Pearson Edexcel

Mark Scheme (Results)

November 2021

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  You must always check the working in the body of the script (and on any diagrams) irrespective of whether the final answer is correct or incorrect and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect: eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = ...$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = ...$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = ...$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = ...$

4. Use of calculators

Unless the question specifically states 'show' or 'prove' accept correct answers from no working. If an incorrect solution is given without any working do not award the Method mark.

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this: eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Multiple attempts at a question.

If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

Question number	Scheme	Marks
1 (a)	$\lfloor \cos 2A = \cos^2 A - \sin^2 A \quad \text{and} \quad \cos^2 A + \sin^2 A = 1 \rfloor$ $\Rightarrow [\cos 2A] = (1 - \sin^2 A) - \sin^2 A$ $\Rightarrow [\cos 2A] = 1 - 2\sin^2 A*$	M1 A1 cso [2]
(b)	$\frac{4-y}{3} = 1 - 2\left(\frac{x+1}{2}\right)^2$ $\Rightarrow \frac{4-y}{3} = 1 - 2\left(\frac{x^2+2x+1}{4}\right) \Rightarrow y = ...$ $\Rightarrow y = \frac{1}{2}(3x^2+6x+5) \ *$	M1 M1 A1* cso [3]
		Total 5 marks

Part	Mark	Notes
(a)	**M1**	Substitutes the identity $\cos^2 A + \sin^2 A = 1$ into the simplified summation formula for cosine to achieve the given identity [for $\cos 2A$]
	A1*	For the correct identity with no errors seen.
(b)	**M1**	For writing $\frac{4-y}{3} = 1 - 2\left(\frac{x+1}{2}\right)^2$ or equivalent at any stage in their working. Allow equivalence to include any attempted expansion of $\left(\frac{x+1}{2}\right)^2$
	M1	For expanding the given expression for sin A to achieve an expression for y in terms of x. Their working must reach y = …. for the award of this mark. Note: The expansion of $\left(\frac{x+1}{2}\right)^2$ must be correct. Allow one error only in re-arrangement.
	A1*	For the correct expression. Note: this is a given answer, there must be no errors in their working for the award of this mark.

Question number	Scheme	Marks
2	$4-x^2=x+2\Rightarrow x^2+x-2=0\Rightarrow (x-1)(x+2)=0\Rightarrow x=1,\ -2$	M1A1
	$V=\pi\int_{-2}^{1}\left(4-x^2\right)^2\ \mathrm{d}x-\pi\int_{-2}^{1}\left(x+2\right)^2\ \mathrm{d}x$	M1
	$V=\pi\int_{-2}^{1}x^4-9x^2-4x+12\ \mathrm{d}x=\pi\left[\frac{x^5}{5}-\frac{9x^3}{3}-2x^2+12x\right]_{-2}^{1}$	M1
	$V=\pi\left\{\left(\frac{1^5}{5}-3\times1^3-2\times1^2+12\times1\right)-\left(\frac{[-2]^5}{5}-3\times[-2]^3-2\times[-2]^2+12\times[-2]\right)\right\}$	M1
	$V=\frac{153\pi}{5}-9\pi$	
	$V=\frac{108\pi}{5}$	A1
	ALT	
	$V=\pi\int_{-2}^{1}\left(4-x^2\right)^2\ \mathrm{d}x-\frac{\pi}{3}\times3^2\times3$	[M1
	$V=\pi\int_{-2}^{1}\left(16-8x^2+x^4\right)\ \mathrm{d}x-[9\pi]=\pi\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_{-2}^{1}-[9\pi]$	M1
	$V=\pi\left\{\left(16\times1-\frac{8(1)^3}{3}+\frac{(1)^5}{5}\right)-\left(16\times(-2)-\frac{8(-2)^3}{3}+\frac{(-2)^5}{5}\right)\right\}-[9\pi]$	M1
	$V=\frac{153\pi}{5}-9\pi$	
	$V=\frac{108\pi}{5}$	A1]
		Total 6 marks

