Pearson Edexcel

Examiners' Report

Principal Examiner Feedback

November 2021

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In Further Pure Mathematics (4PM1)

Paper 01

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Introduction

This was an additional sitting of this specification in extraordinary circumstances, however it appeared that the majority of students were familiar with the content of the specification as generally questions were attempted.

Students should be encouraged to read questions carefully to ensure that they follow instructions such as ‘show’ or guidance on the type of method required ‘use a vector method’, ‘show clear algebraic working’ or ‘use algebraic integration’. They should also pay attention to any requirements relating to the form for the solution.

Question 1

This was generally a well answered question. Most knew how to calculate the sum and product of the roots of the original quadratic and wrote them down clearly. A minority of students attempted to find the roots of the original quadratic equation which the question indicated they should not do.

Where students identified the sum and product of the roots a reasonably common error was the sign of $\alpha + \beta$. Only a few forgot to divide by the coefficient of the quadratic term while doing this. Most were able to find an algebraic expression for the sum and product for the new roots and calculate their value, although errors were seen in manipulating $\frac{1}{\alpha} + \frac{1}{\beta}$.

There were a surprising number of students who did not negate the sign of the sum of the new roots when putting together the new quadratic, having done this correctly for the original equation. A significant number of students failed to gain the last mark, often failing to give integer coefficients or omitting the = 0.

Question 2

The vast majority of students attempted this question. Most were able to identify that they needed to consider $b^2 - 4ac$ for the given quadratic equation. However, many students substituted incorrectly (commonly by identifying c as -2 instead of $-2p$) or made errors when simplifying to a three term quadratic in p often obtaining $p^2 - 18p - 1$ (from using the negative values incorrectly). Where students substituted correctly it was common to see $p^2 + 14p + 1$ with an incorrect inequality sign or an equals sign.

Many students used the quadratic formula to find both solutions to their equation, most showing a clear substitution. The 'completing the square' method was not commonly used. Those students who relied on their calculator for a solution did not always work sufficiently accurately, some even rounding to -14 and 0 instead of the required minimum accuracy of -13.9 and -0.072. Often students with an incorrect quadratic in p were able to show correct working to find the critical values for *their* quadratic.

A very large number of students could not form correct inequalities with their critical values, either by not giving inequalities at all or by giving $-7 - 4\sqrt{3} < p < -7 + 4\sqrt{3}$. Those who gained full marks often successfully used a sketch of their quadratic to help them.

Question 3

Most students attempted to write equations based on the area of the sector and the perimeter of the sector. A significant minority of students were not confident working in radians and therefore attempted to work in degrees but did so incorrectly or inconsistently. Where students did work in radians most remembered to include $2r$ in the perimeter formula, in the instances where students did omit the $2r$ they were generally able to rearrange and substitute but this did not lead to a three-term quadratic as was required.

Many different substitution strategies we used, but the ones to eliminate θ were the most common. In addition to the ones shown on the mark scheme, a common substitution was to substitute $r\theta = 16.4 - 2r$ from the perimeter equation into the area equation, written as $\frac{1}{2}r\left(r\theta\right) = 16.8$ If a correct substitution strategy was identified, and the original equations were correct, then most students were confident in arriving at the correct 3TQ and solving correctly to give 4 and 4.2. A significant number of candidates did not actively reject $r = 4.2$ which was required as the question indicated r was an integer. If a value for r or θ was achieved, most students were then able to make an appropriate substitution and rearrange to find the other unknown.

Question 4

Most students were able to identify that they should use either the product rule or the quotient rule for this question and were able to identify the terms that needed to be differentiated. It was more common to see students using the quotient rule than the product rule. A majority of candidates were able to apply the chain rule to differentiate $\sin 2x$ correctly, but when differentiating $\left(x^2-9\right)^{\frac{1}{2}}$ as part of an attempt at quotient rule it was common to omit the x. In the product rule the most common error was to not take into account the negative index, or to change the index incorrectly.

Where students were able to correctly differentiate many did not correctly show that this could be written as given in the question. Students struggled to deal with the fractional and negative powers correctly.

