Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 15 June 2020

Afternoon (Time: 2 hours) | Paper Reference **4PM1/01R**

Further Pure Mathematics

Paper 1R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all TWELVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here is a formula

$$P = 3 + 2\sin\left(\frac{3\pi t}{8}\right) \qquad 0 \leqslant t \leqslant 12$$

(a) Find the exact value of P when $t = \dfrac{10}{3}$

(2)

(b) Find

(i) the largest value of P

(ii) the smallest value of P

(2)

(c) Find the least value of t for which $P = 4$

(3)

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2 (a) Express $x^2 + 4x - 8$ in the form $(x + a)^2 + b$ where a and b are constants whose values are to be found.

(2)

(b) Use algebra to solve the simultaneous equations

$$y = x^2 + 4x - 8$$
$$y = 2x + 7$$

(5)

The curve C has equation $y = x^2 + 4x - 8$
The straight line L has equation $y = 2x + 7$

Using the same axes and the results of part (a) and part (b),

(c) sketch C and L, showing clearly the coordinates of the turning point of C and the coordinates of the points of intersection of C and L.

(4)

Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 11 marks)

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3 The *n*th term of an arithmetic series is u_n such that

$$u_n = \ln a + (n - 1)\ln b$$

where a and b are positive integers.

Given that $u_2 = \ln 12$ and that $u_5 = \ln 768$

find the value of a and the value of b.

(7)

Question 3 continued

(Total for Question 3 is 7 marks)

4 The curve C has equation

$$y = x^3 - 3x^2 - 24x + 6$$

(a) Use calculus to find the coordinates of each of the stationary points on C.

(4)

(b) Determine the nature of each of these stationary points.
Justify your answers.

(2)

P 6 2 2 8 3 A 0 1 2 4 0

Question 4 continued

(Total for Question 4 is 6 marks)

5 (a) Expand $\sqrt{1-x}$ in ascending powers of x up to and including the term in x^3

Give each coefficient as an exact fraction in its lowest terms.

(3)

(b) Using your expansion with a suitable value of x, obtain an approximation, to 6 decimal places, of $\sqrt{0.92}$

(3)

(c) Hence find an approximation, to 5 decimal places, of $\sqrt{23}$

(2)

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Question 5 continued

(Total for Question 5 is 8 marks)

6 (a) Show that

$$\sin(A + B) + \sin(A - B) = 2\sin A \cos B$$

(2)

(b) Hence express $2\sin 7x \cos x$ in the form $\sin mx + \sin nx$ where m and n are integers, giving the value of m and the value of n.

(1)

(c) Use calculus to evaluate

$$\int_0^{\frac{\pi}{4}} (6\sin 7x \cos x)\,dx$$

(4)

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7 The length of each side of a cube S_1 is increasing at a constant rate of 0.1 m/s.

(a) Find, in m^3/s, the rate of increase of the volume of the cube S_1 when the length of each side of the cube is 2 m.

(4)

The total surface area of a different cube S_2 is increasing at a constant rate of 0.05 m^2/s.

(b) Find in m^3/s, the rate of increase of the volume of the cube S_2 when the length of each side of the cube is 6 m.

(5)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8

$$f(x) = 3x^2 - x + 4$$

$$g(x) = x^2 - px + q$$

The roots of the quadratic equation $f(x) = 0$ are α and β

The roots of the quadratic equation $g(x) = 0$ are $\left(\alpha + \frac{1}{\alpha}\right)$ and $\left(\beta + \frac{1}{\beta}\right)$

Without solving the equation $f(x) = 0$

(a) show that $p = \frac{7}{12}$

(3)

(b) Find the value of q

(4)

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Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9 Showing your working clearly, use algebra to solve the equations

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$

(7)

Question 9 continued

(Total for Question 9 is 7 marks)

10 (a) Solve the equation

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \text{for } 0 \leqslant x \leqslant 2\pi$$

Give your solutions in terms of π, where appropriate.

(3)

(b) Solve the equation

$$3\sin\theta + 5\cos\theta = 0 \qquad \text{for } -360° \leqslant \theta \leqslant 360°$$

Give your solutions to the nearest degree.

(3)

(c) Solve the equation

$$1 + \sin 2y = 2\cos^2 2y \qquad \text{for } -180° \leqslant y \leqslant 0°$$

(5)

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Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 11 marks)

