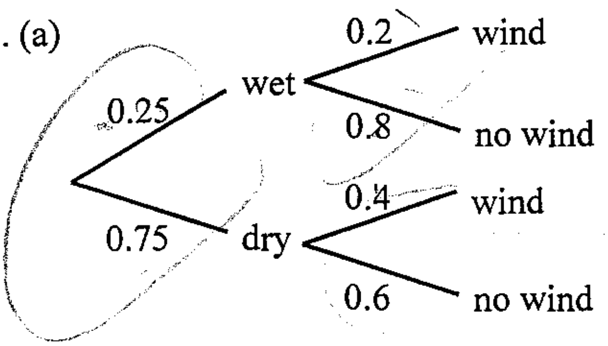
$= \dfrac{1}{2} \times 10 \times 14.62 \sin 70°$

$= 68.69$

Unshaded Area = 68.69 - 2 x 17.5

= 33.7 cm^2

4. (a)



(b) (i) 0.25 x 0.2 = 0.050

(ii) takes place on Tuesday means to be postponed on Monday and on Tuesday it takes place (i.e. not to be postponed on Tuesday).

= 0.050 x (1 – 0.050)

= 0.050 x 0.950 = 0.0475

(c) 0.25 x 0.8 + 0.75 x 0.6 = 0.65

(d) (i) 0.75 x 0.6 x 0.9 = 0.405

(ii) 0.405 x 0.405 x 0.405 = 0.0664

5. (a) A and D

(b) D and F

(c) G, centre (0, -1)

(d) B and E , $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

(e) (i) $\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 2 & 2 & 1\\0 & 0 & 1 & 1\end{pmatrix}=\begin{pmatrix}1 & 2 & 4 & 3\\0 & 0 & 1 & 1\end{pmatrix}$

coordinates of the 4 vertices of the shape H are (1,0) , (2,0) , (4,1) ,(3,1)

(ii) Shear parallel to the x axis.

6. (a)

x	-3	-2	-1	0	1	2	3
y	-27	-8	-1	0	1	8	27

(b) (i) -2.7 (ii) $f^{-1}(x)=1.7 \Rightarrow x=f(1.7)$

x = 5

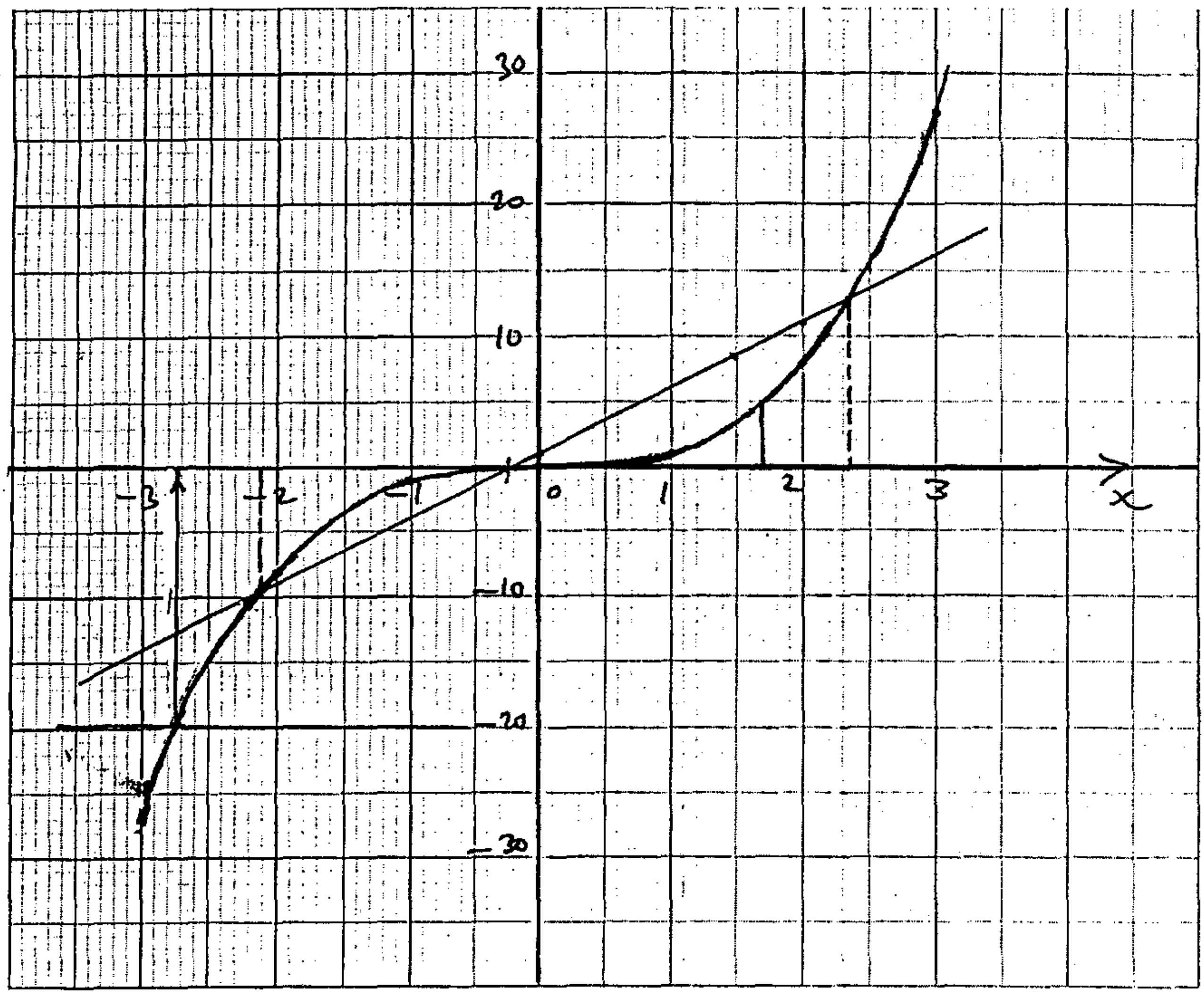
(c) $x^3-5x-1=0 \Rightarrow x^3 = 5x+1$

Line to be drawn = 5x + 1

x = 0 y = 1

x = 2 y = 11

solutions are -2.1 , -0.25 , 2.35



7. (a) (i) In $\Delta' s AMB$ and CMD

$\angle AMB = \angle CMD$ vertically opposite

$\angle BAM = \angle DCM$ angels subtended by the same arc BD.

$\therefore \Delta' s$ are similar.

(ii) $\frac{AM}{CM} = \frac{MB}{MD}$

$\frac{10}{x} = \frac{x}{4}$

$x^2 = 40$ $\quad x = \sqrt{40} = 6.32\,cm$

(b) (i) $\overrightarrow{BM} = \overrightarrow{MC} = p$

(ii) $\overrightarrow{MA} = \frac{5}{2}\overrightarrow{DM} = -\frac{5}{2}q$

(iii) $\overrightarrow{BA} = \overrightarrow{BM} + \overrightarrow{MA}$

$= p - \frac{5}{2}q$

(iv) $\overrightarrow{DC} = \overrightarrow{DM} + \overrightarrow{MC}$

$= -q + p = p - q$

(c) BA is not parallel to DC as there is no relation between Vector $\overrightarrow{BA}$ and $\overrightarrow{DC}$

8. (a) (i) 60 cm ⟶ 46

80 cm ⟶ 67

67 - 46 = 21 trees.

(ii) Median = 64

Lower quartile = 39

Upper quartile = 87

Iaterquartile range = 87 - 39 = 48

(iii)

circumference	mid x class	frequency f	fx
0 – 20	10	0	0
20 – 40	30	26	780
40 – 70	55	30	1650
70 – 100	85	33	2805
100 – 120	110	11	1210
		$\sum f = 100$	$\sum fx = 6445$

$$\text{mean} = \frac{\sum fx}{\sum f} = \frac{6445}{100} = 64.45$$

(iv) modal class is 70 – 100

(b) (i) interval 40 – 70 is represented on the x axis by 3 cm and the frequency of 30 is represented by 30 cm^2, therefore the height is $\frac{30}{3} = 10\,cm$

(ii) 20 – 40 2 cm

26 tree = 26 cm^2

height $x = \frac{26}{2} = 13\ cm$

70 – 100 3 cm

33 tree = 33 cm^2

height $y = \frac{33}{3} = 11\ cm$

100 – 120 2 cm

11 tree = 11 cm^2

height $Z = \frac{11}{2} = 5.5\ cm$

9. (a) Volume = 36 litres = 36000 cm^3

36000 = 50 x 30 x h

$h = \frac{3600}{50 \times 30} = 24\ cm$

(b) Volume = x (5 + x) . 5

= 25 x + $5x^2$

(c) (i) Volume of water displaced up

= 50 x 30 x 1 = 1500

$\therefore 25x + 5x^2 = 1500$

$x^2 + 5x - 300 = 0$

(ii) $x^2 + 5x - 300 = 0$

(x + 20) (x – 15) = 0

x = 15 cm

(iii) Width of block

= 15 cm.

Length of block

= 20 cm.

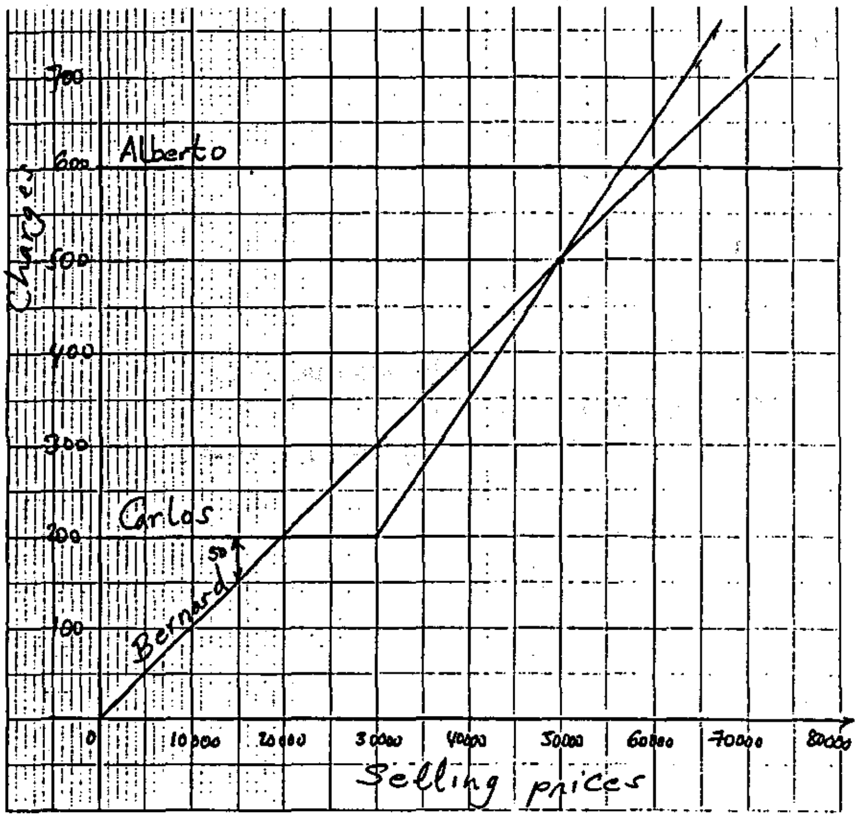
10. (b)(i) Alberto and Bernard graphs intersect

at x = 60000

Answer = \$ 60000

(ii) for prices between 20000 and 50000

(iii) \$ 15000



November 99

Paper 4

1. (a) Anna : Bella : Carla

3 : 2 : 1

$30 (Bella)

Anna share $= \frac{30 \times 3}{2} = \$ 45$

Carla share $= \frac{30 \times 1}{2} = \$15$

(b) (i) Total prize $= 40 + 55 + 25 = 120$

Total shares $= 3 + 2 + 1 = 6$

Amount Anna received $= \frac{120}{6} \times 3 = \$ 60$

Amount Bella received $= \frac{120}{6} \times 2 = \$ 40$

Amount Carla received $= \frac{120}{6} \times 1 = \$ 20$

(ii)

Last year	increase	this year
100	25	125
?		120

value of the prize last year $= \frac{120 \times 100}{125} = \$ 96$

2.(a) (i) $A \cup B = 70$

$x + 11 + (x - 3)^2 = 70$

(ii) $x + 11 + x^2 - 6x + 9 = 70$

$x^2 - 5x - 50 = 0$

(b) (i) $x^2 - 5x - 50 = (x - 10)(x + 5)$

(ii) $x^2 - 5x - 50 = 0$

$(x - 10)(x + 5) = 0$

$x = 10 \quad , \quad x = -5$

(c) (i) $x = 10$ (only positive value is accepted)

(ii) $n(B) = 11 + (x - 3)^2$

$= 11 + (10 - 3)^2 = 11 + 49 = 60$

3(a) *Difference between 20 13 and 05 42 is 14 31 i.e. 14 hours and 31 minutes .*

(b) (i) $\cos C = \dfrac{(470)^2 + (630)^2 - (970)^2}{2 \times 470 \times 630}$

$C = 123°$

(ii) *Bearing of Mendoza from Cordoba*

$= 124 + 123 = 247$

(c) (i) *Total distance BCMB* $= 630 + 470 + 970$

$= 2070$

Total time $= \dfrac{2070}{500} + 1h\ 30\ min + 2h$

$= 7h\ 38.4\ min$

$= 7h\ 38\ min$ *.to the nearest minute*

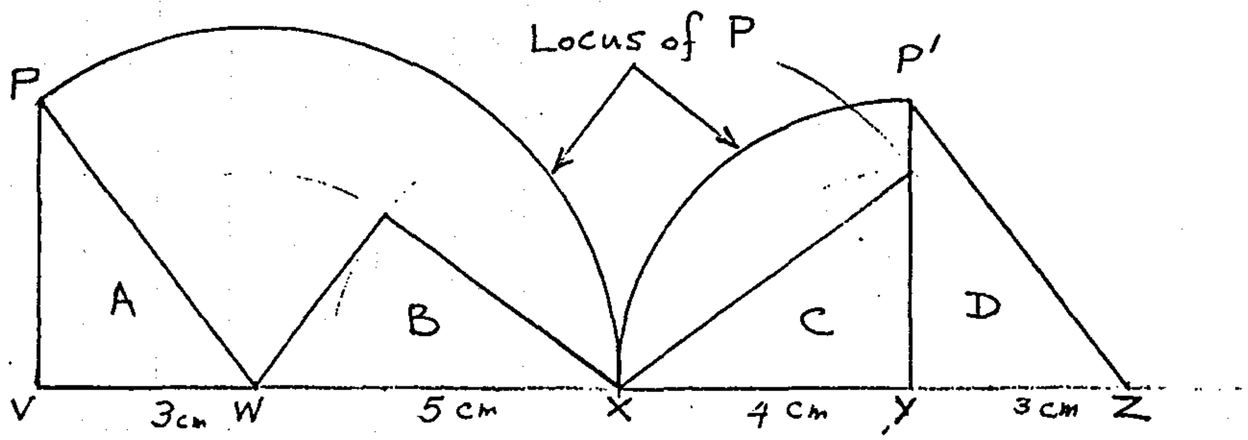
(ii) *Time of arrival*

$= 12\ 40 \quad + \quad 7\ h\ 38\ min$

$= 20\ 18$

sun sets at 20 13 , so the plane will not land before sunset.

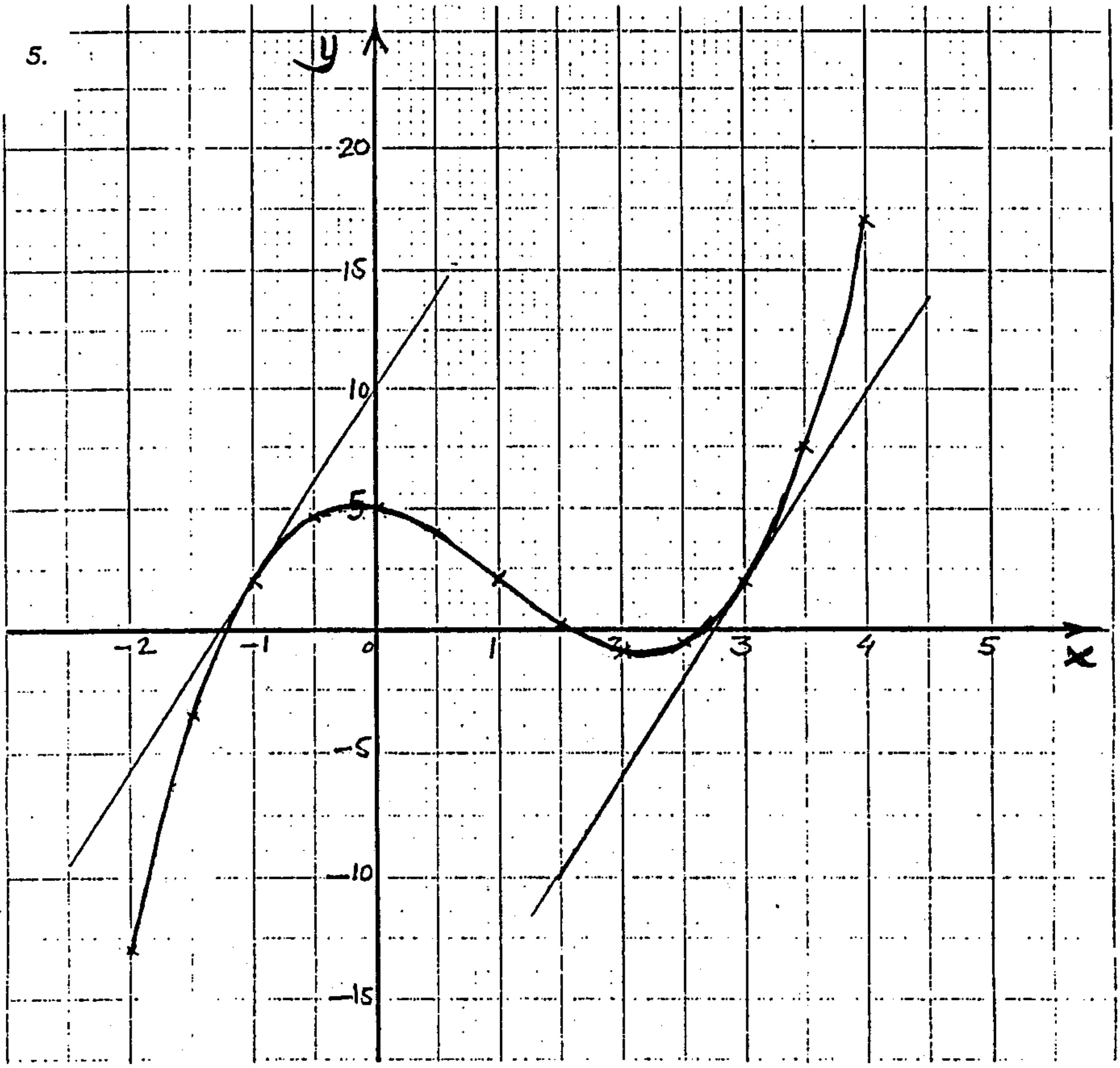
4.



