Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel International GCSE In
Further Pure Mathematics (4PM1)
Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent

    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified.
- If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.
- Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes it clear the method has been used.
- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect: eg algebra. Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.
- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication
from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this: eg in a case of "prove or show....")

Exact answers:
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question		Working	Marks
1	(a)	$-3\times q=-12\Rightarrow q=4$	M1A1 [2]
	(b)	$b^2-4ac<0,\ \ (p-2)^2-4\times1\times'4'<0$ To find critical values; $p^2-4p-12=0\Rightarrow(p-6)(p+2)=0\Rightarrow p=6,\ -2$ (Chooses inside region from their critical values) $-2<p<6$	M1 dM1A1 ddM1 A1 [5]
	Alternative		
	(b)	$b^2-4ac<0,\ \ (p-2)^2-4\times1\times'4'<0$ $(p-2)^2<16\Rightarrow-4<(p-2)<4$ $-2<p<6$	M1 dM1A1 ddM1A1
			Total 7 marks

(a)	M1	Substitute $x=0$ and $f(x)=-12$ and proceed to eliminate the term in *p* to obtain a linear equation in *q*.
	A1	$q=4$ (score M1 A1 if no working shown)
(b)	For the first 3 marks allow $b^2-4ac<0$ or $b^2-4ac=0$ or $b^2-4ac>0$	
	M1	Attempt to use b^2-4ac (<0) with $b=p-2$, $a=1$ and $c=4$ (ft from (a)). Correct formula shown and used or correct substitution.
	dM1	Attempt to solve their quadratic inequality or equation. Depends on previous M mark. See general guidance for solving quadratic equations.
	A1	Correct critical values, $(p=)-2, (p=)\,6$
	ddM1	Correct use of their critical values, showing that the inside region has been selected. Both M marks needed. Allow with $\le$ or $<$ or set language statement.
	A1	$-2<p<6$ must be in terms of *p*. Allow $p\in(-2,6)$ or $-2<p$ AND $p<6$ or $\{-2<p\ \cap\ p<6\}$
Alternative		
(b)	For the first 3 marks allow $b^2-4ac<0$ or $b^2-4ac=0$ or $b^2-4ac>0$	
	M1	Attempt to use $b^2-4ac<0$ with $b=p-2$, $a=1$ and $c=4$ (ft from (a)). Correct formula shown and used or correct substitution.
	dM1	Write in the form $(p-2)^2<16$ and attempt to take square root of both sides (both positive and negative options are needed), or as $(p-2)^2-16<0$ and attempt to factorise $[(p-2)-4][(p-2)+4]<0$ Depends on first M mark.
	A1	Correct critical values, $p=-2,\ p=6$ (Accept if wrong variable is used.)
	ddM1	Correct use of their critical values, showing that the inside region has been selected. Both M marks needed. Allow with $\le$ or $<$ or set language statement.
	A1	$-2<p<6$ (in terms of *p*) with options as above.

Question number	Scheme		Marks
2 (i)	$\frac{a}{1}=-2$	$a=-2$	B1
(ii)	$ax+b=0 \Rightarrow -2\times 4+b=0$	$b=8$	B1ft
(iii)	$-3+c=0$	$c=3$	B1
(iv)	So the equation of S is $y=\frac{8-2x}{x+3}$		
	When x = 0		
	$y=\frac{8-2\times 0}{0+3}=\frac{8}{3}$	$p=\frac{8}{3}$	B1ft [4]
			Total 4 marks

		In parts (i), (ii) and (iii), values must be clearly stated or shown correctly substituted in the equation of the curve.
(ii)	B1ft	ft (i) $b=-4\times$ their a
(iv)	B1ft	ft (ii) and (iii) $p=\frac{\text{their } b}{\text{their } c}$ Accept 2.67 or better, $\left(0,\frac{8}{3}\right)$ or $\frac{8}{3}$ marked on the diagram at the point where the curve crosses the y axis.

