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IGCSE

Further Pure Mathematics

Teacher's guide

Edexcel IGCSE in Further Pure Mathematics (4PM0)

First examination 2011

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Introduction

The Edexcel International General Certificate of Secondary Education (IGCSE) in Further Pure Mathematics is designed for schools and colleges. It is part of a suite of IGCSE qualifications offered by Edexcel.

About this guide

This guide is for teachers who are delivering, or planning to deliver, the Edexcel IGCSE in Further Pure Mathematics qualification. It supports you in delivering the course content and raising student achievement by giving you:

- an example course planner
- example lesson plans
- example questions, with examiner comment
- details of the Assessment Objectives.

Contents

Why choose this qualification?

The Edexcel IGCSE in Further Pure Mathematics has been developed to:

- give a broad overview of mathematical techniques for students who may not study mathematics beyond this level, or for those whose course of study requires a knowledge of mathematical techniques beyond the content of IGCSE Mathematics
- give a course of study for students whose mathematical competence may have developed early
- offer a range of grades from E to A*
- enable students to acquire knowledge and skills with confidence, satisfaction and enjoyment
- enable students to develop an understanding of mathematical reasoning and processes, and the ability to relate different areas of mathematics to each other
- enable students to develop resourcefulness when solving problems
- provide a solid basis for students wishing to progress to Edexcel's AS and Advanced GCE in Mathematics, or equivalent qualifications
- offer papers that are balanced in terms of topics and difficulty.

Go to www.edexcel.com/igcse2009 for more information about this IGCSE and related resources.

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Head Office — London

If you have a question about IGCSE 2009 and are not sure who you need to ask email us on IGCSE2009@edexcel.com or call our Customer Services Team on +44 (0) 1204770696.

Training

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Section A: Qualification content

Introduction

This qualification replaces the legacy Edexcel GCE Alternative Ordinary Level in Pure Mathematics (7362).

You will be pleased to hear that there have been no major changes to the specification content and no changes to the assessment requirements. This minimal amount of change means that you can teach the Edexcel IGCSE in Further Pure Mathematics without having to spend a lot of time updating schemes of work, and that past papers still give students opportunities for examination revision.

The specification content now includes expanded examples and explanation, so that you can be confident about the skills, knowledge and understanding your students need for this qualification.

Information for Edexcel centres

In order to facilitate the transition from the legacy Edexcel GCE Alternative Ordinary Level in Pure Mathematics (7362) to this Edexcel IGCSE in Further Pure Mathematics qualification, changes have been kept to a minimum.

The order of topics is the same and the only new material included is the length of an arc and the area of a sector in radians (*section 10*).

We have included them because the new specification gives clarification of the knowledge and understanding required. In the legacy specification this knowledge and understanding was expected but was not specified.

In addition, some topics that were listed together in the legacy Edexcel GCE Alternative Ordinary Level specification have been listed separately in this qualification. This will make the specification easier for you and your students to use.

Changes to content from Edexcel Alternative Ordinary Level (7362) to this qualification

The table below sets out the relationship of the legacy Edexcel GCE **Alternative Ordinary Level qualification** (7362) to this IGCSE qualification.

Legacy Edexcel qualification content reference	IGCSE qualification content reference	New content
1.1, 1.2	Logarithmic functions and indices 1.2, 1.1	1.3, 1.4 (previously expected but not specified)
2.1, 2.2	The quadratic function 2.1, 2.2, 2.3	
3	Identities and inequalities 3.1, 3.2, 3.3, 3.4, 3.5	3.1 (previously expected but not specified)
4.1, 4.2	Graphs 4.1, 4.2	
5.1, 5.2	Series 5.2, 5.1	
6.1	The Binomial series 6.1	
7.1, 7.2, 7.3, 7.4, 7.5, 7.6	Scalar and vector quantities 7.1, 7.2, 7.3, 7.4, 7.5, 7.6	
8.1, 8.2, 8.3, 8.4, 8.5	Rectangular Cartesian coordinates 8.1, 8.2, 8.4, 8.5	
9.1, 9.2, 9.3, 9.4, 9.5	Calculus 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7	
10.1, 10.2, 10.3, 10.4, 10.5	Trigonometry 10.2, 10.3, 10.4, 10.5	10.1 (new material) 10.6, 10.8 (previously expected but not specified)

Key to references: 10.1 refers to section 10 of the specification, item 1

Changes to assessment from Edexcel Alternative Ordinary Level (7362) to this qualification

The scheme of assessment for the IGCSE in Further Pure Mathematics (4PM0) is the same as that for the legacy Edexcel GCE Alternative Ordinary Level (7362).

Information for centres starting the Edexcel IGCSE for the first time

The Edexcel IGCSE in Further Pure Mathematics gives students the opportunity to develop their knowledge, understanding and skills in the areas of number, algebra and geometry. The qualification builds on previous learning in the Edexcel IGCSE in Mathematics (Specification B or Specification A Higher tier), and provides a basis for students who go on to study mathematics further, for example the Edexcel GCE in Mathematics or other numerate disciplines or professions.

The qualification provides an introduction to calculus, with topics that build on each other to give challenging, engaging material for students to develop their confidence and competence in mathematics.