(c) *Rotation center x ,clockwise by an angle of 143 °*

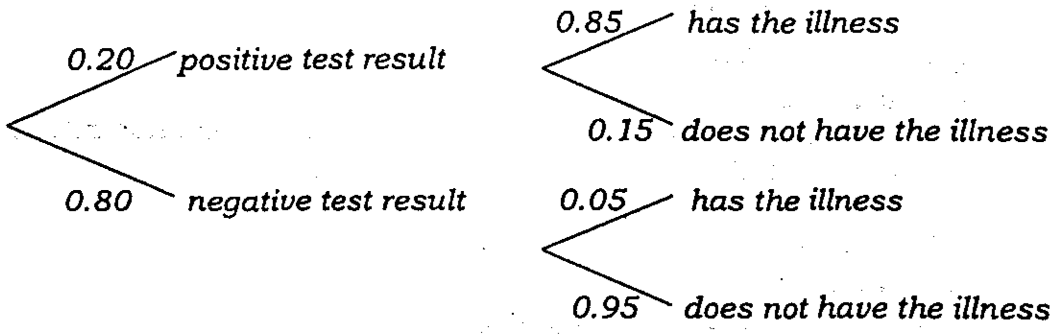
(d) *Translation of* $\begin{pmatrix} 9 \\ 0 \end{pmatrix}$

5.



(b) $f(x) = 0$ intersection with x axis .
$x = 1.6$,2.7
(c) $f(x) = K$ having three solutions
i.e. $y = K$ having three points of intersection
$K =$ 1 , 2 , 3 or 4 (any value of them)
(d) the graph is symmetrical about point (1, 2) , so the graph has a rotational symmetry of order two center point (1, 2)
(e) (i) Gradient of the tangent is given by the gradient of the line joining (-1,2) and (0,10) , gradient $= \frac{10-2}{0-(-1)} = \frac{8}{1} = 8$
(ii) Other point is (3,2)

6.(a)



(b) (i) $0.20 \times 0.85 = 0.17$

(ii) $0.20 \times 0.85 + 0.80 \times 0.05 = 0.21$

(iii) $0.20 \times 0.15 + 0.80 \times 0.05 = 0.07$

(c)(i) probability of positive test × number of people

$= 0.20 \times 10000 = 2000$

(ii) probability of having the illness times the number of people

$= 0.21 \times 10000 = 2100$

7.(a)(i) Area of semicircle BCD

$= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times \left(\frac{1.4}{2}\right)^2$

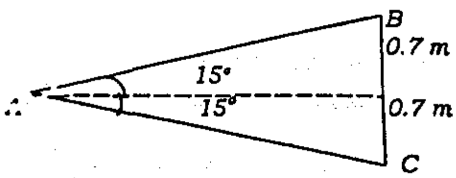
$= 0.7698 \approx 0.77\ m^2$

(ii) $\sin 15^\circ = \frac{0.7}{AC}$

$AC = \frac{0.7}{\sin 15}$

$= 2.7046$

$\approx 2.705\ m$



(iii) Area of $\Delta ABC = \frac{1}{2} AB \times AC \sin 30$

$= \frac{1}{2}(2.705)^2 \times 0.5 = 1.8287$

$\approx 1.83\ m^2$

(iv) Area of the shape ABCD $= 1.83 + 0.77 = 2.6\ m^2$

(b) Area of the small circle $= \pi r^2 = \pi \times (0.3)^2 = 0.2828\ m^3$

Total area of the glass

$= 12 \times 2.6 + 0.2828 = 31.5\ m^2$

(c) Area of the circular window $= \pi r^2 = \pi \times 4^2 = 50.272$

percentage of the window's area which is stone $= \frac{50.272 - 31.5}{50.272} = 37.3\ \%$

8.(a) (i) Line l_1 equation is $x = 2$

(ii) line l_2 equation is $y = 2$

(iii) line $y = mx + n$

$n = 0$ line passes through the origin. To find m take two points on the line (0,0) and (10,5)

$$m = \frac{5-0}{10-0} = \frac{1}{2}$$

(iv) Line $y = px + q$

$q = 12$ y intercept of the line

Line passes through points (0,12) and (8,0)

$$\text{gradient } p = \frac{0-12}{8-0} = \frac{-12}{8} = \frac{-3}{2}, \quad p = -\frac{3}{2}$$

(b)(i) H   (ii) A and E

(c) corners of G are (2,2) (4,2) (6,3), (2,9)

$x+y = 4, 6, 9, 11$ respectively, therefore maximum value of $x + y$ is 11

9.

Amount	mid class (x)	Frequency (f)	fx
0-20	10	50	500
20-40	30	40	1200
40-60	50	48	2400
60-80	70	30	2100
80-120	100	32	3200
		200	9400

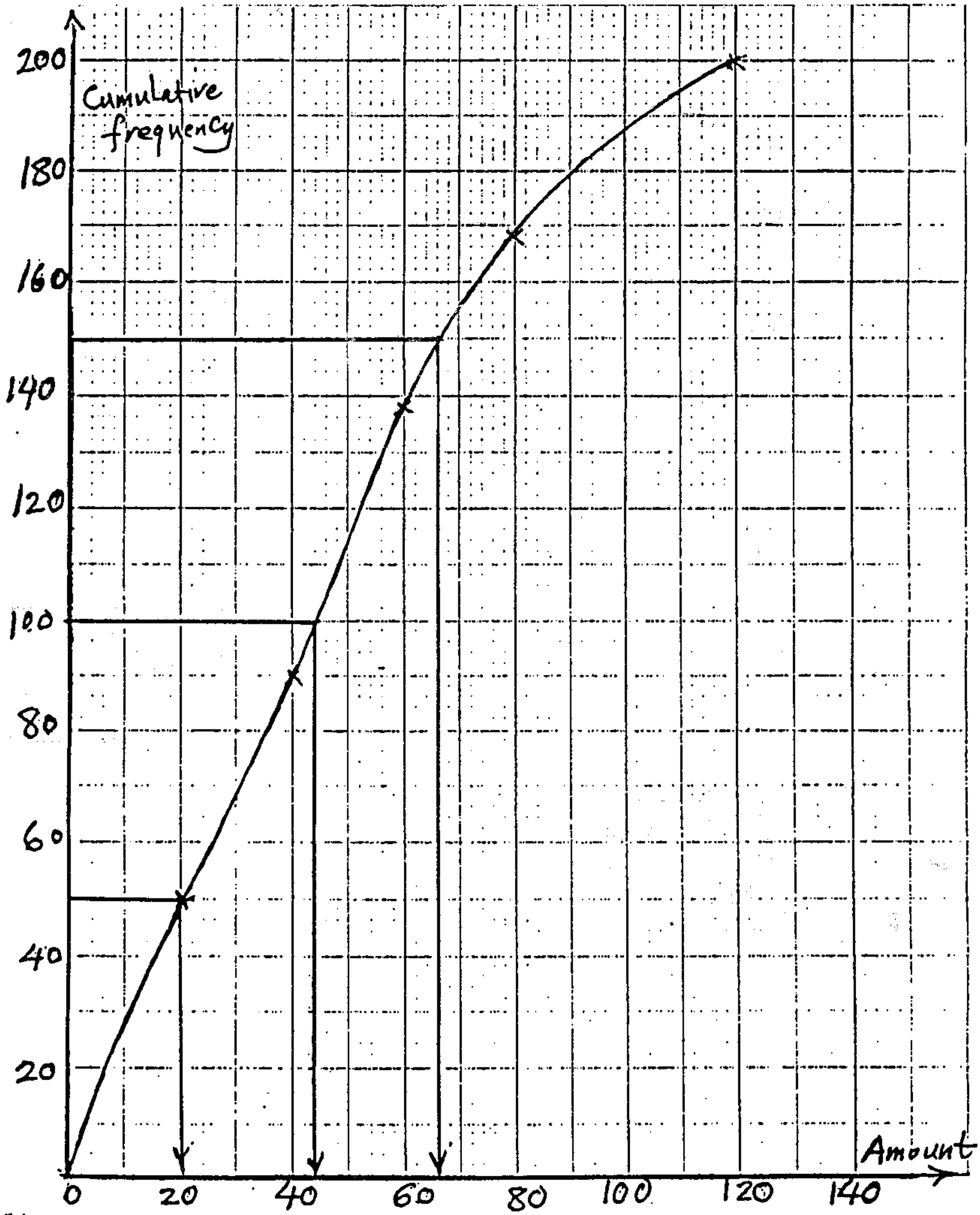
(a)(i) Model class

class with the highest frequency is 0 –2 0

(ii) $\text{Mean} = \dfrac{\sum fx}{\sum f} = \dfrac{9400}{200} = 47$

(iii) It is an estimate as the amount given is in intervals and we take the mid class as the average value of each interval.

(b)



(b)

Amount	Frequency	Cumulative frequency
≤20	50	50
≤40	40	90
≤60	48	138
≤80	30	168
≤120	32	200

(c) *From graph (i) medium = 44*

(ii) lower quartile = 20 upper quartile = 66

(iii) interquartile range = 66 – 20 = 46

10. (a) $AB = \sqrt{10^2 + (2\pi r)^2}$

$= \sqrt{10^2 + (2\pi \times 2.5)^2}$

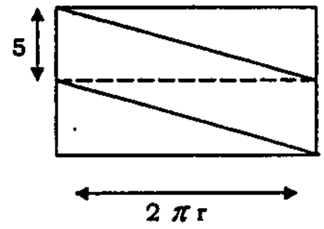
$= 18.6$

(b) Length of string

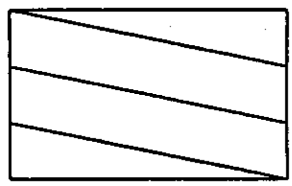
$= 2\sqrt{5^2 + (2\pi \times 2.5)^2}$

$=32.97$

$= 33$ cm



(c) As shown



(d) length of string

$=n\sqrt{\left(\frac{10}{n}\right)^2 + (2\pi \times 2.5)^2}$

$= n\sqrt{\left(\frac{10}{n}\right)^2 + (5\pi)^2}$

Math 0580

June 2000

Paper 4

1- (a) (i) $y = x + 2$

$x = 0$ $\quad$ $y = 2$ $\quad$ P is $(0, 2)$

(ii) $3x + 4y = 22$

$y = 0$ $\quad$ $x = \frac{22}{3} = 7\frac{1}{3}$ $\quad$ Q is $(7\frac{1}{3}, 0)$

(iii) $y = x + 2$ $\quad$ $3x + 4y = 22$

$y - x = 2 \quad \times 3$

$3y - 3x = 6$

added to $4y + 3x = 22$

$7y = 28$ $\quad$ $y = 4$

$y = x + 2$ $\quad$ $4 = x + 2$ $\quad$ $x = 2$

point $(2, 4)$

(b) $y \geq 0$ $\quad$ (1)

$y \leq x + 2$ $\quad$ (2)

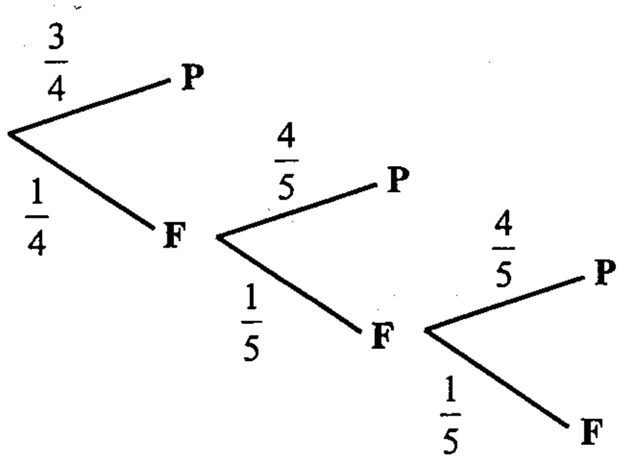
$3x + 4y \leq 22$ $\quad$ (3)

2- (a) $\frac{3}{4} > \frac{2}{3}$ $\quad$ Winston is more likely

(b) $\left(1-\frac{3}{4}\right)\left(1-\frac{2}{3}\right) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

(c) $\frac{3}{4} \times \left(1-\frac{2}{3}\right) + \left(1-\frac{3}{4}\right) \times \frac{2}{3} = \frac{3}{4} \times \frac{1}{3} + \frac{1}{4} \times \frac{2}{3} = \frac{5}{12}$

(d) (i)



(ii) $\frac{1}{4} \times \frac{1}{5} \times \frac{4}{5} = \frac{1}{25}$

(iii) $\frac{3}{4} + \frac{1}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{1}{5} \times \frac{4}{5} = \frac{99}{100}$ Or $1 - \frac{1}{4} \times \frac{1}{5} \times \frac{1}{5} = \frac{99}{100}$

3- (a) Between 1991 and 1992

(b) In 1984 85 grams
In 1994 162 grams
Increase 162 - 85 = 77

Percentage increase = $\frac{77}{85} \times 100 = 90.6\%$

(c) $\frac{144}{162} = \frac{8}{9}$

(d) In 1990 mass of bananas eaten per person per week = 125 grams
Total mass 125 x 497 x 10^6 x 52 (weeks per year)
= 3.2305 x 10^{12} grams
= $\frac{3.2305 \times 10^{12}}{1000 \times 1000}$ Tonnes
= 3.2305 x 10^6 = 3200 000 Tonnes
3.2 million Tonnes.

4- (a) Area of triangle PQR = $\frac{1}{2} \times 5 \times 8 \times \sin 70 = 18.8 cm^2$

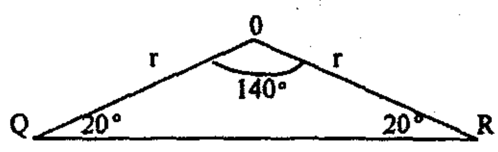
(b) $\overline{QR}^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos 70° = 61.638$
QR = 7.85 cm

(c) ∠ QOR = 2 ∠ QPR = 2 x 70 = 140°

(d) ∠ OQR = $\frac{180 - 140}{2} = 20°$

$\frac{r}{\sin 20°} = \frac{QR}{\sin 140°}$

r = $\frac{7.85 \sin 20}{\sin 140} = 4.177 \approx 4.18 cm$



(e) ∠ QOR = 140°

Length of minor arc QR = $\frac{140}{360} \times 2\pi \times 4.18 = 10.2 cm.$

(f) Reflex angle QOR = 360 - 140 = 220°

5- (a) (i) cos. 295 = 0.423

(ii) sin x and cos. x are negative in the third quadrant i.e. x between 180° and 270°

(b) $d = 5 + 4 \sin 30° t$

(i) at midnight t = 0

$d = 5 + 4 \times 0 = 5$

(ii) at 10 a.m.,

$d = 5 + 4 \sin 30(10) = 1.54$ m

(iii) sin 30t has a greatest value of 1

Greatest depth = 5 + 4 x 1 = 9 m

(iv) sin 30t = 1 when 30t = 90°

t = 3 i.e. 3 a.m.

Also sin (360 + 90) = 1

sin 450 = 1 450 = 30t

t = 15 i.e. 1500 hours i.e. 3 p.m.

Depth of water is greatest at 3 a.m. and at 3 p.m.

(v) Least value of sin 30t is -1

Least depth = 5 + 4 (-1) = 1 m

6- (a) (i) By completing the rectangle using arcs or using set squares point D is found to be (-2 , 7)

Note : For any parallelogram the sums of the opposite coordinates are equal.

