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Examiners' Report
Principal Examiner Feedback

Summer 2019

Pearson Edexcel International GCSE

In Mathematics (4PM1) Paper 1and 2

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Further Pure Mathematics **4PM1 01 and 4PM1 02** **June 2019**

Principal Examiner's Report

This is the first sitting of the new specification, although in fairness the changes from 4PM0 are very minor. Students will notice the inclusion of rationalising the denominator in a surd of the form, $\frac{a}{b+\sqrt{c}}$, and it was pleasing to note that many were adept at applying this method. Some students continue to misunderstand the instruction 'show'. Show questions require **every** step to be seen, so that examiners can be assured that what was required to be proved has indeed been proved.

Presentation of the work was an improvement on some recent sessions. Also, there were very few blank answers this exam series.

<u>4PM1 01</u>

Question 1

In Q1 part (a) virtually all students could factorise the quadratic successfully. For part (b) only a minority used the hint given in (a) by using the factor theorem on $x=-1$ and $x=2$. Quite a lot of time was used by dividing f(*x*) by x^2-x-2 or even separately by *x* + 1 and $x-2$. although most who adopted this method were able to do so correctly. A significant number of students lost the last mark by not providing a conclusion; the most minimal form would have been accepted.

Question 2

This question divided students into two halves; those who knew the method to rationalise such a surd, and those that did not. Of those that did know the method, most succeeded in getting to the correct final answer. Some incorrectly added 25 and 12 on the denominator, others multiplied numerator and denominator by $\left(5-\sqrt{3}\right)$ or $\left(5+2\sqrt{3}\right)$. Those that did not know the correct method often used their calculator to obtain the right answer, and then tried to write a minimal amount of mostly incorrect working scoring no marks, despite it stating clearly in bold in the question to 'show your working clearly'.

Question 3

This question (which did not feature the ambiguous case, often a source of problems for many students in the past) was answered completely correctly by the majority of students. When students did not score full marks, the marks were generally lost in part (b) where they used their answer from part (a) as the angle needed in the sine rule with the two given sides for finding the area.
A small number of students completed part (b) in a roundabout way. Some calculated the missing side before either using that with their answer from part (a) or, in a few cases, attempting to use Heron's formula. Others calculated a perpendicular height and used that. These strategies, as well as being time-consuming, were not always successful as some made mistakes in calculating the missing side or perpendicular, or else introduced premature rounding which led to the loss of the final mark.

Having said that, overall there were relatively few rounding errors in the final answers to part (a) and part (b).

Question 4

Part (a) was largely answered correctly, with only a few students not getting the first M mark for the length of side *OC* or *OD*. The majority who lost marks did not work out the smaller sector to subtract from the larger. Most students knew the correct formula for the area of a sector and of arc length. Possibly not reading the question carefully led to some students taking 100 to be the area of the sector *OAB*, and thus losing all marks in part (a). Some also took the ratio of lengths to be the same ratio of areas and thus stated the area of sector *OAB* = 300, again losing all marks.
Even those who lost marks in (a) got both method marks in (b) by using the correct formula for their θ, and virtually everyone who got part (a) correct proceeded to gain full marks for the whole question.
A few worked in degrees, and some of these were able to score full marks by correctly converting to radians at the end.

Question 5

This question was generally answered well, with most students gaining full marks or 4 out of 5 marks, where the constant was incorrect but correctly followed up in part (b). There were very few non-starters. There seemed to be less use of differentiation in part (b) than in similar questions in the past. Perhaps because the question said 'hence', or maybe just because students are becoming more adept at completing the square and more aware of the information this holds for the nature of the curve. There were very few attempts using the more long-winded and error-prone method of equating coefficients.

Question 6

This question was answered well by the majority of the students. There were a number of mistakes made in forming the expressions for perimeter and area. A common mistake seen was to include two diameters $(4x)$ in the expression for the perimeter. As in question 2, a number of students chose to write out longer expressions than necessary and it was easy for the operation $'\times'$ to be mixed up with the variable '*x*'. A small number of students did not give their answer in the required form $(S = ...)$ and/or did not state the value of *k*, so losing the final A mark.