The course content revisits material in the Higher tier of the Edexcel IGCSE in Mathematics (Specification A) (4MA0), and the Edexcel IGCSE in Mathematics (Specification B) (4MB0). It then builds on this material to introduce new pure mathematics topics.

The qualification is designed to appeal to students who are high achievers in mathematics, and to those who wish to develop their knowledge of mathematics.

The table below shows where the mathematical topics appear in the specification.

Mathematical topic	Specification reference
Quadratic functions	Section 2 – The quadratic function
Indices and surds	Section 1 – Logarithmic functions and indices
Factors of polynomials	Section 3 – Identities and inequalities
Logarithms and exponentials	Section 1 – Logarithmic functions and indices
Straight line graphs	Section 4 – Graphs Section 8 – Rectangular Cartesian coordinates
Circular measure	Section 10 - Trigonometry
Trigonometry	Section 10 - Trigonometry
Binomial expansions	Section 6 – The Binomial Series
Vectors in 2D	Section 7 – Scalar and vector quantities
Differentiation and integration	Section 9 - Calculus
Series and summation	Section 5 - Series

The topics shown below can be found in the specification content for the Edexcel IGCSE in Mathematics (Specification B) (4MB0).

Mathematical topic
Sets
Functions
Simultaneous equations
Matrices

Section B: Assessment

Assessment overview

Paper 1	Percentage	Marks	Time	Availability
4PM0/01 Grades available A* to E All Assessment Objectives tested	50%	100	Two hours	January and June First assessment June 2011
Paper 2	**Percentage**	**Marks**	**Time**	**Availability**
4PM0/02 Grades available A* to E All Assessment Objectives tested	50%	100	Two hours	January and June First assessment June 2011

Assessment Objectives (AO) and weightings

		% in IGCSE
AO1	Demonstrate a confident knowledge of the techniques of pure mathematics required in the specification	**30-40%**
AO2	Apply a knowledge of mathematics to the solutions of problems for which an immediate method of solution is not available and which may involve knowledge of more than one topic in the specification	**20-30%**
AO3	Write clear and accurate mathematical solutions	**35-50%**
	TOTAL	**100%**

Assessment summary

Papers 1 and 2	Description	Knowledge and skills
4PM0/01 4PM0/02	For both papers the following apply: • The two-hour paper is externally assessed. • There will be about 11 questions. • Calculators are allowed. • Questions can be set on any topic in the specification. • All questions should be attempted. • The paper will start with short questions targeting mainly grades C and D. • Longer questions then target several grades, starting with lower grades and becoming progressively more difficult. • Grades A* to D are available, with a 'safety net' grade E for those just failing to achieve grade D. • All Assessment Objectives are tested. • Overview of content: – number – algebra and calculus – geometry and trigonometry. • Equipment required for the examination: electronic calculator, pen, pencil, ruler in centimetres and millimetres.	**Number** Students should be able to apply their numerical skills in a purely mathematical way and to real-life situations. **Algebra and calculus** Students should: • use algebra and calculus to set up and solve problems • develop competence and confidence when manipulating mathematical expressions • construct and use graphs in a range of situations. **Geometry and trigonometry** Students should: • use properties of shapes, angles and transformations • use vectors and rates of change to model situations • coordinate geometry and trigonometry.

Examination questions

This guidance includes four questions of varying length from different areas of the specification content, together with a model answer and the mark scheme for each question. The questions show how a short question may target a single grade, whereas a longer question usually targets several grades, working upwards to grade A. There are also examples of how a question may require knowledge of more than one section of the specification. Each answer is followed by an examiner comment explaining how the mark scheme has been applied to the student's work and including the target grades for each part of the question. There are also comments about the best way of approaching some of the questions, and common errors seen in past examinations.

Using the mark scheme

The mark scheme gives the responses we expect from students. Indicative answers are given but during the standardisation of examiners process the mark scheme is updated and expanded to cover unexpected but nonetheless correct student responses.

General comments on mark schemes

- M marks are 'method marks'. They are awarded for knowing a method and attempting to apply it. Any formulae used must be correct, shown by either quoting the formula in its general form before substituting the required numbers, or by having a completely correct substitution alone.
- A marks are 'accuracy marks'. They can be awarded only if the relevant method mark(s) has been earned.
- B marks are independent of method marks. They are used, for example, when the question requires students to 'write something down' (see *Question 4*) or where a student has made progress in a small part of the question, such as finding the number of terms in the series in *Question 2*.

Vectors

Question 1

Referred to a fixed origin O, the position vectors of the points P and Q are $(6\mathbf{i} - 5\mathbf{j})$ and $(10\mathbf{i} + 3\mathbf{j})$ respectively. The mid point of PQ is R.

(a) Find the position vector of R.

(2)

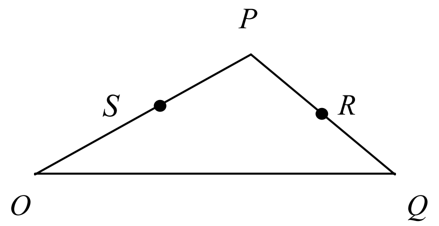
The mid point of OP is S.