Let point D be (x , y),

Then $x + (-1) = -5 + 2$

$x = -2$

and $y + (-1) = 1 + 5$

$y = 7$

point D is (-2 , 7)

(ii) A (-5 , 1) B (-1 , 1)

$AB = \sqrt{(-5+1)^2+(1+1)^2} = \sqrt{20} = 4.47$

(iii) Length of $BC = \sqrt{(2+1)^2+(5+1)^2} = \sqrt{45} = 6.71$

Area of rectangle ABCD = 4.47 x 6.71 = $\sqrt{20} \times \sqrt{45} = 30$

(b) (i) Mid point of AB is (-3 , 0) and the mid point of DC is (0 , 6). The line joining (-3 , 0) and (0 , 6) is the line of reflection

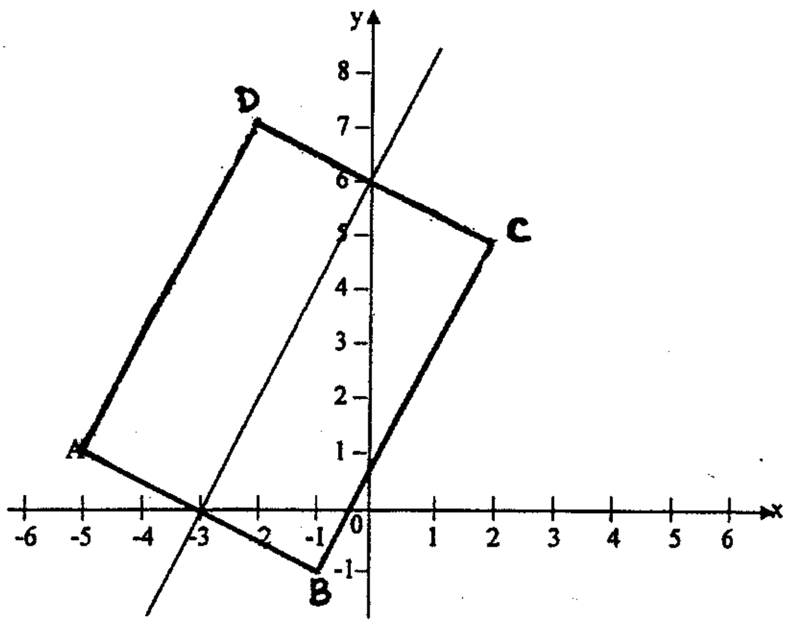
gradient of the line $m = \frac{6-0}{0-(-3)} = 2$ y intercept is 6

equation of AB is $y = mx + c$

$y = 2x + 6$

(ii) Translation
B is (-1 , -1) and
C is (2 , 5)
Translation BC is

$$\begin{pmatrix}2\\5\end{pmatrix}-\begin{pmatrix}-1\\-1\end{pmatrix}=\begin{pmatrix}3\\6\end{pmatrix}$$



(c) (i) M transformers point A to point C $\begin{pmatrix}x^2 & 2x+5\\1 & 10\end{pmatrix}\begin{pmatrix}-5\\1\end{pmatrix}=\begin{pmatrix}2\\5\end{pmatrix}$

x^2 (-5) + (2x + 5) 1 = 2
$-5x^2$ + 2x + 5 - 2 = 0
$5x^2$ - 2x - 3 = 0

(ii) (5x + 3) (x – 1) = 0
x = $-3/5$ x = 1

(iii) M = $\begin{pmatrix}1 & 7\\1 & 10\end{pmatrix}$ $|M|$ = 1 x 10 - 1 x 7 = 3

$$M^{-1}=\frac{1}{3}\begin{pmatrix}10 & -7\\-1 & 1\end{pmatrix}=\begin{pmatrix}\frac{10}{3} & \frac{-7}{3}\\\frac{-1}{3} & \frac{1}{3}\end{pmatrix}$$

7- (a) (i) Area of base ABC = ½ ba

Volume of pyramid ABCD = $\frac{1}{3}\left(\frac{1}{2}ba\right)\times h=\frac{abh}{6}$

(ii) a = 6 b = 5 h = 8

Volume = $\frac{6\times5\times8}{6}=40\,cm^3$

(b) (i) Volume = $\frac{1}{3}[x(x+3)]\times12$

= 4x (x + 3) = $4x^2$ + 12x

(ii) Perimeter = 2 (x + 3) + 2x = 4x + 6

$4x^2$ + 12x = 4x + 6
$4x^2$ + 8x - 6 = 0
$2x^2$ + 4x - 3 = 0

(iii) $2x^2$ + 4x - 3 = 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4 \times 2 \times (-3)}}{2 \times 2} = \frac{-4 \pm \sqrt{40}}{4}$$

$$= \frac{-4 + \sqrt{40}}{4} \quad \text{OR} \quad \frac{-4 - \sqrt{40}}{4}$$

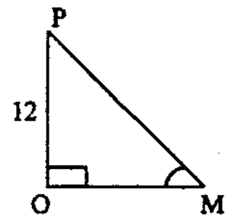
$= 0.58$ OR -2.28 (rejected)

(iv) x = 0.58 cm RS = x + 3 = 3.58 cm

(v) OM = ½ RS = $\frac{3.58}{2} = 1.79$

Tan ∠ PMO = $\frac{12}{1.79} = 6.70$

∠ PMO = 81.5°

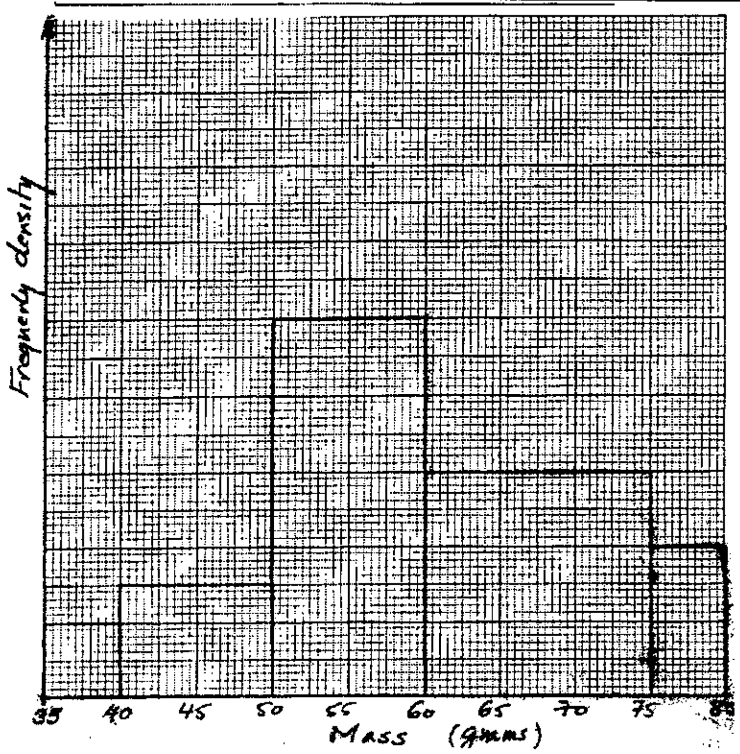


(c) (i) Cone

(ii) Volume of cone = $\frac{1}{3}\pi r^2 h$

8- (a)

Mass	Length of base	Freq.	Area	Height
35 —— 40	40 – 35 = 5 g → 2 cm	20	4 cm^2	$\frac{4}{2} = 2cm$
40 —— 50	50 – 40 = 10 g → 4 cm	60	12 cm^2	$\frac{12}{4} = 3cm$
50 —— 60	60 – 50 = 10 g → 4 cm	200	40 cm^2	$\frac{40}{4} = 10cm$
60 —— 75	75 – 60 = 15 g → 6 cm	180	36 cm^2	$\frac{36}{6} = 6cm$
75 —— 80	80 – 75 = 10 g → 2 cm	40	8 cm^2	$\frac{8}{2} = 4cm$

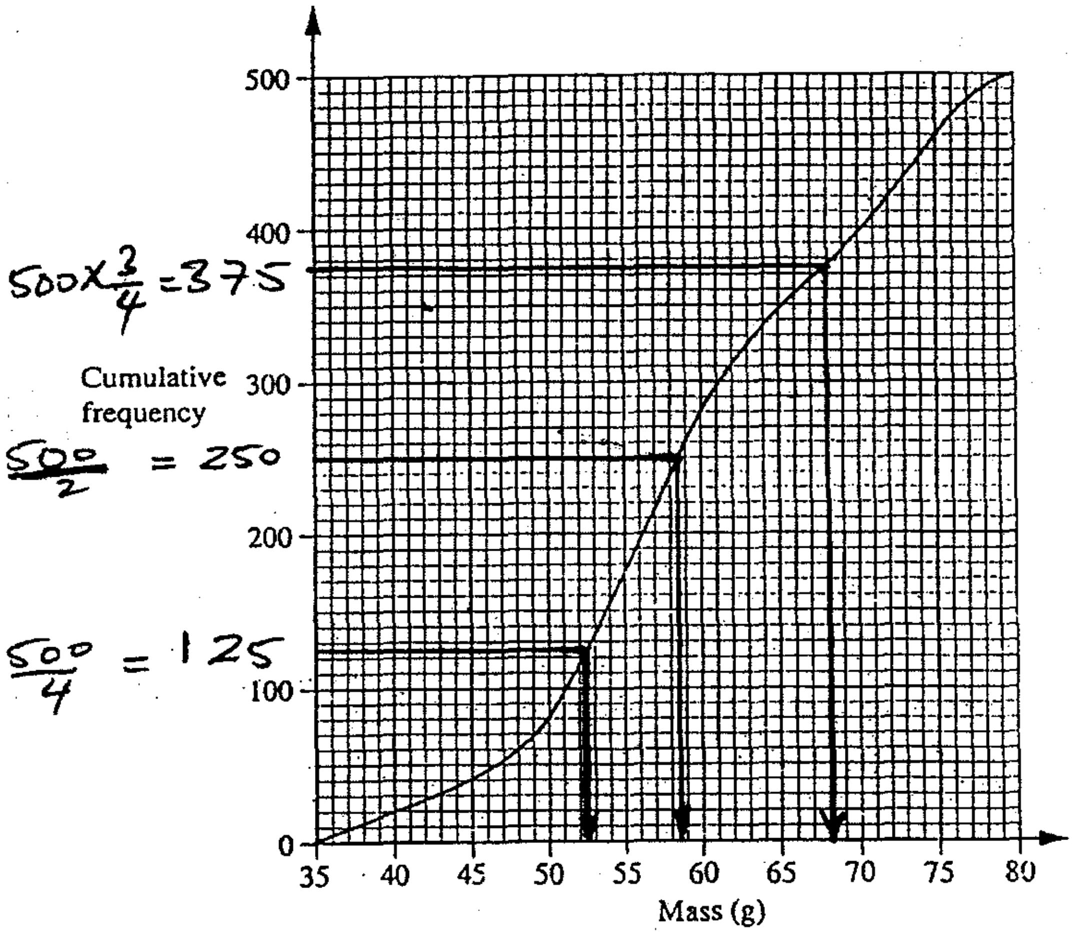


(b)

Mass	Mid class	Freq.	Fx
35 ——— 40	37.5	20	750
40 ——— 50	45	100	2700
50 ——— 60	55	200	11000
60 ——— 75	67.5	180	12150
75 ——— 80	77.5	40	3100
		500	29700

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{29700}{500} = 59.4$$

(c) (i) Number of eggs of mass < 60 is 20 + 60 + 200 = 280



(ii) Median = 58.5 g

(iii) Lower quartile = 52.5

Upper quartile = 68

Interquartile range = 68 - 52.5 = 15.5 g

9- (a) (i) OC = OB = OD = x cm
CF = 90 - 80 = 10 cm
OF = OC - CF = x - 10

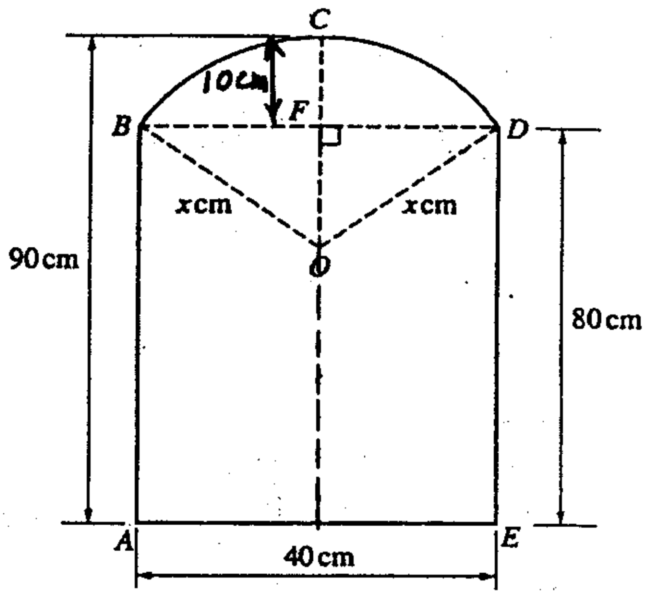
(ii) $\overline{OD}^2 = \overline{OF}^2 + \overline{FD}^2$

$x^2 = (x-10)^2 + (20)^2$

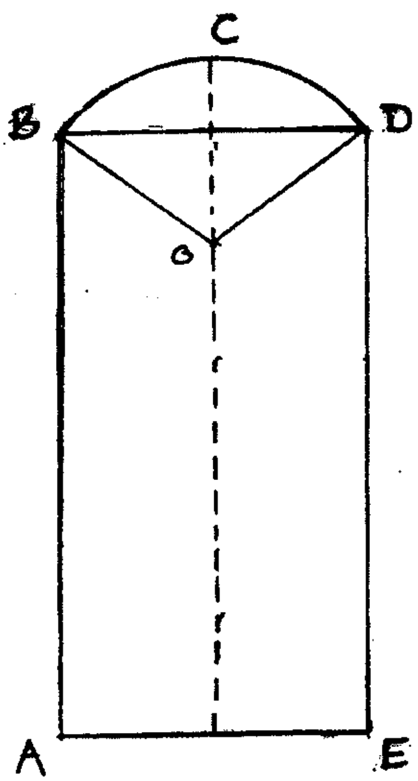
(iii) $x^2 = x^2 - 20x + 100 + 400$

20x = 500

x = 25



(b)



(c) Area of window = Area of sector OBCD + Area of trapezium OBAM + trapezium ODEM.

$\sin \angle BOF = \dfrac{20}{25}$ $\angle BOF = 53.1°$

$\angle BOD = 53.1 \times 2 = 106.2$

Area of sector OBCD = $\dfrac{106.2}{360} \times \pi \times 25^2 = 579.6$

Area of each trapezium = $\dfrac{65+80}{2} \times 20 = 1450$

Area of window = 579.6 + 1450 x 2 = 3479.6 = 3480 cm^2

(d) Volume of glass window $= 3479.6 \times \frac{2}{10} = 695.92$ cm

Mass of glass $= 695.92 \times 6.5$ g

$= 4523.48$ g

$= \frac{4523.48}{1000}$ kg

$= 4.52$ kg

10- (a) $5 = 2 + 3$ OR $7 = 2 + 5$

(b) (i) $16 = 3 + 13 = 5 + 11$

(ii) $38 = 31 + 7$ only

(c) $16 = 11 + 2 + 3$

(d) $5 = 2 + 3$

$7 = 2 + 5$

$9 = 2 + 7$

11 can not be written

Statement is false.

Mathematics 0580

November 2000

Paper 4

1- (a) (i) Labour costs 75000 angle 150 profit 36000

angle x = $\dfrac{36000 \times 150}{75000} = 72°$

(ii) Amount paid for material = $\dfrac{84}{150} \times 75000$ = \$42000

(iii) angle for tax = 360 – (150 + 84 + 72) = 54°

Ratio tax : profit = 54 : 72 = 3 : 4

(b) (i) 78000 = 7.8×10^4

(ii) Increase in labour costs = 78000 – 75000 = 3000

Percentage increase = $\dfrac{3000}{75000} \times 100$ = 4 %

(c)

1993	increase	1999
100	160	260
?		78000

Labour costs in 1993 = 30000

2- (a) $\begin{pmatrix} 3 & 0 & 0 \\ 9 & 5 & 0 \\ 4 & -3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ q \\ r \end{pmatrix} = \begin{pmatrix} p \\ -26 \\ 35 \end{pmatrix}$

3 (1) + 0 + 0 = p   p = 3

g (1) + 5 q + 0 = -26   5 q = -35   q = -7

4 (1) – 3 (q) + 2 r = 35

4 – 3 (-7) + 2 r = 35

2 r = 35 – 25 = 10   r = 5

(b) $M = \begin{pmatrix} t & 6 \\ t & 5t \end{pmatrix}$ $M^{-1} = \begin{pmatrix} -5t & 6 \\ t & -2 \end{pmatrix}$

$MM^{-1} = I$

$\begin{pmatrix} t & 6 \\ t & 5t \end{pmatrix}\begin{pmatrix} -5t & 6 \\ t & -t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

t (-5 t) + 6 t = 1

$-5t^2 + 6t - 1 = 0$

$5t^2 - 6 + 1 = 0$

(5 t – 1) (t – 1) = 0 $t = \dfrac{1}{5}$ t = 1

(c) $(x \quad 2)\begin{pmatrix} x \\ 5 \end{pmatrix} = Kx$

$x^2 + 10 = Kx$ $\qquad$ $x^2 - Kx + 10 = 0$ $\qquad$ $\therefore K = -8$

(i) $x^2 + 8x + 10 = 0$

(ii) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-8 \pm \sqrt{64 - 4 \times 1 \times 10}}{2}$

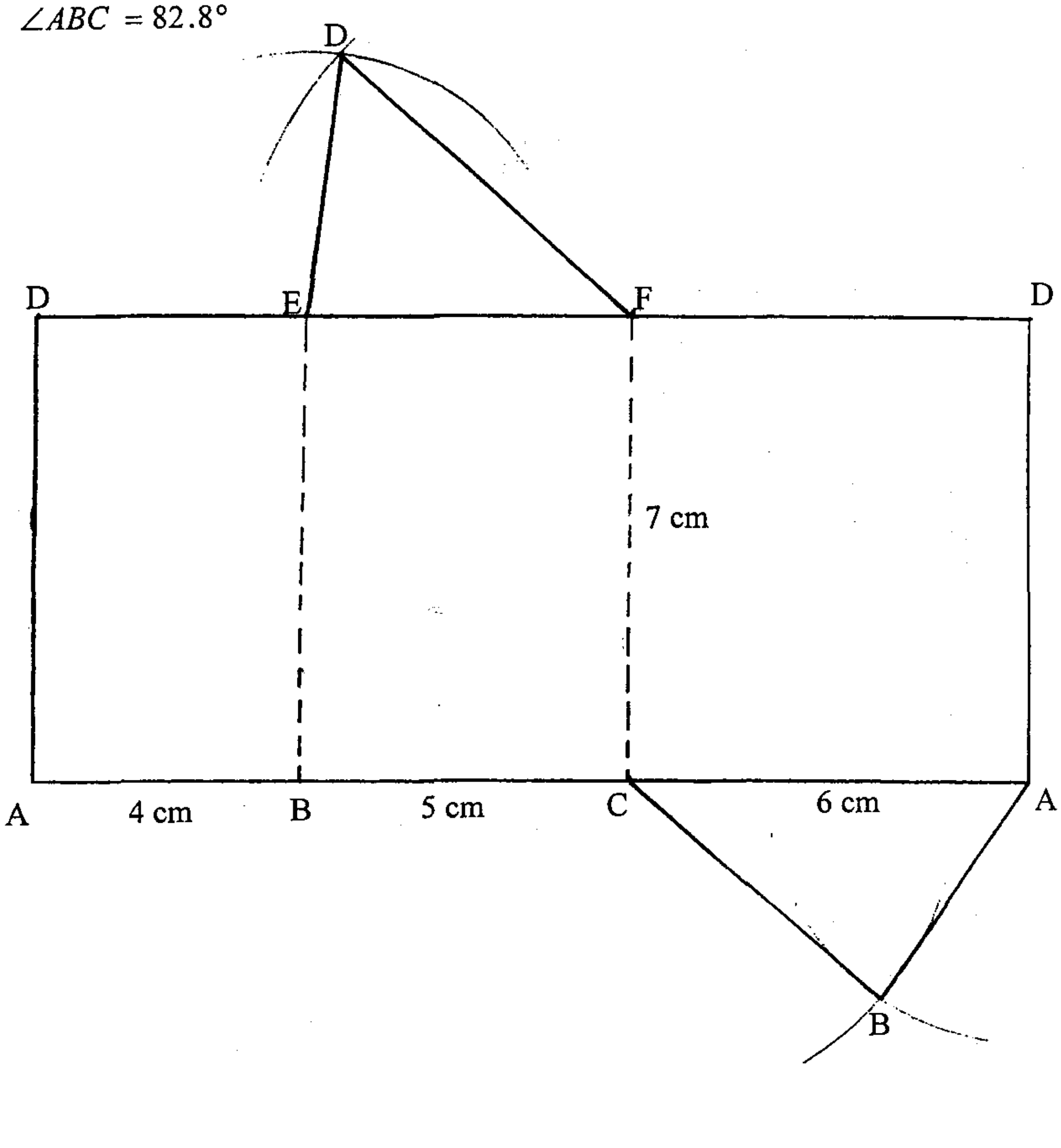
$= \dfrac{-8 \pm \sqrt{24}}{2} = \dfrac{-8 + \sqrt{24}}{2}, \dfrac{-8 - \sqrt{24}}{2}$