Parts (b) and (c) were completed well. Those that attempted to justify that the value of *x* gave a maximum did so by finding the second differential correctly, with graphical methods generally not seen. The question did specify that *x* was given to 4 significant figures and some students lost a mark for leaving their answer as $\frac{45}{\pi}$.
Part (c) was almost always answered correctly if they had found a correct value in part (b), or a method mark given if using an incorrect value from (b).

Question 7

Full marks in both parts (a) and (b) were commonplace. It was especially pleasing to see the vast majority showing all the steps required in part (b). Perhaps students are becoming more aware of the subtext of 'show'. In part (c), despite the method being virtually in part (b), there were still some students spending valuable time by starting from scratch and expanding both sides to arrive at $5\tan y = 11\tan 30^\circ$. A very common mark pattern was M1A1M1A1A0 for missing out, or wrongly calculating the second angle. As in Question 3, rounding errors were rare either in parts (a) or (c).

Question 8

Most students realised that each part of the question was a development of the previous part. However, even those who did start again often went on to produce the correct result in parts (b) and (c).

Part (a) was done well. Students must show their factorisation or use of the quadratic formula when solving quadratic equations, and direct evidence of calculator use was very rare.

In part (b) those who found it hard to see the connection with the first part

later had difficulty turning $5^{(2x+1)}$ into $5.\left(5^x\right)^2$, but the majority made the connection and could thus solve the quadratic. Seeing $9\left(5^x\right) = 45^x$ was a common error. The great majority of students used a substitution to set up and solve the required quadratic, and should be encouraged to use a letter that does not appear in the question, particularly *x*.

Marks were occasionally lost through not converting quadratic solutions back to the underlying variable and by the incorrect rounding of −0.13864...to 3sf.

In part (c) students seemed to find the rearrangement of the two equations difficult, and often did not seem to see the link to the earlier parts of the question. Although most students were able to equate the equations of the curves, they did not have a complete correct attempt to find one of the coordinates. Of those that did, many found (0, 6) but rounding errors led to incorrect value of *y* of 3.95 or 3.99 instead of the required 4, although answers which rounded to 4.00 were accepted for full marks.

Question 9

There were relatively few students scoring full marks for this question, mainly because of part (c).

In part (a) there were varying degrees of success, with the most able students arriving quickly and efficiently at the final answer. Those who struggled generally were not able to pick up the first M mark as they did not know how to change the base, despite the formula being given in the formula sheet on page 2 of the question paper. Many, however, did arrive at a correct 3TQ, and solved it correctly, often using a substitution for $\log_3 p$ or $\log_p 3$. While most then went on to convert into the correct values for *p*, a significant number forgot that they were supposed to be solving for *p*, and just left their values for whatever letter they had chosen to substitute, thus losing the 2 A marks.

For part (b), many students arrived at $k = 2,$ although not all used conversion to log base 4. There was evidence of using their calculator to find $\log_2 3$ and $\log_4 3.$ A few just wrote down the answer.

In part (c), students scoring full marks were in a distinct minority. Many seemed not to appreciate this was a show question, nor that part (b) was there for a purpose, and were not clear in their method for changing the base. Often there was no method. It was common to see $3x\log_2 3 \to 6x\log_4 3$ without any rationale. Where students took their time to explain each step of their working clearly they generally scored full marks with no mistakes being seen in their simplification. Otherwise, they were limited to 1 mark at most for this section, usually the second M mark. Of those successfully changing base and dealing with the powers, some lost the last 2 marks by not grouping the 4 terms convincingly in a way that led directly to the given answer.

Question 10

Possibly because it was towards the end of the paper there were a few blank scripts for this question.

The majority of the students answered part (a) well, as would be expected with the formula now given. The most common mistake was using $\left(x^2\right)$ or (x) rather than $\left(2x^2\right),$ while a few got the coefficient of x^6 wrong, due to simplifying incorrectly.