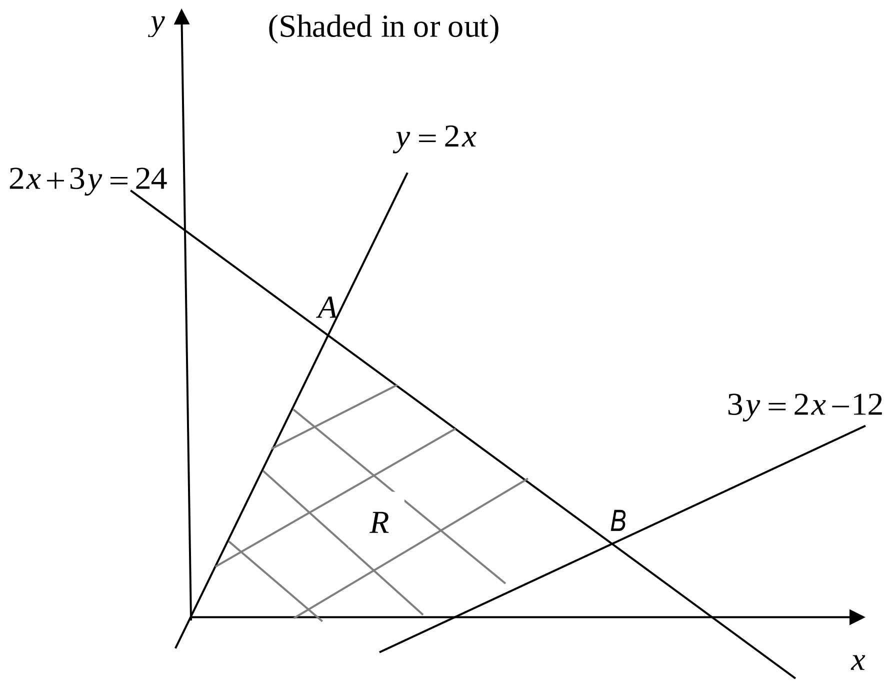
Question number	Scheme	Marks
3	Let l be the length. $\frac{\mathrm{d}l}{\mathrm{d}t}=0.005$	
	$l=10x\Rightarrow\frac{\mathrm{d}l}{\mathrm{d}x}=10$ or $\frac{\mathrm{d}l}{\mathrm{d}t}=10\frac{\mathrm{d}x}{\mathrm{d}t}=0.005\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=0.0005$	B1
	$r=\frac{x}{2},\quad V=\pi\left(\frac{x}{2}\right)^2\times10x\Rightarrow V=\frac{5\pi x^3}{2}$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}x}=\frac{15\pi x^2}{2}$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}l}\times\frac{\mathrm{d}l}{\mathrm{d}t}$ or $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{15\pi\times3^2}{2}\times\frac{1}{10}\times0.005$ or $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{15\pi\times3^2}{2}\times0.0005$	dM1
	$=0.106028...$	
	$\frac{\mathrm{d}V}{\mathrm{d}t}\approx0.11\ (\mathrm{cm}^3/\mathrm{s})$	A1 [6]
Alternative		
	$x=\frac{l}{10}$ or $r=\frac{l}{20}$ stated or used	B1
	$V=\pi\left(\frac{l}{20}\right)^2\times l=\frac{\pi l^3}{400}$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}l}=\frac{3\pi l^2}{400}$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}l}\times\frac{\mathrm{d}l}{\mathrm{d}t}$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{3\pi\times30^2}{400}\times0.005=0.106028...$	dM1
	$\frac{\mathrm{d}V}{\mathrm{d}t}\approx0.11\ \ (\mathrm{cm}^3/\mathrm{s})$	A1 [6]
		Total 6 marks

Apart from x, which is defined in the question, different variables may be used providing that they are consistent throughout the working. Score the best mark using either alternative but not a mixture of both.		
	B1	$\frac{dl}{dx}=10$ or $\frac{dx}{dt}=0.0005$
	B1	Correct expression for the volume of the cylinder. Simplification not needed. Brackets for $\left(\frac{x}{2}\right)^2$ are required unless implied by subsequent working.
	M1	Attempt to differentiate to find $\frac{dV}{dx}$. You need to see a multiple of x^2 from differentiating x^3. ft their V if it is a multiple of x^3 . Mark the differentiation and ignore any error in the label, $\frac{dV}{dx}$, for this mark only.
	M1	State, or use correctly, the chain rule $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dl}\times\frac{dl}{dt}$ or $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$
	dM1	Substitute to find a numerical expression for $\frac{dV}{dt}$ dep on both M marks. ft their $\frac{dV}{dx},\frac{dl}{dx},\frac{dx}{dt}$ if clearly stated.
	A1	$\frac{dV}{dt}\approx 0.11$ **given to 2SF**

Alternative		
	B1	$x=\frac{l}{10}$ or $r=\frac{l}{20}$ stated or used
	B1	Correct expression for the volume of the cylinder. Simplification not needed. Brackets for $\left(\frac{l}{20}\right)^2$ are required unless implied by subsequent working.
	M1	Attempt to differentiate to find $\frac{dV}{dl}$. You need to see a multiple of l^2 from differentiating l^3. ft their V if it is a multiple of l^3 . Mark the differentiation and ignore any error in the label, $\frac{dV}{dl}$, for this mark only.
	M1	State, or use correctly, the chain rule $\frac{dV}{dt}=\frac{dV}{dl}\times\frac{dl}{dt}$
	dM1	Substitute to find a numerical expression for $\frac{dV}{dt}$ dep on both M marks. ft their $\frac{dV}{dl}$ if clearly stated.
	A1	$\frac{dV}{dt}\approx 0.11$ **given to 2SF**