When using the quotient rule, those with the most success wrote the numerator as a single fraction and then correctly divided to get the final correct denominator. When using the product rule, those who had the most success wrote each differentiated term as their own fraction first and then simplified into a single fraction: dealing with it with just indices didn't work out too well.

Lots of incorrect answers for the final two steps showed the incorrect manipulation of the numerator where one of the terms was a fraction, they would think that $\dfrac{a+\frac{b}{c}}{d}=\dfrac{a+b}{cd}$

resulting in $$\frac{2\cos(2x)\left(x^2-9\right)^{\frac{1}{2}}-\dfrac{x\sin(2x)}{\left(x^2-9\right)^{\frac{1}{2}}}}{\left(x^2-9\right)}=\frac{2\cos(2x)\left(x^2-9\right)^{\frac{1}{2}}-x\sin(2x)}{\left(x^2-9\right)\left(x^2-9\right)^{\frac{1}{2}}}$$

Question 5

This question proved challenging for many students with a significant number of responses showing only minimal understanding of logarithms. It was common for students to be able to deal with the root or the change of base, but to make no further progress with the question. A common method was to change log 9 to log 3 squared. Although from here students were unable to complete this log change and a common wrong answer was $2\log_3(x+3)$.

Students who successfully dealt with the root and change of base (either working in base 3 or in base 9) where not always successful in combining the logs and incorrectly added instead of multiplied. Others incorrectly split the terms, for example incorrectly writing $\log_9(x+3)$ as $\log_9 x + \log_9 3.$ Some students merely rewrote the equation and omitted logs before they had combined terms appropriately or treated $\log_3$ as a multiplier with statements such as let $\log_3 = a$ observed.

A common successful strategy avoided taking the power of $\frac{1}{2}$ out of the first log and candidates successfully combined to reach $\log\left(\sqrt{x-5}\right)\left(\sqrt{x+3}\right) = ...$ others chose to work in base 9 and quickly reached $\log_9(x-5)+\log_9(x+3)-1=0.$

Where students had followed a correct method to reach a single logarithm on the left hand side of the equation they were often successful at removing logarithms to achieve a three term quadratic and then go on to solve this by a correct method. A small number of students knew to remove logarithms at this point but were unable to change the constant to 3 squared or 9. Where students successfully solved a correct three term quadratic there were a significant proportion that did not recognise the need to reject $x = -4$.

Question 6

A majority of candidates recognised the need to use the chain rule for this question and were often successful in completing the required calculation. Occasionally correct working was combined with a minor error in calculation leading to an incorrect final value. Some students recognised the need to use the chain rule, but incorrectly used $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ with $\frac{dr}{dt} = 3$ rather than $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$.

Where students did not recognise the need to use the chain rule, they were often able to identify that they needed to differentiate and would differentiate one or both of the volume and surface area correctly. Some students believed they were differentiating with respect to t, wrote $\frac{dV}{dt}$ and $\frac{dA}{dt}$ instead of $\frac{dV}{dr}$ and $\frac{dA}{dr}$ and then struggled to make further progress.

There were some instances where candidates would just substitute 10 into the formulae for the surface area and volume, and not know what to do with it.

Question 7

Part (a) The majority of students were able to show the required result. In some cases, students made errors in writing the cosine rule and assumed that rearranging would give the given result.

Part (b) There were a number of different approaches to this question in the responses seen. The most common approach was to use the formula $\frac{1}{2}ab\sin C$ for area of a triangle to find a value for $\sin\theta$, , use this to obtain θ and then substitute into the result from part (a) in order to find a value of *k*. Many candidates using this approach did not recognise that there were two possible values for θ to consider which would give the two possible values of *k* and instead gave the answer ± 7.48 failing to recognise that the length could not be negative.

Some students wrote θ as $\cos^{-1}\left(\frac{6^2+8^2-k^2}{2\times6\times8}\right)$ and substituted this into $\frac{1}{2}ab\sin C$ as θ.

Students who took this approach were generally reasonably successful, although only finding one value of θ and hence of *k* was still an issue. Some students who appeared to be attempting this route incorrectly omitted the inverse cosine and merely substituted $\frac{6^2+8^2-k^2}{2\times6\times8}$ for θ.

