Pearson Edexcel

Examiners' Report

Principal Examiner Feedback

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Paper 02R

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Introduction

There were some very good responses in this paper with some candidates demonstrating a very high level of mathematical ability.

Questions which discriminated particularly well between differing abilities were questions.

- Q5(b) and (c) [geometric series with logarithms]
- Q6 [differentiation and tangents/perpendiculars]
- Q8(b) [connected rates of change]
- Q9 (a) and (d) [3 dimensional trigonometry]
- Q10(b) [areas under curves using calculus]
- Q11 (a) and (b) [vectors].

Question 1

This question proved to be an accessible start to this question with every candidate finding the correct set of values of x in part (a).

(b) Every candidate solved the quadratic equation correctly to find the critical values. Many candidates found the correct range of values of x using a correct inequality, however those few candidates who did not define the correct set of values, generally did not draw a simple sketch or number line to indicate the required region. Candidates should be encouraged to use a sketch to give a visual aid in any question involving inequalities.

(c) Approximately half of this group of candidates could not combine the two regions and here is where a sketch would have made defining this region obvious.

Question 2

(a) Most candidates achieved all three available marks in this part of the question. The most popular method was to find the gradient of $\frac{1}{2}$ and then use the formula $y - y_1 = m\left(x - x_1\right)$ with either set of coordinates to find the equation of a line. Some candidates use $y = mx + c$ which is a perfectly acceptable method, but centres might note that candidates must achieve a value for c and put the equation together before the M mark can be awarded.

The formula $\frac{y-y_1}{y_1-y_2}=\frac{x-x_1}{x_1-x_2}$ was not seen although a correct substitution would have automatically scored M1M1.

(b) We were testing section 8A in the specification here, and quite a few candidates seemed unable to apply a simple Pythagoras theorem to find the lengths of *AB* and *AC* with some candidates not reading the question carefully and finding the length of *BC*.

(c) There were two approaches to this part of the question and both were used by those candidates who attempted this question in roughly equal proportion.

The first method was to find the equation of the perpendicular using a gradient of – 2 and the coordinates of $C\ (-3, 7)$ to give $y=-2x+1$ and by substituting $x = 3$ finding the value of p, or otherwise use the gradient to find the value of p directly. Although there were a few sign errors seen, most candidates who attempted this part of the questions were largely successful.

Question 3

(a) Virtually every student found the correct derivative of the given function in this very routine question. Candidates are clearly well versed in using Product Rule. It is worth adding that unless the question specifically asks for an answer in a given form, it is not necessary to simplify once the derivative has been found. Some candidates made subsequent simplification errors on an initially correct product rule, but we did not penalise them for this and full credit was given for a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ seen with any subsequent errors ignored.

(b) Again, most students were able to achieve at least 2 out of three marks for using Quotient rule, although it is noted that there were more errors made in this part than in part (a). The same comment regarding needless simplification applies here as well.

Question 4

This question on roots of quadratic equations was very approachable and many candidates scored well in this question.

(a) Nearly every candidate achieved all four marks and provided a clear demonstration that $\alpha^2 + \beta^2 = 1$.

(b) This part caused problems for a few because some candidates started again without using either the same algebraic principles as in part (a) or indeed the information that $\alpha^2 + \beta^2 = 1$. Centres should impress on their students, that very frequently, part (b) relies on something found in part (a) especially as in this question, part (a) was a show question.

(c) Although there were many correct equations with integer coefficients, the following common errors were seen:

- not making the equation = 0 (which will always lose the final A mark)
- not reading the demand in the question to give integer coefficients. We saw a few correct equations written as $x^2 + \frac{7}{2}x + \frac{81}{16} = 0$
- and finally not using $-\frac{b}{c}$ for the coefficient of the *x* term.

<u>Question 5</u>

This question on Geometric Series was a good discriminator of ability with parts (b) and (c) causing all sorts of problems.

(a) Almost every candidate managed to find the sum to infinity of *G*.

(b) The demand in the question was to **show** that $U_6 = \frac{3^6}{2^{13}}$.

The most popular response was: $U_6 = ar^5 = 12\left(\frac{3}{8}\right)^5 = \frac{729}{8192} = \frac{3^6}{2^{13}}$ which scored M1 for the correct *n*th term and then M0A0. Candidates should realise at this level of mathematics that we will not award three marks for entering numbers into a calculator. Our expectation (realised by a minority of candidates was that we required 12 and 8 written in the form of prime factors and then to simplify (correctly) to obtain the required answer.