(b) Prove that SR is parallel to OQ.

(3)

(Total 5 marks)

Model answer

(a) 	
$\overrightarrow{OR} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ} = (6\mathbf{i} - 5\mathbf{j}) + \frac{1}{2}(10\mathbf{i} + \mathbf{3j} - (6\mathbf{i} - 5\mathbf{j}))$	M1
$= 8\mathbf{i} - \mathbf{j}$	A1
(b) $\overrightarrow{SR} = \overrightarrow{OR} - \overrightarrow{OS} = 8\mathbf{i} - \mathbf{j} - \frac{1}{2}(6\mathbf{i} - 5\mathbf{j}) = 5\mathbf{i} + \frac{3}{2}\mathbf{j} = \frac{1}{2}\overrightarrow{OQ}$	M1A1
$\therefore SR$ is parallel to OQ	A1

Examiner comments

a) A typical C grade student should be able to complete this part of the question. The M mark is earned for attempting to find $\overrightarrow{OR}$ by using the sides of ΔOPQ as shown above, or by using the formula $\frac{1}{2}(\overrightarrow{OP} + \overrightarrow{OQ})$. The A mark is awarded for a completely correct answer.

An A grade student would probably write the answer down by using the formula for dividing a line in a given ratio (in this case 1:1) and would be awarded one mark for each correct component.

b) This part is aimed at the A grade student. The M mark is awarded for an attempt to find $\overrightarrow{SR}$, using $\overrightarrow{OR} - \overrightarrow{OS}$.

The first A mark is for obtaining $\overrightarrow{SR} = \frac{1}{2}\overrightarrow{OQ}$ (or equivalent) and the second mark for a correct conclusion to the question.

$\sum$ notation

Question 2

Evaluate $\sum_{r=7}^{50}(3r-5)$

(Total 4 marks)

Model answer

$\sum_{r=7}^{50}(3r-5) = 16 + 19 + 22 + \ldots\ldots 145$	B1(44)
$= \frac{44}{2}[16 + 165]$	M1A1
$= 3542$	A1

Examiner comments

$\sum$ notation questions look 'innocent' but it is rare for students below A grade to complete them successfully. Very few list the first few terms so they can see how the series looks; if they did this they would probably realise that they were simply dealing with an arithmetic series.

The B mark is awarded for obtaining the correct number of terms, whether or not they are used in a formula. The M mark is for applying the formula for the sum of an arithmetic series (in either form – $\frac{n}{2}(a+l)$ or $\frac{n}{2}(2a+(n-1)d)$) with 44 or 43 terms. The first A mark is for putting correct numbers in the bracket. The second mark is for obtaining the correct answer of 3542.

Roots of a quadratic equation

Question 3

$\mathrm{f}(x) = 3x^2 - 6x + p$. where p is a constant.

The equation $\mathrm{f}(x) = 0$ has roots α and β. Without solving the equation $\mathrm{f}(x) = 0$

(a) form a quadratic equation, with integer coefficients, which has roots $(\alpha + \beta)$ and $\frac{1}{\alpha + \beta}$,

(4)

(b) show that the quadratic equation with roots $\frac{\alpha+\beta}{\alpha}$ and $\frac{\alpha+\beta}{\beta}$ can be written in the form $px^2 - qx + q$ and find the value of the integer q.

(4)

Given that 3 is a root of the equation found in part (b), find

(c) the value of p,

(2)

(d) the other root of the equation.

(2)

(Total 12 marks)

Model answer

(a) $\alpha + \beta = 2 \quad (\alpha+\beta) + \frac{1}{\alpha+\beta} = 2 + \frac{1}{2} = \frac{5}{2}$	M1A1
$x^2 - \frac{5}{2}x + 1 = 0 \quad 2x^2 - 5x + 2 = 0$	M1A1
(b) $\frac{\alpha+\beta}{\alpha} + \frac{\alpha+\beta}{\beta} = \frac{\alpha\beta+\beta^2+\alpha^2+\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{12}{p}$	M1A1
Prod. $= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{12}{p}$, Eqn. $x^2 - \frac{12}{p}x + \frac{12}{p} = 0$, $q = 12$	B1, B1
(c) $x = 3: \quad 9 - \frac{36}{p} + \frac{12}{p} = 0, \quad 9 = \frac{24}{p} \quad p = \frac{8}{3}$	M1, A1
(d) sum $= \frac{12}{p} = \frac{9}{2}$ other root $= \frac{9}{2} - 3 = \frac{3}{2}$	M1A1

Examiner comments

C grade students are usually successful in the initial parts of these questions – in this case parts (a) and (b). Parts (c) and (d) are aimed at A grade students.

a) The first M mark is awarded for obtaining the sum of the roots of the original equation and attempting the sum of the roots of the new equation. The first A mark is awarded for obtaining the correct result for this sum. The second M mark is for using

$$x^2 - (\text{sum of roots})x + \text{product}$$

and is earned even if '= 0' is omitted. For the second A mark, however, the '= 0' must be present and the coefficients must now be integers, as required in the question. Any integer multiple of the equation shown is acceptable for this mark.

b) The M mark here is for a correct expression in terms of α and β for the sum of the roots of the required equation. The A mark is for $\frac{12}{p}$. The first B mark is for a correct product of the roots and the second for $q = 12$. The equation necessarily includes p so integer coefficients were not asked for, and are actually impossible to obtain.

c) The most efficient way to do this part of the question is to substitute $x = 3$ in the equation obtained in (b), earning the M mark, and then solve the resulting equation in p, earning the A mark if this is correct. However, using any other valid method would gain the M mark and obtaining a correct value for p would gain the A mark.

d) Using the sum (or product) of the roots with the value of p obtained in (c) to obtain the other root earns the M mark. Obtaining a correct value for the second root earns the A mark.