Other students worked with the formula $\frac{1}{2}ab\sin C$ for area of a triangle, obtained a value for $\sin\theta$ and used the identity $\sin^2\theta + \cos^2\theta = 1$ to obtain $\cos\theta$ before solving for *k*. Students using this approach were more successful at finding both values of *k*.

Other students attempted to use Hero's formula or to work with the cosine ratio with Pythagoras' theorem to find an expression for the relative height of the right-angled triangle and then use this to give an expression for $\sin\theta$ in terms of *k* which was then used with the formula $\frac{1}{2}ab\sin C$ for area of a triangle. Both of these methods were used successfully by a small number of students, but in other cases students struggled with the algebraic manipulation required in their chosen route.

A small minority of students incorrectly used $\cos\theta$ rather than $\sin\theta$ in their attempt to use the formula for area of a triangle.

Question 8

The vast majority of students attempted this question, although some did not progress beyond attempting parts (a) and (b).

Part (a) The majority of students were able to correctly answer part (a) by giving $\overrightarrow{OB}$ in terms of **a** and **b**. Where students were not able to answer this part of the question correctly they generally struggled with the remainder of the parts often omitting the later parts completely.

Part (b) This part of the question was answered correctly by a majority of students. Many of the students were able to write down a correct answer by reading straight from the diagram without describing a path for it. Some students struggled with this part of the question either giving an incorrect route and answer or managing a correct route but failing to simplify correctly or using an incorrect $\overrightarrow{OC}$.

Part (c) The number of responses for this part of the question forward was lower than for parts (a) and (b), however there were still a significant proportion of students who were able to use $\overrightarrow{OA} + \overrightarrow{AB} + \frac{2}{3}\overrightarrow{BC}$ to achieve the correct answer for their previous vectors. A small number of solutions used the alternative $\overrightarrow{OC} - \frac{1}{3}\overrightarrow{BC}$. Failure to interpret the ratio correctly was rare, but a minority did use $\overrightarrow{OA} + \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$.

Part (d) This part of the question proved most challenging for students and many did not make an attempt or made attempts that did not make progress towards a solution.

Where students made a meaningful attempt at this part of the question they were often able to form the two simultaneous equations. Some students did not know how to solve for their unknowns and so did not reach the required ratio. Where students were aware of the method to use they were often able to correctly solve for their unknowns and arrive at the ratio of 1:4.

A small minority of students did not read the question and so did not attempt to use vectors in their approach.

(e) A minority of students attempted this part of the question. Where successful attempts were seen students used a wide variety of different approaches including some very succinct geometrical reasoning.

Question 9

Part (a) Students were generally able to demonstrate the required result by using one of the two formulae for the sum of n terms of an arithmetic series. It was more common to see the use of the first term and common difference approach with the values of a and d clearly stated and then correctly substituted into a valid formula and the given result shown.

A minority of students used the standard summation results to answer this part of the question. Where this was seen it was generally correct.

Part (b) This part of the question was not answered well. It was rare to see the result from part (a) used correctly to provide the difference of two summations. Often when the result from part (a) was used, candidates simply substituted $n = 20$ or $n = 11$ once. The most common approach was to recalculate the first term and then use the standard formulas to calculate the answer. Unfortunately, where this approach was used, many students still assumed there were 20 terms.

Part (c) A majority of students were able to make an attempt at this part of the question. If the correct expression for u_{n+1} was found, then this generally led to a fully correct solution. Some students used $5r-1$ to obtain their expression for u_{n+1}, however others used the first term and common difference. Some students incorrectly used S_{n+1} instead of u_{n+1}. Taking a correctly substituted equation, accurately converting it to a three term quadratic equation, and solving it was handled well by most. Where correct working was seen a number of candidates left both values of *n*, not recognising that one was negative and non-integer and therefore not suitable given the question posed. A minority of students tried to use the result found in part (b) and gained no credit.

Question 10

A small number of students did not attempt this question.

Part (a) This part of the question was attempted by most students. The majority of attempts were based on solving the trigonometric equation which was generally correct and students were generally then able to correctly show the coordinate *M*. Some worked in degrees and then changed their answers to radians which was acceptable in this case. There were a number of students who did not give the coordinates for *N*, or who attempted to find the coordinates for *N* but did so incorrectly.