Question number	Scheme	Marks
4 (a)	$v=\int(6t-12)\ \mathrm{d}t=3t^2-12t\ \ (+c)$ When t = 0, v = 0 so c = 0 $v=3t^2-12t=3\times 2^2-12\times 2=-12\ \ (\text{m/s})$	M1A1 A1cso [3]
(b)	$v=0,\ \ 3t^2-12t=0\Rightarrow 3t(t-4)=0\Rightarrow T=4$	M1A1 [2]
(c)	$s=\int(3t^2-12t)\ \mathrm{d}t=t^3-6t^2\ \ (+k)$ $d=\left\|\left[t^3-6t^2\right]_0^4\right\|+\left[t^3-6t^2\right]_4^8=32+160=192\ \ (\text{m})$	M1 M1A1 [3]
Alternative		
(c)	$s=\int(3t^2-12t)\ \mathrm{d}t=t^3-6t^2\ \ (+k)$ At time t = 0, distance = 0, k = 0 t = 4 $\quad s=4^3-6\times 4^2=64-96=-32$ t = 8 $\quad s=8^3-6\times 8^2=512-384=128$ Total distance travelled = 32 +32 +128 = 192 (m)	M1 M1 A1
		Total 8 marks

(a)	M1	Attempt to integrate $6t-12$, must see power of t increased by one in either term and no decrease in powers. Do not accept $v=t(6t-12)$.
	A1	Correct result of integration. Constant is not required.
	A1cso	Correct velocity. Negative sign is required. Must show or state constant = 0.
(b)	M1	$3t^2-12t=0$ and attempt to solve. ft their integration if M1 awarded in (a).
	A1	T = 4 Allow t = 4. Do not allow if T = 0 is also given as an option.
(c)	M1	Attempt to integrate their velocity providing it includes at least one term in t^2 or higher powers. Must see power of t increased by one in at least one term and no decrease in powers. Do not accept $s=t\times$ their v.
	M1	Attempt to use limits of 0 to 4 and 4 to 8 and then add the magnitudes of the two values. ft their T = 4 if M1 awarded in (b).
	A1	192
Alternative		
(c)	M1	Attempt to integrate their velocity providing it includes at least one term in t^2 or higher powers. Must see power of t increased by one in at least one term and no decrease in powers. Do not accept $v=t(6t-12)$.
	M1	Substitute and attempt to find distance when t = 4 and when t = 8. Working to confirm k = 0 must be shown. ft their T = 4 if M1 awarded in (b).
	A1	192

Question number	Scheme	Marks
5 (a)	$2x+3y=24, \quad y=2x, \quad 3y=2x-12$	M1 A1A1 [3]
(b)		B1 [1]
(c)	Greatest value of F is at A, intersection of lines $y=2x$ and $2x+3y=24$ Greatest value of F is at (3, 6) Greatest value of F = $2\times3+5\times6=36$	M1 A1 A1ft [3]
		Total 7 marks

(a)	M1	Attempt to find two points on any one of the straight lines. Look for two correct points (stated, plotted or implied by a correct line), or for any incorrect point from working which shows an appropriate substitution of one variable with an attempt to solve for the second variable.
	A1	For any two correct lines.
	A1	For three correct lines.
(b)	B1	Correct region, identified by shading in or outside the region. Label R is not required but it needs to be correct if it is shown.
(c)	M1	Identify A as the point with coordinates that give the greatest value of F, or find the value of F at A and B, ft their graph, or draw line $2x+5y=k$ for at least one value of k.
	A1	Correct coordinates (3, 6) for the point identified to give the greatest value of F. May be read from graph or implied by working.
	A1ft	(F =) 36 ft their coordinates for A. Note that this mark is shown as B1 on ePEN.

Question number	Scheme	Marks
6 (a)	$\sqrt{9-x}=3\left(1-\frac{x}{9}\right)^{\frac{1}{2}} \Rightarrow p=3,\quad q=-\frac{1}{9}$	B1B1 [2]
(b)	$3\left(1-\frac{x}{9}\right)^{\frac{1}{2}}=$ $3\left\{1+\left(\frac{1}{2}\right)\left(-\frac{x}{9}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{x}{9}\right)^2}{2!}+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{9}\right)^3}{3!}+\ldots\right\}$	M1
	$=3\left(1-\frac{x}{18}-\frac{x^2}{648}-\frac{x^3}{11664}\right)=3-\frac{x}{6}-\frac{x^2}{216}-\frac{x^3}{3888}$	A1A1 [3]
(c)	$\frac{31}{4}=9-x \Rightarrow x=\frac{5}{4}$	B1
	$\sqrt{\frac{31}{4}}\approx 3-\frac{1.25}{6}-\frac{1.25^2}{216}-\frac{1.25^3}{3888}=2.783930523\ldots\approx 2.78393$	M1A1 [3]
		Total 8 marks