(c) The most common marking pattern (aside from those few candidates who scored full marks) in this part was M1 for the correct use of ar^{n-1} to obtain , and subsequently M0M0M0A0 as candidates had difficulty with the logs, particularly $\log_2 12$.

There is in fact, a very neat alternative method [seen a few times] using prime factors as follows:

$$U_n = 12\left(\frac{3}{8}\right)^{n-1} = 2^2 \times 3 \times \frac{3^{n-1}}{2^{3(n-1)}} = \frac{3^n}{2^{3n-5}}$$

$$\Rightarrow \log_2 U_n = \log_2\left(\frac{3^n}{2^{3n-5}}\right) = n\log_2 3 - \log_2(3n-5)$$

Question 6

Success in Question 6 fell into two distinct groups of candidates; those (the majority) who achieved full marks, and those who scored one or two marks only, seemingly unable to even start the question.

(a) Overall this was a very routine question with the only possible complication being the derivative of curve C $y = 4\sqrt{x}$. Of those candidates unable to differentiate this function, the most popular error was to give: $\frac{dy}{dx} = 2x^{-\frac{3}{2}}$ suggesting that more work on simple indices may be required by some candidates.

Some candidates did not find a value for the gradient but substituted in an algebraic expression which we would not accept unless it was evaluated at some stage.

(b) A lack of success in part (a) meant that success in part (b) was also very unlikely.

(c) The base of triangle ATN was the *x*-axis meaning that finding the area was simplicity in itself with merely find the length of the base $(17 - -9)$ and using the *y* value from the coordinate of *A* which is (9, 12). The area *A* = $\frac{1}{2} \times 12 \times (17 - -9) = 156$ square units, and yet the majority of candidates default as a matter of course to using determinants which they do not always understand. Common errors we see with this method are:

- Not finishing and starting with the same coordinates.
- Not dividing by 2 to obtain the final area
- Incorrect processing.

The absence of a sketch in most scripts [which should be an essential start in any question on coordinate geometry] no doubt added to the lack of clarity on what was a very simple area.

Question 7

Although this paper is not strictly ramped in difficulty as is for example 4MA1, we place the straightforward questions towards the beginning of the paper with the demand broadly increasing as the paper progresses. This question was the first of the discriminators of ability.

(a) In this part, we were expecting candidates to apply the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$ to eliminate $\sin^2\theta$ and obtain an equation just in terms of $\cos\theta$.

There were two ways of obtaining the required value of $\cos\theta$:

1. Multiply across by $\cos\theta - 2$ to form a 3TQ in $\cos\theta$
2. Notice that $4 - \cos^2\theta = (2 - \cos\theta)(2 + \cos\theta)$ and divide through by $\cos\theta - 2$

Cancelling through factors in trigonometry can of course be very risky as there is the possibility that candidates will lose solutions, but in this case the solution $\cos\theta = 2$ is not feasible anyway, so this was generally the more successful strategy to pursue.

Some candidates lost the final A mark by not rejecting the unfeasible solution and gave the final answer as $\cos\theta = -\frac{1}{2},\quad 2$.

(b) These examiner reports frequently remind centres that in most questions part (b) uses the result from part (a) and so it was the case here with a surprising number of candidates starting from scratch only to find that $\cos 3x = -\frac{1}{2}$ which they could have just written down for the first M mark as it follows directly from part (a).

It is crucial to find all the feasible values [and even more than are required] of firstly 3*x* **before** dividing these values by 3. Quite a few candidates who otherwise produced good solutions throughout the question did not find sufficient values of 3*x* only going as far as 240° without realising that 480° was also required.

Question 8

Questions on rates of change, which appear in almost every 4PM1 paper are often not well answered and every series, a large group of candidates do not seem to not where to start. On the other hand, the other approximately half of the entry were very well versed in this type of question and scored all 7 marks using efficient mathematical methods.

This was a generously scaffolded question asking candidates firstly to find a value for r when $t = 4$ seconds, and then to find the rate at which the radius of the circular stain was increasing.
(a) At least half of the entry were unable to start the question at all and could did not interpret the information in the stem of the question correctly.
There is always going to some preparatory work required before Chain Rule can be applied and in this case, candidates were expected to find that the area of the stain after 4 seconds was $4 \times 1.5 = 6$ and then equate this to the formula for the area of a circle to find a value for r.
This part of the question was worth 3 marks.
(b) Most candidates differentiated the formula for the area of a circle to obtain $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ and for some candidates this was the only mark they scored.
We did not award a mark just for writing down an expression for a relevant Chain Rule to find $\frac{\mathrm{d}r}{\mathrm{d}t}$, but required candidates to apply their Chain Rule for the award of the mark.
The final two marks required substitution of the given $\frac{\mathrm{d}A}{\mathrm{d}t}$ and the reciprocal of their $\frac{\mathrm{d}A}{\mathrm{d}r}$ to find the rate at which the radius of the stain was changing.