Mark	Notes
M1	For equating the equation of the curve and the straight line, form a 3TQ with an acceptable attempt to solve the equation to find the x coordinates of the **two** points of intersection. [See General Guidance for the definition of an attempt] This mark is awarded for a complete method.
A1	For $x = 1$ and -2
Method 1	
M1	Uses the **correct** form for the volume of rotation. Ft their x coordinates used correctly. Everything must be correct for this mark including π
M1	For an attempt to integrate their expression for the volume of rotation for either the curve or the line (or even a combined expression). Do not accept a mixture of integration and differentiation. [See general guidance for minimum requirements for integration]. Award even if their expression is not squared. Ignore the absence of π or limits for this mark.
M1	For attempting to evaluate their integrated expression (which must be a changed expression) with their limits the correct way round. Substitution must be seen. Ignore the absence of π and ft their values of x for this mark
A1	For the correct exact value of $\frac{108\pi}{5}$ or exact equivalent.
Method 2	
M1	Uses the **correct** form for the volume of rotation of the curve minus the volume of the cone. The volume of the cone is: $\frac{1}{3}\times\pi\times('1'+2)^2\times('1'-'-2')$ Ft their x coordinates for the limits and for the dimensions of the cone. Everything must be correct for this mark including π
M1	For an attempt to integrate their expression for the curve. Do not accept a mixture of integration and differentiation. [See general guidance for minimum requirements for integration]. Award even if their expression is not squared, Ignore the absence of π or limits for this mark.
M1	For attempting to evaluate their integrated expression (which must be a changed expression) with their limits the correct way round. Substitution must be seen. Ignore the absence of π and ft their values of x for this mark
A1	For the correct exact value of $\frac{108\pi}{5}$ or exact equivalent.

Question number	Scheme	Marks
3 (a)	$c=2,\ \ d=3$	B1B1 [2]
(b)	$a-bx=0 \Rightarrow x=\frac{a}{b}=\frac{5}{4} \Rightarrow a=5,\ b=4$	M1A1 [2]
(c)	When the curve crosses the y-axis $x = 0$ $y=\frac{5-4\times 0}{2\times 0-3}=-\frac{5}{3} \Rightarrow p=-\frac{5}{3}$	M1A1 [2]
(d)	$y=-2$	B1ft [1]
		Total 7 marks

Part	Mark	Notes
(a)	**B1**	For either c = 2 **OR** d = 3
	B1	For both c = 2 **AND** d = 3
(b)	**M1**	For setting the numerator = 0, making x the subject and equating the result to $\frac{5}{4}$
	A1	For both $a=5$ and $b=4$
	ALT	
	M1	Substitutes $x=\frac{5}{4}$ into the equation, sets the numerator = 0 and finds the value of $\frac{a}{b}$
	A1	For both $a=5$ and $b=4$
(c)	**M1**	For setting the value of x = 0 and finding a value for y ft their values of a and d
	A1	For the value of $p=-\frac{5}{3}$
(d)	**B1ft**	For $y=-2$ Ft their values of b and c such that $y=\frac{-b}{c}$

Question number	Scheme	Marks
4 (a)	$f(x) = 2(x-3)^2 - 5$ $\Rightarrow f(x) = 2(x^2 - 6x + 9) - 5 = 2x^2 - 12x + 13$ $p = -12,^* \ q = 13$ **ALT** $\frac{dy}{dx} = 4x + p = 0$ at $(3, -5)$ $\Rightarrow 4 \times 3 + p = 0 \Rightarrow p = -12^*$ $-5 = 2 \times 3^2 - 12 \times 3 + q \Rightarrow q = 13$	M1M1 M1A1 cso [4] M1 M1 M1A1 cso [4]
(b)	Minimum since coefficient of x^2 is positive	B1 [1]
(c)	$\frac{dy}{dx} = 4x - 12 \Rightarrow$ when $x = 1$ $\frac{dy}{dx} = -8$ So gradient of normal = $\frac{1}{8}$ When $x = 1$ $y = 3$ $y - '3' = \frac{1}{'8'}(x - 1)$ Eg. $y - 3 = \frac{1}{8}(x - 1)$, $x - 8y + 23 = 0$ oe	B1 B1ft M1 M1 A1, A1 [6]
		Total 11 marks