$= -1.55$ or -6.45

3- (a) $\cos B = \dfrac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} = 0.125$

$\angle ABC = 82.8°$

(b)



(c) (i) From the net draw, the height of $\triangle$ DEF is measured = 3.95 cm

area of $\triangle$ DEF = $\frac{1}{2}$ x 5 x 3.95 = 9.875

(Exact value of area = $\frac{1}{2}$ x 5 x 4 sin 82.8 = 9.92)

Total surface area = 9.875 x 2 + (4 + 5 + 6) x 7

= 124.75 = 125 cm^2

(ii) Volume = area of $\triangle$ DEF x length

= 9.875 x 7 = 69.1 cm^3

(using area = 9.92 , volume = 69.4 cm^3

4- (a) $h = 20t - 5t^2 + 1$

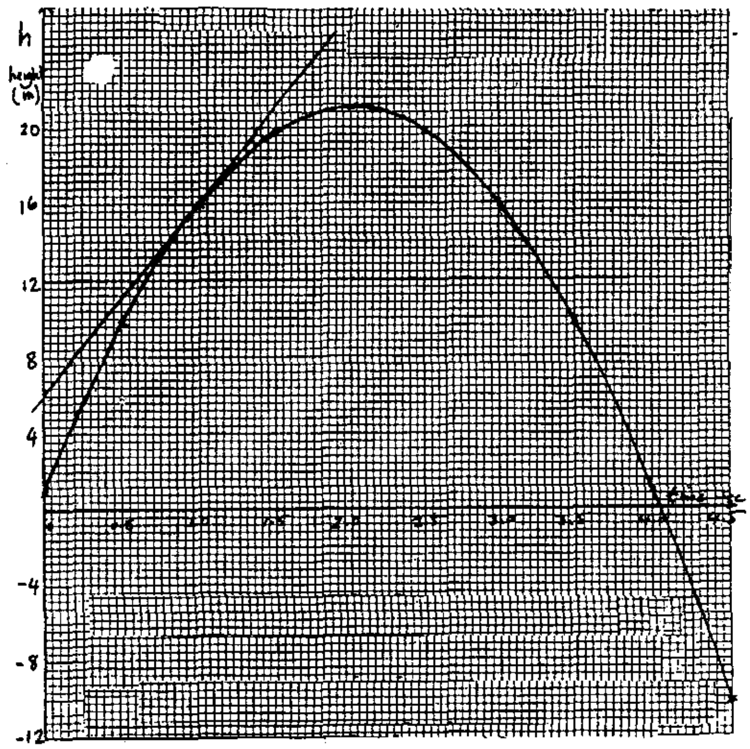
t = 2.5 $\quad$ h = 20 (2.5) – 5 $(2.5)^2$ + 1 = 19.75

t = 4 $\quad$ h = 20 (4) – 5 $(4)^2$ + 1 = 1

t = 4.5 $\quad$ h = 20 (4.5) – 5 $(4.5)^2$ + 1 = -10.25

a = 19.75 $\quad$ b = 1 $\quad$ c = -10.25

(b)



(c) (i) h = 0 at t = 4.05 sec

(ii) h = 12 at t = 0.65 and 3.35

stone is more than 12 m for 3.35 – 0.65 $\quad$ i.e for 2.7 sec

(iii) During first 3 sec stone rises from 1 m to 21 m then drops to 16 m

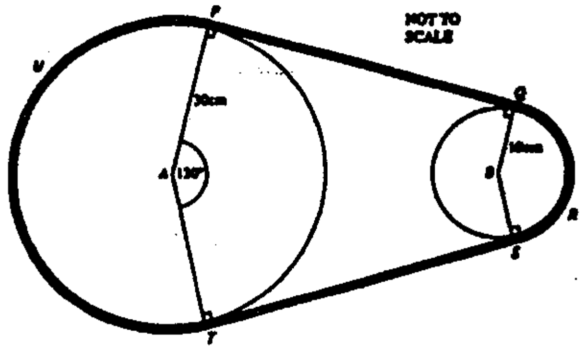
Total distance moved = 21 – 1 + 21 – 16 = 25

(d) (i) Tangent at (1, 16) passes through (0, 6)

gradient = $\frac{16-6}{1-0} = 10$

(iii) Gradient is a measure of speed (velocity) its units is m / s

5-



(a) all angles between targents the radii

(b) $\angle$ PUT $= \frac{1}{2}\angle$ PAT $= \frac{1}{2} \times 130 = 65°$

$\angle$ QBS $= 360 - 130 = 230°$

$\angle$ QRS $= \frac{1}{2}\angle$ QBS $= \frac{1}{2}(230) = 115°$

(c) Radius of larger wheel is 3 times smaller wheel. Circumference is 3 times speed of the larger wheel is $\frac{1}{3}$ speed of smaller wheel

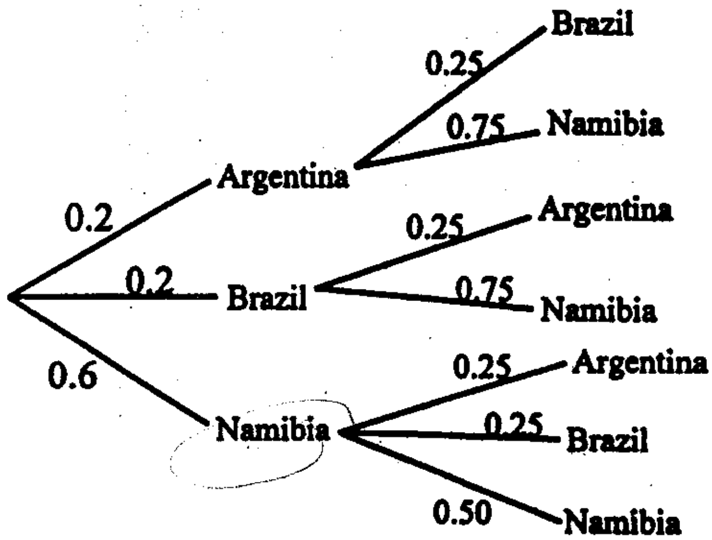
$= \frac{1}{3} \times 12 = 4$ revolution in the same direction i.e clockwise.

(d) $Ax = 30 - 10 = 20$ cm

$$\cos\left(\frac{130}{2}\right) = \frac{Ax}{AB} = \frac{20}{AB}$$

$$AB = \frac{20}{\cos 65} = 47.3$$

6- (a)



(b) (i) $0.6 \times 0.5 = 0.3$

(ii) $0.2 \times 0.25 + 0.2 \times 0.25 = 0.1$

(iii) $0.2 \times 0.75 + 0.6 \times 0.25 + 0.6 \times 0.25 = 0.45$

(c) Prob. = prob. of Argentina then Namibia then Brazil
OR Namibia then Argentina then Brazil
OR Namibia then Namibia then Brazil

$$= 0.2 \times 0.75 \times \frac{1}{3} + 0.6 \times 0.25 \times \frac{1}{3} + 0.6 \times 0.5 \times \frac{1}{3} = 0.2$$

7- (a) Translation $T = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

(b) The small angle of the triangle α

$$\tan \alpha = \frac{1.4}{4.8} = 0.2917$$

$$\alpha = 16.3°$$

Angle of rotation $\theta = 90 - 16.3 = 73.7°$

(c) (i) factor $K = \frac{7}{5} = 1.4$

(ii) By joining the corresponding points and extending them to meet at point (10, 0) a = 10

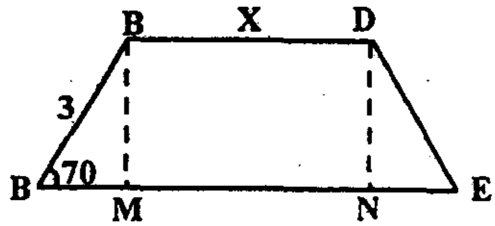
(d) (i) Area $= \frac{1}{2} \times 4.8 \times 1.4 = 3.36 \ cm^2$

(ii) gradient $m = \frac{1.4}{4.8} = \frac{14}{48} = \frac{7}{24}$

equation of the hypotenuse of triangle is $y = \frac{7}{24} x$

(e) Ratio of areas of similar triangles is the square of ratio of sides.
Ratio of sides $= \sqrt{64} = 8$
Length of hypotenuse of triangle T is 5
Length of hypotenuse of the enlargement is $8 \times 5 = 40$ cm

8- (a) (i) $\cos 70 = \frac{AM}{3}$



AM = 1.026

also NE = 1.026

$BD = MN = 7.5 - 1.026 \times 2 = 5.448$

$$Bx = \frac{1}{2} BD = \frac{5.448}{2} = 2.724 \text{ cm}$$

(ii) $\sin \angle BCX = \frac{2.724}{2.8}$

$\angle BCX = 76.6°$

(b) (i) Area of triangle BCD = $\frac{1}{2}$ 2.8 x 2.8 x sin (76.6 x 2) = 1.77 cm^2

(ii) Sin 70 = $\frac{BM}{3}$ BM = 2.819

Area of the trapeium = $\frac{5.448 + 7.5}{2} \times 2.819$ = 18.25

(iii) Area of major sector BCD = $\frac{(360-2\times 76.6)}{360} \times \pi \times 2.8 = 14.15$

(iv) Total area of the cloud = 1.77 + 18.25 + 14.15 + $\pi \times (1.5)^2$

= 41.24 = 41.2 cm^2

9- (a)

	n = 2	n = 1	n = 0	n = -1	n = -2
Ahmed	49	16	1	4	25
Bumni	125	27	1	-1	-27
Cesar	256	16	0	16	256
Dan	256	16	1	$\frac{1}{16}$	$\frac{1}{256}$

(b) (i) Cesar expression

(ii) Dan expression

(c) $(2n)^{3+1} = 2^4 n^4 = 16n^4$

$\therefore$ a = 16 b = 4

$(3+1)^{2n} = (4)^{2n} = (4^2)^n = 16^n$

c = 16

(d) $1(n)^2 + 3$

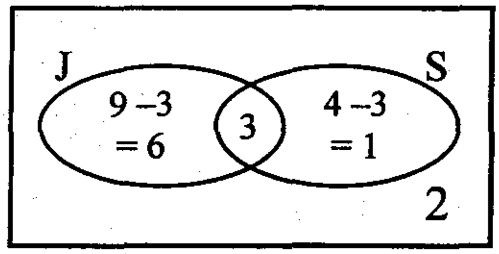
Mathematics 0580

June 2001

Paper 4

1- (a) (i) number who have already been to both countries $= \frac{1}{4} \times 12 = 3$

(ii)



(iii) $n(J \cup S) = 6 + 3 + 1 = 10$

(b) 1000 Riyals $= \frac{1000}{5.28} = 189.39$

Ahmed	:	Yousef	:	Ibrahim	Total
2	:	3	:	1	6
?					189.39

Amount Ahmad kept for himself $= \frac{2 \times 189.39}{6} = 63.1$

i.e 63 Dinars

2- (a) (i) $LR^2 = 1200^2 + 750^2 - 2 \times 1200 \times 750 COS 110$

LR = 1618

(ii) $\frac{1618}{\sin 110} = \frac{750}{\sin B\hat{L}R}$

$\sin B\hat{L}R = 0.4356$ $\angle BLR = 25.8 = 26°$

(b) (i) Bearing of L from B = 180 + 53 = 233°

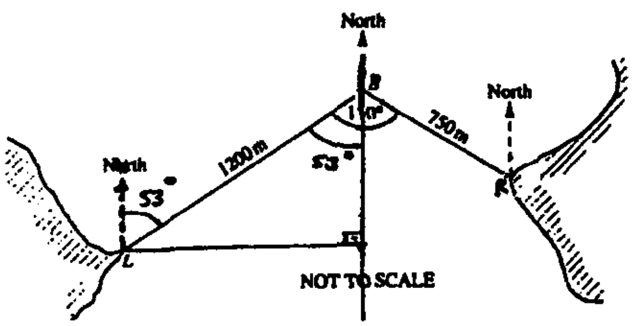
(ii) Bearing of R from B = 233 − 110 = 123°

Bearing of B from R = 180 + 123 = 303°

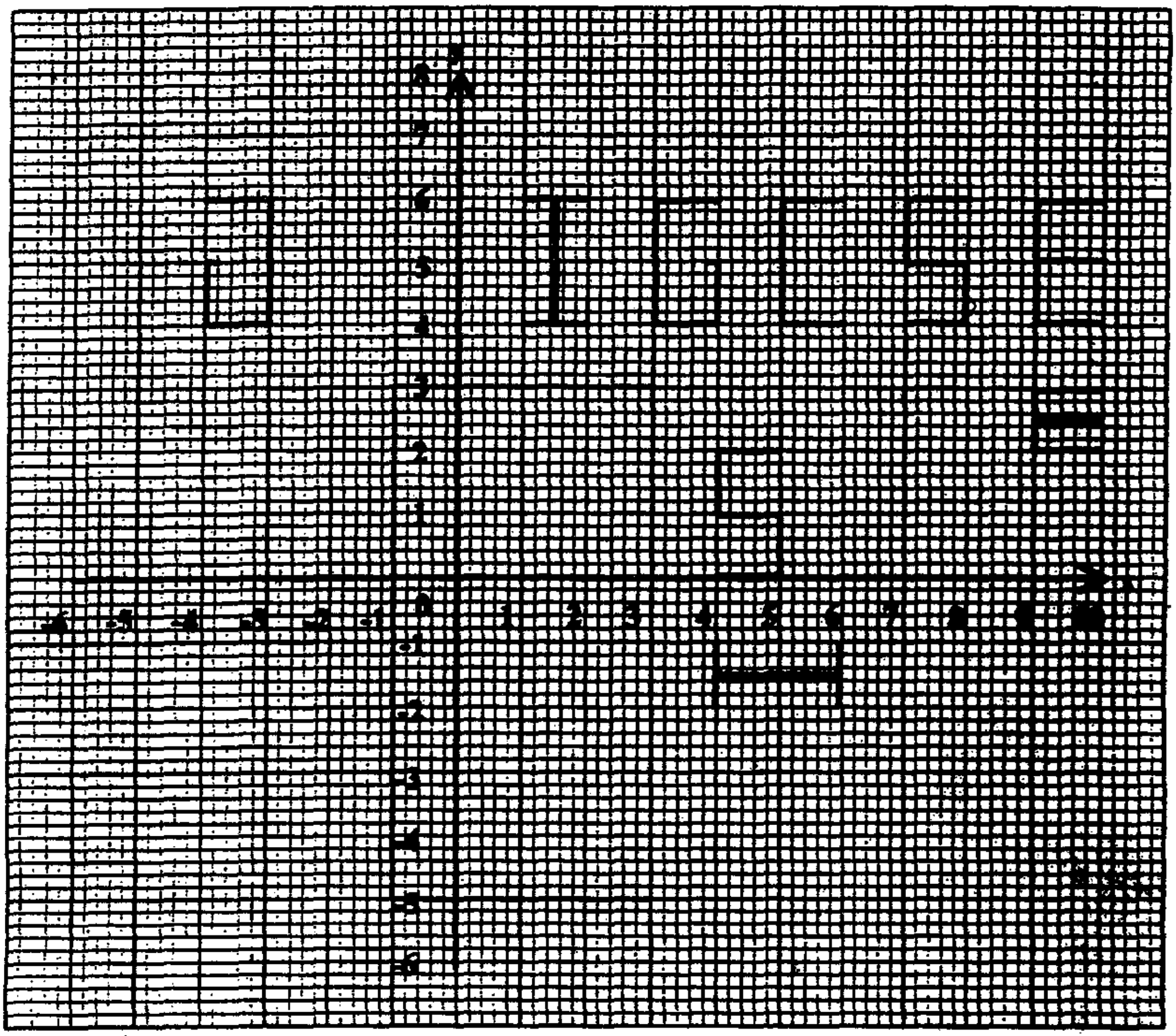
(c) shortest distance

= 1200 sin 53

= 958.4 = 958



3- (a) , (b)



(c) (i) Rotation 90° clockwise about the origin

$$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix}$$

matrix is $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$

(ii) $M^{-1} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

M^{-1} is Rotation 90° anticlockwise center origin.