11

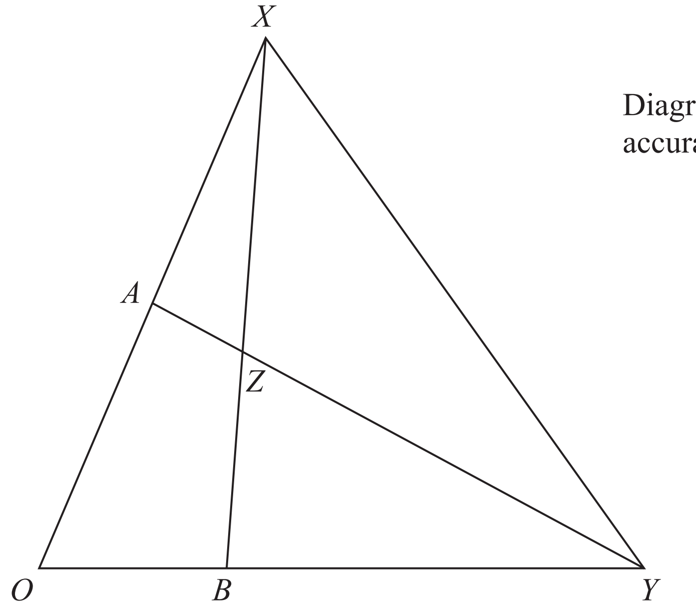

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a triangle *OXY*

$$\overrightarrow{OX} = 2\mathbf{a} \text{ and } \overrightarrow{OY} = 3\mathbf{b}$$

A is the midpoint of *OX* and *B* is the point on *OY* such that $OB : BY = 1 : 2$
The lines *XB* and *AY* intersect at *Z*.

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of **a** and **b**

(1)

(b) Using a vector method, find $\overrightarrow{OZ}$ as a simplified expression in terms of **a** and **b**

(9)

The point *M* on *XY* is such that *O*, *Z* and *M* are collinear.

(c) Find $\overrightarrow{OM}$ as a simplified expression in terms of **a** and **b**

(3)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 13 marks)

12

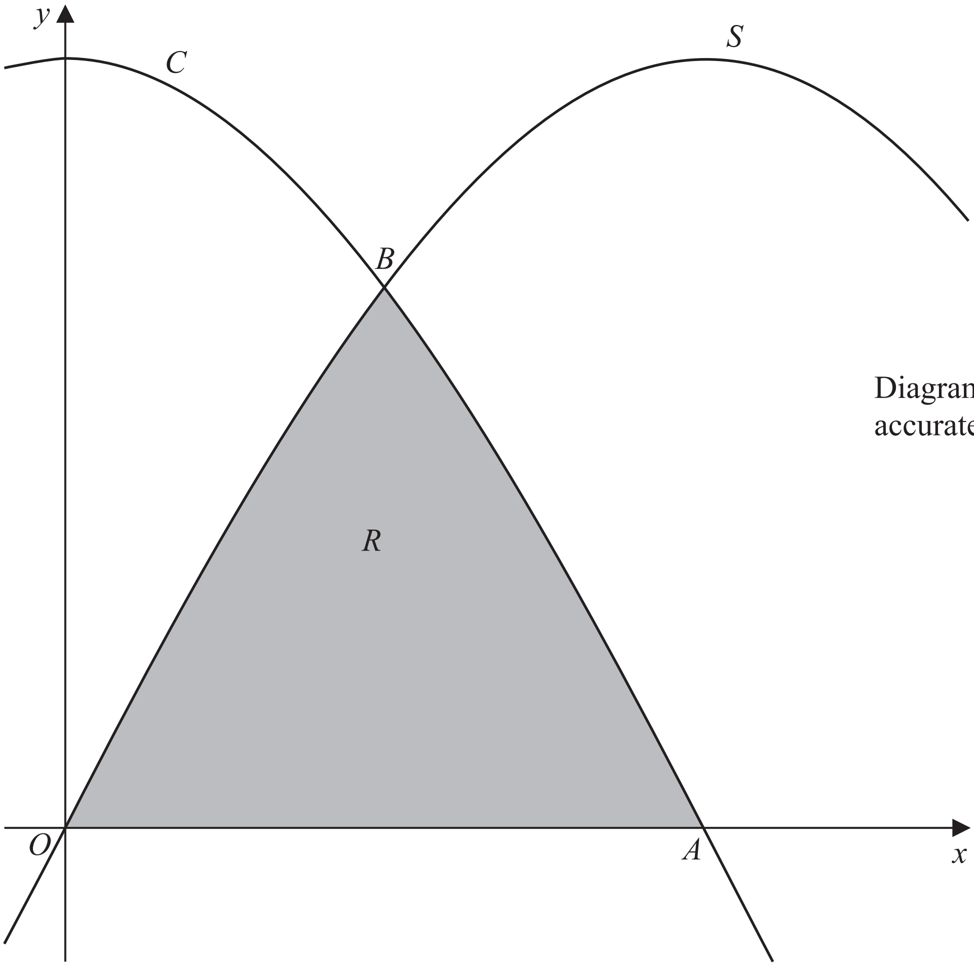

Figure 2

The region R, shown shaded in Figure 2, is bounded by the x-axis, the curve S with equation $y = 2\sin x$ and the curve C with equation $y = 2\cos x$.
As shown in Figure 2, C crosses the x-axis at the point A.

(a) Write down the x coordinate of A.

(1)

As shown in Figure 2, C and S intersect at the point B.

(b) Find the x coordinate of B.

(2)

(c) Using calculus, find the area of the shaded region R.
Give your answer in the form $a - \sqrt{b}$ where a and b are integers.

(4)

Question 12 continued

Question 12 continued

(Total for Question 12 is 7 marks)

TOTAL FOR PAPER IS 100 MARKS

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