Part	Mark	Notes
(a)	**M1**	For using the coordinate of the stationary point and forming the expression: $f(x)=2(x-3)^2-5$ This must be correct for this mark
	M1	Expands their $2(x-3)^2-5$ to form a 3TQ
	M1	For equating coefficients with the given equation to find the value of p and q
	A1	For $p=-12,*$ $q=13$ **NB:** The value of p is given in the question.
	ALT – Uses differentiation	
	M1	For a correct differentiation of the expression for y This must be correct for this mark
	M1	For equating their $\frac{dy}{dx}=0,$ substituting in x = 3 and attempting to find a value for p
	M1	For substituting the given value of p into $y=2x^2+px+q$ using the coordinates $(3,-5)$ to find the value of q. Explicit substitution of the value of p into the equation must be seen if p has not been found using differentiation. [M0M0M1A0 can be scored]
	A1	For $p=-12,*$ $q=13$ **NB:** The value of p is given in the question.
(b)	**B1**	For the correct statement: Minimum since coefficient of x^2 is positive
	ALT	
	B1	For $\frac{d^2y}{dx^2}=4\Rightarrow$ positive, hence minimum. A correct conclusion is required.
(c)	**B1**	For the value of the gradient when x = 1 $[m=-8]$
	B1ft	For the gradient of the normal which must have come from differentiation. Ft their -8
	M1	For the value of y when x = 1 using their value of q
	M1	For a complete equation of the line in any form using their normal and their value of y Allow this mark for an equation of the line with x = 1 and y = ‘3’ with their normal, provided there is evidence that the normal comes from the negative reciprocal of their value of the tangent. For this mark the gradient of the tangent need not have come from differentiation.
	A1	For the correct equation in any form.
	A1	For the correct equation in the required form or any multiples of the coefficients.

Question number	Scheme	Marks
5	$\frac{dy}{dx} = e^{-2x} - 2xe^{-2x} = e^{-2x}(1-2x) \Rightarrow \delta y = e^{-2x}(1-2x)\delta x$	M1A1A1
	$\frac{\delta y}{y} \times 100 = \frac{e^{-2x}(1-2x)\delta x}{xe^{-2x}} \times 100 = \frac{e^{-2x}(1-2x) \times 0.03x}{xe^{-2x}} \times 100$	dM1ddM1
	% change in $y = 3(1-2x)[\%]$	A1
		Total 6 marks

Mark	Notes
M1	For an attempt at Product Rule. The definition of an attempt is as follows. • Both terms must be differentiated as follows: $\frac{d(x)}{dx} = 1 \qquad \frac{d(e^{-2x})}{dx} = \pm Be^{-2x}$ • The correct form of product rule must be used. $u\frac{dv}{dx} + v\frac{du}{dx}$ or the other way around.
A1	For a fully correct product rule $\frac{dy}{dx} = e^{-2x} - 2xe^{-2x}$
A1	For $\delta y = e^{-2x}(1-2x)\delta x$ or $\delta y = (e^{-2x} - 2xe^{-2x})\delta x$ or $\delta y = (e^{-2x} - 2xe^{-2x}) \times \frac{3x}{100}$ or $\delta y = (e^{-2x} - 2xe^{-2x}) \times 0.03x$ oe
dM1	For writing $\frac{\delta y}{y} = \frac{e^{-2x}(1-2x)\delta x}{xe^{-2x}}$ or $\frac{\delta y}{y} = \frac{(e^{-2x} - 2xe^{-2x})\delta x}{xe^{-2x}}$ Ft their differentiated expression provided it is of the form: $\delta y = (Ae^{-2x} - Bxe^{-2x})\delta x$ This mark is dependent on the first M mark.
ddM1	For substituting in the value of $\delta x = 0.03x$ and attempting to simplify the **percentage change** to the required form. i.e. $\frac{\delta y}{y} \times 100 = \frac{e^{-2x}(1-2x) \times 0.03x}{xe^{-2x}} \times 100$ or $\frac{\delta y}{y} \times 100 = \frac{(e^{-2x} - 2xe^{-2x}) \times 0.03x}{xe^{-2x}} \times 100$ This mark is dependent on the previous two M marks The minimum required for an attempt is a correct cancellation of xe^{-2x} from the numerator and denominator.
A1	For the correct answer in the required form $y = 3(1-2x)[\%]$

Question number	Scheme	Marks
6 (a)	8 (m) (must be positive)	B1 [1]
(b)	$\frac{\mathrm{d}s}{\mathrm{d}t} = 3t^2 - 8t - 16$ $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ when P is stationary $\Rightarrow 3t^2 - 8t - 16 = 0$ $3t^2 - 8t - 16 = (3t+4)(t-4) = 0 \Rightarrow t = 4$	M1A1 M1A1 [4]
(c)	$\frac{\mathrm{d}^2s}{\mathrm{d}t^2} = 6t - 8$ $6t - 8 = 10 \Rightarrow t = 3$	B1ft M1A1 [3]
		Total 8 marks