Graphs and normals

Question 4

A curve has the equation $y = \dfrac{2x-1}{4x+2}$, $x \neq -\tfrac{1}{2}$.

(a) Write down an equation for the asymptote which is parallel to

(i) the x-axis

(ii) the y-axis.

(2)

(b) Find the coordinates of the points where the curve crosses the coordinate axes.

(2)

(c) Sketch the curve, showing clearly the asymptotes and the coordinates of the points where the curve crosses the coordinate axes.

(3)

The curve intersects the y-axis at the point P.

(d) Find an equation for the normal to the curve at P.

(5)

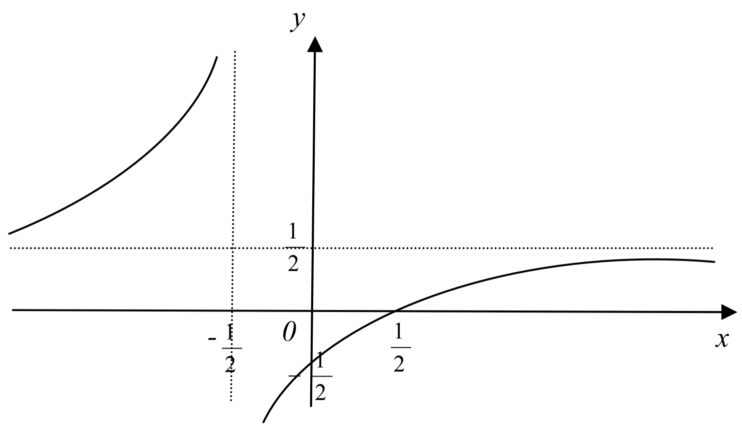
The normal at P meets the curve again at Q.

(e) Find the coordinates of Q.

(5)

(Total 17 marks)

Model answer

(a) (i) $y = \frac{1}{2}$ (ii) $x = -\frac{1}{2}$	B1 B1
(b) $x = 0 \Rightarrow y = -\frac{1}{2}$ or $(0, -\frac{1}{2})$	B1
$y = 0 \Rightarrow x = \frac{1}{2}$ or $(\frac{1}{2}, 0)$	B1
(c) 	B1 (2 branches) B1 (asymptotes) B1 (intersects.)
(d) $\frac{dy}{dx} = \frac{2(4x+2) - 4(2x-1)}{(4x+2)^2}$	M1A1
$x = 0$ $\frac{dy}{dx} = \frac{8}{4} = 2$, grad. normal $= -\frac{1}{2}$	M1,A1
Eqn. normal: $y + \frac{1}{2} = -\frac{1}{2}x$	B1
(e) $-\frac{1}{2}x - \frac{1}{2} = \frac{2x-1}{4x+2}$	M1
$-(4x+2)(x+1) = 4x - 2$	
$-(4x^2 + 6x + 2) = 4x - 2$	
$4x^2 + 10x = 0$	A1
$2x(2x+5) = 0$	M1
$(x = 0)$, $x = -\frac{5}{2}$ $y = \frac{-5-1}{-10+2} = \frac{3}{4}$	
Q is $\left(-\frac{5}{2}, \frac{3}{4}\right)$	A1A1

Examiner comments

Parts (a) and (c) of this question are targeted at C grade students and part (b) is a basic request which all students should complete correctly. Parts (d) and (e) are more challenging and are likely to be completed successfully by A grade students only.

a) 'Write down' implies that no workings need be shown. (i) is found by ignoring the constants – 1 and +2 on the right-hand side and (ii) is obtained by taking the condition $x \neq -\frac{1}{2}$ and replacing the $\neq$ with =. Many students unnecessarily cover a page with workings to obtain these two equations.

b) Many students will write down some workings here, although it is not necessary to do so in order to gain the marks.

c) Students are expected to know the shape of a curve of this type and in particular to know that it has two branches. The lines found in (a) divide the plane into four 'quadrants'. If the two branches, with approximately the correct shape, are shown in the correct 'quadrants' the first B mark is earned. The second B mark is awarded for labelling the asymptotes by means of numbers where they cross the axes or equations on them. However, if the curve is not asymptotic to these lines the mark will not be given. The third B mark is for showing the coordinates of the points where the graph crosses the coordinate axes on the diagram.

d) The first M mark is for attempting to apply the quotient rule to differentiate the curve equation. An A mark is awarded if the differentiation is correct, but it does not need to be simplified. The second M mark is for finding the gradient of the tangent at *P* (which does not need to be numerically correct as this is a method mark). The second A mark is for the gradient of the normal at *P* (which must be numerically correct as this is an accuracy mark). An equation for the normal is needed. The question does not require integer coefficients, or any particular order for the terms, so any correct equation is acceptable for the B mark.

e) Eliminating y between the equation of the curve and the equation of the normal is the essential first step here and gains the first M mark. Simplifying this equation to obtain a correct two-term quadratic earns the first A mark. If the student then attempts to solve their two- or three-term quadratic by factorising, using the formula or completing the square, the second M mark is awarded. The final two A marks are for the x and y-coordinates of Q. The case $x = 0$ has been shown here as a solution of the quadratic, but as this is the x-coordinate of P it does not need to be shown (why it is in brackets).