4- (a) $m = \frac{5-2}{2-8} = \frac{3}{-6} = -\frac{1}{2}$

equation of AB is

$y = mx + c$

$y = -\frac{1}{2}x + 6$

(b) $AB = \sqrt{(2-8)^2 + (5-2)^2} = \sqrt{45}$

$= 6.71$

(c) point A (2 , 5) is the mid point of BC where B (8 , 2) and C (x , y)

$2 = \frac{8+x}{2}$ $\quad$ x = - 4

$5 = \frac{2+4}{2}$ $\quad$ y = 8

point C is (- 4 , 8)

(d) Area of $\Delta ABD = \frac{1}{2} BD$ x height $\qquad$ height = 3

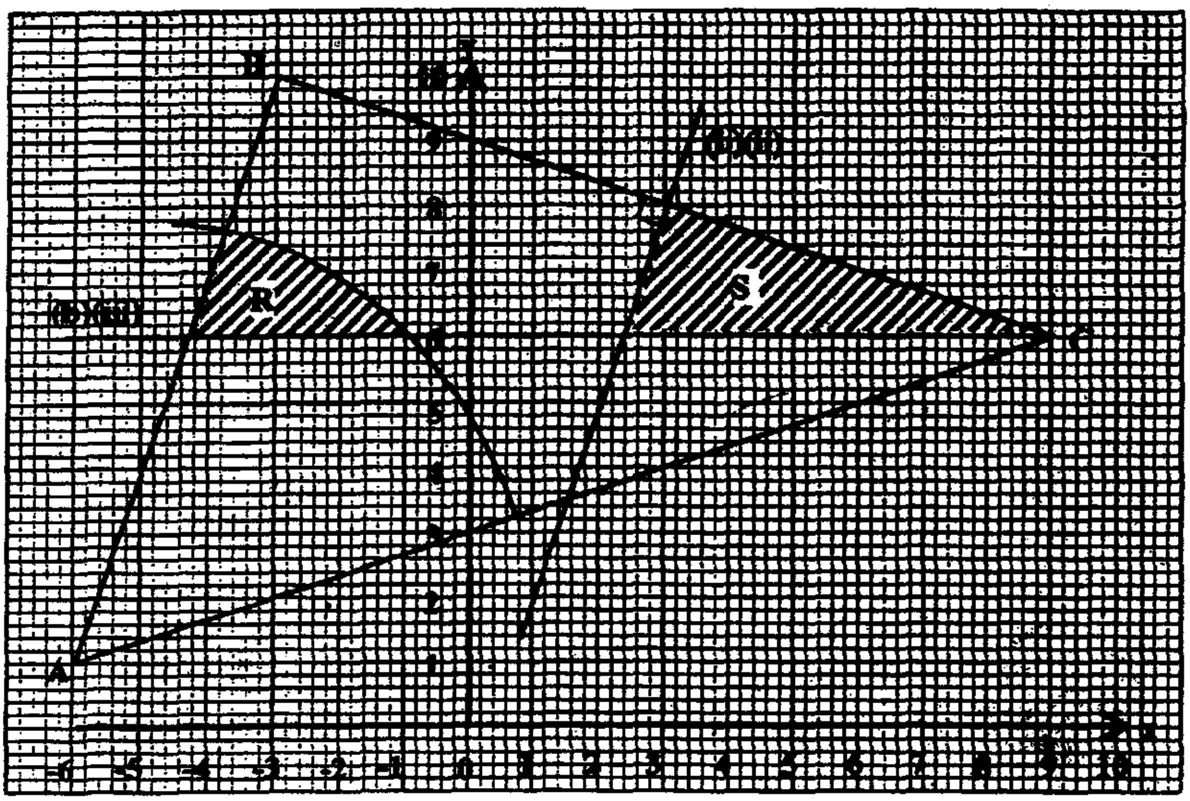
$15 = \frac{1}{2} BD \times 3$

BD = 10

∴ point D enither (- 2 , 2) Or (18 , 2)

possible value of x are - 2 and 18

5-



6- (a) $y = x + 4$

(b) (i) $(x-7)^2 = 1 + y$

$x^2 - 14x + 49 = 1 + (x+4)$

$x^2 - 15x + 44 = 0$

(ii) $(x-4)(x-11) = 0$

$x = 4 \qquad x = 11$

(c) Since test marked out of 12 so possible value of x is 4.
Monica scored 4 and sandra scored 8.

7- (a) (i) $p(3, 3) = \frac{1}{25}$

(ii) $p[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)] = \frac{5}{25}$

(iii) $p[(2, 4), (3, 3), (4, 2)] = \frac{3}{25}$

(iv) $p[(4, 2), (2, 4), (3, 3), (3, 4), (4, 3), (4, 4)] = \frac{6}{25}$

(v) $p[(0, 1), (1, 0), (0, 2), (2, 0), (0, 3), (3, 0), (0, 4), (4, 0), (1, 1), (1, 2),$
$(1, 3), (1, 4), (2, 1), (3, 1), (4, 1)] = \frac{15}{25} = \frac{3}{5}$

(b) (i) $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
(ii) $\{0, 1, 2, 3, 4, 6, 8, 9, 12, 16\}$
(iii) $\{5, 7\}$

8- (a) (i) $180 - 84 = 96°$

(ii) $\frac{84}{360} \times 60 = 14$

(iii) $\frac{270}{360} \times 60 = 45$

(iv) Widths of class intervals are 25, 25, 25, 20. The largest frequency density will be obtained for the smallest class width, i.e. for interval 150 – 170

(b) (i) mode is O

(ii) median order is $\frac{60+1}{2} = 30\frac{1}{2}$

30 th is 1 and 31 th is 2, so the median is $1\frac{1}{2}$

(iii) mean $= \frac{\sum fx}{\sum f}$

$\sum fx = 0 \times 16 + 1 \times 14 + 2 \times 3 + 3 \times 9 + 4 \times 7 + 5 \times 6 + 6 \times 5$

$= 135$

$\sum f = 60$

mean $= \frac{135}{60} = 2.25$

9- (a) (i) Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 6^2 \times 14 = 527.78 = 528 \ cm^2$

(ii) $OC = \sqrt{10^2 - 6^2} = 8$ cm

$SC = SO + OC = 10 + 8 = 18$ cm

(iii) Volume $= \frac{1}{3}\pi H^2 (3R - H)$

$= \frac{1}{3}\pi \times 18^2 \ (3 \times 10 - 18)$

$= 4071.5 = 4070 \ cm^3$

(iv) Volume not occupied by the cone $= 4070 - 528 = 3542$

percertage = 87 %

(b) (i) $OS = R = 3$

$OT = 3 - 1 = 2$

$OC = h - OT = 2r - 2$

(ii) $\overline{OC}^2 = 9 - r^2$

(iii) $(2r-2)^2 = 9 - r^2$

$4r^2 - 8r + 4 = 9 - r^2$

$5r^2 - 8r - 5 = 0$

(iv) $r = \frac{8 \pm \sqrt{8^2 - 4(5)(-5)}}{10}$

$= \frac{8 \pm \sqrt{164}}{10} = 2.08 , - 0.48$

(v) $r = 2.08$ $h = 2r = 4.16$ cm

10- (a) (i) B. (ii) G. (iii) F (iv) E

(v) A. (vi) C.

(b) $y = x^2$

Mathematics 0580

November 2001

Paper 4

1. (a) (i) $\frac{8}{7}$ x 11424 = 13056

(ii) 13056 x $\frac{100}{40}$ = 32640

(b) Number of people who did out vote = 42320 – 32640 = 9680

percentage = $\frac{9680}{42320}$ x 100 = 22.9 %

(c) (i) $\frac{3}{11}$ x 572 = 156

(ii) number in blue party = $\frac{6}{11}$ x 572 = 312

number who were not in the blue party = 572 – 312 = 260

difference = 312 – 260 = 52

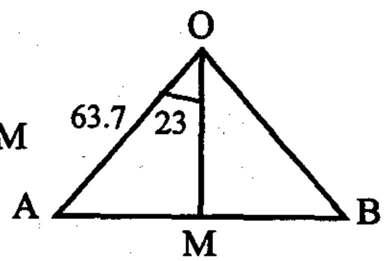
2. (a) arc AWB = $\frac{46}{360} \times 2\pi \times 63.7$ = 51.1 cm

(b) $AB^2 = (63.7)^2 + (63.7)^2 - 2x(63.7)(63.7)\cos 46°$

AB = 49.8 cm

(c) Length of the perpendiculer from O to AB = OM

cos 23° = $\frac{OM}{63.7}$ OM = 58.6



(d) Greatest depth = 63.7 – 58.6 = 5.1 cm or 51 mm

3. (a) (i) In Δ's MNO and QPO

$\angle M = \angle Q$ alternate

$\angle P = \angle N$ alternate

$\angle MON = \angle QOP$ vertically opposite

(ii) Δ's are similar, sides are proportional

$$\frac{NO}{PO} = \frac{MO}{QO} \qquad \frac{4}{y+1} = \frac{5}{2y-2}$$

i.e $\frac{2y-2}{5} = \frac{y+1}{4}.$

(iii) $4(2y-2) = 5(y+1)$

$8y - 8 = 5y + 5$

$3y = 13 \qquad y = \frac{13}{3} = 4\frac{1}{3}$

(iv) $NP = 4 + (y+1) = 4 + 4\frac{1}{3} + 1 = 9\frac{1}{3}$

(b) (i) $\sin 30 = \frac{1}{2}$

(ii) $\sin 30 = \frac{AC}{AB}$

$\frac{1}{2} = \frac{(x-3)^2}{30-4x}$

$2(x-3)^2 = 30 - 4x$

$2x^2 - 12x + 18 = 30 - 4x$

$2x^2 - 8x - 12 = 0$

$x^2 - 4x - 6 = 0$

(iii) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times (-6)}}{2}$

$\frac{4 \pm \sqrt{40}}{2} = 5.16$ or -1.16

(iv) $AC = (x-3)^2 \qquad x = 5.16 \qquad AC = (5.16 - 3)^2 < 10$

$x = -1.16 \qquad AC = (x-3)^2 = 17.3 \qquad AC > 10$

$AB = 30 - 4x = 30 - 4(-1.16) = 34.6$

4. (a) (i) P $\rightarrow$ (6 , 0)

(ii) $\begin{pmatrix}6\\2\end{pmatrix}+\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}5\\5\end{pmatrix}$

Q $\rightarrow$ (5 , 5)

(iii) R $\rightarrow$ (2 , 6)

(b) (i) Enlargement center (0 , 2) scale factor $\frac{1}{2}$

(ii) Ratio of area $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

i.e. 1 : 4 $\quad \therefore$ n = 4

(c) Area of $\Delta POR = \frac{1}{2} \times 2 \times 2 = 2cm^2$

Area of the stretched triangle = 3 x 2 = 6 cm^2

(d) (i) Inverse is $\frac{1}{2-(-3)}\begin{pmatrix}1 & -3\\1 & 2\end{pmatrix}=\frac{1}{5}\begin{pmatrix}1 & -3\\1 & 2\end{pmatrix}=\begin{pmatrix}\frac{1}{5} & -\frac{3}{5}\\\frac{1}{5} & \frac{2}{5}\end{pmatrix}$

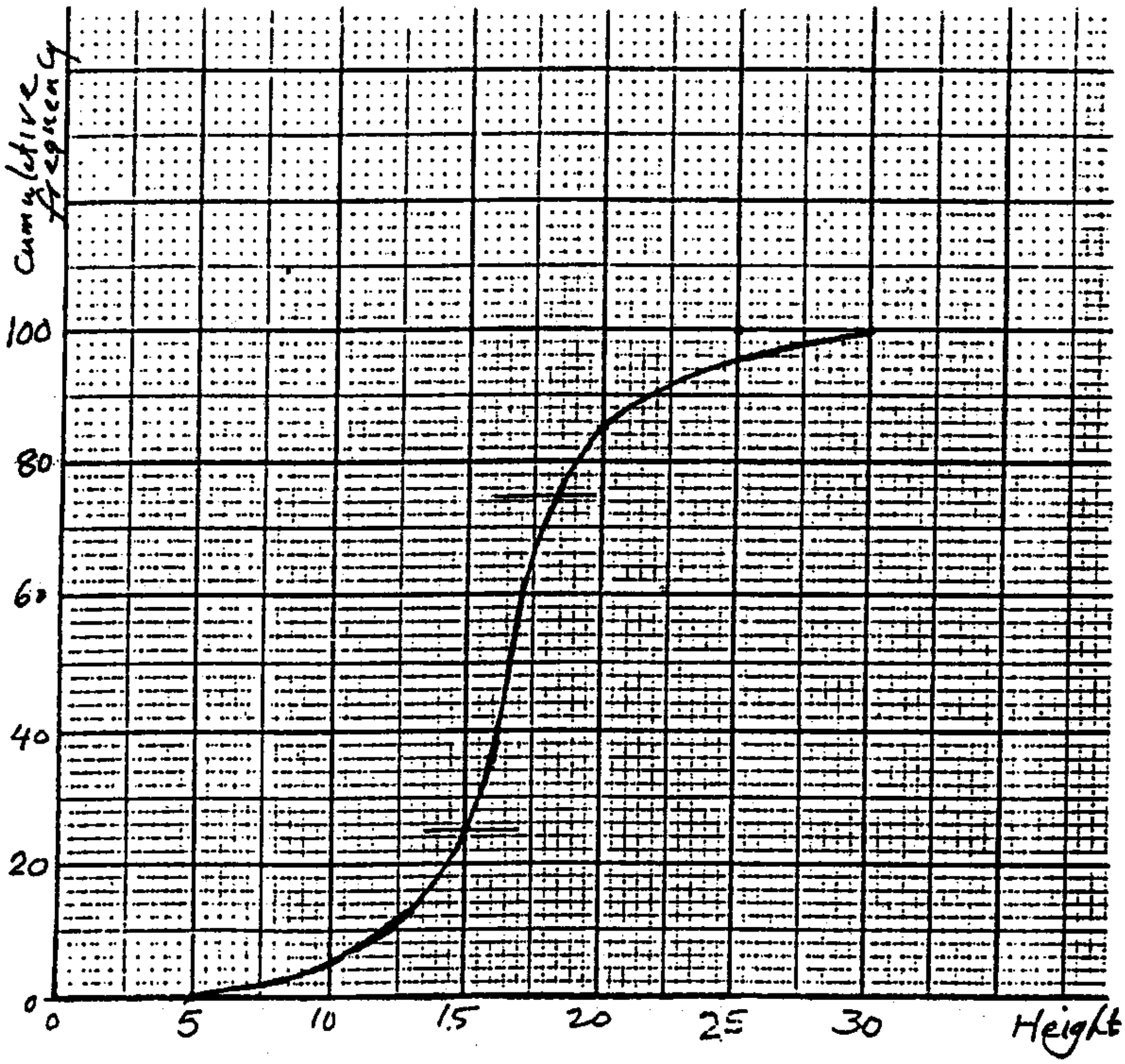
(ii) $M\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}6\\2\end{pmatrix}$

$$\therefore \begin{pmatrix}x\\y\end{pmatrix}=M^{-1}\begin{pmatrix}6\\2\end{pmatrix}=\begin{pmatrix}\frac{1}{5} & -\frac{3}{5}\\\frac{1}{5} & \frac{2}{5}\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}$$

$$=\begin{pmatrix}\frac{6}{5} & -\frac{6}{5}\\\frac{6}{5} & +\frac{4}{5}\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}$$

Coordinates of w are (0 , 2)

5.



(a) (ii) Median = 16.5
Lower quartile = 15
Upper quartile = 18.5
Interquartile range = 18.5 – 15 = 3.5

(iii) Number of plants in the 15 – 20 group = 85 – 25 = 60

(b) (i) Mean = $\dfrac{2.5\times10+7.5\times20+12.5\times45+17.5\times23+25\times2}{10+20+45+23+2} = 11.9$

(ii) $5 < h \leq 10$

(iii) $\dfrac{90}{100}$ x 100 = 90, class is $15 < h \leq 20$

(c) (i) Total less than 10 in both groups = 5 + 10 + 20 = 35
Probability = $\dfrac{35}{200} = \dfrac{7}{40}$

(ii) $\dfrac{5}{35} = \dfrac{1}{7}$

6. (a) (i) $f(1.5) = 1.1$

(ii) $g(0) = 0.85$

(iii) If $g(x) = y$ then $g^{-1}(y) = x$

$\therefore y = (-0.5)$ $\quad\quad \therefore x = -1.05$

(b) (i) $f(x) = g(x)$ at -1.5 and at 0.75.

$\therefore f(x) > g(x)$ means the curve is above the straight line.