Part (b) was very poorly done in comparison. Some students forgot to square root at the right point and often students seemed to know the method but could not execute it correctly, with common answers involving $\frac{1}{2}.$

The great majority of students understood what was required in part (c). In multiplying the two brackets quite a lot of students left off one of the terms in x^6 . Only a minority understood how the answer to part (c) had to be written despite it being stated in the question, and so did not write it as required. Students need to appreciate that an expansion in powers of *x* requires only one term in each of the powers.

Part (d) was done well in virtually all responses, with most students gaining both marks, with any earlier errors generally not affecting this part of the question.

Although many students had earlier mistakes in their answers to either (a) or (c) they were still able to achieve 3 marks in part (e), for the correct integration of their expression and substituting in 0.5.

Studetns who had got to 1.0551 may have lost the last M and A marks if there was not enough evidence to show that they had substituted 0.5 (as a minimum) in their integrated expansion. This is one of the requirements when directed to use algebraic integration.

Question 11

Parts (a) and (b) were generally answered well with many students achieving full marks. These were standard questions and students knew to differentiate for the gradient of the tangent and to use $m_1 \times m_2 = -1$ for the normal. Most students used the more efficient method of $y - y_1 = m(x - x_1)$ to secure an M and A mark quickly because it is a correct equation, albeit not in the required form. Fewer seem to be using $y = mx + c$ than in the past; with this method a value of *c* must be reached before any marks are awarded.

In part (c) a significant minority made progress calculating the area. Use of $\frac{1}{2} \times \text{base} \times \text{height}$ with the base being the distance along the *x*-axis was the most common and successful route. Some students overcomplicated things by calculating the length of the two sides not along the *x*-axis and using the fact that there was a right angle at *P*. The determinant method was not common and integration under the two lines and using the correct limits was rarely successful. In these cases, the absence of a sketch generally led students to a complex solution where, the simple fact that the base of the triangle was in fact the *x*-axis, was missed.

Most students scored nothing for part (d). Without a clear idea of the shape that was being revolved, very few students were able to put together a complete strategy to answer this question. A tiny minority of successful students were able to see 'in their heads' what was required without a diagram, but for everyone else it was absolutely essential, and unfortunately frequently missing. In the absence of a decent sketch, it was all too often a matter of integrating everything in sight and hoping for the best. There was a great deal of confusion over what limits to use and even which curve or straight line to integrate. Many students produced a page of working for which no marks could be awarded because they had combined the curve and the line. Few students spotted that they could use the formula for the volume of a cone. However, many knew how to use V = $\pi \int y^2\, dx$ and some who had no overall strategy were able to pick up 2 marks in (d) by applying this to the curve as these marks could be gained without using the correct limits and even the absence of π was condoned.

4PM1 02

Question 1

Part (a) was generally done well. The most common error was the addition of two vectors rather than subtraction.

The concept of a unit vector in part (b) did not appear to be well understood by at least half of the students. They generally multiplied their $\overrightarrow{AB}$ by 2 or used Pythagoras theorem and did not go any further which lost them 2 marks. Some knew they had to find the modulus and divide by it.

Column vectors were rarely used.

Question 2

Students were generally able to write down $\frac{\mathrm{d}A}{\mathrm{d}t} = 8$ to gain at least the first B1, though sometimes the notation used was dubious. There were a few cases of using a formula for the surface area of a sphere or cylinder with students either having a misconception that the question was referring to a 3D shape or simply not knowing the correct formula.

If students didn't score the first M1 then they often didn't score the second either – again usually an error with the formula.

Many correct chain rules were seen, with the majority using appropriate variables, and often applied correctly (dependant on how successful they'd been with the previous marks).

Rounding was very accurate and there were few scripts seen where rounding was not as demanded.

Question 3

Almost all students made progress with this question, with very few simply substituting values into the expression for v.

The majority of students could do part (a) of the question with ease but a small number of students did not know what acceleration meant.