Section C: Planning and teaching

Course planner

The following outline course planner lists the main teaching objectives which need to be covered in order to meet the specification requirements. It is based on a one-year course with two main teaching terms and one very short term, which is mainly for examination revision. The work for each term allows some time for testing, can be built into a course plan. Most topics are not covered successively, as students benefit from variety, and you will need to briefly revisit work previously learned in order to progress through a topic. The planner suggests an order of work. You will need to determine the finer detail in order to meet the needs of your students.

This course planner should not be seen as an alternative to the specification – the two should be used in conjunction.

It is assumed that all students have knowledge of the Edexcel IGCSE in Mathematics (Specification A) (4MA0) (Higher tier) or the Edexcel IGCSE in Mathematics (Specification B) (4MB0).

Term 1

Content area	Topic	Activities
1	1.3, 1.4 Simple manipulation of surds including rationalising the denominator.	Now included in the specification – previously a little understanding was assumed but none specified.
2	2.1, 2.2 Quadratic expressions and equations.	Factorising, completing the square, solving equations to include the formula. Use of the discriminant to investigate the nature of the roots, real roots only.
3	3.1, 3.2 Simple algebraic division. Factor and remainder theorems.	Division by linear terms only is required. This could include division by a quadratic that factorises into two linear brackets and requires two divisions. To include factorising cubic expressions when one factor has been given and solving a cubic equation with at least one rational real root.

Content area	Topic	Activities
8	All of 8 – rectangular Cartesian coordinates The distance between two points. The point dividing a line in a given ratio. Gradient of a line joining two points. Equation of a straight line. Parallel and perpendicular lines.	All the formulae should be learned (but proofs will not be required). $y = mx + c$ and $y - y_1 = m(x - x_1)$ are needed. Also $ax + by = c$ should be recognised as a straight line.
5	5.1, 5.2 (part) Σ notation and the arithmetic series.	Formulae should be learned (but proofs will not be required).
10	10.1, 10.2 Radian measure, including arc length and area of a sector. The three basic trigonometric ratios of angles of any magnitude (in degrees or radians) and their graphs.	Formulae should be learned (but proofs will not be required). Arc length and area in radian measure are new to the specification. Exact values for the sine, cosine and tangent of 30°, 45° and 60° (and radian equivalents). Use of these to find the trigonometric ratios of related values such as 120°, 300°
10	10.3 Trigonometric problems in two or three dimensions.	Knowledge of 10.2 needed. Include angles between a line and a plane and between two planes.
4	All of 4 – Graphs Graphs of polynomials and rational functions with linear denominators. The solution of equations by graphical methods.	Asymptotes parallel to the coordinate axes only. Non-graphical iterative methods are not required.
9	9.1 (part) Differentiation and integration of sums and multiples of powers of x.	Differentiation by first principles will not be examined. Integration of $\frac{1}{x}$ is excluded.

Content area	Topic	Activities
1	1.1, 1.2 The functions a^x and $\log_b x$ (where b is a natural number greater than 1). Use and properties of indices and logarithms, including change of base.	A knowledge of the graphs of these functions is required but differentiation is not. Formulae will not be given in the examination. See the specification for further detail.
5	5.2 (part) The geometric series.	Formulae should be learned (but proofs will not be required). The sum to infinity of a convergent series with the condition for convergence is included.

Term 2

Content area	Topic	Activities
7	All of 7 – Vectors Addition and subtraction of coplanar vectors and multiplication by a scalar. Components and resolved parts of a vector. Magnitude of a vector and unit vectors. Position vectors. Application to simple properties of geometrical figures.	To include the use of the vectors **i** and **j**. Properties involve collinearity, parallel lines and concurrency. The position vector of a point dividing the line *AB* in the ratio *m:n* is included.
10	10.4, 10.5, 10.6 Sine and cosine rule and area of a triangle in the form $\frac{1}{2}ab\sin C$. The identities $\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$.	Formulae should be learned (but proofs will not be required). $\tan\theta = \frac{\sin\theta}{\cos\theta}$ will be given when needed.