(a)	B1	$p=3$ or $\sqrt{9}$ or $9^{\frac{1}{2}}$	May be shown in their $3\left(1-\frac{x}{9}\right)^{\frac{1}{2}}$ (allow isw)
	B1	$q=-\frac{1}{9}$	
(b)	M1	Attempt to use the binomial expansion for their $(1+qx)^{\frac{1}{2}}$. Must have first term 1, three more terms with ascending powers of x, 2 or 2! and 6 or 3! Seen, and their $\left(-\frac{x}{9}\right)$ used at least once. No simplification needed. Ignore terms beyond x^3	
	A1	Two algebraic terms correct in the expansion of $3\left(1-\frac{x}{9}\right)^{\frac{1}{2}}$. Must be single fractions, not necessarily in lowest terms. Ignore terms beyond x^3	
	A1	All four terms correct and in lowest terms. Ignore terms beyond x^3	
(c)	B1	Identify $x=\frac{5}{4}$ as the value needed to make $\sqrt{9-x}=\sqrt{\frac{31}{4}}$	
	M1	Substitute their x into each of the algebraic terms of their expansion of $\sqrt{9-x}$	
	A1	2.78393 **given to 5DP**.	

Question number	Scheme	Marks
7 (a)	(i) $\frac{U_7}{U_3}=\frac{ar^6}{ar^2}=r^4 \Rightarrow \frac{16}{4}=r^4 \Rightarrow 4=r^4 \Rightarrow 2=r^2 \Rightarrow r=\sqrt{2}*$	M1 A1cso
	(ii) $ar^2=4 \Rightarrow a=\frac{4}{\left(\sqrt{2}\right)^2}=2$	A1 [3]
Alternative		
(a)	(i) $ar^2=4$, $ar^6=16 \Rightarrow a\left(\frac{4}{a}\right)^3=16 \Rightarrow a^2=4 \Rightarrow a=2$	M1
	$r^2=\frac{4}{2} \Rightarrow r=\sqrt{2}*$	A1cso
	(ii) $a=2$	A1
(b)	$500<2\times\sqrt{2}^{(n-1)} \Rightarrow 250<\sqrt{2}^{(n-1)}$ or $500<2^{\frac{1}{2}(n+1)}$	M1
	$\lg 250<(n-1)\lg\sqrt{2}$ or $\lg 500<\frac{1}{2}(n+1)\lg 2$	
	$n-1>\frac{\lg 250}{\lg\sqrt{2}} \Rightarrow n-1>15.93...$ or $\frac{1}{2}(n+1)>\frac{\lg 500}{\lg 2} \Rightarrow \frac{1}{2}(n+1)>8.96...$	M1 M1A1 [4]
	$n>16.93... \Rightarrow n=17$	
(c)	$S_{20}=\frac{2\left(\left(\sqrt{2}\right)^{20}-1\right)}{\sqrt{2}-1}=\frac{2046}{\sqrt{2}-1}$	M1A1
	$\frac{2046}{\left(\sqrt{2}-1\right)}\times\frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}=2046\sqrt{2}+2046=2046\left(1+\sqrt{2}\right)$	M1A1 [4]
		Total 11 marks

(a)(i)	M1	$ar^2=4$ and $ar^6=16$
	A1 cso	Correct working to eliminate a and show that $r=\sqrt{2}$ (Answer given) Must see $r^4=4$.
(ii)	A1	a = 2 (NB: This is not a B mark; must have scored the first M mark.)
Alternative		
(a)(i)	M1	$ar^2=4$ and $ar^6=16$
	A1 cso	Correct working to find a and use it to show that $r=\sqrt{2}$ (Answer given) Must see a correct substitution to eliminate r which is processed correctly to give a = 2, and $r^2=\frac{4}{2}$ or $r^6=\frac{16}{2}$.
(ii)	A1	a = 2 (Must have scored the first M mark.)