In part (b) most students realised that they needed to integrate to get an expression for the displacement and of these very few made errors in the integration. However, after this point there was often confusion with how to deal with the constant term that sometimes arose from their work. The constant of integration was frequently omitted. Those who used definite integration and substituted 6 into their expression rarely made any errors. Others integrated correctly but then found the distance at one second intervals and added these values together, hinting perhaps at a misconception that integration provides the distance covered in the most recent second. There was also a significant number who substituted 3.

Question 4

This question was well answered by most students with many gaining full marks. It was nice to see few students trying to use the sine rule and many drawing a sketch to help them use the cosine rule correctly. Most problems occurred when manipulating the algebra from a correctly substituted cosine rule to a correct three term quadratic equation. The accuracy mark lost in this step was often compounded by students using their calculator to then write down incorrect solutions. If they had shown the use of the formula, they could then still secure a further method mark. Most students actively rejected the negative solution, which was required for the last accuracy mark.

Question 5

This question was a good source of marks for students and the vast majority seemed well prepared for this type of question. There were occasional slips with algebra because they didn't multiply every term by x or y depending on the substitution. Some seemed to rush the factorising to solve the quadratic and slip up but this was unusual. The alternative method was very seldom seen.

Question 6

Most students who attempted part (a)(i) were successful. The most successful approach resulted in the final answer being factorised which was not required here but was used fully when tackling part (a)(ii). Rarely was the alternative method seen/used and it was pleasing to see that very few students had a negative sign between their two terms.

Most students gained at least one mark in (ii) as they correctly substituted their solution to part (i) into the given equation. The most successful solution had an aspect of factorising either shown in part (i) or correctly factorised here - if they factorised in part (i) they almost always gained full marks. The most common approach was working from the LHS to the RHS although occasionally expanding both sides and showing both sides were equal was seen. If students arrived at the correct solution in part (i) and did not gain full marks here it was usually due to incorrect algebra rather than a lack of understanding.

In part (b) most students used the quotient rule accurately however this proved to be more challenging than part (a). Some students showed poor differentiation of sin5x; commonly seen unchanged, as 5cos$x$ or -cos5x but rarely –5cos5x. Equally, in a number of responses the denominator was not squared, perhaps because it was originally a squared term. Similarly, there were a noticeable number of responses where the terms in the numerator were the wrong way round. The product rule was rarely seen but usually successfully implemented (this alternative method was seen more than the alternative method in part (a). It was disappointing to see errors in the quotient rule as this is now on the formula sheet provided.

Question 7

Most students made progress with parts (a) and (b) by listing terms of each sequence or using the formula. Very few seemed able to deduce the required answers directly from the formulae given. A number of students could only get the values of the first terms and struggled to find the common difference and common ratio suggesting they may not be adequately familiar with the summation notation.

Many correct answers to part (c) were seen, but a very common error was to assume A_{14} and G_n were the fourteenth and nth terms of their respective series, rather than the sums.

For those who did use the appropriate summation formula, a common error was to expand $4(3^n - 1)$ to obtain $12^n - 4$, which cost them the last two marks.

A small number of students avoided use of formulae by simply listing and adding terms, which for this question was a profitable strategy, as the required value of n was small. Those who reached the equation $3^n = 243$ had no problem solving the equation, by using logs or writing 243 as a power of 3.

Question 8

Part (a) was attempted by almost all students and most were successful in gaining all four marks available. There were a few instances of working in decimals but most worked in surds and correctly gave the answer in simplified surd form. Efficient and clear working was seen here. There was a small number of students who displayed a lack of knowledge of Pythagoras' Theorem.

Part (b) was well attempted by most students, however, they did not always take the most efficient route. Many successful solutions did not spot the isosceles triangle and did two (or in some cases three) separate angle calculations. Students often used the cosine rule followed by the sine rule to find the angles.

For part (c) only the more perceptive students spotted that the triangle was in a semi-circle. Some students appeared to have some understanding of this, but they often used *AB* or *AC* as their diameter.

For those who had been successful with part (c) this part was almost trivial but in most cases there was no response at all when part (c) had not been answered correctly.