Content area	Topic	Activities
9	9.1 (part), 9.2 Differentiation and integration of $\sin ax, \cos ax, e^{ax}$. Differentiation of a product, quotient and simple cases of a function of a function.	Proofs of the results will not be required.
9	9.3, 9.4 Applications of calculus to simple linear kinematics and to determination of areas and volumes. Stationary points of a curve.	Understanding how displacement, velocity and acceleration are related. Includes the area between a line and a curve. The volumes will be obtained only by revolution about the coordinate axes. Justification of maxima and minima must be known.
2	2.3 Simple examples involving functions of the roots of a quadratic equation $(\alpha \text{ and } \beta)$.	Includes forming an equation with specified roots.
3	3.3, 3.4, 3.5 Simultaneous solution of one linear and one quadratic equation in two variables. Linear and quadratic inequalities including graphical representation of linear inequalities in two variables.	Simple linear programming problems may be set.

Content area	Topic	Activities
6	6 (all) Use of the Binomial Series $(1+x)^n$	*n* may be (i) a positive integer or (ii) rational, with $\lvert x \rvert < 1$ (validity condition required in (ii)). The expansion of $(a+bx)^n$ may be used but questions will not require it. The series for $(1+x)^n$ will not be given in the examination.
9	9.5, 9.6 Practical problems on maxima and minima. The equations of tangents and normals to the curve $y = \mathrm{f}(x)$.	Much of Unit 9 is needed and also the equation of a straight line (*8.4*) and perpendicular lines (*8.5*).
10	10.7, 10.8 Use of the basic addition formulae of trigonometry. The solution of simple trigonometric equations for a given interval.	The formulae for $\sin(A+B)$, $\cos(A+B)$ and $\tan(A+B)$ will be given when needed. Examples of the types of equations are shown in the specification.

Term 3

Content area	Topic	Activities
9	9.7 Applications of calculus to rates of change and connected rates of change.	Includes small changes and the use of $\partial y \approx \frac{\mathrm{d}y}{\mathrm{d}x}\partial x$.
Remaining time	Revision and practice using past examination papers.	With so little change in the specification content, papers set for the legacy Edexcel Alternative Ordinary in Pure Mathematics (7362) qualification can be used as revision practice.

Teaching ideas

This section is designed to help with lesson planning by giving outline teaching plans for two topics in the specification content. The amount of student practice that will need to be incorporated will depend on the amount of time available; with short lessons the work to be covered may span more than one lesson. These topics have been chosen as many students find them difficult in the examinations.

Topic 1: Simple examples involving functions of the roots of a quadratic equation

(Specification reference 2.3)

Aims

1. To evaluate expressions given in terms of α and β where α and β are the roots of the equation

$$ax^2 + bx + c = 0.$$

2. To form a new equation with roots which are expressed in terms of α and β.

Prerequisite

Basic algebra of quadratic functions.

Outline lesson plan

Start by establishing $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$:

When the equation $ax^2 + bx + c = 0$ has roots α and β, we have

$$(x-\alpha)(x-\beta) \equiv \left(ax^2 + bx + c\right) \times \frac{1}{a}$$

$$x^2 - (\alpha+\beta)x + \alpha\beta \equiv x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$\Rightarrow \quad \alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

So for the equation $5x^2 + 3x - 7 = 0$, $\alpha + \beta = -\frac{3}{5}$ and $\alpha\beta = -\frac{7}{5}$.

Functions of the roots are established by algebraic methods. **The roots of the given equation should not be evaluated**.

The required function should be re-arranged so that it is expressed in terms of $\alpha + \beta$ and $\alpha\beta$.

Remember $(\alpha+\beta)^2 \equiv a^2 + 2\alpha\beta + \beta^2$. Use this to evaluate $\alpha^2 + \beta^2$:

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

Also $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

Using the equation $5x^2+3x-7=0$

$$\alpha^2+\beta^2=\left(-\frac{3}{5}\right)^2-2\left(-\frac{7}{5}\right)=\frac{9}{25}+\frac{14}{5}=\frac{79}{25},\ \alpha^2\beta^2=\left(-\frac{7}{5}\right)^2=\frac{49}{25}.$$

Example question:

The roots of the equation $5x^2+3x-7=0$ are α and β. Without solving this equation, find a quadratic equation with roots α^3 and β^3

Students could now practise evaluating some expressions themselves, using different given equations.

Cubic expressions can be evaluated by use of

$$(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)\equiv\alpha^3+\beta^3$$
$$(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)\equiv\alpha^3-\beta^3$$

(These are normally built into examination questions when needed.)

eg: $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$$=(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$$

and for $5x^2+3x-7=0$,

$$\alpha^3+\beta^3=\left(-\frac{3}{5}\right)\left[\left(-\frac{3}{5}\right)^2-3\left(-\frac{7}{5}\right)\right]=\left(-\frac{3}{5}\right)\left[\frac{9}{25}+\frac{21}{5}\right]=-\frac{3}{5}\times\frac{114}{5}=-\frac{342}{25}$$

Returning to the original equation: $x^2-(\alpha+\beta)x+\alpha\beta=0$ can be expressed as

$$x^2-(\text{sum of roots})x+\text{product of roots}=0$$

To obtain a new equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$

1. Evaluate $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ and $\frac{1}{\alpha^2\beta^2}$

   ie the sum and product of the roots of the new equation.