$\therefore x < -1.5$

(ii) $f(x) = 0$ $\quad\quad x = -1.75, 0, 1.75$

(iii) $-2 < K < 2$

(c) Gradient of tangent at $x = 0.75$

(Tangent at $x = 0.75$ is the same line drawn representing $g(x)$)

Gradient $= \dfrac{1.8}{1.4} = 1.3$

(d) $1 - f(x) = 0 \longrightarrow f(x) = 1$

Draw the line $y = 1$ to intersect graph of $f(x)$ at three points

$x_1 = -1.85$ to -1.9

$x_2 = 0.35$

$x_3 = 1.5$ to 1.55

7. (a) (i) Volume = volume of cone + cylinder + hemisphere

$$= \frac{1}{3}\pi \times 3^2 \times 4 + \pi 3^2 \times 7 + \frac{1}{2} \times \frac{4}{3}\pi \times 3^3$$

$$= 292 \ cm^3$$

(ii) Surface area = surface area of cone + cylinder + hemisphere

$$= \pi \times 3 \times \sqrt{3^2 + 4^2} + 2\pi \times 3 \times 7 + 2\pi 3^3$$

$$= 236 \ cm^2$$

(b) (i) Volume $= \dfrac{1}{3}\pi x^2(x) \times 4 + \pi x^2(x) + \dfrac{2}{3}\pi x^3 = 2\pi x^3$

(ii) $x = 5$ $\quad\quad$ Volume $= 2\pi(5)^3 = 785 \ cm^3$

(c) Ratio of masses equal ratio of volumes for the same material

Ratio mass of cone : mass of cylinder

$= \dfrac{1}{3} : 1 = 1 : 3$

Mass of hemisphere equal the sum of masses of both cone and cylinder, therefore ratio mass of hemisphere : mass of cylinder : mass of cone

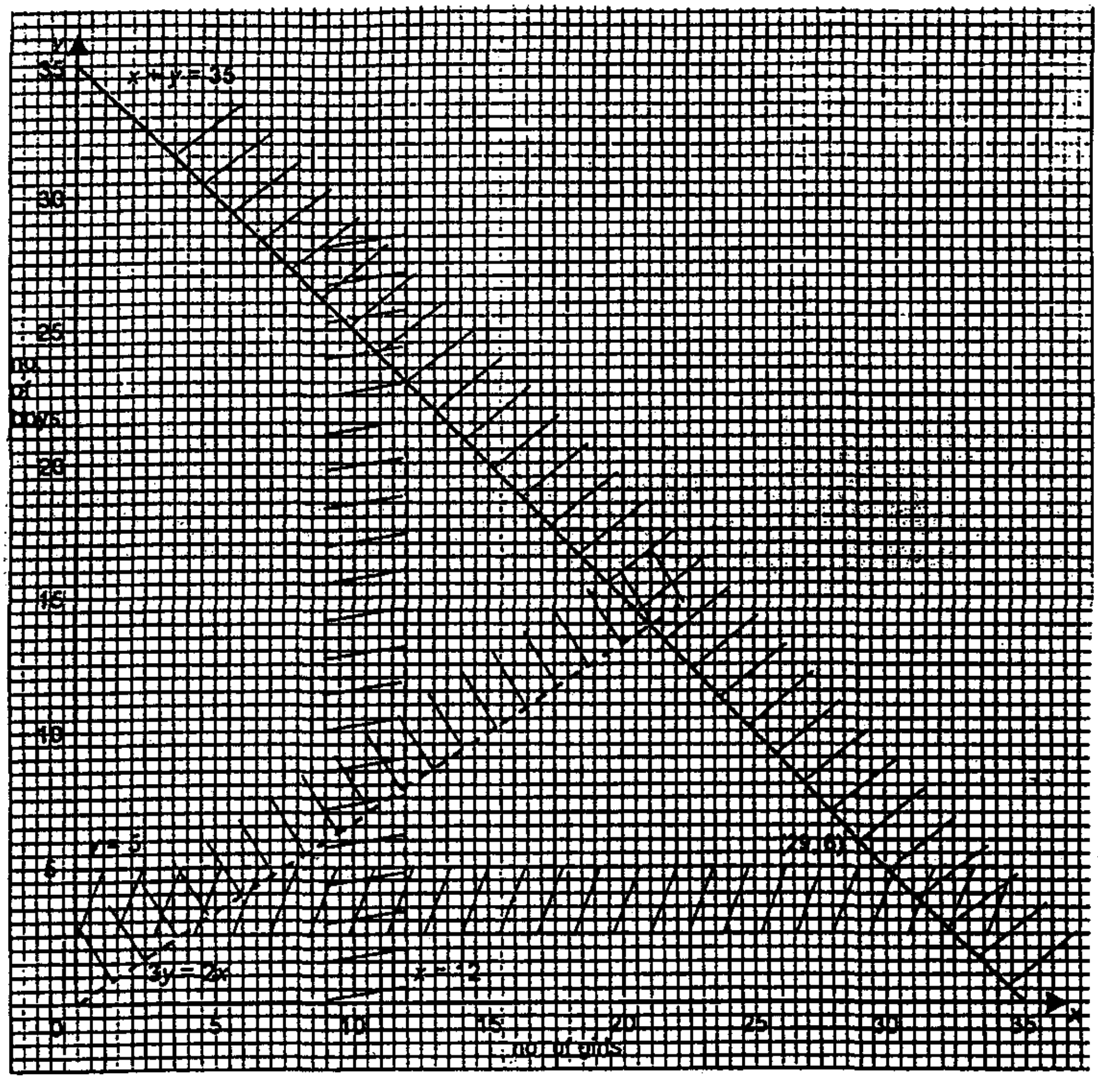
4 : 3 : 1

8. (a) (i) $x > 1.5\,y \longrightarrow x > \frac{3}{2}\,y$

$\therefore \quad y < \frac{2}{3}\,x$

(ii) $x > 12, \qquad y > 5$

$x + y \leq 35$



(c) The point giving maximum possible cost is (29 , 6)
Maximum cost = 29 x 25 + 6 x 20 = 845

9. (a) 7, 10, 15, 22, (22 +9 = 31), (31 + 11 = 42)

(b) $\frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \ldots\ldots\ldots\ldots\ldots\ldots, \frac{51}{103}$

(d) 17, 13, 9, 5, (5 – 4 = 1), (1 – 4 = -3)

n^{th} term = 17 + (n –1) (-4)

= 17 – 4n +4

= 21 – 4n +4

= 21 – 4n

Mathematics

Paper 4
June 2002

1. a) i) Time of working = 1700 – 800 = 9 hours

   Time for writing = $\frac{4}{2+5+4+1} \times 9 = 3$ hours

   ii) Time having lunch = $\frac{1}{12} \times 9 = \frac{3}{4}$h = 45min

   b)

	A	B	C
	2	5	3
		855	

   i) Amit earns = $\frac{855 \times 2}{5}$ = \$ 342

   ii) Chris earns = $855 \times \frac{3}{5}$ = \$ 513

   c) Bernard earning in 52 weeks = 855 × 52 = 444 60

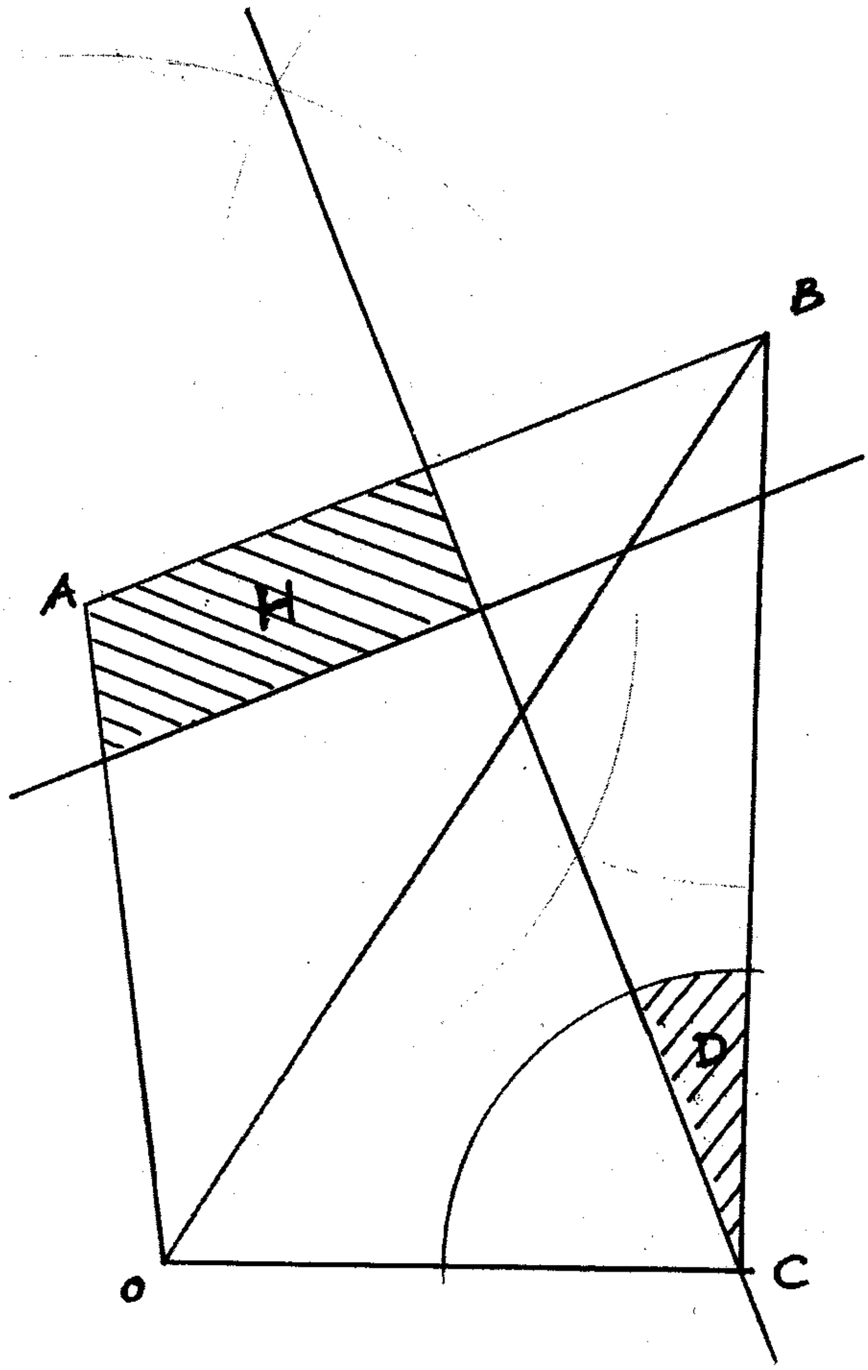
   Fraction of his earnings he saved = $\frac{2964}{44460} = \frac{1}{15}$

   d)

Last year	Increase	This year
100	40	140
?		3500

   Chris saving last year = $\frac{3500 \times 100}{140}$ = 2500

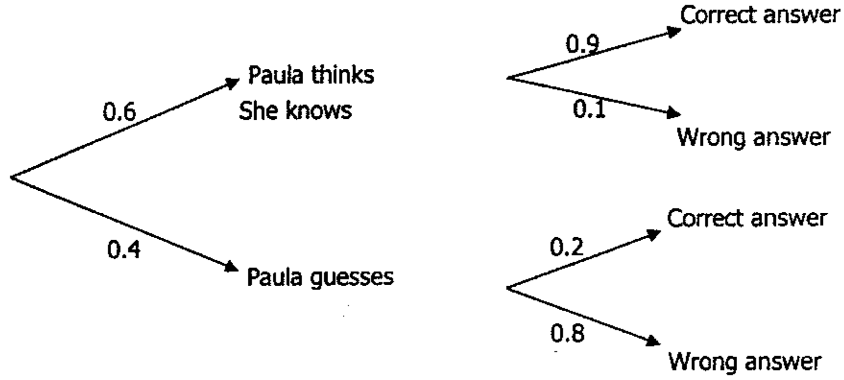
2. (a)



(b) (i) Distance OC = 7.7 x 10 = 77 m
(ii) $\angle$ OAB = 104°

(c) Bearing of A from B is 360 – 104 = 256°

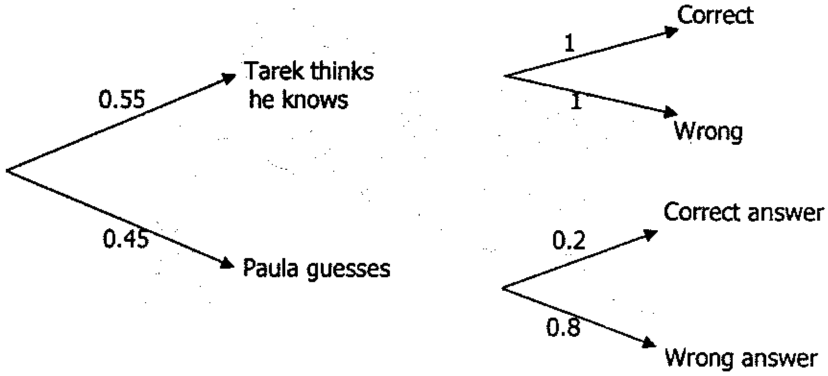
3. a)



b) i) $0.6 \times 0.9 = 0.54$

ii) $0.6 \times 0.9 + 0.4 \times 0.2 = 0.54 + 0.08 = 0.62$

c)



ii) $0.55 \times 1 + 0.45 \times 0.2 = 0.55 + 0.09 = 0.64$

d) i) $100 \times 0.62 = 62$

ii) $100 \times 0.64 = 64$

4. a) $a = 90°$ tangent and radius

$b = 90°$ tangent and radius

$c = 180 - 42 = 138°$

$d = ½ \times 138 = 69°$

From x two tangents are drawn XA = XD

$\therefore \angle XAD = 45° \Rightarrow e = 45°$

b) Congruent

c) i) $\tan 21° = \frac{CA}{GA} = \frac{54}{GA}$

$GA = \frac{54}{\tan 21} = 140.67 = 141cm$

ii) $GX = GA + AX = 141 + 54 = 195cm$

iii) $\cos 42° = \frac{GX}{GW}$

$GW = \frac{195}{\cos 42} = 262$ cm

iv) $GA = GB = 141$

$BW = 262 - 141 = 121$ cm

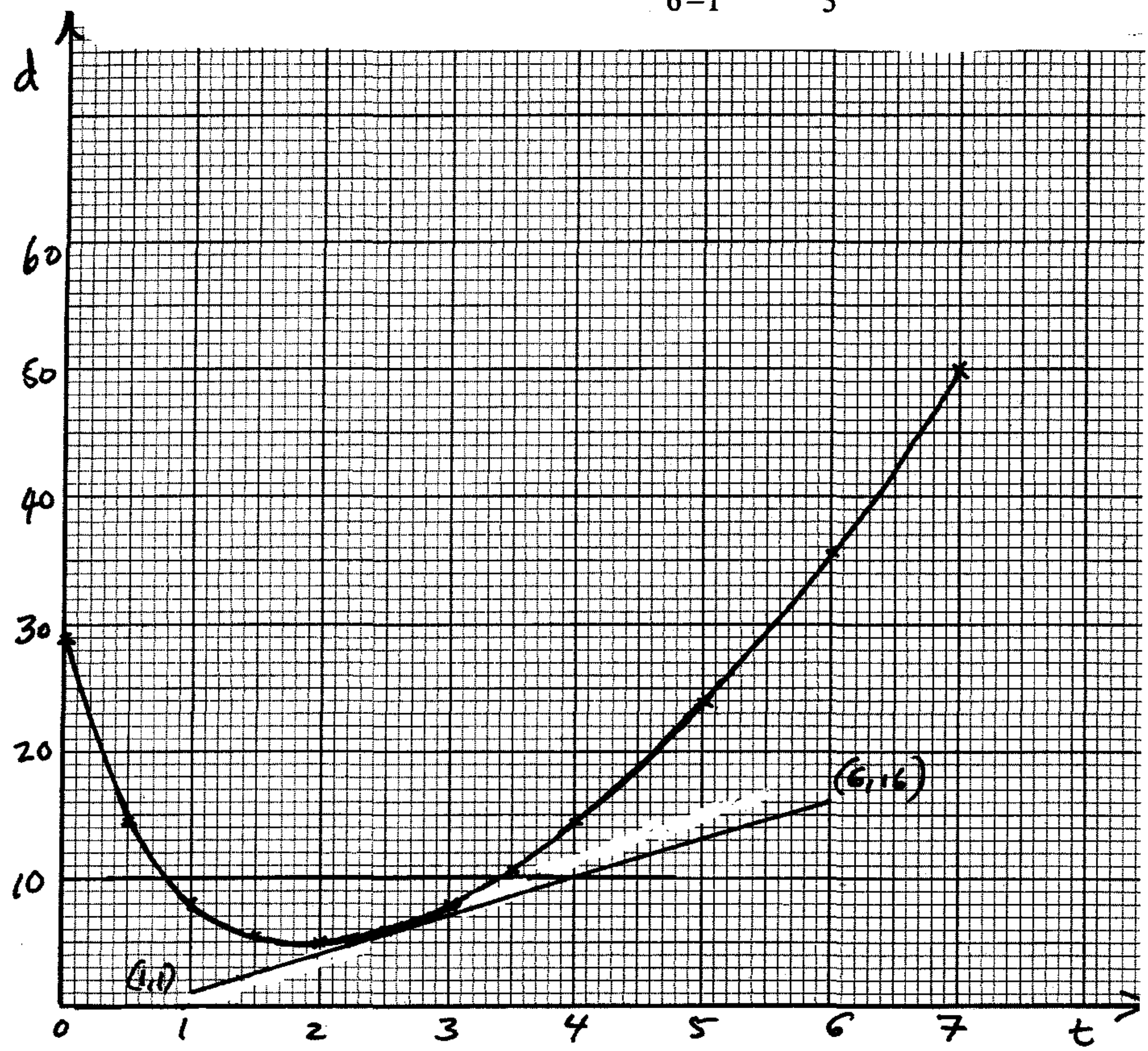
5. (a) d = $(t+1)^2 + \frac{48}{t+1} - 20$

t = 0	d = 29	p = 29
t = s	d = 24	q = 24
t = 7	d = 50	r = 50

(b) d = 12 t = 3.7 min

(d) d = 10 t = 0.8 and 3.4
d is less than 10 m for 3.4 - 0.8 = 2.6 min

(e) Speed is the gradient
at t = 2.5 gradient = $\frac{16-1}{6-1}$ = $\frac{15}{5}$ = 3 m / min



6. a) i) $PQ = 12 - 2x$

$(PQ)^2 = (PA')^2 + (A'Q)^2$

$(12 - 2x)^2 = x^2 + x^2$

$\therefore\ 2x^2 = (12 - 2x)^2$

ii) $2x^2 = 144 - 48x + 4x^2$

$2x^2 - 48x + 144 = 0$

$x^2 - 24x + 72 = 0$

iii) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{24 \pm \sqrt{24^2 - 4 \times 1 \times 72}}{2}$

$= \dfrac{24 \pm \sqrt{288}}{2}$

$= 20.49$ or 3.51

b) i) Possible answer is 3.51

Perimeter = $16 \times 3.51 = 56.2$ cm.

ii) Area = Area of a square (12 by 12) + area of 4 Δ

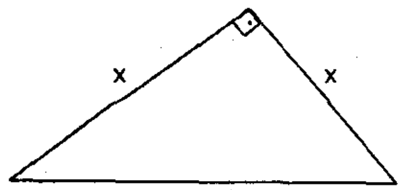
Area of the square = $12 \times 12 = 144$

Area of one Δ = $\frac{1}{2} x^2$

$= \frac{1}{2} (3.51)^2 = 6.18$ cm^2

Area of the 16 sided figures

$= 144 + 6.18 \times 4 = 169$ cm^2



7. a) i) Rotation centre origin 90° clockwise

ii) Reflection on the line $y = x$

iii) Enlargement center origin scale factor 2.

b) Translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

c) i) Reflection on the line $y = -x$

ii) (-4, 2)

d) i) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Matrix = $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

ii) F under M $\to$ C

C under R $\to$ A

New position of F after the transformation RM is A.