Question 9

The first two marks in part (a) were gained by the majority of students – good integration including a constant was seen. A significant minority of students lost the next two marks, the most common error was to substitute in $(0,0)$ rather than $\left(-2,-\frac{28}{3}\right)$ meaning a maximum score of 2/4 was achievable. If they had achieved the first three marks, the final mark was lost in a minority of scripts due to the lack of a conclusion.

Part (b) (i) was often poorly answered mainly due to students failing to show that $f'(x) = 0$ and assuming because they were calculating using the second derivative this did not need to be done. Differentiation was excellent and using it to determine the nature of the points was also done very well, but students often failed to achieve those marks due to not having tested for turning points at $x = 1$ and $x = 2$.

Some students started 'from scratch' by solving $f'(x) = 0$ and the alternative scheme could be applied. These students tended to do much better and more frequently than not gained all the marks for this part of the question.

In part (b) (ii) almost all students were able to find the correct y coordinates.

Part (c) was another well answered question – some students had already found $x = -2$ in part (b) and so had little to do to achieve the 3 marks. Also, given that they had already differentiated in (b) there was little to do in this question if they had been successful earlier! A minority of students lost the last two marks as they failed to identify the y coordinate.

Question 10

Most students had a good understanding of this topic and there were few blanks or attempts to find the roots by solving the equation. They could virtually all identify $\alpha + \beta = -3$ and $\alpha\beta = -5$ The questions in parts (a) and (b) that required the use of the commonly used identities $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ and $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ were answered well. Finding an expression for $\alpha^4 + \beta^4$ in part (a) was answered less successfully. The most commonly used correct method set it equal to $(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$ and used the previous result for $\alpha^2 + \beta^2$. A minority of students attempted to expand $(\alpha + \beta)^4$ using the binomial expansion but rarely managed to reduce it to a form ready for substitution.

Factorising $\alpha^4 - \beta^4$ in part (c) was problematic for many. Some students tried to produce an expression ready for substitution; some who did not factorise fully here, did so in part (d). It may be that students were confused by being asked to factorise something in a question on this topic.

Most students who correctly answered part (c) correctly substituted values for full marks in (d). As mentioned above, a few who were not successful in (c) were also able to answer part c) here and arrive at the correct answer. Unfortunately, the marks for part (c) could only be given for work seen in part (c).

For part (e), most students who answered (d) correctly recognised to solve $\alpha^4 + \beta^4 = 311$ from (a) with $\alpha^4 - \beta^4 = -57\sqrt{29}$ from (d). A significant number who were unsuccessful with part (d) still recognised the need to solve the equations simultaneously and so gained a method mark.

Question 11

This question proved to be an excellent discriminator. Many students scored well in parts (a), (b) and (c) but then began to encounter problems in parts (d) and (especially) in (e). Hence the question differentiated between the reasonable student, (well-done parts 11 (a) – (c), poor attempts at (d) and probably none at (e)), the good student (well-done parts 11 (a) – (d) and probably none at (e)) and the strongest students (full marks for Q11).

Some students made little or no attempt to answer this question. Solutions to (a) and (b) were often combined with students finding the length of *VA* first and then using that to obtain the height which was a valid method and could score full marks for both parts. Some students showed that *AC* = $16\sqrt{2}$ and then stated that $\frac{1}{2}$*AC* = $8\sqrt{2}$ and triangle *VAC* is isosceles and angle $VAC = 45^0$ so the height was also $8\sqrt{2}$. The great loss in part (c) was using Pythagoras with a plus sign in triangle *AXD* where *X* is the foot of the perpendicular from *D* to *VA*. A minority equated the area of the triangle using half base times height with *DX* as the height so

(0.5) *VA* x *DX* = (0.5) *AD* x altitude from *V* to *AD*

Many students could not identify the required angle in part (d). Some made an attempt to calculate the angle *VAB*. However, only the best students could identify the angle required in part (e). These students usually achieved full marks.
Those who drew diagrams were better able to identify the correct triangle to use for each part of the question whilst those who did not even annotate the given diagram generally made many errors.

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