(b)	M1	$500 < 2\times\left(\sqrt{2}\right)^{(n-1)}$ Accept an equation or >. Accept $500 < 2\times\sqrt{2}^{n-1}$ ft their value for a
	M1	$n-1 > \dfrac{\lg 250}{\lg\sqrt{2}}$ or $\dfrac{1}{2}(n-1) > \dfrac{\lg 250}{\lg 2}$ or $\dfrac{1}{2}(n+1) > \dfrac{\lg 500}{\lg 2}$ or $\lg_{\sqrt{2}} 250 < n-1$ Accept equation or >, and logs with any base. ft $500 < 2\times\left(\sqrt{2}\right)^{n}$
	M1	Evaluate and divide logs and attempt to find a value (at least 1DP) or numerical expression for *n*. Ignore incorrect inequality sign. eg $n > 16.9$ or $n = 16.9$ or $n > 1+15.93$ or $n = 1+2\times 7.96$ ft $500 < 2\times\left(\sqrt{2}\right)^{n}$
	A1	$n = 17$ Do not award after incorrect inequalities e.g. $n < 16.9$
Alternative		
(b)	M1	$500 < 2\times\left(\sqrt{2}\right)^{(n-1)}$ Accept an equation or >. Accept $500 < 2\times\sqrt{2}^{n-1}$ ft their value for a
	M1	Attempt to use trial and improvement, showing two trials that can be used to confirm the required value of *n*, eg $\left(\sqrt{2}\right)^{15} = 181.0...$ and $\left(\sqrt{2}\right)^{16} = 256$ or $2\left(\sqrt{2}\right)^{15} = 362.0...$ and $2\left(\sqrt{2}\right)^{16} = 512$ or $2^8 = 256$ and $2^9 = 512$ ft $500 < 2\times\left(\sqrt{2}\right)^{n}$ if clearly stated
	M1	Use the higher power from these trials in an appropriate equation eg $n-1 = 16$ or $\dfrac{1}{2}(n+1) = 9$ and attempt to find a value for *n*. ft $500 < 2\times\left(\sqrt{2}\right)^{n}$ if clearly stated
	A1	$n = 17$
(c)	M1	Attempt to use $S_n = \dfrac{a(1-r^n)}{(1-r)}$ with their a, $r = \sqrt{2}$ and $n = 20$ Allow notation or sign errors if formula is stated, otherwise the substitution must be correct.
	A1	$S_{20} = \dfrac{2046}{\sqrt{2}-1}$ or $S_{20} = -\dfrac{2046}{1-\sqrt{2}}$ Correct expression with numerator evaluated.
	M1	Attempt to rationalise the denominator. Must use $\sqrt{2}+1$ and attempt to simplify the denominator.
	A1	$2046(1+\sqrt{2})$ or $2046(\sqrt{2}+1)$ Accept p = 2046. Award full marks for a correct answer if first M1 has been awarded.

Question number	Scheme	Marks
8 (a)	$BN = 10\cos 60^{\circ}$ $= 5$ (cm)*	M1 A1cso [2]
(b)	$FN^2 = 10^2 - 5^2 = 75$ or $FN = 10\sin 60 = 5\sqrt{3}$ or $FN = 5\tan 60$ $NE = \sqrt{12^2 + 75} = 14.7986... \approx 14.8$ (cm)	M1 M1A1 [3]
(c)	Let R be the midpoint of FN and P be the point on AD that corresponds to point N on BC. $\angle RPN = \tan^{-1}\left(\dfrac{\sqrt{75}/2}{12}\right) = 19.84167...^{\circ} \approx 19.8^{\circ}$	 M1A1 [2]
(d)	Let the perpendicular from Y to line BC meet BC at Z $AY = \sqrt{AZ^2 + ZY^2} = \sqrt{12^2 + 6.5^2 + \left(\dfrac{\sqrt{75}}{2}\right)^2} = \sqrt{205}$ $FC = \sqrt{FN^2 + NC^2} = \sqrt{75 + 3^2} = \sqrt{84} \Rightarrow FY = \sqrt{21}$ So $EY = \sqrt{12^2 + 21} = \sqrt{165}$ $\angle AYE = \cos^{-1}\left(\dfrac{205 + 165 - 100}{2 \times \sqrt{205} \times \sqrt{165}}\right) = 42.7745...^{\circ} \approx 42.8^{\circ}$	 M1A1 M1 M1 M1A1 [6]
		Total 13 marks

(a)	M1	Use cosine or sine in triangle BNF, or any complete method. $BN = 10\cos 60^{\circ}$ or $\cos 60^{\circ} = \dfrac{BN}{10}$ or $\dfrac{BN}{\sin 30} = \dfrac{10}{\sin 90}$ or $BN = 10\sin 30$ or $FN = 10\sin 60 = 5\sqrt{3}$ and $BN^2 = 10^2 - (5\sqrt{3})^2$ (may use any single letter instead of BN)
	A1 cso	$BN = 5$ (cm) Correct answer from correct working. (Answer given.) Must show a calculation in the form $BN = \ldots$ e.g. $BN = 10\cos 60$
(b)	M1	Use Pythag or trig to find a numerical expression for FN or FN^2 eg $FN^2 = 10^2 - 5^2$ $(= 75)$ or $FN = 10\sin 60$ $(= 5\sqrt{3} = 8.66...)$ or $FN = 5\tan 60$
	M1	Use Pythag with their FN to find a numerical expression for NE, eg $NE = \sqrt{12^2 + (5\sqrt{3})^2}$ $(= \sqrt{219})$
	A1	$NE = 14.8$ (cm) **given to 3SF**.
(c)	M1	Use tangent with their FN, eg $\tan RPN = \left(\dfrac{\frac{1}{2}\sqrt{75}}{12}\right)$ or any complete method.
	A1	19.8° **given to 1DP**.