Question 9

(a) Surprisingly, quite a few candidates could not manage to find an exact value for $\cos\alpha$ by applying simple Pythagoras theorem. The most popular blind alley that candidates followed was to use the identity; $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{2}{3} \Rightarrow \cos\alpha = \frac{3\sin\alpha}{2}$ and thereafter being unable to proceed.

(b) The vast majority of candidates were able to apply three dimensional Pythagoras (or broken down into two parts) and find the correct height of the pyramid

(c) Likewise with this part, many candidates were also able to show that $\tan\theta = \frac{2}{3}$ by using their correct value of h with half of AB.

(d) The final part of this question requiring candidates to find the angle between two planes [specification reference 10C] was very easy indeed when candidates either drew the appropriate triangle on the given sketch, or sketched one of their own, because it was obvious to see that the required triangle was isosceles, and the next calculation was very simple indeed. It is interesting to note, that successful candidates drew themselves thumbnail sketches in nearly every case to aid them in their thought process and could therefore very easily see which angle was required. Whereas candidates who were not successful or did not attempt the part of the question at all, did not either draw a sketch or annotate the given diagram.

The last two questions were accessible to only the most able candidates and proved to be a good discriminator of ability at this level

Question 10

(a) Many candidates found the correct exact x coordinates of A and B , but many went on to complete needless work and also found the exact y coordinates which were not required and for which no credit could be given.

(b) Those candidates who made an attempt at part (b) knew that integration was required and went on to subtract the area under curve C_2 from C_1 between the points A and B. In actual fact this was quite an easy trigonometrical integration because the area is defined by

$A=\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x+1)\,\mathrm{d}x-\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\cos x+1)\,\mathrm{d}x \Rightarrow A=\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)\,\mathrm{d}x$ leading to

$\left[-\cos x-\sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ which was a straightforward evaluation for the award of M1M1M1A1 for the correct area of $2\sqrt{2}$.

Area R_2 was slightly trickier to find with candidates requiring the area under the curve C_2 between $\frac{5\pi}{4}$ and $\frac{3\pi}{2}$ and adding the area under the curve C_1 between π and $\frac{5\pi}{4}$.

The latter area was very seldom found correctly.

The correct ratio of $R_1 : R_2$ in the required form, which required some manipulation with surds was found by only one candidate.

Question 11

In common with other questions where a diagram is required, it is nearly always the case that those candidates who annotate the given sketch or draw small thumbnail sketches of the part of the diagram which is relevant to the part the question they are working on are generally more successful than those candidates who do not.

As the last question in this paper, it is to be expected that able candidates are stretched, although those who are less able can still score some marks in the earlier of the question.

(a) Although to find a vector for $\overrightarrow{OP}$ requires a strategy that may be beyond some candidates, it is fairly obvious that the vectors $\overrightarrow{AC}$, $\overrightarrow{AM}$ and $\overrightarrow{AB}$ are all required and two marks were to be gained for finding all three, using the vector $\overrightarrow{OA}$ with the given ratios for the vector $\overrightarrow{OB}$.

After this, it was necessary to find the vector $\overrightarrow{NB}$ which very few candidates found and therefore could make no further progress in this question.

One of the problems a number of candidates faced was to choose two paths to the vector $\overrightarrow{OP}$. Unfortunately, quite a few candidates who attempted this question followed the following two paths; $\overrightarrow{OP} = \overrightarrow{OA} + \lambda \overrightarrow{AM}$ and then $\overrightarrow{OP} = \overrightarrow{OM} + \mu \overrightarrow{MA}$. This strategy could never be successful because $\lambda = 1 - \mu$ and so candidates were not using two unique parameters. It was necessary to use one of these latter vectors together with $\overrightarrow{OP} = \overrightarrow{OB} + k\,\overrightarrow{NB}$ [for which as has been mentioned before the vector $\overrightarrow{NB}$ was required].

(b) Virtually no candidate even attempted this part for which were required any two of the following vectors; $\overrightarrow{CP} = \frac{1}{3}\mathbf{a} - \frac{1}{9}\mathbf{b}$ or $\overrightarrow{PQ} = \frac{1}{6}\mathbf{a} - \frac{1}{18}\mathbf{b}$ or $\overrightarrow{CQ} = \frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$ and then showing that firstly they are all in the same direction [as they are multiples of each other] and secondly, pass through a common point.

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