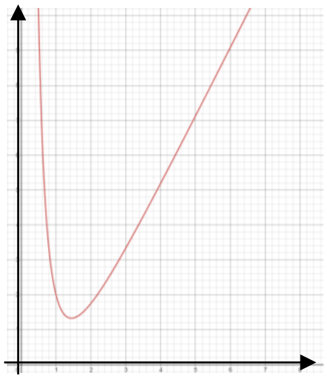
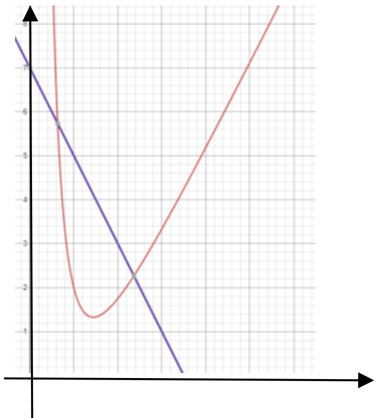
Part	Mark	Notes
(a)	**B1**	For 8 (m) only
(b)	**M1**	For an attempt to differentiate the expression for s $\left[v \text{ or } \frac{\mathrm{d}s}{\mathrm{d}t}\right] = 3t^2 - 8t - 16$ See General guidance for the definition of an attempt with no terms integrated.
	A1	For the correct expression for v or $\frac{\mathrm{d}s}{\mathrm{d}t}$
	M1	For setting v or $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ and solving their 3TQ to find at least one value of t.
	A1	For $t = 4$ If two values of t are given $\left(t = -\frac{4}{3}\right)$ do not award this mark
(c)	**B1ft**	For the correct expression for the acceleration. Ft their $\frac{\mathrm{d}s}{\mathrm{d}t}$ The differentiation must be correct for this mark.
	M1	For setting their differentiated expression = 10 and solving for t
	A1	For $t = 3$

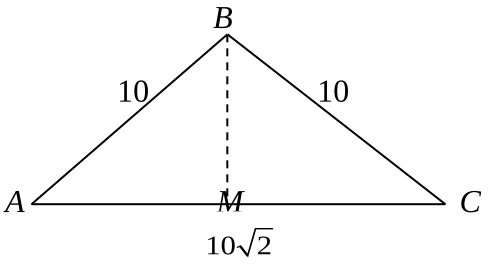
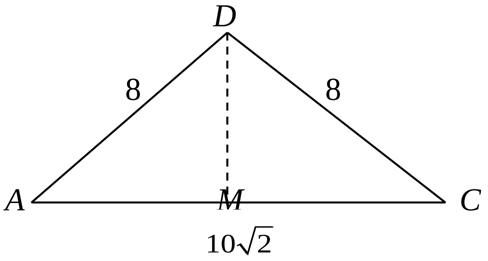
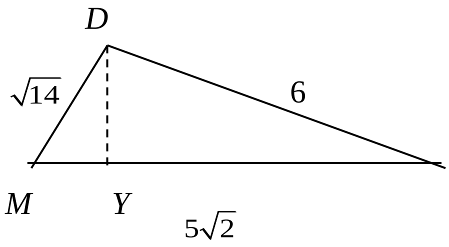
Question number	Scheme	Marks
7 (a)	$\frac{61}{6}=\frac{a\left(1-r^3\right)}{1-r} \quad \rightarrow \mathbf{1}$	M1
	$\frac{125}{6}=\frac{a}{1-r} \quad \rightarrow \mathbf{2}$	M1
(i)	$1\div 2 \Rightarrow \frac{\frac{61}{6}}{\frac{125}{6}}=\frac{\frac{a\left(1-r^3\right)}{1-r}}{\frac{a}{1-r}} \Rightarrow \frac{61}{125}=1-r^3 \Rightarrow r=\frac{4}{5}*$	dM1A1 cso
(ii)	$\frac{125}{6}=\frac{a}{1-\frac{4}{5}} \Rightarrow a=\frac{25}{6}$	M1A1 [6]
	ALT	
	$a+ar+ar^2=a\ 1+r+r^2\ =\frac{61}{6}$ and $\frac{a}{1-r}=\frac{125}{6}$	[M1M1
	$\Rightarrow \frac{125}{6}\ 1-r\ \ 1+r+r^2\ =\frac{61}{6} \Rightarrow 125\ 1-r^3\ =61$	dM1
(i)	$\Rightarrow 1-r^3=\frac{61}{125} \Rightarrow r^3=\frac{64}{125} \Rightarrow r=\frac{4}{5}*$	A1 cso
(ii)	$\frac{125}{6}=\frac{a}{1-\frac{4}{5}} \Rightarrow a=\frac{25}{6}$	M1A1]
(b)	$19.8<\frac{\frac{25}{6}\left(1-0.8^n\right)}{1-0.8} \Rightarrow \frac{19.8\times 6\times 0.2}{25}<1-0.8^n$	M1
	$\frac{594}{625}<1-0.8^n \Rightarrow 0.8^n<\frac{31}{625}$	
	$n\lg(0.8)<\lg\left(\frac{31}{625}\right)*$	A1 cso [2]
(c)	$n>\frac{\lg\left(\frac{31}{625}\right)}{\lg(0.8)} \Rightarrow n>13.461... \Rightarrow n=14$	M1A1 [2]
		Total 10 marks