(d)	M1	Complete method to find *AY* eg $AY=\sqrt{(12^2+6.5^2)+\left(\frac{\sqrt{75}}{2}\right)^2}$ $(=\sqrt{205}=14.31...)$ (allow their *FN*)
	A1	$AY=\sqrt{205}$ (= 14.31…) (may be implied by subsequent working)
	M1	Complete method to find *FY* eg $FC=\sqrt{75+3^2}\left(=\sqrt{84}\right)\Rightarrow FY=\frac{1}{2}\sqrt{84}\ (=\sqrt{21})$ (allow their *FN*) eg $FC=\sqrt{8^2+10^2-2\times8\times10\cos60}\left(=\sqrt{84}\right)\Rightarrow FY=\frac{1}{2}\sqrt{84}\ (=\sqrt{21}=4.58...)$
	M1	Method to find *EY* using their *FY* (correct or from a correct method) eg $EY=\sqrt{12^2+21}\ (=\sqrt{165}=12.84...)$
	M1	Substitute their values for *AY* and *EY* into the cosine rule used in triangle *AYE*. $\cos AYE=\left(\frac{205+165-100}{2\times\sqrt{205}\times\sqrt{165}}\right)$ oe
	A1	$\angle AYE=42.8^{\text{o}}$ **given to 1DP**. Allow more than 1DP if 1DP was penalised in part (c).

Question number	Scheme	Marks
9	Intersections of curve and line: $x^2-2=-2x+13$ or $y=\left(\frac{y-13}{2}\right)^2-2$ $\Rightarrow x^2+2x-15=0$ $\Rightarrow (x+5)(x-3)=0$ or $\Rightarrow y^2-30y+161=0$ $\Rightarrow x=(-5),\ 3$ $\quad\Rightarrow (y-7)(y-23)=0$ when $x=3, y=7$ $\quad\Rightarrow y=7,\ (23)$	M1 M1 A1
	<u>Volume of cone</u> Using formula: Volume of cone $=\frac{1}{3}\times\pi\times3^2\times(13-7)=18\pi$ or using integration: Volume of cone $=\pi\int_7^{13}\left(\frac{13}{2}-\frac{y}{2}\right)^2\,\mathrm{d}y=\frac{\pi}{4}\left[169y-13y^2+\frac{y^3}{3}\right]_7^{13}=18\pi$	B1
	<u>Volume of solid generated by rotating the curve:</u> When $x=0, y=-2$ Volume $=\pi\int_{-2}^{7}(y+2)\,\mathrm{d}y=\pi\left[\frac{y^2}{2}+2y\right]_{-2}^{7}$ $=\pi\left\{\left(\frac{49}{2}+14\right)-\left(\frac{(-2)^2}{2}+(-4)\right)\right\}=\frac{81}{2}\pi$ Total volume $=18\pi+\frac{81}{2}\pi=\frac{117}{2}\pi$ oe	M1M1 ddM1 A1 A1 [9]
		Total 9 marks

	M1	Attempt to solve $y+2x=13$ with $y=x^2-2$ and obtain a three term quadratic in *x* or *y*. ($x^2+2x-15=0$ or $y^2-30y+161=0$) (Condone = 0 missing.)
	M1	Attempt to solve (see general principles for marking quadratics).
	A1	$y = 7$
	B1	18π correct volume of cone, in terms of π.
	M1	Correct integral for volume generated by rotating curve. Volume $=\pm\pi\int_{(-2)}^{(7)}(y+2)\ (\mathrm{d}y)$ Ignore limits and condone d*y* omitted or with the wrong variable.
	M1	Evidence of completing integration (see general principles for integration). Ignore limits. π may be missing. If $y + 2$ is combined with another expression in an attempt to add or subtract two volumes, you may accept evidence of integration for the combined expression.
	dd M1	Dep on previous two M marks. Substitute both correct limits $y=-2$ and $y=7$ into the result of their integration. The full substitution should be shown unless the answer is $\frac{81}{2}\pi$ oe.
	A1	$\frac{81}{2}\pi$ oe, given in terms of π. A correct answer from the correct integration implies M1 for a correct substitution of limits.
	A1	$\frac{117}{2}\pi$ oe, correct total volume, in terms of π.