8. a) i) Area = $\frac{20}{360}\Pi r^2$

$= \frac{20}{360} \times \Pi \times 6^2 = 6.28 \text{ cm}^2$

ii) Arc of sector = $\frac{20}{360} \times 2\Pi r$

$= \frac{20}{360} \times 2\Pi \times 6 = 2.09 \text{ cm}$

b) i) Volume = Area $\times$ height

$= 6.28 \times 5 = 31.4 \text{ cm}^3$

ii) Total surface area $= (6 + 6 + 2.09) \times 5 + 6.28 \times 2$

$= 83.0 \text{ cm}^2$

c) $\Pi r^2 h$ = Const

$h = \frac{\text{Const}}{r^2}$

D is correct, as h is inversely proportional to r^2.

ii) r doubled

h is $\frac{1}{(2)^2} = \frac{1}{4}$

9. a) Median is 7½, means the average of the middle numbers is 7½, one is 7 and the other is 8.

The mode is 8, means, 8 is repeated twice (at least). So the numbers in order are;

3 4 7 8 8 z (still missing)

Mean is 7, so $\frac{3+4+7+8+8+z}{6} = z$

$30 + z = 42$

$z = 12$

So, x, y and z are 7, 8, 12.

b) i) Total amount $= 5 \times 15 + 15m + 30n$ (5, 15 and 30) are the mid values.

Total amount $= 75 + 15m + 30n$

ii) Mean = 13

$\frac{75 + 15m + 30n}{15 + m + n} = 13$

$75 + 15m + 30n = 195 + 13m + 13n$

$2m + 17n = 120$

iii) Area represent frequency in histograms, so $m + n = 15$

iv) Solving simultaneously

$m + n = 15$ (1)

$2m + 17n = 120$ (2)

(1) x – 2 $\quad -2m - 2n = -30$

$15n = 90$

$n = 6 \Rightarrow m = 9$

Mathematics

Paper 4
November 2002

1. a) i) Time the race started = 15 h 17 min – 0 h 33 min = 14 44

 ii) Speed = $\frac{\text{Distance}}{\text{Time}}$

 Distance = 10 000m = 10 km

 Time = 33 min = $\frac{33}{60}$ = 0.55h

 Speed = $\frac{10}{0.55}$ = 18.2 km/h

 iii) Time = 0 h 33 min 0 sec – 0 h 0 min 51.2 sec = 0 h 32 min 8.8 sec

 Time = 32 min 8.8 sec

 b) 95% of 80m = $\frac{95}{100} \times 80$ = 76m

 c) Mona Pamela

 100 110

 ? 6.16

 Mona jump = $\frac{6.16 \times 100}{110}$ = 5.6m

2. a) i) BE = $\sqrt{5^2 + 6^2} = \sqrt{25 + 36} = \sqrt{61}$

 ii) DB = BE = $\sqrt{61}$

 iii) DA = AF = $\sqrt{5^2 + 8^2} = \sqrt{25 + 64} = \sqrt{89}$

 b) Cos $\angle$B = $\frac{(\sqrt{61})^2 + (10)^2 - (\sqrt{89})^2}{2\sqrt{61}\ (10)} = \frac{72}{20\sqrt{61}}$

 $\angle$DBA = 62.6°

 c) Area of Δ DBA = ½ DB × BA sin B = ½ $\sqrt{61}$ × 10 sin 62.6 = 34.7 cm^2

 d) Total surface area

 = Area of Δ ABC + Area of Δ ADB + Area of Δ ACF + Area of Δ BCE

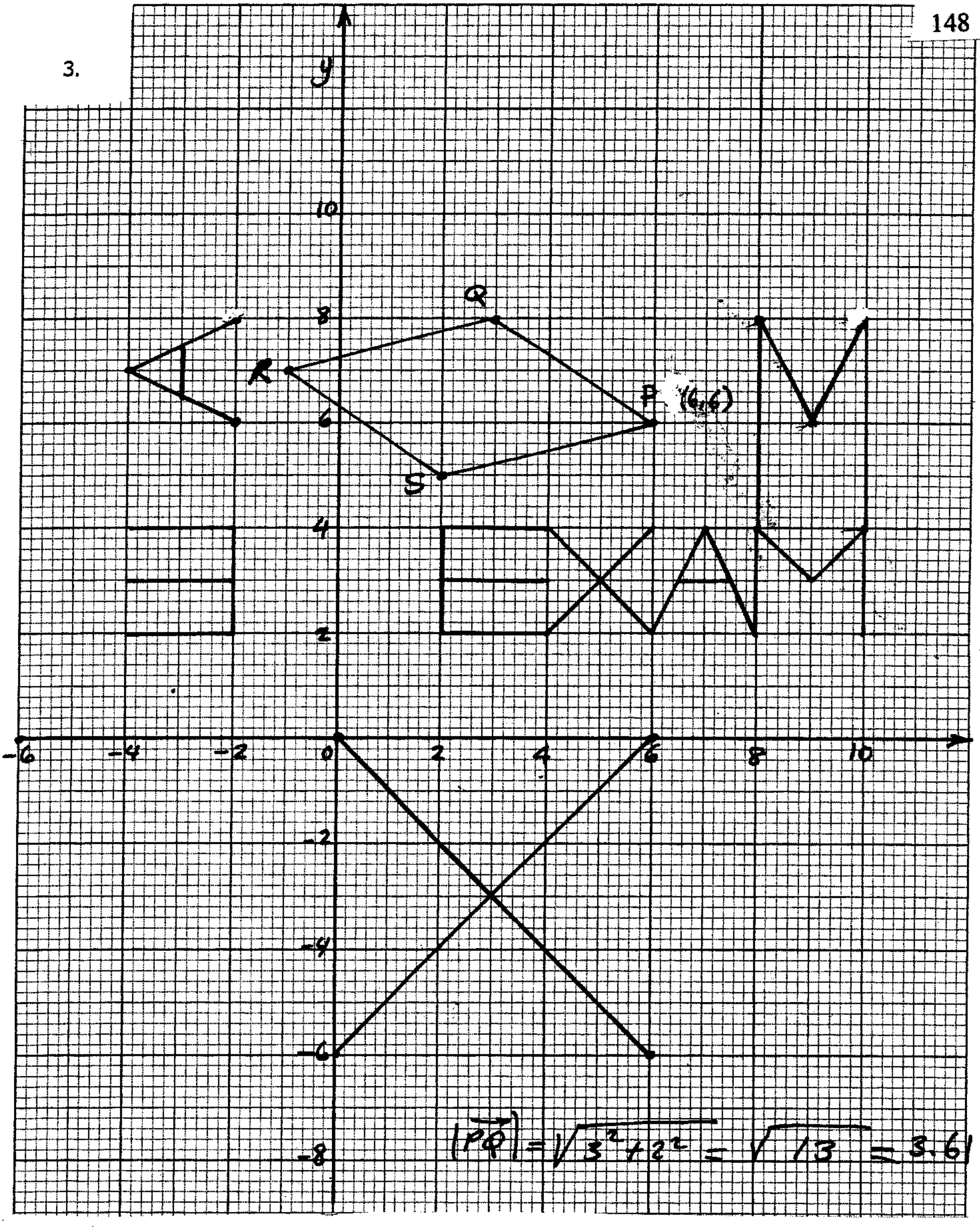
 = ½ × 8 × 6 + 34.7 + ½ × 8 × 5 + ½ × 6 × 5 = 93.7 cm^2

 e) Base area = area of Δ ABC

 = ½ × 6 × 8 = 24 cm^2

 Volume = $\frac{1}{3}$ × 24 × 5 = 40 cm^3

3.



$|\overrightarrow{PQ}| = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.61$

4. a) i) $\frac{5}{10} = \frac{1}{2}$

ii) $\frac{4}{10} = \frac{2}{5}$

iii) $\frac{7}{10}$

iv) $\frac{2}{10}$

b) i) $\frac{4}{10} \times \frac{4}{10} = \frac{4}{25}$

ii) $\frac{4}{10} \times \frac{6}{10} \times 2 = \frac{48}{100} = \frac{12}{25}$

iii) Zero

iv) Less than 4, **i.e.** 1+ 1, 1 + 2, 2 + 1.

$\frac{1}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{1}{10} = \frac{3}{100}$

v) The possible selections to make the product a square number,

$1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, 5 \times 5, 6 \times 6, 7 \times 7, 8 \times 8, 9 \times 9, 10 \times 10, 1 \times 4, 4 \times 1, 2 \times 8, 8 \times 2, 1 \times 9, 9 \times 1, 4 \times 9, 9 \times 4.$

Probability $= 18 \times \frac{1}{10} \times \frac{1}{10} = \frac{18}{100} = 0.18$

5. a) x = 3.5

$f(x) = \frac{24}{(3.5)^2} + (3.5)2 = 14.2$

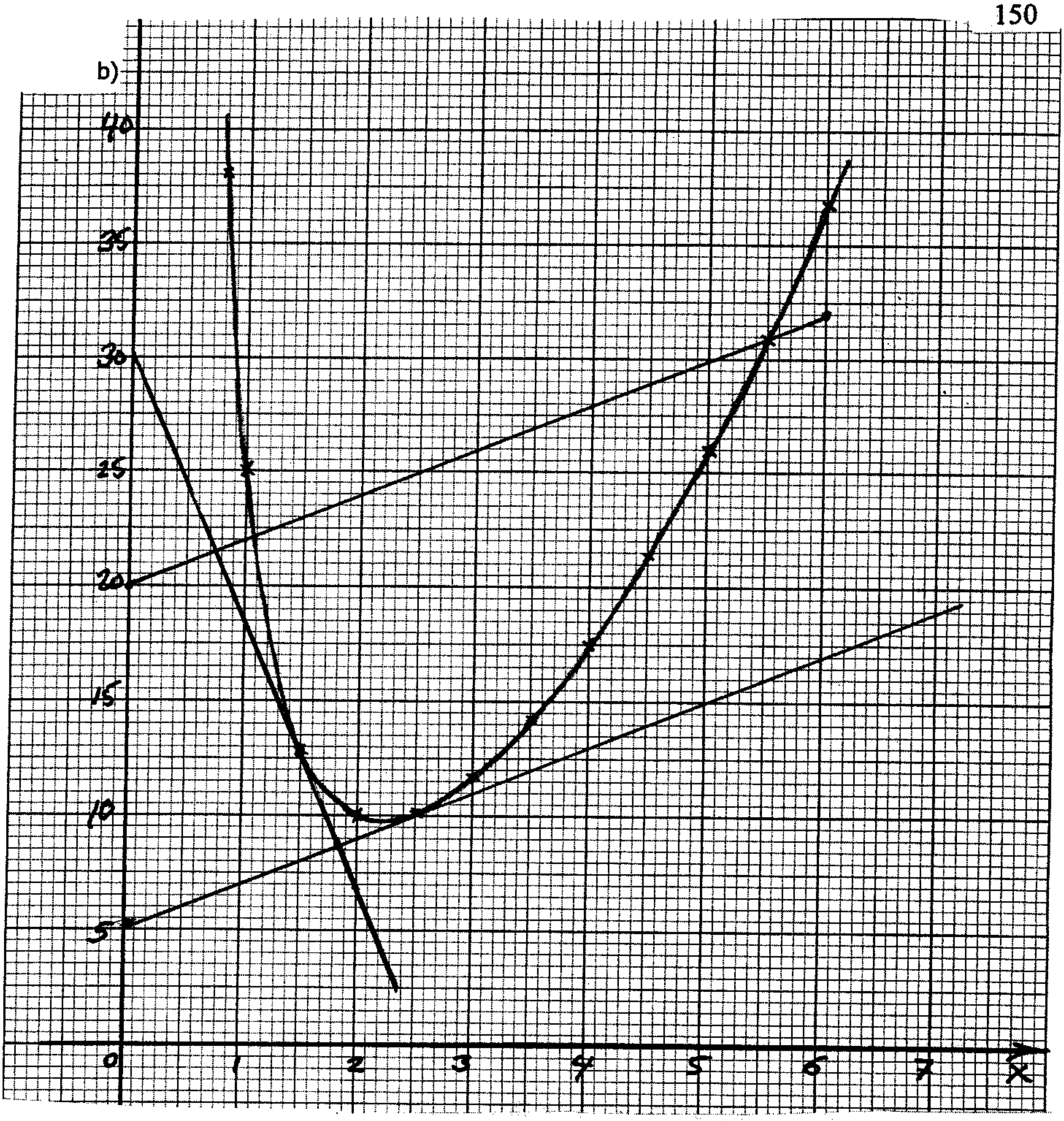
x = 4

f (x) = 17.5

x = 4.5

f (x) = 21.4

b)



c) Gradient at point (1.5, 12.9). Using the two points on the line (0, 30) (2, 7)

Gradient = $\frac{30-7}{0-2}$ = -11.5

d) ii) m = $\frac{32-20}{6-0}$ = 2

Equation is y = 2x + 20

iii) x = 1.1, y = 5.5

v) y = 2x + 5

6. a) i) Bukki age = $x + 5$
Claude age = $2x$
ii) On Jan 1st 2002
Ashraf $x + 2$
Bukki $x + 7$
Claude $2x + 2$
iii) $(x + 2)(2x + 2) = (x + 5)^2$
$2x^2 + 6x + 4 = x^2 + 10x + 25$
$x^2 - 4x - 21 = 0$
iv) $(x - 7)(x + 3) = 0$
$x = 7$
$x = -3$ (rejected)
v) Claude age on 1st Jan 2002 = $2x + 2 = 2 \times 7 + 2 = 16$
b) i) $h^2 + 8h - 17 = 0$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$h = \frac{-8 \pm \sqrt{64 - 4 \times 1(-17)}}{2} = \frac{-8 \pm 11.489}{2} = 1.74 \text{ m}$$

ii) 174 cm

7. a) i) $\frac{36}{108} \times 360 = 120$
ii) $120 - 36 = 84$
iii) $4 + 5 + 3 = 12$

Number got grade B = $\frac{4}{12} \times 84 = 28$

Number got grade C = $\frac{5}{12} \times 84 = 35$

Number got grade D = $\frac{3}{12} \times 84 = 21$

iv) Angle of grade B = $\frac{28}{120} \times 360 = 28 \times 3 = 84^\circ$

Angle of grade C = $\frac{35}{120} \times 360 = 105^\circ$

Angle of grade D = $\frac{21}{120} \times 360 = 63^\circ$

v) Ratio = 36 : 28 = 9 : 7
b) For the interval 0 – 20, area = 20 cm^2;
20 cm^2 equal \$25
Area scale 1 cm^2 = \$ 1.25

For the interval 20 – 30, area = 16 cm^2
Frequency = 16 × 1.25 = 20 — p = 20
For the interval 30 – 40, area = 8 cm^2
Frequency = 8 × 1.25 = 10 — q = 10
For the interval 40 – 70, area = 12 cm^2
Frequency = 12 × 1.25 = 15 — r = 15

8. a) i) $V_A = \pi(3r)^2 h = 9\pi r^2 h$
$V_B = \pi(r)^2 3h = 3\pi r^2 h$
$V_C = \pi(3r)^2 3h = 27\pi r^2 h$
ii) Ratio $9\pi r^2 h : 3\pi r^2 h : 27\pi r^2 h$
3 : 1 : 9
iii) Pot C as $\frac{3r}{r} = \frac{3h}{h} = 3$ (**I.e.** ratio of diameters equal ratio of heights)
iv) Ratio of area of similar pots = $(3)^2 = 9$
Surface area of pot C = 9s
b) i) Total surface area = $2\pi r h + \pi r^2$
$= 2\pi \times 15 \times 20 + \pi \times 15^2$
$= 600\pi + 225\pi = 825\pi$
$= 2592 = 2590$ cm^2
ii) 30 m^2 = 30 × 100 × 100 = 300 000 cm^2
Number of pots = $\frac{300000}{2592} = 115.7$
∴ Number of pots = 115

9. a) i) 10^{th} term is 10 — n^{th} term is n
ii) 10^{th} term is 16 — n^{th} term is n + 6
iii) 10^{th} term is 26 — n^{th} term is 2 (n + 3)
b) i) Next term is 5 (16 – 11) — 10^{th} term is 10 (26 – 16)
ii) n^{th} term is n (2n + 6 – n – 6) = n (n) = n^2

Mathematics 0580

June 2003

Paper 4

1- (a) Total cost = 197 x 10 + 95 x 16
= 1970 + 1520 = 3490 $

(b) 4018 = 157 x 10 + 16 n
16 n = 4018 − 1570
16 n = 2448
$n = \frac{2448}{16} = 153$
n = 153

(c) x + y = 319 ⟶ (1)
10 x + 16 y = 3748 ⟶ (2)
(1) x - 10 -10 x - 10 y = - 3190
(2) 10 x + 16 y = 3748
adding 16 y = 558
$y = \frac{558}{6} = 93$
x + y = 319
x = 319 − 93 = 226
x = 226 y = 93

(d) New cost = $16 - (16 \times \frac{15}{100}) = 13.6$ $

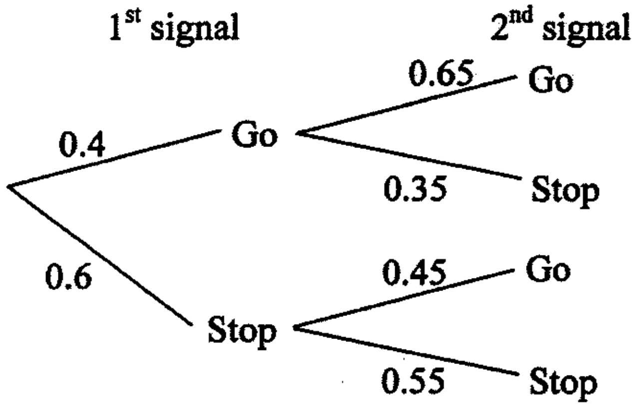
(e) 125 ⟶ 10 $ old cost = $\frac{100 \times 10}{125} = 8$ $
100 ⟶ ?

2- (a) $\cos x = \frac{AB^2 + AC^2 - BC^2}{2(AB)(AC)} = \frac{77^2 + 55^2 - 120^2}{2 \times 77 \times 55}$
x = 130°

(b) $\frac{55}{\sin y} = \frac{60}{\sin 45}$ $\sin y = \frac{55 \sin 45}{60}$ y = 40.4°

(c) (i) bearing of A from C = 180 + 45 = 225°
(ii) bearing of B from A = 360 − (x − 45)
= 275°

3- (a)



(b) (i) P (both are Go) = 0.4 x 0.65 = 0.26

(ii) P (one is Go) = 0.4 x 0.35 + 0.6 x 0.45 = 0.14 + 0.27 = 0.41

(iii) P (No two Stop) = 1 – (0.6 x 0.55) = 0.67

(c) (i) Time = $\frac{D}{S}$ = $\frac{12}{40}$ = 0.3 hour = 0.3 x 60

= 18 min.