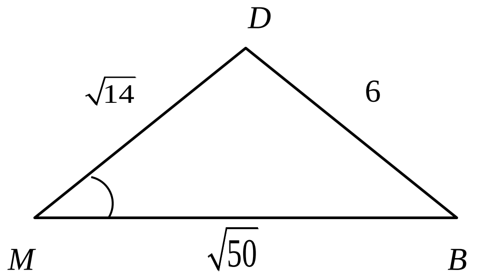
Part	Mark	Notes
(a)(i)	**M1**	Forms a correct equation for the sum of the first 3 terms or the sum to infinity. Either $\frac{61}{6}=\frac{a\left(1-r^3\right)}{1-r}$ or $\frac{125}{6}=\frac{a}{1-r}$ These formulae must be correct as they are given in the Formulae sheet

(i)	**M1**	For both $\frac{61}{6}=\frac{a(1-r^3)}{1-r}$ and $\frac{125}{6}=\frac{a}{1-r}$ correct
	dM1	For a valid method to eliminate *a* from both equations by division or substitution and attempting to re-arrange to find a value for r^3. Eg., $\frac{125}{6}=\frac{a}{1-r}\Rightarrow a=\frac{125(1-r)}{6}\ \Rightarrow \frac{61}{6}=\frac{\left[\frac{125(1-r)}{6}\right](1-r^3)}{1-r}\Rightarrow r^3=1-\frac{61}{125}$ This mark is dependent on **both** previous M marks.
	A1*	For $r=\frac{4}{5}$ This value is given, there must be no errors for the award of this mark
	ALT	
	M1	For either $a+ar+ar^2=\left[a(1+r+r^2)\right]=\frac{61}{6}$ or $\frac{125}{6}=\frac{a}{1-r}$ correct
	M1	For both $a+ar+ar^2=\left[a(1+r+r^2)\right]=\frac{61}{6}$ and $\frac{125}{6}=\frac{a}{1-r}$ correct
	dM1	For a valid method to eliminate *a* from both equations by division or substitution and attempting to re-arrange to find a value for r^3. E.g. $\Rightarrow \frac{125}{6}(1-r)(1+r+r^2)=\frac{61}{6}\Rightarrow 125(1-r^3)=61\Rightarrow r^3=1-\frac{61}{125}$ This mark is dependent on **both** previous M marks.
	A1	For $r=\frac{4}{5}$ This value is given, there must be no errors for the award of this mark
(ii)	**M1**	For substituting $\frac{4}{5}$ into either **their** $\frac{61}{6}=\frac{a(1-r^3)}{1-r}$ or **their** $\frac{125}{6}=\frac{a}{1-r}$ or **their** $a+ar+ar^2=\frac{61}{6}$ (where their expression for a sum has been seen earlier) to find a value of *a*
	A1	For $a=\frac{25}{6}$
(b)	**M1**	For using the **correct** formula for the sum of a geometric series with $r=0.8$ [oe] and their *a*, setting up an inequality (allow < or > for this mark) using the value of 19.8 and attempting to achieve the given result.
	A1*	For the correct inequality as shown with no errors as this is a given result. $n\lg(0.8)<\lg\left(\frac{31}{625}\right)$
(c)	**M1**	For solving the given inequality in *n* using logarithms. They must achieve a value for *n* for this mark. Allow use of <, >, or = for this mark.
	A1	For $n=14$