Question number	Scheme	Marks
10 (a)	$\cos 2A = \cos^2 A - \sin^2 A = \cos^2 A - \left(1-\cos^2 A\right) = 2\cos^2 A - 1*$	M1A1cso [2]
(b)	$\cos 4A = 2\cos^2 2A - 1$ $= 2\left(2\cos^2 A - 1\right)^2 - 1$ $= 2\left(4\cos^4 A - 4\cos^2 A + 1\right) - 1$ $= 8\cos^4 A - 8\cos^2 A + 1*$	M1 M1 M1 A1cso [4]
(c)	Let $\dfrac{\theta}{4} + \dfrac{\pi}{24} = x$ $\cos^2 x\left(\cos^2 x - 1\right) = \cos^4 x - \cos^2 x$ $\Rightarrow \cos^4 x - \cos^2 x = \dfrac{\cos 4x - 1}{8}$ $4\left(\dfrac{\theta}{4} + \dfrac{\pi}{24}\right) = \theta + \dfrac{\pi}{6}$ $\dfrac{\cos\left(\theta + \frac{\pi}{6}\right) - 1}{8} = -\dfrac{1}{16} \Rightarrow \cos\left(\theta + \dfrac{\pi}{6}\right) = \dfrac{1}{2}$ $\Rightarrow \theta + \dfrac{\pi}{6} = \dfrac{\pi}{3},\ \dfrac{5\pi}{3},\ \dfrac{7\pi}{3} \Rightarrow \theta = \dfrac{\pi}{6},\ \dfrac{3\pi}{2}$	M1 M1 M1 M1A1 [5]
Alternative (with x defined as above)		
(c)	$\cos^4 x - \cos^2 x = -\dfrac{1}{16} \Rightarrow 8\cos^4 x - 8\cos^2 x + 1 = 1 - \dfrac{8}{16}$ $\cos 4x = \dfrac{1}{2} \Rightarrow 4x = \theta + \dfrac{\pi}{6} = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}, \cdots \Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{3\pi}{2}$	M1 M1M1 M1A1
Alternative (with x defined as above)		
(c)	$16\left(\cos^2 x\right)^2 - 16\cos^2 x + 1 = 0$ $\cos^2 x = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4\times 16\times 1}}{2\times 16} = \dfrac{2\pm\sqrt{3}}{4}$ $\cos x = \sqrt{\dfrac{2\pm\sqrt{3}}{4}}$ $(= \pm 0.9659...\text{ or } \pm 0.2588...)$ $x = 0.2617..., 1.3089..., 1.8325..., 2.8787...(= \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}...)$ $\theta = 4\left(x - \dfrac{\pi}{24}\right) = \dfrac{\pi}{6}, \dfrac{3\pi}{2}$	M1 M1 M1 M1A1
(d)	$\mathrm{f}\left(A\right) = \dfrac{\cos 4A}{2} + \dfrac{1}{2}$ $\left[\dfrac{\sin 4A}{8} + \dfrac{A}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \left(\dfrac{\sin 2\pi}{8} + \dfrac{\pi}{4}\right) - \left(\dfrac{\sin \frac{2\pi}{3}}{8} + \dfrac{\pi}{12}\right) = \dfrac{\pi}{4} - \dfrac{\sqrt{3}}{16} - \dfrac{\pi}{12} = \dfrac{\pi}{6} - \dfrac{\sqrt{3}}{16}$	M1 dM1 ddM1 A1 [4]
		Total 15 marks

(a)	M1	Use $\cos(A+A)=\cos A\cos A-\sin A\sin A$ (may go straight to $\cos 2A=\cos^2 A-\sin^2 A$) and then try to replace $\sin^2 A$ by $1-\cos^2 A$ Allow with any other single variable.
	A1 cso	Obtain $\cos 2A=2\cos^2 A-1$ with no errors in the working. Must be stated with *A* as the variable.
(b)		Allow with any other single variable for all M marks. Allow one sign error removing brackets and ft for all M marks.
	M1	$\cos 4A=2\cos^2 2A-1$ or $\cos 4A=\cos^2 2A-\sin^2 2A$ Obtain an expression in terms of cos 2*A* and/or sin 2*A*.
	M1	eg $\cos 4A=2\left(2\cos^2 A-1\right)^2-1$, $\cos 4A=\left(2\cos^2 A-1\right)^2-4\left(1-\cos^2 A\right)\cos^2 A$ Obtain a correct expression in cos *A*. Eliminate terms in sin *A* and sin 2*A*.
	M1	Expand brackets. Numerical factors may remain. e.g. $\cos 4A=2\left(4\cos^4 A-2\cos^2 A-2\cos^2 A+1\right)-1$ e.g. $\cos 4A=4\cos^4 A-4\cos^2 A+1-4\cos^2 A+4\cos^4 A$
	A1 cso	Obtain the given result with no errors in the working. Must be stated with *A* as the variable.
(c)	M1	Expand brackets and use the result from part (b) to express the LHS of the equation in terms of 4*x*, $\dfrac{\cos 4x-1}{8}$, where $x=\dfrac{\theta}{4}+\dfrac{\pi}{24}$ **Alternative** Arrange the equation in the form $8\cos^4 x-8\cos^2 x+1=1-\dfrac{8}{16}$ Allow a single numerical or sign error in the constants for both methods.
	M1	Rearrange the equation to give $\cos\left(\theta+\dfrac{\pi}{6}\right)=\dfrac{1}{2}$ or oe Note that this mark is shown as B1 on ePEN.
	M1	Find any correct positive value for $4\left(\dfrac{\theta}{4}+\dfrac{\pi}{24}\right)$, given in radians. eg $\dfrac{\pi}{3}$
	M1	Find any correct positive value for θ, given in radians. eg $\dfrac{\pi}{6}$
	A1	Both correct answers with no extras, given in radians as multiples of π. Ignore answers outside the given range.