2. Substitute the values obtained in $x^2-(\text{sum of roots})x+\text{product of roots}=0$.

(*NB: Many students forget the* '= 0', *so emphasis the need for this!*)

Topic 2: An application of calculus to area

(Specification reference 9.3)

Aims

1. To be able to find the area between a curve and a straight line efficiently.

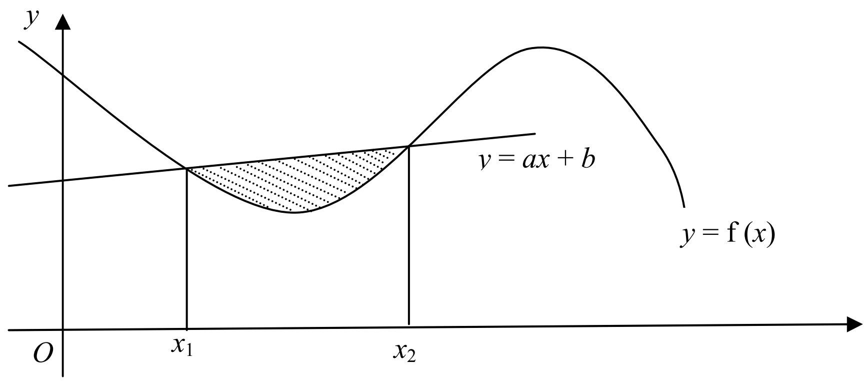
2. Justification of the formula:

area between a line and a curve $= \int_{x_1}^{x_2} \left|\text{line} - \text{curve}\right| \mathrm{d}x$.

Prerequisite

Definite integration including the finding of an area under a curve.

Outline teaching plan



Start with the simplest case, ie where the area is totally above the *x*-axis, as in the above diagram.

Area under the curve $= \int_{x_1}^{x_2} \mathrm{f}\left(x\right) \mathrm{d}x$

Area under the line $= \int_{x_1}^{x_2} \left(ax + b\right) \mathrm{d}x$

Shaded area $= \int_{x_1}^{x_2} \left(ax + b\right) \mathrm{d}x - \int_{x_1}^{x_2} \mathrm{f}\left(x\right) \mathrm{d}x = \int_{x_1}^{x_2} \left(ax + b - \mathrm{f}\left(x\right)\right) \mathrm{d}x$

ie area = $\int_{x_1}^{x_2} \left(\text{line} - \text{curve}\right) \mathrm{d}x$

Now work through a specific example.

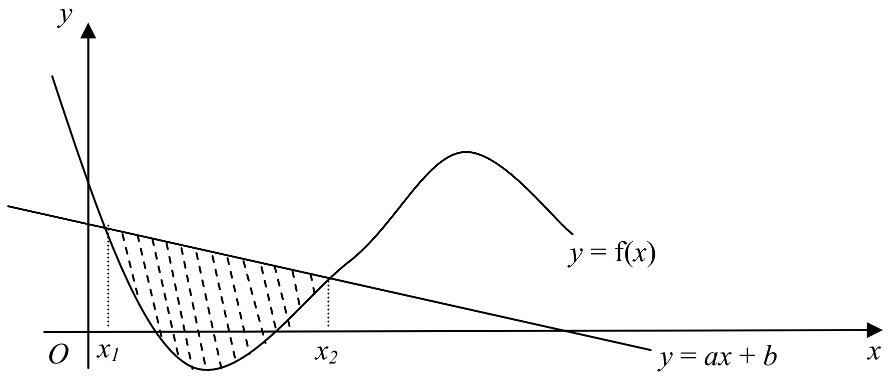
For example

The curve with equation $y = x^2 - 3x + 4$ and the line with equation $y = 2x + 4$ intersect at the points $\left(0, 4\right)$ and $\left(5, 14\right)$. Find the finite area enclosed between this line and curve.

Point out to students that sometimes they will need to find the x-coordinates of the points of intersection of the line and the curve before starting to calculate the area. (The y-coordinates are not normally needed.)

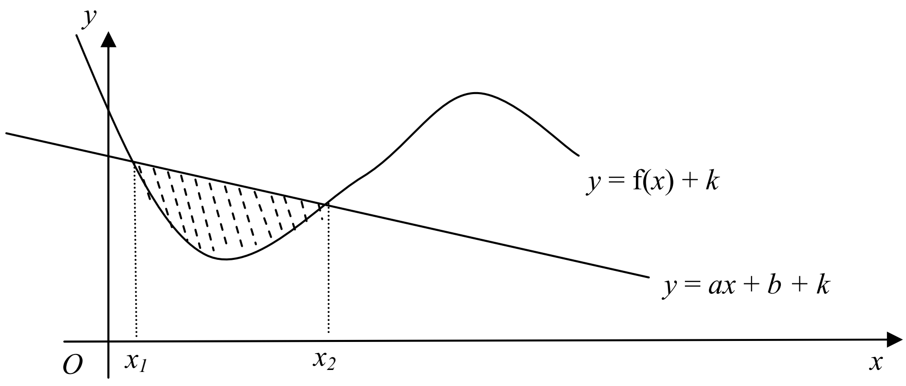
Students could now attempt one or two examples themselves.

Next, consider the case where part of the area is below the x-axis.



Translate both the line and the curve upwards by $+k$, where k is chosen to be large enough to bring the entire shaded area above the x-axis.