Question number	Scheme	Marks
8 (a)	<table><tr><td>x</td><td>0.5</td><td>0.75</td><td>1</td><td>1.5</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>10</td><td>**3.83**</td><td>2</td><td>**1.33**</td><td>**1.75**</td><td>**3.33**</td><td>5.19</td><td>7.12</td></tr></table>	B2 [2]
(b)	Graph drawn: All points to within half of a square All points joined together in a smooth curve	B1ft B1ft [2]
(c)	$2x+\frac{3}{x^2}-3=ax+b \Rightarrow 2x^3+3-3x^2=ax^3+bx^2$ $\Rightarrow x^3(2-a)-x^2(3+b)+3 \equiv 4x^3-10x^2+3$ $2-a=4 \Rightarrow a=-2$ $3+b=10 \Rightarrow b=7$ So line is $y=7-2x$ $x=0.6,\ \ 2.4$ [Accept x = 0.7 and x = 2.3] **ALT** $\frac{4x^3-10x^2+3}{x^2}=0 \Rightarrow 4x-10+\frac{3}{x^2}=0$ $2x+\frac{3}{x^2}=10-2x \Rightarrow 2x+\frac{3}{x^2}-3=7-2x$ so line is $y=7-2x$ $x=0.6,\ \ 2.4$ [Accept x = 0.7 and x = 2.3]	M1 M1A1 M1A1 [5] {M1 M1A1 M1A1 [5]}
		Total 9 marks

Part	Mark	Notes
(a)	B1	For two out of 4 values rounded correctly. Penalise rounding only once for awrt the required values.
	B1	For all four values correctly rounded
(b)	B1ft	For **their** points plotted correctly to within half of a square.
	B1ft	For their points joined up in a smooth curve. 
(c)	M1	For setting $2x+\frac{3}{x^2}-3=ax+b$ and attempting to simplify to the form $x^3(2-a)-x^2(3+b)+3$ where the coefficient of each of x^3 and x^2 is in terms of a and/or b
	M1	For equating coefficients and attempting to find the value of a and b
	ALT	
	M1	For dividing through by x^2 to give $4x-10+\frac{3}{x^2}=0$
	M1	For adding 7 and subtracting $2x$ from both sides to give on the LHS $2x+\frac{3}{x^2}-3$
	A1	For the correct straight line $y=7-2x$
	M1	For their line drawn provided it is of the form $y=A-2x$ where $A\neq 0$  The line intersects the coordinates axes at (0, 7) and (3.5, 0)
	A1	For both values of 0.6 and 2.4 Accept 0.7 and 2.3 - without working these roots score no marks. **NB**: Calculator roots are 2.366 and 0.634

Question number	Scheme	Marks
9 (a)	$AC=\sqrt{10^2+10^2}=\sqrt{200}=10\sqrt{2}$	M1A1 [2]
(b)	 $BM=\sqrt{10^2-\left(5\sqrt{2}\right)^2}=5\sqrt{2}$	M1A1 [2]
(c)	 $DM=\sqrt{8^2-\left(5\sqrt{2}\right)^2}=\sqrt{14}$ $\angle DMB=\cos^{-1}\left(\frac{14+50-36}{2\times\sqrt{14}\times5\sqrt{2}}\right)=58.0519...^{\circ}\approx58.1^{\circ}$	M1A1 M1A1 [4]
(d)	Let perpendicular from point D to BM be the point Y  Vertical height of shape *ABCD* In triangle *DMY* (above) Height $= DY=\sqrt{14}\sin58.0519^{\circ}=3.1749...\approx3.17$ (cm)	M1A1 [2]
		Total 10 marks

Part	Mark	Notes
(a)	**M1**	For using Pythagoras theorem correctly on triangle *ABC* to find *AC*
	A1	For $AC = 10\sqrt{2}$
(b)	**M1**	For using Pythagoras theorem or any appropriate trigonometry correctly on triangle *ABC* to find *BM*
	A1	For the correct length $BM = 5\sqrt{2}$ **NB:** Allow for the answer just seen without any working as the triangle *ABC* is isosceles.
(c)	**M1**	For using Pythagoras theorem to find the length *DM*
	A1	For $DM = \sqrt{14}$ or accept awrt 3.74
	M1	For using cosine rule to find the required angle *DMB* 
	A1	For awrt 58.1º
(d)	**M1**	For using any appropriate trigonometry to find the required length. Ft their angle *DMB*
	A1	For awrt 3.17 (cm) Accept 3.18 (cm)