(ii) Time will be 18 + 6 = 24 min.

S = $\frac{D}{T}$ = $\frac{12 \times 60}{24}$ = 30 Km / hour.

(d) (i) T = $\frac{D}{S}$ = $\frac{15}{40}$ x 60 = 22.5 min.

(ii) P (stops at both signals) = 0.6 x 0.55 = 0.33

4- (b) f (x) = 0 i.e. y = 0 intersection with x axis

From the graph x_1 = - 3.5 , x_2 = 0 , x_3 = 3.5

(c) y = g (x) , g (x) = x + 1

$-4 \le x \le 4$

x	- 4	0	4
g (x)	- 3	1	5

(d) (i) g (1) = 1 + 1 = 2

(ii) f g (1) = f (2) = - 8

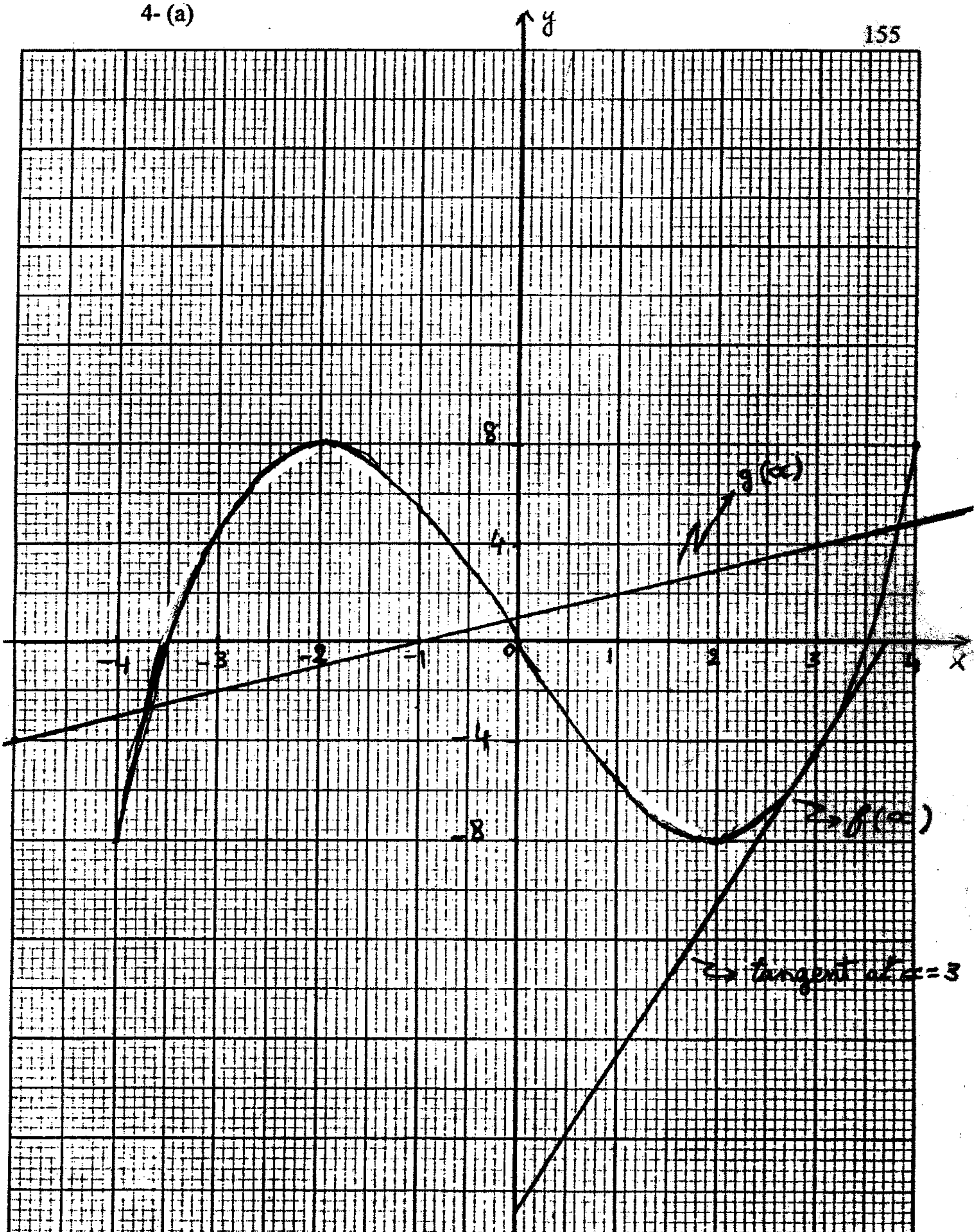
(iii) $g^{-1}(4)$ = 3 for the graph of g (x) , y = 4 x = 3

(iv) f (x) = g (x) intersection of the two graph

From graph positive value of x = 3.8

(e) Gradient = $\frac{23}{3.7}$ = 6.2 (any answer from 5 to 10 acceptable)

4- (a)



5- (a) Area $= \frac{1}{2} \times 10 \times 10 \sin 60$

$= 43.3 \text{ cm}^2$

(b) $2\pi r = 10$

$r = \frac{10}{2\pi} = 1.59 \text{ cm}$

(c) (i) Diagram 1 is a pyramid (with triangular base)

Surface area = Area of all faces

$= 4 \times 43.3 = 173 \text{ cm}^2$

(ii) Diagram 2 is a cylinder

Volume = area of circle x height

$= (\pi r^2).(10)$

$= \pi (1.59)^2 (10) = 79.4 \text{ cm}^3$ (from 79.4 – 79.6)

(iii) Diagram 3 is a cone

$h^2 = l^2 - r^2$

$= 10^2 - (1.59)^2$ $\qquad h = 9.87 \text{ cm}$

6- (a) (i) Volume $= (2x).(x+4).(x+1)$

(ii) $V = 2x(x^2 + x + 4x + 4)$

$= 2x(x^2 + 5x + 4) = 2x^3 + 10x^2 + 8x$

(b) (i) Dimensions of the box :

$(2x - 2) = 2(x - 1)$

$(x + 4 - 2) = (x + 2)$

$(x + 1 - 1) = x$ (because of open top)

(ii) Volume of inside box $= 2(x-1)(x+2)\,x$

$= 2x(x^2 + 2x - x - 2)$

$= 2x(x^2 + x - 2)$

$= 2x^3 + 2x^2 - 4x$

∴ Volume of wood = outside volume – Inside volume

$= (2x^3 + 10x^2 + 8x) - (2x^3 + 2x^2 - 4x)$

$= 2x^3 + 10x^2 + 8x - 2x^3 + 2x^2 - 4x$

$= 8x^2 + 12x$

(c) $V_{(wood)} = 1980 \text{ cm}^3$

(i) $8x^2 + 12x = 1980$

$8x^2 + 12x - 1980 = 0$

$2x^2 + 3x - 495 = 0$

$$x = \frac{-3 \pm \sqrt{9 + (4 \times 2 \times 495)}}{4} = \frac{-3 \pm 63}{4} = 15 \text{ or } (-16.5 \text{ rejected})$$

$x = 15 \text{ cm}$

(ii) External dimensions are : $2x = 30$ cm

$(x + 4) = 19$ cm

$(x + 1) = 16$ cm

7- (a) (i) $\overrightarrow{OS} = \overrightarrow{OA} + \overrightarrow{AF} + \overrightarrow{FS}$

$= a + a + a$

$= 3a$

(ii) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$

(iii) $\overrightarrow{CD} = a$

(iv) $\overrightarrow{OR} = \overrightarrow{OF} + \overrightarrow{FR}$

$= 2a + 2b$

(v) $\overrightarrow{CF} = \overrightarrow{CO} + \overrightarrow{OF}$

$= -2b + 2a = 2(a - b)$

(b) $|a| = 5$ (i) $|b| = |a| = 5$

(ii) $|a - b| = |\overrightarrow{BA}| = |a| = 5$ triangle OAB equilateral

(c) (i) Enlargment centre O with scale factor 3

(ii) Reflection in the line CF

(d) (i) The star has 6 lines of symmetry

(ii) Angle of rotation $= \frac{360}{6} = 60°$

8- (a) (i) Modal class $60 < x \leq 80$

(ii)

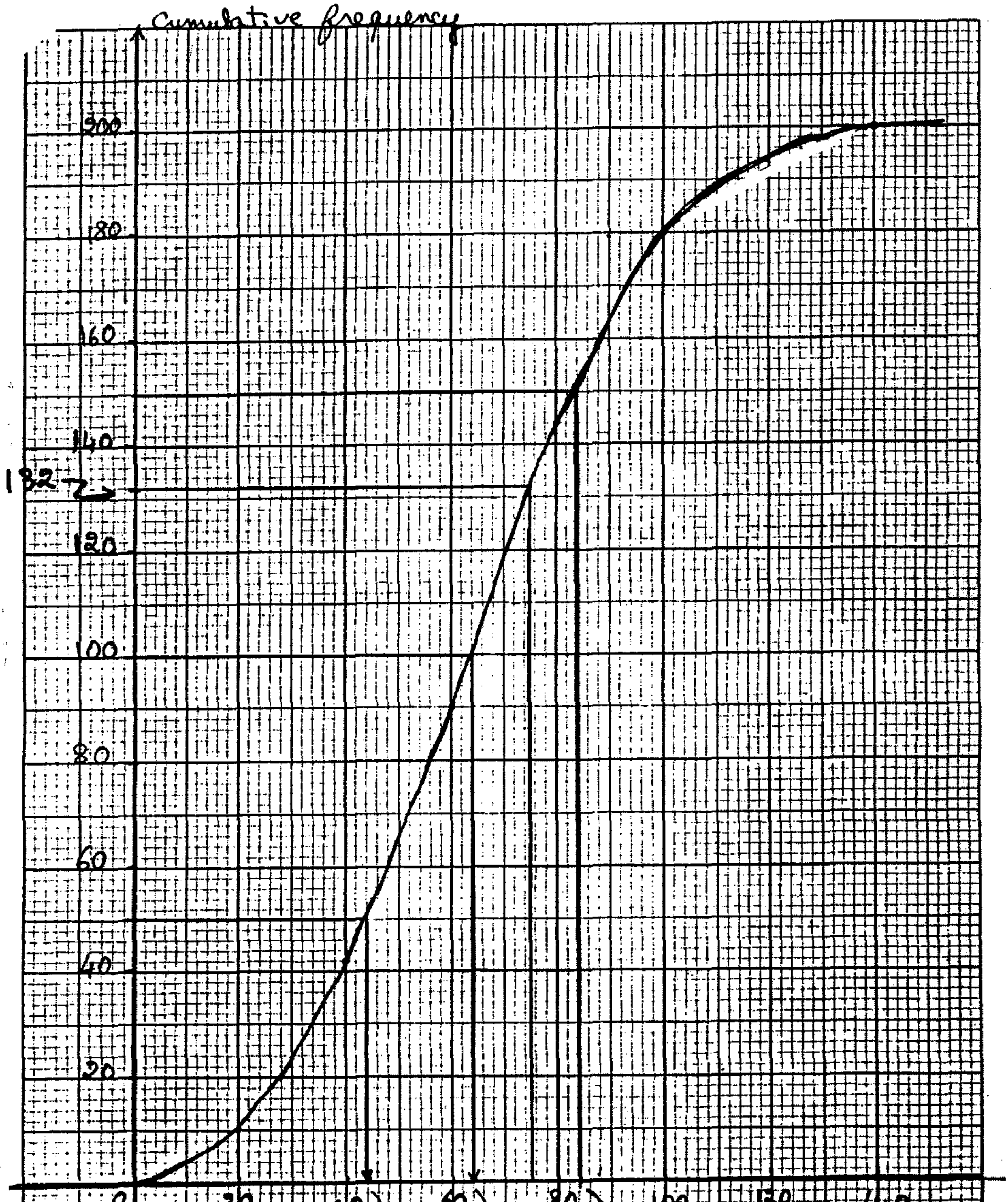
Amount	Mid-value	Frequency	Product
0 - 20	10	10	100
20 – 40	30	32	960
40 – 60	50	48	2400
60 – 80	70	54	3780
80 – 100	90	36	3240
100 – 140	120	20	2400
		$\sum = 200$	$\sum = 12880$

Mean $= \frac{12880}{200} = 64.4$

(b) (i) cumulative Frequency table :

Amount	cumulative frequency
≤ 20	10
≤ 40	42
≤ 60	90
≤ 80	144
≤ 100	180
≤ 140	200

8- (b) (ii)


Cumulative frequency
200
180
160
140
182
120
100
80
60
40
20
0
20
40
60
80
100
120
140
Lower
Median
Upper

(c) From Graph :

(i) Median = 64

(ii) Upper quartile = 84

(iii) lower quartile = 44

In terquartile range = 84 − 44 = 40

(iv) 132 shopper spent less than 75

∴ 200 − 132 = 68 shoppers spent at least 75 $

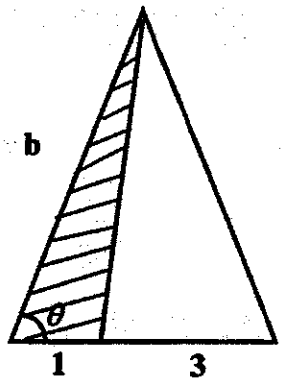
9- (a) <u>Diagram 1 :</u>

Shaded area $= A_s = \frac{1}{2}\left(\frac{a}{4}\right)b \sin\theta$

$= \frac{1}{8}ab\sin\theta$

Total area $= A_T = \frac{1}{2}ab\sin\theta$

$\frac{A_s}{A_T} = \frac{1/8}{1/2} = \frac{1}{4} = 25\,\%$



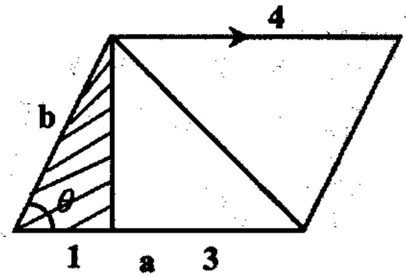
<u>OR</u> triangles are of the same height ratio of areas equal ratio of bases shaded area $= \frac{1}{4}$ total area $= 25\,\%$

<u>Diagram 2 :</u>

$A_s = \frac{1}{2}\frac{a}{4}b\sin\theta = \frac{1}{8}ab\sin\theta$

$A_T = a\,b\sin\theta$

$\frac{A_s}{A_T} = \frac{1/8}{1} = \frac{1}{8} = 12.5\%$



<u>OR</u> shaded triangle is $\frac{1}{4}$ large triangle which is $\frac{1}{2}$ parallagram

Area $= \frac{1}{8}$ total area

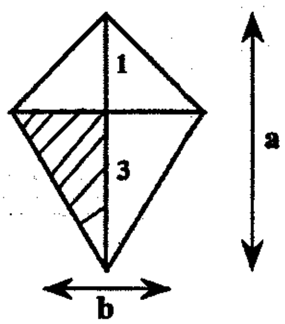
$= \frac{1}{8} \times 100 = 12.5\ \%$

<u>Diagram 3 :</u>

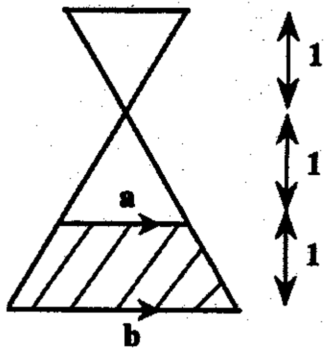
$A_s = \frac{1}{2}\left(\frac{b}{2}\right)\left(\frac{3a}{4}\right) = \frac{3}{16}ab$

$A_T = \frac{1}{2}\ ab$

$\frac{A_s}{A_T} = \frac{3/16}{1/2} = \frac{3}{8} = 37.5\%$



<u>Diagram 4 :</u> ⟶ $b = 2a$

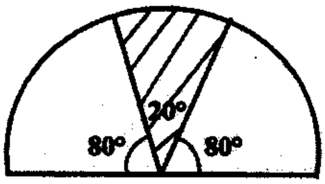


$$A_s = \frac{1}{2}(a+b) \times 1 = \frac{1}{2}(a+b) = \frac{3}{2}a$$

$$A_T = \frac{1}{2}(a+b) + 2\left[\frac{1}{2}a \times 1\right]$$

$$= \frac{3}{2}a + a = \frac{5}{2}a$$

$$\frac{A_s}{A_T} = \frac{3/2}{5/2} = \frac{3}{5} = 60\,\%$$



(b) $$A_s = \frac{\theta}{360}(\pi r^2)$$

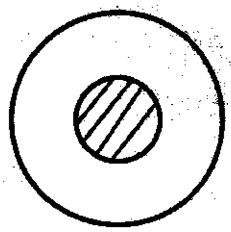
$$= \frac{20}{360}(\pi r^2)$$

$$A_s = \frac{1}{18}\pi r^2$$

$$A_T = \frac{\pi r^2}{2}$$

$$\frac{A_s}{A_T} = \frac{1/18}{1/2}$$

$$\boxed{A_s = \frac{1}{9}A_T}$$

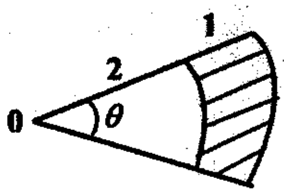


$$A_s = \pi r^2 = \pi \qquad (r_s = 1)$$

$$A_T = \pi\,(5)^2 = 25\,\pi$$

$$\frac{A_s}{A_T} = \frac{1}{25}$$

$$\boxed{A_s = \frac{1}{25}A_T}$$



$$A_s = \frac{\theta}{360}\pi(3)^2 - \frac{\theta}{360}\pi(2)^2$$

$$= \frac{5\pi}{360}\theta$$

$$A_T = \frac{9\pi}{360}\theta$$

$$\frac{A_s}{A_T} = \frac{5}{9}$$

$$\boxed{A_s = \frac{5}{9}A_T}$$