A minority of students attempted this question by substituting $\frac{7\pi}{18}$ into the given equation and demonstrating that the coordinates of *M* were correct. Students taking this approach generally did not give the coordinates for point *N*.

Part (b) This part of the question was answered well by the majority of students. Some were able to write down the coordinates without working or with only minimal working.

Where students included working, they were generally able to differentiate correctly and obtain the correct maximum point with some going further and differentiating again to demonstrate that this was, in fact, a maximum. Those that correctly obtained $\frac{\pi}{6}$ were generally successful in substituting this into the original equation to obtain 1.5.

A minority of students made errors in their attempt to differentiate. Others decided to work in degrees, which was acceptable for the method, sometimes they converted to radians appropriately for their answer and sometimes they did not.

Part (c) This part of the question was attempted by the majority of students. Where students had a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ they were generally able to obtain the correct gradient and use this correctly to obtain an equation for the tangent. There were some students who had an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ and could still gain credit for their working if substitution of the appropriate values was shown. A small minority found a correct gradient, but then attempted to find the equation of the normal rather than the tangent. Some students obtained a correct equation of the tangent but did not give it in the required form. Other errors included omitting the x or π from their final equation or failing to include = 0.

Part (d) This part of the question was not well answered. Many students attempted the integration, but only a small number achieved the correct answer of 1.46 and included the appropriate supporting working.

In attempting this part students often struggled to give a correct strategy to deal with the negative area. The integration itself was generally correct, but then many did not show the substitution of their limits into the changed expression with substitution of 0 being missed as a common error. It is worth noting that this part of the question asked students to use algebraic integration, so they needed to show the substitution of limits into their integrated expression rather than just giving a final answer.

Question 11

This question was attempted by almost all students with the majority realising that they needed to integrate $\mathrm{f}'(x)$ as given in the question.

Part (a) The majority of students realised the need to integrate $\mathrm{f}'(x)$ and integrated each of the terms correctly, although a common error was to omit the '+ c' when doing this. It was common to see students finding f (4) and equating this to 0 and then $\mathrm{f}(-1)$ and equating this to 25. It was rare to see students using -4 or 1 in their substitutions. Students who had included '+ c' were then generally aware of the need to solve the equations they had obtained simultaneously in order to find $a = 6$. In some cases, students did not show sufficient working for the solution of the simultaneous equations for the 'show that' requirement of the question. Some students had minor errors in either the formation of, or solution of, the simultaneous equations and so their calculations did not lead to $a = 6$ although many assumed this result and stated it even where their working did not lead to it.

Where students had omitted the '+ c' they generally were unable to make further progress with the question as they did not obtain the required simultaneous equations. In a small number of cases the student appeared to have identified that there was an error in their previous working and so put a + 24 on their working in order to make $\text{f}(-1) = 25$ having assumed that $a = 6$ which did not give rise to the simultaneous equations required. Students should be aware that in 'show that' questions they should not assume the result and work forwards from that point.

Where students did not recognise the need to integrate there were often attempts at f (4) and equating this to 0 and $\text{f}(-1)$ and equating to 25, but these incorrectly used $\text{f}'(x)$ which did not allow for progress towards the required solution.

Part (b) Those students who had recognised the need to integrate in part (a) were often able to make progress with part (b). If students had omitted the '+ c' in part (a) then they sometimes identified that there should be a constant term, but more often worked without a constant term. The majority of students with an integrated $\text{f}(x)$ attempted to identify the quadratic that multiplies $x = -4$ to give their cubic with both polynomial division and comparison of coefficients between f (x) and $(x-4)(ax^2+bx+c)$ being seen. Some students were able to write down the quadratic with minimal working.

Once a correct quadratic was obtained most students were able to factorise this correctly and give the roots of f (x). Some students did not show working for this and in this case, this was required because of the instruction in the question to 'use algebra' which meant that merely stating roots from a calculator was not sufficient.

When students had omitted the '+ c' then they were often able to attempt to find the quadratic and attempt to factorise the quadratic they obtained.

Students who did not integrate sometimes attempted to divide by $x - 4$, but more commonly attempted to solve the quadratic $f'(x)$ but as this had not been obtained from a cubic this did not gain credit.

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