Alternative		
(c)	M1	Write the equation as a three term quadratic. e.g. $16\left(\cos^2 x\right)^2 - 16\cos^2 x + 1 = 0, \left(x = \frac{\theta}{4} + \frac{\pi}{24}\right)$, $A^2 - A + \frac{1}{16} = 0, \left(A = \cos^2\left(\frac{\theta}{4} + \frac{\pi}{24}\right)\right)$
	M1	Any valid method to solve their quadratic equation. Note that this mark is shown as B1 on ePEN.
	M1	Any correct value for $x = \frac{\theta}{4} + \frac{\pi}{24}$ in the range $0 \le x \le \pi$, given in radians, either as a decimal or as a multiple of π.
	M1	Either value of θ given in radians, $\theta = \frac{\pi}{6}$ or $\theta = \frac{3\pi}{2}$ or $\theta = 0.523...$ or $\theta = 4.712...$
	A1	Both $\theta = \frac{\pi}{6}$ and $\theta = \frac{3\pi}{2}$ given as multiples of π.
(d)	M1	Use result from part (b) to write $\mathrm{f}(A) = k(\cos 4A \pm 1)$.
	dM1	Integrate their $\mathrm{f}(A)$ correctly, dep on M1 awarded. Ignore limits.
	ddM1	Substitute both given limits, dep on both M marks awarded.
	A1	$\frac{\pi}{6} - \frac{\sqrt{3}}{16}$ *or* $\frac{1}{6}\pi - \frac{1}{16}\sqrt{3}$ simplified in this format. Do not isw.

Question number	Scheme	Marks
11 (a)	(i) $2\alpha\beta = 3 \Rightarrow q = \alpha\beta \Rightarrow q = \dfrac{3}{2}$	B1
	(ii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta,\ \left[\alpha^2+\beta^2 = \dfrac{k^2-6k-3}{4}\right]$	
	$(\alpha+\beta)^2 = \dfrac{k^2-6k-3}{4}+3 = \dfrac{k^2-6k+9}{4} = \dfrac{(k-3)^2}{4}$	M1A1
	$\Rightarrow \alpha+\beta = \dfrac{k-3}{2} \Rightarrow p = \dfrac{k-3}{2}$	M1A1 [5]
(b)	$7\left(\dfrac{3}{2}\right) = 3\left(\dfrac{k-3}{2}\right) \Rightarrow 7 = k-3 \Rightarrow k = 10$	M1A1 [2]
(c)	$\alpha+\beta = \dfrac{10-3}{2} = \dfrac{7}{2}$	M1
	Sum: $\dfrac{\alpha}{\alpha+\beta} + \dfrac{\beta}{\alpha+\beta} = \dfrac{\alpha+\beta}{\alpha+\beta} = 1$	B1
	Product: $\dfrac{\alpha}{\alpha+\beta} \times \dfrac{\beta}{\alpha+\beta} = \dfrac{\alpha\beta}{(\alpha+\beta)^2} = \dfrac{\frac{3}{2}}{\left(\frac{7}{2}\right)^2} = \dfrac{6}{49}$	M1
	Equation: $x^2 - x + \dfrac{6}{49}\ (=0)$	M1
	$49x^2 - 49x + 6 = 0$	A1 [5]

(a) (i)	B1	$q = \dfrac{3}{2}$
(ii)	M1	State or use any correct arrangement of $(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$ and substitute $\alpha\beta = \dfrac{3}{2}$ and $\alpha^2+\beta^2 = \dfrac{k^2-6k-3}{4}$ oe p may be seen instead of $\alpha+\beta$ at any stage of the working. Condone $-p = \alpha+\beta$ for this mark only.
	A1	Any correct expression for $(\alpha+\beta)^2$ or $4(\alpha+\beta)^2$ in terms of k
	M1	Correct method to find an expression for p in terms of k from their expression for $(\alpha+\beta)^2$ or $4(\alpha+\beta)^2$. Ignore negative square root. Must identify $p = \alpha+\beta$.
	A1	A correct expression for p in terms of k. The question does not demand simplification, but if it is left as a square root there must be an indication that p must be the positive root (ie no $\pm$ present).

(b)	M1	Substitute $\alpha\beta = \frac{3}{2}$ and their expression for $\alpha + \beta$ from (a)(ii) into $7\alpha\beta = 3(\alpha + \beta)$ and obtain an equation in k
	A1	k = 10 Do not accept if there is an additional root, such as $k = -4$.
(c)	M1	Find a value for $\alpha + \beta$. Need not be simplified. Follow through their k and their expression for p.
	B1	Show or state sum of roots = 1 (stated or explicitly used).
	M1	Write product of roots as $\frac{3/2}{(7/2)^2}$, ft their $\alpha + \beta = \frac{7}{2}$.
	M1	Use equation is $x^2 -$ (sum of roots) $+$ (product of roots) $(= 0)$ (= 0 may be missing). There must be some attempt to substitute values, not just expressions in terms of α and β. May be implied by an equation with fractions correctly cleared.
	A1	$49x^2 - 49x + 6 = 0$ Correct equation with integer coefficients. Must have = 0.