Question number	Scheme	Marks
10 (a)	$\dfrac{3}{\sqrt{9-3x}}=\dfrac{1}{\sqrt{9}}\times\dfrac{3}{\sqrt{1-\dfrac{3x}{9}}}=\dfrac{1}{\sqrt{1-\dfrac{x}{3}}}=\left(1-\dfrac{x}{3}\right)^{-\frac{1}{2}}$ *	B1B1* cso [2]
(b)	$\left(1-\dfrac{x}{3}\right)^{-\frac{1}{2}}=1+\left(-\dfrac{1}{2}\right)\left(-\dfrac{x}{3}\right)+\dfrac{\left(-\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right)\left(-\dfrac{x}{3}\right)^2}{2!}+\dfrac{\left(-\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right)\left(-\dfrac{5}{2}\right)\left(-\dfrac{x}{3}\right)^3}{3!}$	M1
	$\left(1-\dfrac{x}{3}\right)^{-\frac{1}{2}}=1+\dfrac{x}{6}+\dfrac{x^2}{24}+\dfrac{5x^3}{432}$	A1A1 [3]
(c)	$3\text{f}(x)=\dfrac{3(1+2x)}{\sqrt{9-3x}}=\dfrac{3(1+2x)}{3\sqrt{1-\dfrac{x}{3}}}=(1+2x)\left(1-\dfrac{x}{3}\right)^{-\frac{1}{2}}$	B1
	$3\text{f}(x)=(1+2x)\left(1+\dfrac{x}{6}+\dfrac{x^2}{24}+\dfrac{5x^3}{432}\right)$	M1
	$3\text{f}(x)=(1+2x)\left(1+\dfrac{x}{6}+\dfrac{x^2}{24}+\dfrac{5x^3}{432}\right)=1+\dfrac{13}{6}x+\dfrac{3}{8}x^2+\dfrac{41}{432}x^3$	M1A1 [4]
(d)	$\int_{0.1}^{0.2}\dfrac{3+6x}{\sqrt{9-3x}}\,\text{d}x=\int_{0.1}^{0.2}\left(1+\dfrac{13}{6}x+\dfrac{3}{8}x^2+\dfrac{41}{432}x^3\right)\text{d}x$	
	$=\left[x+\dfrac{13}{12}x^2+\dfrac{3}{24}x^3+\dfrac{41}{1728}x^4\right]_{0.1}^{0.2}$	M1A1ft
	$\int_{0.1}^{0.2}\dfrac{1+2x}{\sqrt{9-3x}}\,\text{d}x=\dfrac{1}{3}\left[\begin{array}{l}\left(0.2+\dfrac{13}{12}\times0.2^2+\dfrac{3}{24}\times0.2^3+\dfrac{41}{1728}\times0.2^4\right)\\-\left(0.1+\dfrac{13}{12}\times0.1^2+\dfrac{3}{24}\times0.1^3+\dfrac{41}{1728}\times0.1^4\right)\end{array}\right]=0.044\ 470\ 2$	dM1A1 [4]
	Total 13 marks	

Part	Mark	Notes
(a)	**B1**	For taking out the common factor of 9 from the denominator to achieve $\dfrac{3}{\sqrt{9-3x}}=\dfrac{1}{\sqrt{9}}\times\dfrac{3}{\sqrt{1-\dfrac{3x}{9}}}$ **NB** The common factor of 9 must be seen for this mark.

	B1*	For simplifying to the required form $\frac{3}{\sqrt{9-3x}}=\left(1-\frac{x}{3}\right)^{-\frac{1}{2}}$
(b)	**M1**	For an attempt to find the binomial expansion for the given expression. • The expansion must begin with 1 • The denominators must be correct (ie. 2! And 3!) on the third and fourth terms. • The power of x must be correct (ie. $\left(-\frac{x}{3}\right)$ $\left(-\frac{x}{3}\right)^2$ and $\left(-\frac{x}{3}\right)^3$ with the correct corresponding denominators.
	A1	For one algebraic term correct and simplified
	A1	For the whole expansion correct up to x^3 Ignore extra terms beyond these. $\left(1-\frac{x}{3}\right)^{-\frac{1}{2}}=1+\frac{x}{6}+\frac{x^2}{24}+\frac{5x^3}{432}$
(c)	**B1**	For writing down $3\text{f}(x)=(1+2x)\left(1-\frac{x}{3}\right)^{-\frac{1}{2}}$
	M1	For replacing $\left(1-\frac{x}{3}\right)^{-\frac{1}{2}}$ with their $\left(1+\frac{x}{6}+\frac{x^2}{24}+\frac{5x^3}{432}\right)$ provided their expansion has at least two terms one of which is an algebraic term.
	M1	For an attempt to expand $(1+2x)\left('1+\frac{x}{6}+\frac{x^2}{24}+\frac{5x^3}{432}'\right)$ up to and including the term in x^3. Accept the expansion of $(3+6x)\left('1+\frac{x}{6}+\frac{x^2}{24}+\frac{5x^3}{432}'\right)$
	A1	For the correct expansion $3\text{f}(x)=1+\frac{13}{6}x+\frac{3}{8}x^2+\frac{41}{432}x^3$ ignore extra terms
(d)	**M1**	For an attempt to integrate their expansion of 3f(x) or f(x) which must have as a minimum two terms, one of which is a term in x, x^2 or x^3 Ignore limits for this mark. They may multiply by $\frac{1}{3}$ at this stage so accept e.g $\left[\frac{x}{3}+\frac{13}{36}x^2+\frac{3}{72}x^3+\frac{41}{5184}x^4\right]$
	A1ft	For a correct integrated expression ft their expression.
	dM1	For substituting both values of 0.2 and 0.1 the correct way around seen. This mark can be implied by the value of 0.044 470 2 seen. If their expansion from (c) has not been divided by 3 for the integration, it must be divided by 3 here for the award of this mark.
	A1	For the correct value of 0.044 470 2 **NB**: The calculator value is 0.044 470 77