This translation simply adds $+k$ to each y-coordinate on the line and the curve. The x-coordinates are unchanged and so the limits for the integrals are unchanged.



The shape of the curve is unchanged so the area between the line and the curve is unchanged.

$$\text{The required area } = \int_{x_1}^{x_2} (ax+b+k)\,dx - \int_{x_1}^{x_2} (f(x)+k)\,dx$$

$$= \int_{x_1}^{x_2} \left[(ax+b+k)-(f(x)+k)\right] dx$$

$$= \int_{x_1}^{x_2} \left[(ax+b)-(f(x))\right] dx$$

$$= \int_{x_1}^{x_2} (\text{line } - \text{ curve})\,dx$$

In cases where the curve is above the line, area $= \int_{x_1}^{x_2} (\text{curve} - \text{line})\,\mathrm{d}x$.

In all cases: Area between a line and a curve $= \int_{x_1}^{x_2} |\text{line} - \text{curve}|\,\mathrm{d}x$.

(*If you do not want to introduce the idea of translating the line and the curve upwards to justify the use of the formula in all cases, you could divide the required area into parts in order to calculate the area and also calculate by the formula, showing the result is the same. Asking students to attempt one for themselves would confirm the accuracy.*)

Appendices

Appendix 1 – Suggested resources

Please note that while resources are correct at the time of publication, they may be updated or withdrawn from circulation. Website addresses may change at any time.

Books

For this Edexcel IGCSE qualification, the following books could be used as teaching aids. Although they are designed for AS/A2 Level in the UK, they are equally useful for IGCSE students at this level.

The list is not exhaustive.

The books listed are not recommended by Edexcel neither are they mandatory for IGCSE qualifications.

The internet is a valuable tool for research and learning.

Backhouse J K, Houldsworth S T P and Horril P J F – *Pure Mathematics: A First Course* (Longman, 1991)

Bostock L and Chandler S – *Mathematics: The Core Course for Advanced Level* (Nelson Thornes, 2000)

Emanuel R and Wood J – *Longman Advanced Maths AS Core for Edexcel and A2 Core for Edexcel* (Longman, 2006)

Pledger K et al – *Edexcel AS and A2 Modular Mathematics Units C1 to C4* (Heinemann, 2008-9)

Sadler A J and Thorning D W S – *Understanding Pure Mathematics* (Oxford University Press, 1987)

Smedley R and Wiseman G – *Introducing Pure Mathematics* (Oxford University Press, 2001)

Websites

www.math.com

www.mathsnet.net

Appendix 2 – Formulae

This appendix gives the formulae that students are expected to remember. The formulae will not be included in the examination papers.

Logarithmic functions and indices

$\log_a xy = \log_a x + \log_a y$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a x^k = k \log_a x$

$\log_a \frac{1}{x} = -\log_a x$

$\log_a a = 1$

$\log_a x = \frac{\log_b x}{\log_b a}$

$\log_a 1 = 0$

$\log_a b = \frac{1}{\log_b a}$

Quadratic equations

$ax^2 + bx + c = 0$ has roots given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

When the roots of $ax^2 + bx + c = 0$ are α and β then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

and the equation can be written $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Series

Arithmetic series: *n*th term $= a + (n-1)d$

Sum to *n* terms $= \frac{n}{2}\{2a + (n-1)d\}$

Geometric series: *n*th term $= ar^{n-1}$

Sum to *n* terms $= \frac{a(r^n - 1)}{r - 1}$

Sum to infinity $= \frac{a}{1-r}$ $\quad |r| < 1$

Binomial series

for $n \in \mathbb{N}$ $|x| < 1$, $n \in \mathbb{Q}$,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots$$

Coordinate geometry

The gradient of the line joining two points (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by

$$d^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2$$

The coordinates of the point dividing the line joining (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ are $\left(\dfrac{nx_1 + mx_2}{m+n}, \dfrac{ny_1 + my_2}{m+n}\right)$

Calculus

Differentiation:

function	derivative
x^n	nx^{n-1}
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$
e^{ax}	ae^{ax}
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
$f(g(x))$	$f'(g(x))g'(x)$

Integration:

function	integral	
x^n	$\frac{1}{n+1}x^{n+1}+c$	$n \neq -1$
$\sin ax$	$-\frac{1}{a}\cos ax + c$	
$\cos ax$	$\frac{1}{a}\sin ax + c$	
e^{ax}	$\frac{1}{a}e^{ax} + c$	

Area and volume

Area between a curve and the x axis = $\int_a^b y \, dx, y \geq 0$

$\left|\int_a^b ydx\right|, y < 0$

Area between a curve and the y-axis = $\int_c^d x.dy \; x \geq 0$

$\left|\int_c^d x.dy\right|, x < 0$

Area between g(x) and f(x) = $\int_a^b |g(x) - f(x)| \, dx$

Volume of revolution $\int_a^b \pi y^2 \, dx$ or $\int_c^d \pi x^2 \, dy$

Trigonometry

Radian measure: length of arc $= r\theta$

area of sector $= \frac{1}{2}r^2\theta$

In triangle ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos^2\theta + \sin^2\theta = 1$

area of a triangle = $\frac{1}{2}ab\sin C$

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