Question number	Scheme	Marks
11 (a)	$\frac{y-5}{5--5}=\frac{x-7}{7--3}\quad [\Rightarrow y=x-2]$ oe	M1A1 [2]
(b)	$m_{BC}=\frac{5-1}{7-p}=\frac{4}{7-p},\ m_{AC}=\frac{1-(-5)}{p-(-3)}=\frac{6}{p+3}$ $m_{BC}\times m_{AC}=-1\Rightarrow\frac{4}{7-p}\times\frac{6}{p+3}=-1\Rightarrow\frac{4}{7-p}=-\left(\frac{p+3}{6}\right)$ $\Rightarrow -24=-p^2+4p+21\Rightarrow p^2-4p-45=0$ $\Rightarrow (p-9)(p+5)=0\Rightarrow p=-5,\ (9)$	M1A1A1 M1 dM1 M1A1* [7]
(c)	Using formula; $\left(\frac{(7\times-5)-(3\times7)}{7-3},\frac{(7\times1)-(3\times5)}{7-3}\right)=(-14,-2)$	B1B1 [2]
(d) (i) (ii)	$AC=\sqrt{(1-[-5])^2+(-5-[-3])^2}=\sqrt{40}=2\sqrt{10}$ $BD=\sqrt{(7-[-14])^2+(5-[-2])^2}=\sqrt{490}=7\sqrt{10}$ Area = $\frac{1}{2}\times2\sqrt{10}\times7\sqrt{10}=70\ (\text{units}^2)$	B1 M1 M1A1 [4]
		Total 15 marks

Part	Mark	Notes
(a)	**M1**	For forming an equation of a straight line
	A1	Accept a correct line in any form following simplification of denominators; for example, accept $\frac{y-5}{10}=\frac{x-7}{10}$ or $y-5=x-7$ or $y+5=x+3$
(b)	**M1**	For attempting to find an expression for the gradient of *BC* **or** the gradient of *AC* $m_{BC}=\frac{5-1}{7-p}$ or $m_{AC}=\frac{1-(-5)}{p-(-3)}$ Accept maximum one slip in either expression for this mark.
	A1	For an expression for the gradient of *BC* **or** *AC* fully correct
	A1	For an expression for the gradient of *BC* **and** *AC* fully correct Need not be simplified in either case
	M1	For setting their gradient of *BC* = negative reciprocal of their gradient of *AC*
	dM1	For attempting to form a 3TQ in *p*: $\left[p^2-4p-45=0\right]$ Dependent on the previous M mark.
	M1	For solving their 3TQ by any method. See General Guidance.
	A1*	For $p=-5$
(c)	**B1**	For either $x=-14$ or $y=-2$
	B1	Both coordinates correct $(-14,-2)$ given as coordinates only.
(d)(i)	**B1**	For the correct length of *AC* $\left[=2\sqrt{10}\right]$
(ii)	**M1**	For the correct method to find length *BD* $\left[=7\sqrt{10}\right]$ Ft their $x=-14$ or $y=-2$
	M1	For a correct method using $A=\frac{1}{2}\times\text{base}\times\text{height}$ using their *AC* and *BD*
	A1	For the correct area of $70\ (\text{units}^2)$
	ALT – Uses determinants	
	M1	For stating $A=\frac{1}{2}\begin{pmatrix}-3 & 7 & -14 & -3\\ -5 & 5 & -2 & -5\end{pmatrix}$ oe
	M1	For evaluating their determinant $A=\frac{1}{2}\left(\left[-3\times5+7\times-2+-14\times-5\right]-\left[7\times-5+-14\times5+-3\times-2\right]\right)$ oe
	A1	Area = $70\ (\text{units}^2)$ Which must be a positive value

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